The time reparameterization soft mode

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Talk online: sachdev.physics.harvard.edu
Annals of Physics, \textbf{418}, 168202 (2020)
Challenge for theory:

A model of a metal in which the resistivity, \( \rho \), obeys

\[
\lim_{T \to 0} \frac{d\rho}{dT} \neq 0
\]

\[
\rho(T) = \rho(0) + AT + \ldots, \quad T \to 0.
\]
2 key ingredients

1. Emergent gauge symmetry and fractionalization

\[ c_\alpha = f_\alpha b^\dagger \]

\[ f_\alpha \rightarrow e^{i\phi} f_\alpha , \quad b \rightarrow b e^{i\phi} \]
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1. Emergent gauge symmetry and fractionalization

\[ c_\alpha = f_\alpha b^\dagger \]
\[ f_\alpha \to e^{i\phi} f_\alpha, \quad b \to b e^{i\phi} \]

2. Time reparameterization symmetry

\[ \tau \to f(\tau) \]

Present in models of non-Fermi liquids:
quantum matter at variable density
without quasiparticle excitations
- Gauge fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm \vec{k}_0$.

- Expand fermion kinetic energy at wavevectors about $\pm \vec{k}_0$. In Landau gauge $\vec{A} = (a, 0)$.
\[ \mathcal{L}[\psi_{\pm}, a] = \]
\[ \psi_+^{\dagger} \left( \partial_\tau - i \partial_x - \partial_y^2 \right) \psi_+ + \psi_-^{\dagger} \left( \partial_\tau + i \partial_x - \partial_y^2 \right) \psi_- \]
\[ - a \left( \psi_+^{\dagger} \psi_+ - \psi_-^{\dagger} \psi_- \right) + \frac{1}{2g^2} \left( \partial_y a \right)^2 \]

Fermi surface coupled to a gauge field

Fermi surface coupled to a gauge field

\[ \mathcal{L} = \psi^+_+ (\partial_\tau - i\partial_x - \partial_y^2) \psi_+ + \psi^+_- (\partial_\tau + i\partial_x - \partial_y^2) \psi_- - a \left( \psi^+_+ \psi_+ - \psi^+_- \psi_- \right) + \frac{1}{2g^2} (\partial_y a)^2 \]

Simple scaling argument for \( z = 3/2 \).
Under the rescaling \( x \to x/s, \ y \to y/s^{1/2}, \) and \( \tau \to \tau/s^z, \) we find invariance provided

\[
\begin{align*}
a & \to a \ s \\
\psi & \to \psi \ s^{(2z+1)/4} \\
g & \to g \ s^{(3-2z)/4}
\end{align*}
\]

So the action is invariant provided \( z = 3/2 \).

Simple scaling argument for $z = 3/2$.

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So the action is invariant provided $z = 3/2$.

Fermi surface coupled to a gauge field

\[
\mathcal{L} = \psi_+^\dagger (\square - i\partial_x - \partial_y^2) \psi_+ + \psi_-^\dagger (\square + i\partial_x - \partial_y^2) \psi_- \\
- a \left( \psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} (\partial_y a)^2
\]

Because the bare time derivatives are irrelevant, the critical theory has an emergent time reparameterization symmetry:

\[
\begin{align*}
\tau & \to f(\tau) \\
\tau d\tau & \to f'(\tau) d\tau \\
x & \to [f'(\tau)]^{1/z} x \\
y & \to [f'(\tau)]^{1/(2z)} y \\
A & \to [f'(\tau)]^{1/z} A \\
\psi & \to [f'(\tau)]^{-(2z+1)/(4z)} \psi
\end{align*}
\]

Work in progress with Haoyu Guo and Aavishkar Patel
1. SYK criticality +
   *time reparameterization soft mode*

2. SYK lattice models

3. *Fractionalization* and SYK criticality in *t-J* models with random exchange
1. SYK criticality + 
   *time reparameterization soft mode*

