Quantum entanglement and the phases of matter

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Foundations of quantum many body theory:

1. Ground states connected adiabatically to independent electron states
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2. Boltzmann-Landau theory of quasiparticles
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Metals
Quantum entanglement:

EPR pair: Non-local correlations between quantum measurements due to superposition between many-electron states

Hydrogen molecule
Quantum entanglement:
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1. Ground states connected adiabatically to independent electron states

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Metals

**Modern phases of quantum matter:**

1. *Ground states disconnected from independent electron states: many-particle entanglement*

2. *Boltzmann-Landau theory of quasiparticles*

**Famous example:**

The fractional quantum Hall effect of electrons in two dimensions (e.g. in graphene) in the presence of a strong magnetic field. The ground state is described by Laughlin’s wavefunction, and the excitations are *quasiparticles* which carry fractional charge.
Modern phases of quantum matter:

1. Ground states disconnected from independent electron states: many-particle entanglement

2. No quasiparticles
Quantum matter without quasiparticles:

1. Ground states disconnected from independent electron states: many-particle entanglement

2. No quasiparticles

- Superfluid-insulator transition of ultracold bosonic atoms in an optical lattice
- Graphene
- Strange metals in high temperature superconductors
- Quark-gluon plasma
- Charged black hole horizons in anti-de Sitter space
Quantum matter without quasiparticles:

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Superfluid-insulator transition


Ultracold $^{87}$Rb atoms - bosons
\[ U \gg t \]

Insulator (the vacuum)
at large repulsion between bosons

\[ |\text{Ground state}\rangle = \prod_i b_i^\dagger |0\rangle \]

On-site repulsion between bosons = \( U \)
Tunneling amplitude between sites = \( t \)
Excitations of the insulator:

Particles $\sim \psi^\dagger$

On-site repulsion between bosons $= U$
Tunneling amplitude between sites $= t$
$U \gg t$

Excitations of the insulator:

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Holes $\sim \psi$
$U \gg t$

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On-site repulsion between bosons $= U$
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Excitations of the insulator:

\[ U \gg t \]

Holes \( \sim \psi \)

On-site repulsion between bosons = \( U \)
Tunneling amplitude between sites = \( t \)
Superfluid at small repulsion between bosons

\[ |\text{Ground state}\rangle = \left[ \sum_i b_i^\dagger \right]^N |0\rangle \]

On-site repulsion between bosons = \( U \)
Tunneling amplitude between sites = \( t \)
The diagram illustrates a phase transition between a superfluid and an insulator. The critical point, marked as $\lambda_c$, indicates the boundary between these two states. The parameter $\lambda \sim U/t$ is used to characterize the proximity to this critical point, with $U$ being the on-site interaction energy and $t$ the hopping parameter.
\( \Psi \rightarrow \text{a complex field representing the Bose-Einstein condensate of the superfluid} \)

\[ \langle \Psi \rangle \neq 0 \quad \text{Superfluid} \]

\[ \langle \Psi \rangle = 0 \quad \text{Insulator} \]

\( \lambda_c \)

\( \lambda \sim U/t \)
\[
S = \int d^2 r dt \left[ |\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]
\]

\[
V(\Psi) = (\lambda - \lambda_c) |\Psi|^2 + u (|\Psi|^2)^2
\]

**Long-range quantum entanglement and no quasiparticles:**
A conformal field theory
in 2+1 spacetime dimensions:
a CFT3

\[
\langle \Psi \rangle \neq 0 \quad \text{Superfluid}
\]

\[
\langle \Psi \rangle = 0 \quad \text{Insulator}
\]

\[
\lambda \sim U/t
\]
Quantum critical

Superfluid

Insulator

$T_K T$

$0$

$\lambda_c$

$\lambda \sim U/t$
“Boltzmann” theory of Nambu-Goldstone phonons and vortices

Boltzmann theory of particles/holes

Quantum critical

Superfluid

Insulator

$T$

$0$

$\lambda_c$

$\lambda \sim U/t$
CFT3 at $T>0$

Quantum critical

Superfluid

Insulator

$T_{KT}$

$\lambda_c$

$\lambda \sim U/t$
CFT3 at $T > 0$

Quantum matter without quasiparticles
CFT3 at $T > 0$

Shortest possible local thermal equilibration time $\sim \frac{\hbar}{k_B T}$

Quantum matter without quasiparticles:

