Competing orders in the high temperature superconductors: implications of recent experiments

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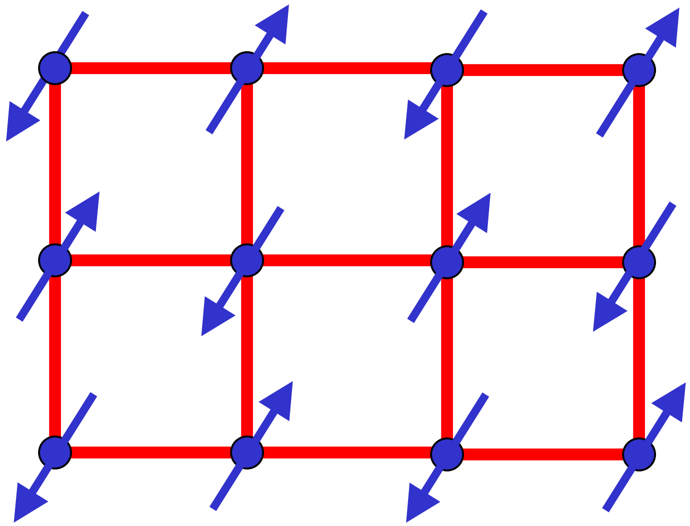
Science **286**, 2479 (1999).

Transparencies on-line at
http://pantheon.yale.edu/~subir

Parent compound of the high temperature superconductors: La_2CuO_4

Square lattice antiferromagnet

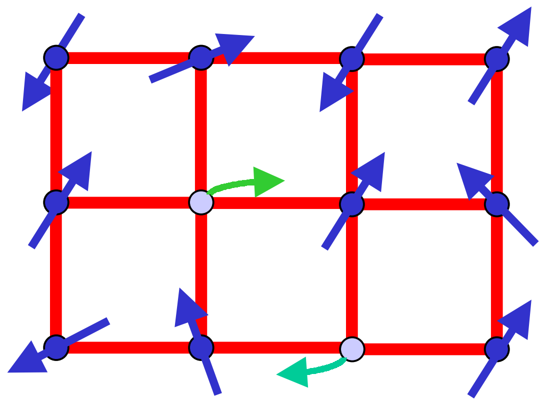


$$H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

Ground state has long-range magnetic (Neel) order

$$\left\langle \vec{S}_i \right\rangle = (-1)^{i_x+i_y} N_0 \neq 0$$

Introduce mobile carriers of density δ by substitutional doping of out-of-plane ions *e.g.* $La_{2-\delta}Sr_{\delta}CuO_4$



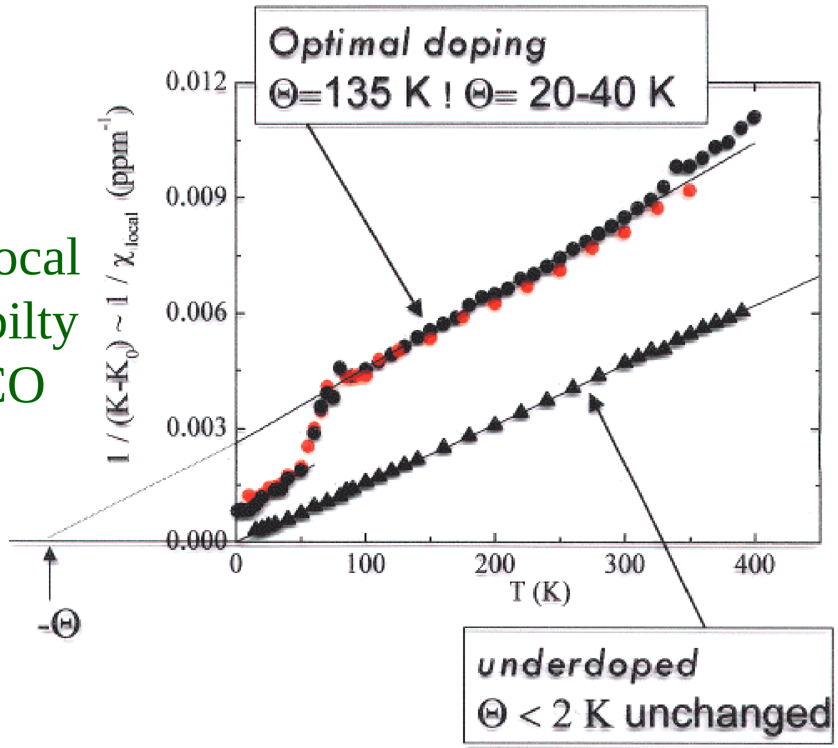
Exhibits superconductivity below a high critical temperature T_c