2. SYK lattice models

3. *Fractionalization* and SYK criticality in *t-J* models with random exchange
The complex SYK model

\[ H = \frac{1}{(2N)^{3/2}} \sum_{a,b,c,d=1}^{N} U_{ab;cd} c_{a}^{\dagger} c_{b} c_{c} c_{d} - \mu \sum_{a} c_{a}^{\dagger} c_{a} \]

\[ c_{a} c_{b} + c_{b} c_{a} = 0 \quad , \quad c_{a} c_{b}^{\dagger} + c_{b}^{\dagger} c_{a} = \delta_{ab} \]

\[ Q = \frac{1}{N} \sum_{a} c_{a}^{\dagger} c_{a} \]

\( U_{ab;cd} \) are independent random variables

with \( \overline{U_{ab;cd}} = 0 \) and \( |U_{ab;cd}|^2 = U^2 \)

\[ G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \sum = \]

S. Sachdev and J. Ye, PRL \textbf{70}, 3339 (1993)
The complex SYK model

Key properties

1. There is a quantum critical state, without quasiparticle excitations, for a range of charge densities around $Q = 1/2$.

S. Sachdev and J. Ye,
PRL 70, 3339 (1993)
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2. There is a non-zero extensive entropy as $T \to 0$

$$\lim_{T \to 0} \lim_{N \to \infty} \frac{S}{N} = S_0(Q) \neq 0$$

This entropy is not due to an exponentially large ground degeneracy. Instead, it reflects an exponentially small many-body level spacing $\sim e^{-NS_0}$ down to the ground state.
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3. Thermal equilibration in a ‘Planckian time’ $\sim \hbar/(k_B T)$

A. Eberlein, V. Kasper, S. Sachdev, and J. Steinberg, PRB *96*, 205123 (2017)
The complex SYK model

Key properties

4. The leading low temperature behavior of many observables is controlled by a time reparameterization soft mode. A Schwarzian action for this soft mode is implied by an emergent SL(2,R) symmetry. Specifically, the entropy is \( S(T)/N = S_0(Q) + \gamma T \), where \( \gamma \) proportional to the co-efficient of the Schwarzian.

A. Kitaev, KITP talk (2015)
J. Maldacena and D. Stanford, PRD 94, 106002 (2016)
The complex SYK model

\[ G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau) \]

\[ \Sigma(z) = \mu - \frac{1}{A} \sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}} \]
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\[ \Sigma(z) = \mu - \frac{1}{A} \sqrt{z} + \ldots, \quad G(z) = \frac{A}{\sqrt{z}} \]

At frequencies \( \ll U \), the \( i\omega + \mu \) can be dropped, and without it equations are invariant under the reparametrization and gauge transformations. The singular part of the self-energy and the Green’s function obey

\[
\int_0^\beta d\tau_2 \Sigma_{\text{sing}}(\tau_1, \tau_2) G(\tau_2, \tau_3) = -\delta(\tau_1 - \tau_3)
\]

\[
\Sigma_{\text{sing}}(\tau_1, \tau_2) = -U^2 G^2(\tau_1, \tau_2) G(\tau_2, \tau_1)
\]
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5. Maximal quantum Lyapunov exponent for the out-of-time-order correlator (OTOC):

$$\langle c^\dagger_a(t)c_b(0)c_a(t)c^\dagger_b(0) \rangle = C_0 + C_1 \left( \frac{e^{\lambda t}}{N} \right) + \ldots$$

with $\lambda = 2\pi k_B T/\hbar$.

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J. Maldacena and D. Stanford, PRD 94, 106002 (2016)

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$$\left\langle c_{a}^\dagger(t)c_{b}(0)c_{a}(t)c_{b}^\dagger(0) \right\rangle = C_0 + C_1 \left( \frac{e^{\lambda t}}{N} \right) + \ldots$$

with $\lambda = 2\pi k_B T/\hbar$.