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Graphene

Same “Hubbard” model as for ultracold atoms, but for electrons on the honeycomb lattice
Graphene

Hole Fermi surface

Electron Fermi surface

$\mu < 0$

$\mu > 0$
Graphene

M. Müller, L. Fritz, and S. Sachdev, PRB 78, 115406 (2008)
M. Müller and S. Sachdev, PRB 78, 115419 (2008)
Graphene

Predicted strange metal

$T(K)$

Quantum critical
Dirac liquid

Hole Fermi liquid

Electron Fermi liquid

$\mu > 0$

$\mu < 0$

M. Müller, L. Fritz, and S. Sachdev, PRB 78, 115406 (2008)
M. Müller and S. Sachdev, PRB 78, 115419 (2008)
Fermi liquids: quasiparticles moving ballistically between impurity (red circles) scattering events
Fermi liquids: quasiparticles moving ballistically between impurity (red circles) scattering events

Strange metals: electrons scatter frequently off each other, so there is no regime of ballistic quasiparticle motion. The electron “liquid” then “flows” around impurities
Thermal and electrical conductivity with quasiparticles

- Wiedemann-Franz law in a Fermi liquid:

\[
L_0 = \frac{\kappa}{\sigma T} \approx \frac{\pi^2 k_B^2}{3e^2} \approx 2.45 \times 10^{-8} \frac{W}{\Omega K^2}.
\]
Transport in Strange Metals

For a strange metal with a “relativistic” Hamiltonian, hydrodynamic, holographic, and memory function methods yield Lorentz ratio $L = \kappa/(T\sigma)$

$$L = \frac{v_F^2 \mathcal{H} \tau_{\text{imp}}}{T^2 \sigma_Q} \frac{1}{(1 + e^2 v_F^2 Q^2 \tau_{\text{imp}}/(\mathcal{H} \sigma_Q))^2}$$

$Q \rightarrow$ electron density; $\mathcal{H} \rightarrow$ enthalpy density
$\sigma_Q \rightarrow$ quantum critical conductivity
$\tau_{\text{imp}} \rightarrow$ momentum relaxation time from impurities.

Note that for a clean system ($\tau_{\text{imp}} \rightarrow \infty$ first), the Lorentz ratio diverges $L \sim 1/Q^4$, as we approach “zero” electron density at the Dirac point.

M. Müller and S. Sachdev, PRB 78, 115419 (2008)
Graphene

Quantum critical
Dirac liquid

Hole Fermi liquid

Electron Fermi liquid

$T(K)$

$\frac{n}{10^{12}/m^2}$

M. Müller, L. Fritz, and S. Sachdev, PRB 78, 115406 (2008)
M. Müller and S. Sachdev, PRB 78, 115419 (2008)
Graphene

$T(K)$

Quantum Critical

Directed

Hole
Fermi liquid

Electron
Fermi liquid

Predicted strange metal

$\sim \sqrt{n} (1 + \lambda \ln \frac{n}{\sqrt{2}})$

$\mu > 0$

$\mu < 0$

$T(K)$

$\frac{n}{10^{12}/m^2}$

M. Müller, L. Fritz, and S. Sachdev, PRB 78, 115406 (2008)

M. Müller and S. Sachdev, PRB 78, 115419 (2008)
Graphene

Quantum critical
Dirac liquid

Hole
Fermi liquid

Electron
Fermi liquid

M. Müller, L. Fritz, and S. Sachdev, PRB 78, 115406 (2008)
M. Müller and S. Sachdev, PRB 78, 115419 (2008)
Realization of the Dirac fluid in graphene requires that to keep a well-defined temperature profile the breakdown of the WF law can be observed. This high temperature limit occurs when the thermal energy be larger than the local chemical potential, and even when the sample is globally neutral, it is locally doped to form electron-hole puddles with finite potential.