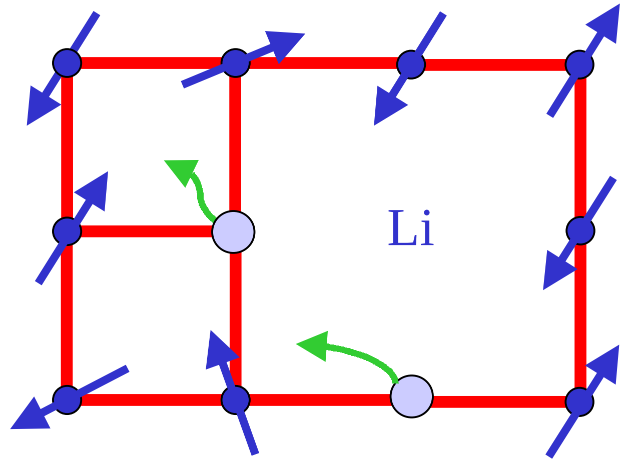
Almost all $T \rightarrow 0$ properties can be understood in the framework of a standard BCS theory in which the electrons form spin-singlet, *d*-wave Cooper pairs. However, many $T > T_c$ properties are anomalous.

$$\text{As } T \to 0, \quad \langle S_i \rangle = 0 \quad \text{and} \quad \chi_{\text{spin}} = 0$$

Measurement of spin susceptibility near non-magnetic (Zn/Li) impurities

Inverse local susceptibilty in YBCO





^{7}Li NMR below T_c

J. Bobroff, H. Alloul, W.A. MacFarlane, P. Mendels, N. Blanchard, G. Collin, and J.-F. Marucco, Phys. Rev. Lett. **86**, 4116 (2001).

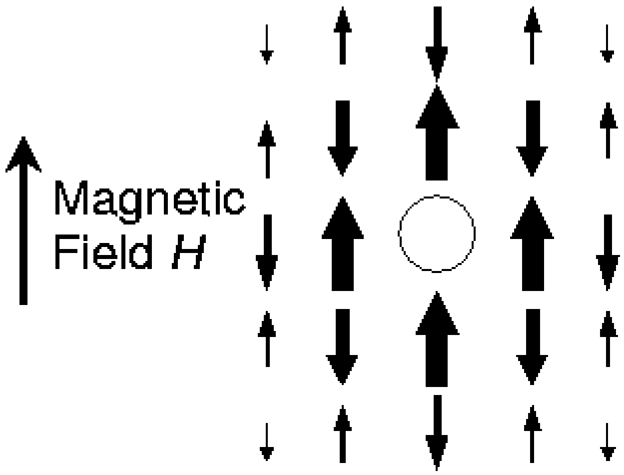
Measured $\chi_{\text{impurity}}(T \to 0) = \dfrac{S(S+1)}{3k_B T}$ with $S = 1/2$ in underdoped sample.

Not expected from BCS theory, which predicts $\chi_{\text{impurity}}(T \to 0) \neq \infty$ for a non-magnetic impurity with strong potential scattering.