6. For spinful fermions, spin correlations decay as

$$\left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim 1/|\tau|$$

S. Sachdev and J. Ye, PRL 70, 3339 (1993)
1. SYK criticality +
    \textit{time reparameterization soft mode}

2. SYK lattice models

3. \textit{Fractionalization} and SYK criticality in \textit{t-J} models with random exchange
A strange metal: lattice of SYK islands

\[ H = \frac{1}{(2N)^{3/2}} \sum_i \sum_{a,b,c,d=1}^{N} U_{i,ab;cd} c_{ia}^\dagger c_{ib}^\dagger c_{ic} c_{id} - t \sum_{\langle ij \rangle} \sum_a c_{ia}^\dagger c_{ja} \]

Random interaction within each island \( U \).

Amplitude to hop between islands \( t \).

\[ G(k, i\omega) = \frac{1}{i\omega - \epsilon_k - \Sigma(k, i\omega)} \]

\[ \Sigma = U \]

See also Antoine Georges and Olivier Parcollet
PRB 59, 5341 (1999)

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL 119, 216601 (2017);
Pengfei Zhang, PRB 96, 205138 (2017); Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, PRX 8, 031024 (2018); Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, PRX 8, 021049 (2018)
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Amplitude to hop between islands \( t \).

Model yields SYK criticality and resistivity

\[ \rho \sim T \]

for \( t^2/U \lesssim T \lesssim U \)

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL 119, 216601 (2017);
Pengfei Zhang, PRB 96, 205138 (2017); Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, PRX 8, 031024 (2018); Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, PRX 8, 021049 (2018)

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   in *$t$-$J$ models with random exchange*
The \( t-J \) model

\[
H = -\frac{t}{\sqrt{z}} \sum_{\langle ij \rangle} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{z}} \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j
\]

We consider the hole-doped case, with no double occupancy.

\[
\alpha = \uparrow, \downarrow, \quad \{c_{i\alpha}, c_{j\beta}^\dagger\} = \delta_{ij} \delta_{\alpha\beta}, \quad \{c_{i\alpha}, c_{j\beta}\} = 0
\]

\[
\vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \hat{\sigma}_{\alpha\beta} c_{i\beta}, \quad \sum_{\alpha} c_{i\alpha}^\dagger c_{i\alpha} \leq 1, \quad \sum_{\alpha} \langle c_{i\alpha}^\dagger c_{i\alpha} \rangle = 1 - p
\]

We take the large \( z \) limit in a lattice with co-ordination number \( z \) and \( J_{ij} \) random, \( \overline{J_{ij}} = 0, \overline{J_{ij}^2} = J^2 \)

\[
|0\rangle \quad c_{\uparrow}^\dagger |0\rangle \quad c_{\downarrow}^\dagger |0\rangle
\]
Fractionalization in the $t$-$J$ model

$$H = -\frac{t}{\sqrt{z}} \sum_{\langle ij \rangle} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{z}} \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

Each site has 3 states which we map to the ‘superspin’ space of a boson $b$ (the holon) and a fermion $f_\alpha$ (the spinon):

\[
\begin{align*}
    b^\dagger & |v\rangle \\
    f^\dagger_\uparrow & |v\rangle \\
    f^\dagger_\downarrow & |v\rangle
\end{align*}
\]

\[
\begin{align*}
    c_\alpha &= f_\alpha b^\dagger \\
    \vec{S} &= \frac{1}{2} f^\dagger_\alpha \sigma_{\alpha\beta} f_\beta \\
    f^\dagger_\alpha f_\alpha + b^\dagger b &= 1
\end{align*}
\]

U(1) gauge invariance, $b \rightarrow be^{i\phi}$, $f_\alpha \rightarrow f_\alpha e^{i\phi}$
Fractionalization in the $t$-$J$ model

$$H = -\frac{t}{\sqrt{z}} \sum_{\langle ij \rangle} f_{i\alpha}^\dagger f_{j\alpha} c_i b_j b_j^\dagger + \frac{1}{\sqrt{z}} \sum_{\langle ij \rangle} \frac{J_{ij}}{4} f_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} f_{i\beta} \cdot f_{j\gamma}^\dagger \vec{\sigma}_{\gamma\delta} f_{j\delta}$$