Red dots: data
Blue line: value for $L = L_0$
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The breakdown of the WF law can be observed. These two temperatures set the experimental window in which the DF and electron-electron scattering rate becomes comparable to the electron-phonon scattering rate becomes comparable to the local chemical potential, and even when the sample is globally neutral, it is locally doped to form electron-hole puddles with finite potential. To minimize disorder, the monolayer graphene samples are encapsulated in hexagonal boron nitride (hBN) used in this report are.

The minimum density (green) aligns with the temperature axis to the right. Solid black lines correspond to a linear fit of log($\kappa$) versus $T$, we find the thermal energy be larger than the local chemical potential, and even when the sample is globally neutral, it is locally doped to form electron-hole puddles with finite potential. To minimize disorder, the monolayer graphene samples are encapsulated in hexagonal boron nitride (hBN) used in this report are.

Red dots: data
Blue line: value for $L = L_0$
Graphene

Quantum critical
Dirac liquid

Hole Fermi liquid

Electron Fermi liquid

Impurity scattering dominates

M. Müller, L. Fritz, and S. Sachdev, PRB 78, 115406 (2008)
M. Müller and S. Sachdev, PRB 78, 115419 (2008)
Wiedemann-Franz Law Violations in Experiment

Strange metal in graphene

J. Crossno et al., Science 351, 1058 (2016)

Phonon-limited

Disorder-limited

Wiedemann-Franz obeyed
Strange metal in graphene

Wiedemann-Franz violated!
two-terminal to keep a well-defined temperature profile

...used in this report are encapsulated in hexagonal boron

...temperatures set the experimental window in which the

...channels. This high temperature limit occurs when the

...potential, and even when the sample is globally neutral, it

...be larger than the local chemical po-

...sample S2 (inset 1E).

...neutrality point.

...FIG. 1.

...To minimize disorder, the monolayer graphene samples

...Temperature and density dependent electrical and thermal conductivity. (A)

...Ω

...0.6

...1.2

...10

...A

...B

...FIG. 3.

...F

...H

...e

...C

...h

...n

...κ

...L

...min

...T

...m

...|l

...|M

...n

...J. Crossno et al., Science 351, 1058 (2016)

...S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB 76, 144502 (2007)

...Lorentz ratio \( L = \frac{\kappa}{T\sigma} \)

\[
L = \frac{v_F^2 \mathcal{H} \tau_{\text{imp}}}{T^2 \sigma_Q} \left( 1 + \frac{e^2 v_F^2 Q^2 \tau_{\text{imp}}}{(\mathcal{H} \sigma_Q)^2} \right)
\]

\( Q \rightarrow \) electron density; \( \mathcal{H} \rightarrow \) enthalpy density

\( \sigma_Q \rightarrow \) quantum critical conductivity

\( \tau_{\text{imp}} \rightarrow \) momentum relaxation time from impurities
Strange metal in graphene

Negative local resistance due to viscous electron backflow in graphene

Strange metal in graphene

Negative local resistance due to viscous electron backflow in graphene

D. A. Bandurin\textsuperscript{1}, I. Torre\textsuperscript{2,3}, R. Krishna Kumar\textsuperscript{1,4}, M. Ben Shalom\textsuperscript{1,5}, A. Tomadin\textsuperscript{6}, A. Principi\textsuperscript{7}, G. H. Auton\textsuperscript{5}, E. Khestanova\textsuperscript{1,5}, K. S. Novoselov\textsuperscript{5}, I. V. Grigorieva\textsuperscript{1}, L. A. Ponomarenko\textsuperscript{1,4}, A. K. Geim\textsuperscript{1}, M. Polini\textsuperscript{3,6}