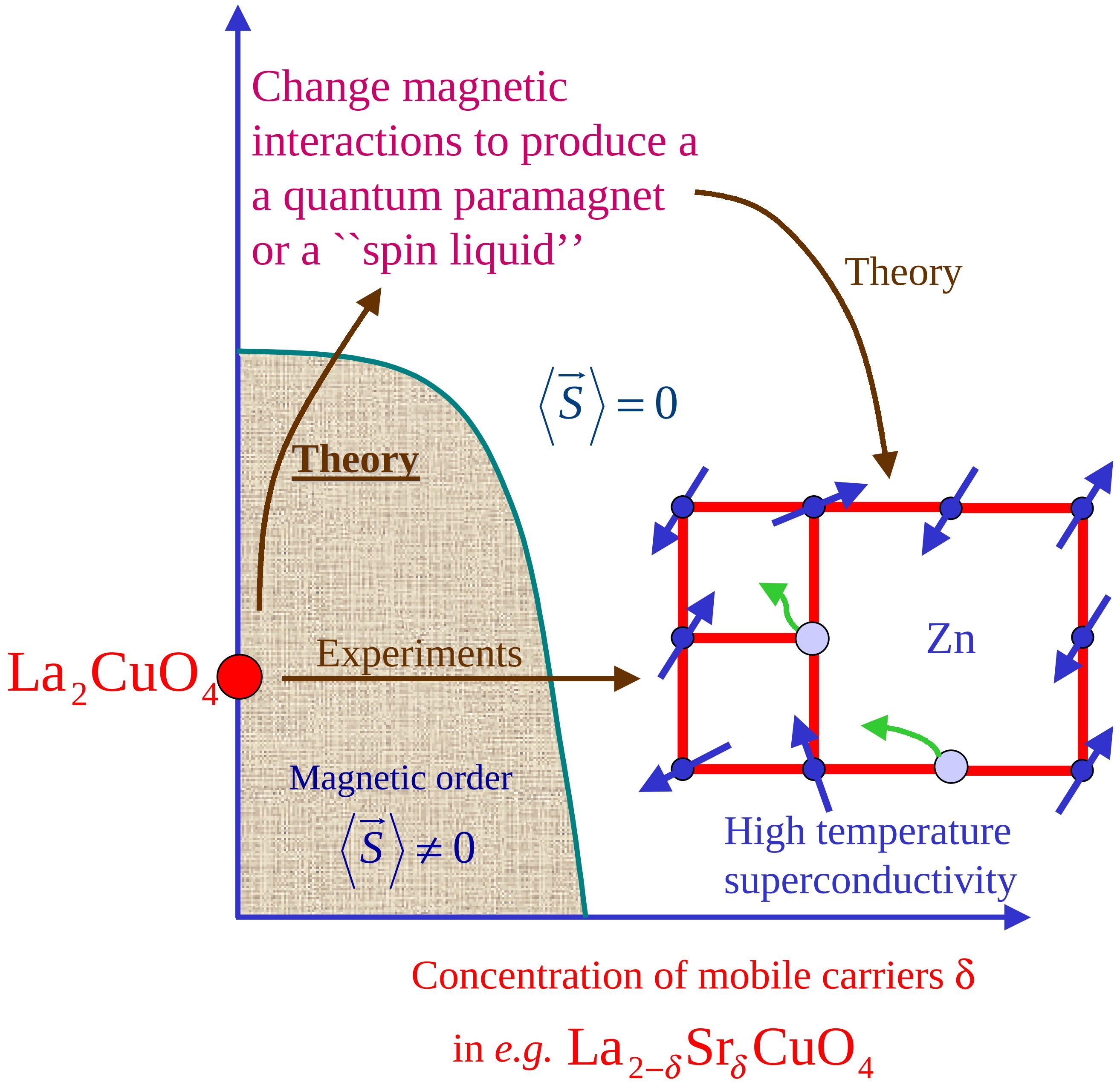
Zn impurity in $YBa_2Cu_3O_{6.7}$

Moments measured by analysis of Knight shifts

M.-H. Julien, T. Feher, M. Horvatic, C. Berthier, O. N. Bakharev, P. Segransan, G. Collin, and J.-F. Marucco, Phys. Rev. Lett. **84**, 3422 (2000); also earlier work of the group of H. Alloul and the original experiment of A.M Finkelstein, V.E. Kataev, E.F. Kukovitskii, and G.B. Teitel'baum, Physica C **168**, 370 (1990).



Berry phases of precessing spins do not cancel between the sublattices in the vicinity of the impurity: net uncancelled phase of S=1/2



Change magnetic
interactions to produce a
a quantum paramagnet
or a ``spin liquid''
Theory
$\langle \vec{S} \rangle = 0$
Theory
La_2CuO_4
Experiments
Zn
Magnetic order
$\langle \vec{S} \rangle \neq 0$
High temperature
superconductivity
Concentration of mobile carriers δ
in *e.g.* $La_{2-\delta}Sr_{\delta}CuO_4$

Outline

I. Why do non-magnetic impurities acquire a $S=1/2$ moment ?

   A. Insulating quantum paramagnets

   B. Doped antiferromagnets

II. Effect on Zn impurities on $S=1$ spin exciton.

   Comparison of theory with neutron scattering experiments.