Each site has 3 states which we map to the ‘superspin’ space of a boson $b$ (the holon) and a fermion $f_{\alpha}$ (the spinon):

$$b^\dagger |v\rangle, \quad f_{\uparrow}^\dagger |v\rangle, \quad f_{\downarrow}^\dagger |v\rangle$$

The physical electron ($c_{\alpha}$) and spin ($\vec{S}$) operators are rotations in this SU(1|2) superspin space; both $t$ and $J$ terms in $H$ are quartic in terms of fractionalized particles.

$$c_{\alpha} = f_{\alpha} b^\dagger$$

$$\vec{S} = \frac{1}{2} f_{\alpha}^\dagger \sigma_{\alpha\beta} f_{\beta}$$

$$f_{\alpha}^\dagger f_{\alpha} + b^\dagger b = 1$$

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$$\begin{align*}
\uparrow & \quad |v\rangle \\
\downarrow & \quad |v\rangle \\
\uparrow & \quad |v\rangle \\
\end{align*}$$

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  c_\alpha & = b_\alpha f^\dagger \\
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$\text{U}(1)$ gauge invariance,

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  b_\alpha & \rightarrow b_\alpha e^{i\phi}
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Fractionalization in the \( t-J \) model

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Each site has 3 states which we map to the ‘superspin’ space of a boson \( b \) (the holon) and a fermion \( f_\alpha \) (the spinon):

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\begin{align*}
&f^\dagger \ket{v} \\
&b^\dagger \ket{v} \\
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\]

The physical electron \((c_\alpha)\) and spin \((\vec{S})\) operators are rotations in this SU(2|1) superspin space; both \( t \) and \( J \) terms in \( H \) are \textit{quartic} in terms of fractionalized particles.

\[SU(1|2) \equiv SU(2|1)\]

\[
\begin{align*}
c_\alpha &= b_\alpha f^\dagger \\
\vec{S} &= \frac{1}{2} b_\alpha^\dagger \sigma_{\alpha\beta} b_\beta \\
\end{align*}
\]

\[
\begin{align*}
b_\alpha^\dagger b_\alpha + f^\dagger f &= 1 \\
U(1) \text{ gauge invariance,} & \quad f \to f e^{i\phi}, \quad b_\alpha \to b_\alpha e^{i\phi}
\end{align*}
\]
Deconfined quantum critical point/phase

\[ \langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{|\tau|} \]

\[ \langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \rangle \sim \frac{1}{\tau} \]
Deconfined quantum critical point/phase

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\]

Zeroth order, \( p_c = 1/3 \)

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX 10, 021033 (2020)
\( t-J \) phase diagram: RG using either SU(2|1) or SU(1|2)

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Disordered Fermi liquid.
Condense holon $b$, $f_\alpha$ carrier density $1 + p$

SU(1|2) theory

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX 10, 021033 (2020)
$t$-$J$ phase diagram: RG using either SU(2$|$1) or SU(1$|$2)

SU(2$|$1) theory

Metallic spin glass.
Condense spinon $b_\alpha$, 
$f$ carrier density $p$

$\langle \tilde{S}_i(\tau) \cdot \tilde{S}_i(0) \rangle \sim \text{constant}$

$\langle c_{i\alpha}(\tau) c_{i\alpha}^{\dagger}(0) \rangle \sim \frac{1}{\tau}$

SU(1$|$2) theory

Disordered Fermi liquid.
Condense holon $b$, 
$f_\alpha$ carrier density $1 + p$

$\langle \tilde{S}_i(\tau) \cdot \tilde{S}_i(0) \rangle \sim \frac{1}{\tau^2}$

$\langle c_{i\alpha}(\tau) c_{i\alpha}^{\dagger}(0) \rangle \sim \frac{1}{\tau}$

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX 10, 021033 (2020)
Fractionalization in the $t$-$J$ model