Figure 1. Viscous backflow in doped graphene. (a,b) Steady-state distribution of current injected through a narrow slit for a classical conducting medium with zero \(\nu\) (a) and a viscous Fermi liquid (b). (c) Optical micrograph of one of our SLG devices. The schematic explains the measurement geometry for vicinity resistance. (d,e) Longitudinal conductivity \(\sigma_{xx}\) and \(R_V\) for this device as a function of \(n\) induced by applying gate voltage. \(I = 0.3\ \mu\text{A}; L = 1\ \mu\text{m.}\) For more detail, see Supplementary Information.
Graphene: “a metal that behaves like water”
Quantum matter without quasiparticles:

- Superfluid-insulator transition of ultracold bosonic atoms in an optical lattice
- Graphene
- Strange metals in high temperature superconductors
- Quark-gluon plasma
- Charged black hole horizons in anti-de Sitter space
Figure: K. Fujita and J. C. Seamus Davis

$\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$
Proposed a SU(2) gauge theory for transition for quantum-criticality at optimal doping, as the origin of strange metal (SM) behavior at higher $T$. 

The diagram shows a phase diagram with $T(K)$ on the y-axis and $\rho$ on the x-axis, highlighting the regions for PG, DW, SM, FL, dSC, and dSC + DW. The arrow indicates the transition at $\rho = 0.20$.
Quark-gluon plasma
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Tensor network representation of entanglement at quantum critical point

$d$-dimensional space

depth of entanglement

Tensor network representation of entanglement at quantum critical point

Emergent direction of an equivalent theory of quantum gravity

Brian Swingle, arXiv:0905.1317
String theory near a D-brane

Emergent direction of AdS4
Infinite-range (SYK) model of a strange metal

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell} c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$Q = \frac{1}{N} \sum_i c_i^\dagger c_i$$

$J_{ij;k\ell}$ are independent random variables with $\overline{J_{ij;k\ell}} = 0$ and $|J_{ij;k\ell}|^2 = J^2$

$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL 70, 3339 (1993)

Infinite-range (SYK) model of a strange metal

\[ H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;kl} c_i^{\dagger} c_j c_k c_{\ell} - \mu \sum_i c_i^{\dagger} c_i \]

\[ c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^{\dagger} + c_j^{\dagger} c_i = \delta_{ij} \]

\[ Q = \frac{1}{N} \sum_i c_i^{\dagger} c_i \]

A fermion can move only by entangling with another fermion: the Hamiltonian has “nothing but entanglement”.

S. Sachdev and J. Ye, PRL 70, 3339 (1993)
Infinite-range strange metals

Local fermion density of states

$$\rho(\omega) = -\text{Im } G(\omega) \sim \begin{cases} \omega^{-1/2}, & \omega > 0 \\ e^{-2\pi\mathcal{E}}|\omega|^{-1/2}, & \omega < 0. \end{cases}$$

\( \mathcal{E} \) encodes the particle-hole asymmetry

While \( \mathcal{E} \) determines the low energy spectrum, it is determined by the total fermion density \( Q \):

$$Q = \frac{1}{4} (3 - \tanh(2\pi\mathcal{E})) - \frac{1}{\pi} \tan^{-1} (e^{2\pi\mathcal{E}}).$$

Analog of the relationship between \( Q \) and \( k_F \) in a Fermi liquid.

\[ H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{i,j;k,\ell} c_i^\dagger c_j^\dagger c_k c_\ell \]

\[ Q = \frac{1}{N} \sum_i \langle c_i^\dagger c_i \rangle. \]

Local fermion density of states
\[ \rho(\omega) \sim \begin{cases} 
\omega^{-1/2}, & \omega > 0 \\
 e^{-2\pi\mathcal{E}} |\omega|^{-1/2}, & \omega < 0.
\end{cases} \]