III. Effect of magnetic field on antiferromagnetic order in superconductor.

   Comparison of theory with neutron scattering experiments.

IV. Conclusions

I. Why do non-magnetic impurities acquire a *S=1/2* moment ?

I.A Insulating quantum paramagnets

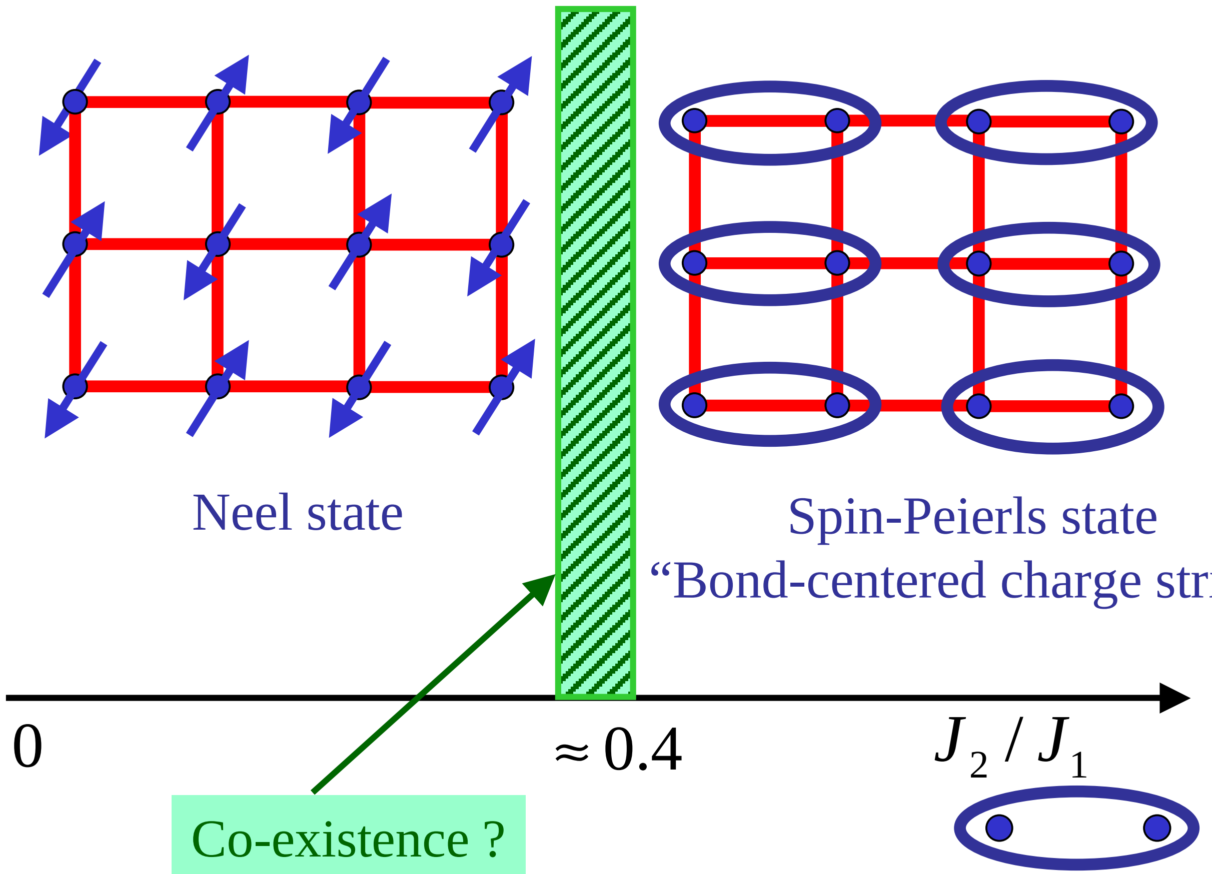
$$H = \sum_{i<j} J_{ij} \; \vec{S}_i \cdot \vec{S}_j$$