Large $M$ limit of SU($M'|M$) theory

Assuming the bosons are not condensed, we obtain SYK-like equations for the boson and fermion Green’s functions:

\[
G_b(i\omega_n) = \frac{1}{i\omega_n + \mu_b - \Sigma_b(i\omega_n)}
\]

\[
\Sigma_b(\tau) = -t^2 G_f(\tau) G_f(-\tau) G_b(\tau)
\]

\[
G_f(i\omega_n) = \frac{1}{i\omega_n + \mu_f - \Sigma_f(i\omega_n)}
\]

\[
\Sigma_f(\tau) = -J^2 G_f^2(\tau) G_f(-\tau) + k t^2 G_f(\tau) G_b(\tau) G_b(-\tau)
\]

Here $\mu_f$ and $\mu_b$ are chemical potentials chosen to satisfy

\[
\langle f^\dagger f \rangle = \frac{1}{2} - kp \quad , \quad \langle b^\dagger b \rangle = p.
\]
Fractionalization in the $t$-$J$ model

Large $M$ limit of SU($M'|M$) theory

The critical solution which is self-consistent in both the $t$ and $J$ terms has $\Delta_b = \Delta_f = 1/4$, implying

$$\langle c_\alpha(\tau)c_\alpha^\dagger(0) \rangle \sim \begin{cases} 
\frac{A_+}{|\tau|}, & \tau > 0 \\
-\frac{A_-}{|\tau|}, & \tau < 0
\end{cases}, \quad \langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim \frac{1}{|\tau|}.$$ 

RG computations show that these results for the exponents of gauge-invariant operators are expected to be exact beyond the large $M$ limit.

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX 10, 021033 (2020)
Hidden magnetism at the pseudogap critical point of a high temperature superconductor

Mehdi Frachet\textsuperscript{1}\textdagger, Igor Vinograd\textsuperscript{1}\textdagger, Rui Zhou\textsuperscript{1,2}, Siham Benhabib\textsuperscript{1}, Shangfei Wu\textsuperscript{1}, Hadrien Mayaffre\textsuperscript{1}, Steffen Krämer\textsuperscript{1}, Sanath K. Ramakrishna\textsuperscript{3}, Arneil P. Reyes\textsuperscript{3}, Jérôme Debray\textsuperscript{4}, Tohru Kurosawa\textsuperscript{5}, Naoki Momono\textsuperscript{6}, Migaku Oda\textsuperscript{5}, Seiki Komiya\textsuperscript{7}, Shimpei Ono\textsuperscript{7}, Masafumi Horio\textsuperscript{8}, Johan Chang\textsuperscript{8}, Cyril Proust\textsuperscript{1}, David LeBoeuf\textsuperscript{1}\textasteriskcentered, Marc-Henri Julien\textsuperscript{1}\textasteriskcentered

\begin{center}
\textbf{Quasi-static magnetism in the pseudogap state of La\textsubscript{2-x}Sr\textsubscript{x}CuO\textsubscript{4}.} Temperature – doping phase diagram representing $T_{\text{min}}$, the temperature of the minimum in the sound velocity, at different fields. Since superconductivity precludes the observation of $T_{\text{min}}$ in zero-field, the dashed line (brown area) represents the extrapolated $T_{\text{min}}(B=0)$. While not exactly equal to the freezing temperature $T_{\text{f}}$ (see Fig. 2), $T_{\text{min}}$ is closely tied to $T_{\text{f}}$ and so is expected to have the same doping dependence, including a peak around $p = 0.12$ in zero/low fields (ref. 2). Onset temperatures of charge order are from ref. 33 (squares) and 35 (hexagons).
\end{center}
At the critical point/phase of the $t$-$J$ model, the Fermi liquid-like behavior of the electron Green’s function

$$\langle c_{i\alpha}(\tau)c_{i\alpha}^\dagger(0) \rangle \sim \frac{1}{\tau}$$

leads to a non-zero residual resitivitiy, $\rho(0) \neq 0$. 


At the critical point/phase of the $t$-$J$ model, the Fermi liquid-like behavior of the electron Green’s function

$$
\langle c_{i\alpha}(\tau)c_{i\alpha}^{\dagger}(0) \rangle \sim \frac{1}{\tau}
$$

leads to a non-zero residual resistivity, $\rho(0) \neq 0$.