Known ‘equation of state’ determines \( \mathcal{E} \) as a function of \( Q \)
\[ H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{i;j;k;\ell} c_i \dagger c_j \dagger c_k c_\ell \]

\[ Q = \frac{1}{N} \sum_{i} \langle c_i \dagger c_i \rangle. \]

Local fermion density of states

\[ \rho(\omega) \sim \begin{cases} \omega^{-1/2}, & \omega > 0 \\ e^{-2\pi\varepsilon} |\omega|^{-1/2}, & \omega < 0. \end{cases} \]

Known ‘equation of state’ determines \( \mathcal{E} \) as a function of \( Q \)

Microscopic zero temperature entropy density, \( S \), obeys

\[ \frac{\partial S}{\partial Q} = 2\pi \mathcal{E} \]
Holographic gravity theory

Start with simplest theory of Einstein gravity and Maxwell electromagnetism

\[ S_{EM} = \int d^{d+2}x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( \mathcal{R} + \frac{d(d+1)}{L^2} - \frac{R^2}{g_F^2} F^2 \right) \right] \]
Holographic gravity theory

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Solve equations of motion in the presence of a \( d \)-dimensional flat boundary with charge density \( Q \)

AdS-Reissner-Nordstrom

Quantum matter on the boundary with a variable charge density \( Q \) of a global \( U(1) \) symmetry.

A. Chamblin, R. Emparan, C.V. Johnson, and R. C. Myers, PRD 60, 064018 (1999)
Charged black branes

AdS-Reissner-Nordstrom

Electric flux

Quantum matter on the boundary with a variable charge density $Q$ of a global U(1) symmetry.

Realizes a strange metal: a state with an unbroken global U(1) symmetry with a continuously variable charge density, $Q$, at $T = 0$ which does not have any quasiparticle excitations.
Quantum fields on charged black branes

$\text{AdS}_2 \times R^d$

$ds^2 = \left( d\zeta^2 - dt^2 \right)/\zeta^2 + d\vec{x}^2$

Gauge field: $A = (\mathcal{E}/\zeta) dt$

$\zeta = \infty$

Dirac fermion $\psi$ of mass $m$

$\text{AdS}_2$ boundary Green's function of $\psi$ at $T = 0$

$\text{Im} G(\omega) \sim \begin{cases} \omega^{-(1-2\Delta)}, \omega > 0 \\ e^{-2\pi \mathcal{E}} |\omega|^{-(1-2\Delta)}, \omega < 0. \end{cases}$

where the fermion scaling dimension $\Delta$ is a function of $m$

$\mathcal{E}$ encodes the particle-hole asymmetry

T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh, PRD 83, 125002 (2011)
\[ H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;kl} c_i^\dagger c_j^\dagger c_k c_\ell \]

\[ Q = \frac{1}{N} \sum_{i} \langle c_i^\dagger c_i \rangle. \]

Local fermion density of states

\[ \rho(\omega) \sim \begin{cases} \omega^{-1/2}, & \omega > 0 \\ e^{-2\pi \varepsilon |\omega|^{-1/2}}, & \omega < 0. \end{cases} \]

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Microscopic zero temperature entropy density, \( S \), obeys
\[ \frac{\partial S}{\partial Q} = 2\pi \mathcal{E} \]

Einstein-Maxwell theory + cosmological constant

Horizon area \( \mathcal{A}_h \):
\[ \text{AdS}_2 \times R^d \]
\[ ds^2 = (d\zeta^2 - dt^2)/\zeta^2 + d\vec{x}^2 \]
Gauge field: \( A = (\mathcal{E}/\zeta) dt \)

\[ \zeta = \infty \]
\[ \zeta \]
\[ \mathcal{L} = \overline{\psi} \Gamma^\alpha D_\alpha \psi + m \overline{\psi} \psi \]

Local fermion density of states
\[ \rho(\omega) \sim \begin{cases} \omega^{-1/2}, & \omega > 0 \\ e^{-2\pi\varepsilon} |\omega|^{-1/2}, & \omega < 0. \end{cases} \]

‘Equation of state’ relating \( \mathcal{E} \) and \( Q \) depends upon the geometry of spacetime far from the \( \text{AdS}_2 \)
\[ H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij; k\ell} c_i^{\dagger} c_j^{\dagger} c_k c_\ell \]

\[ Q = \frac{1}{N} \sum_i \langle c_i^\dagger c_i \rangle . \]

Local fermion density of states
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Microscopic zero temperature entropy density, \( S \), obeys
\[ \frac{\partial S}{\partial Q} = 2\pi \mathcal{E} \]

Black hole thermodynamics (classical general relativity) yields
\[ \frac{\partial S_{\text{BH}}}{\partial Q} = 2\pi \mathcal{E} \]

Einstein-Maxwell theory + cosmological constant

Horizon area \( \mathcal{A}_h \);
\( \text{AdS}_2 \times R^d \)
\[ ds^2 = (d\zeta^2 - dt^2)/\zeta^2 + d\vec{x}^2 \]
Gauge field: \( A = (\mathcal{E}/\zeta)dt \)

Boundary area \( \mathcal{A}_b \);
charge density \( Q \)

Local fermion density of states
\[ \rho(\omega) \sim \begin{cases} 
\omega^{-1/2}, & \omega > 0 \\
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\end{cases} \]

'Equation of state' relating \( \mathcal{E} \) and \( Q \) depends upon the geometry of spacetime far from the \( \text{AdS}_2 \)

A. Sen hep-th/0506177; S. Sachdev PRX 5, 041025 (2015)
\[ H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij; k\ell} c_i^\dagger c_j^\dagger c_k c\ell \]

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Known ‘equation of state’ determines \( \mathcal{E} \) as a function of \( Q \)

Microscopic zero temperature entropy density, \( S \), obeys
\[ \frac{\partial S}{\partial Q} = 2\pi\varepsilon \]

Evidence for AdS\(_2\) gravity dual of \( H \)

\[ \frac{\partial S_{BH}}{\partial Q} = 2\pi\varepsilon \]

S. Sachdev, PRL 105, 151602 (2010); PRX 5, 041025 (2015)
A bound on quantum chaos:

- The “Lyapunov exponent” for chaos, $\lambda_L$, is given by out-of-time-order correlators, and for quantum systems near equilibrium, it obeys the bound $\lambda_L \leq 2\pi k_B T/\hbar$.

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- This bound is saturated by holographic theories with Einstein gravity. This makes it similar to the $\eta/s > 1/(4\pi)$ bound.

  S. H. Shenker and D. Stanford, arXiv:1306.0622
A bound on quantum chaos:

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- This bound is saturated by holographic theories with Einstein gravity. This makes it similar to the $\eta/s > 1/(4\pi)$ bound.

  S. H. Shenker and D. Stanford, arXiv:1306.0622

- The bound is also saturated by the SYK model

  A. Kitaev, unpublished
Entangled quantum matter without quasiparticles

- Superfluid-insulator transition of ultracold bosonic atoms in an optical lattice
- Graphene
- Strange metals in high temperature superconductors
- Quark-gluon plasma
- Charged black hole horizons in anti-de Sitter space
Entangled quantum matter without quasiparticles

- No quasiparticle excitations
- Shortest possible “collision time”, or more precisely, fastest possible local equilibration time $\sim \frac{\hbar}{k_B T}$
- Continuously variable density, $Q$ (conformal field theories are usually at fixed density, $Q = 0$)
- Theory built from hydrodynamics/holography/memory-functions/strong-coupled-field-theory
- Exciting experimental realization in graphene.
- Future work: detection of hydrodynamic flow in other strange metals