However, the critical state is not a Fermi liquid, as indicated by the slow decay of the spin correlations

$$
\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{|\tau|}
$$

Moreover, in a Fermi liquid, we expect $\rho(T) - \rho(0) \sim T^2$, which also does not hold here.
Time reparameterization soft mode

The leading corrections to the SL(2,R) invariant critical Green’s function arise from the time reparameterization soft mode, and these take the form

$$\left\langle c_{i\alpha}(\tau)c_{i\alpha}^\dagger(0)\right\rangle \sim \frac{\pi T}{\sin(\pi T\tau)} \left(1 + \alpha_G \frac{T}{J} \Phi_{\text{non-conformal}}(T\tau)\right)$$

where $\Phi_{\text{non-conformal}}(T\tau)$ is a computable (in the large $M$ limit) scaling function, and $\alpha_G$ is universally proportional to the co-efficient $\alpha_S$ of the Schwarizan action for the time reparameterization mode.

J. Maldacena and D. Stanford, PRD 94, 106002 (2016)
Finally, computing the resistivity from this Green’s function via the Kubo formula, we find

\[ \rho(T) = \rho(0) \left( 1 + 8\alpha_G \frac{T}{J} + \ldots \right) \]

Random $t$-$J$-$U_H$ model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} c_i^{\dagger} c_j + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_i^{\uparrow} n_i^{\downarrow}$$

$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_i^{\dagger} \vec{\sigma}_{\alpha\beta} c_i^{\beta}, \quad n_i^{\alpha} = c_i^{\dagger} c_i^{\alpha},$$

$J_{ij}$ random, $\overline{J_{ij}} = 0, \overline{J_{ij}^2} = J^2$

$t_{ij}$ random, $\overline{t_{ij}} = 0, \overline{t_{ij}^2} = t^2$

$U_H > 0$ non-random
Random $t$-$J$-$U_H$ model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$


$$n_{i\uparrow} + n_{i\downarrow} = 1$$

doping $p = \langle 1 - n_{i\uparrow} - n_{i\downarrow} \rangle$

Spin glass
Insulator
Random $t$-$J$-$U_H$ model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} \, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$
Random \( t-J-U_H \) model

\[
H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} c_i^{\dagger} c_j + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_i \uparrow n_i \downarrow
\]

Disordered Fermi liquid \( \langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim \frac{1}{\tau^2} \)

SYK-like criticality \( \langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim \frac{1}{|\tau|} \)

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX 10, 021033 (2020)
Random $t$-$J$-$U_H$ model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} \, c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$

Spin glass

Disordered Fermi liquid $\langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim \frac{1}{\tau^2}$

Metal-insulator transition with SYK criticality

SYK-like criticality $\langle \vec{S}(\tau) \cdot \vec{S}(0) \rangle \sim \frac{1}{|\tau|}$

$doping$ $p = \langle 1 - n_{i\uparrow} - n_{i\downarrow} \rangle$

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX 10, 021033 (2020)
Linear resistivity and Sachdev–Ye–Kitaev (SYK) spin liquid behavior in a quantum critical metal with spin–1/2 fermions

Peter Cha, Nils Wentzell, Olivier Parcollet, Antoine Georges, Eun–Ah Kim

Critical spin correlations: 
\[ \left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|} \]

Resistivity \( \rho \sim T \) to the lowest \( T \) at the critical point in a large-dimension model
Challenge for theory:

A model of a metal in which the resistivity, \( \rho \), obeys

\[
\lim_{T \to 0} \frac{d\rho}{dT} \neq 0
\]

\[
\rho(T) = \rho(0) + AT + \ldots, \quad T \to 0.
\]
2 key ingredients

1. Emergent gauge symmetry and fractionalization

\[ c_\alpha = f_\alpha b^\dagger \]
\[ f_\alpha \rightarrow e^{i\phi} f_\alpha, \quad b \rightarrow b e^{i\phi} \]

2. Time reparameterization symmetry

\[ \tau \rightarrow f(\tau) \]

Present in models of non-Fermi liquids:
quantum matter at variable density
without quasiparticle excitations