Square lattice with first (J_1) and second (J_2) neighbor exchange interactions

N. Read and S. Sachdev, Phys. Rev. Lett. **62**, 1694 (1989).

O. P. Sushkov, J. Oitmaa, and Z. Weihong, Phys. Rev. B **63**, 104420 (2001).

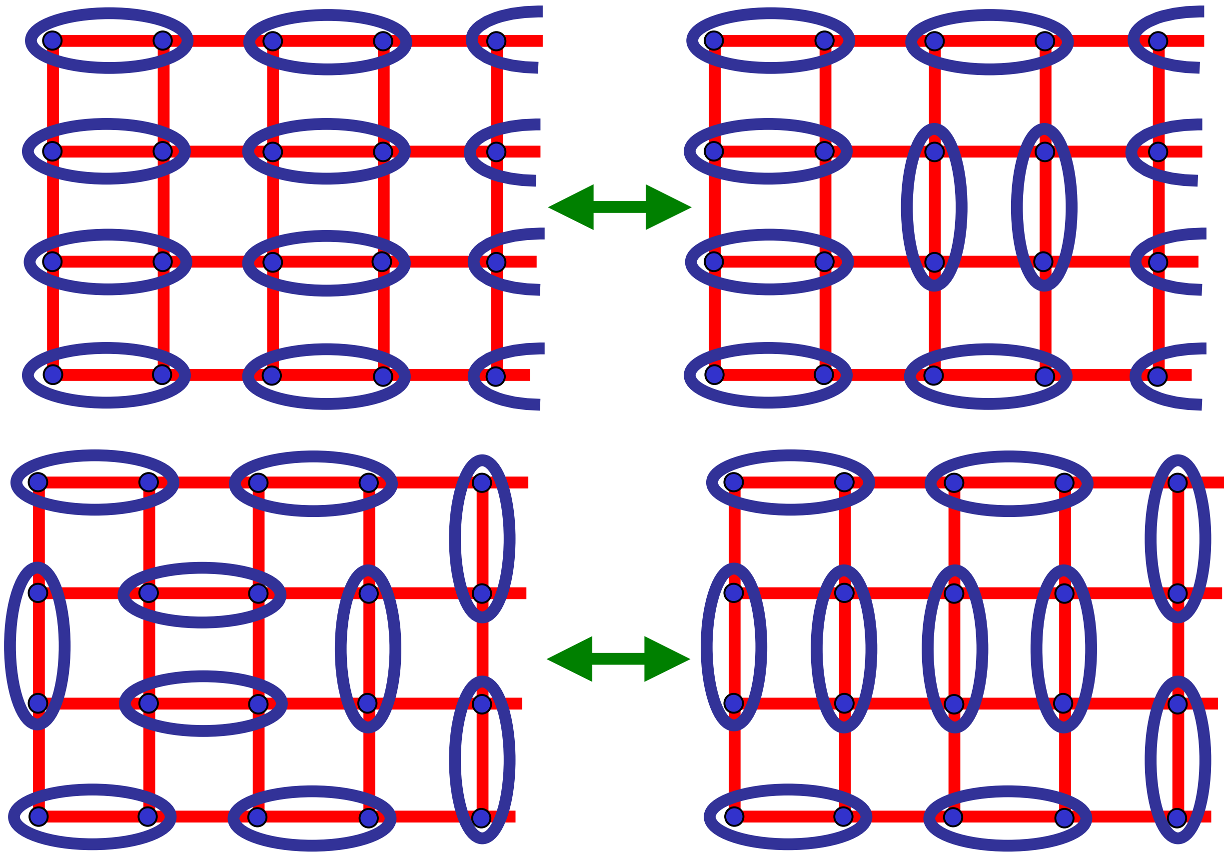
M.S.L. du Croo de Jongh, J.M.J. van Leeuwen, W. van Saarloos, Phys. Rev. B **62**, 14844 (2000).



$$= \frac{1}{\sqrt{2}} \left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle \right)$$

Quantum dimer model –

D. Rokhsar and S. Kivelson Phys. Rev. Lett. **61**, 2376 (1988)



Quantum “entropic” effects prefer one-dimensional striped structures in which the largest number of singlet pairs can resonate. The state on the upper left has more flippable pairs of singlets than the one on the lower left.

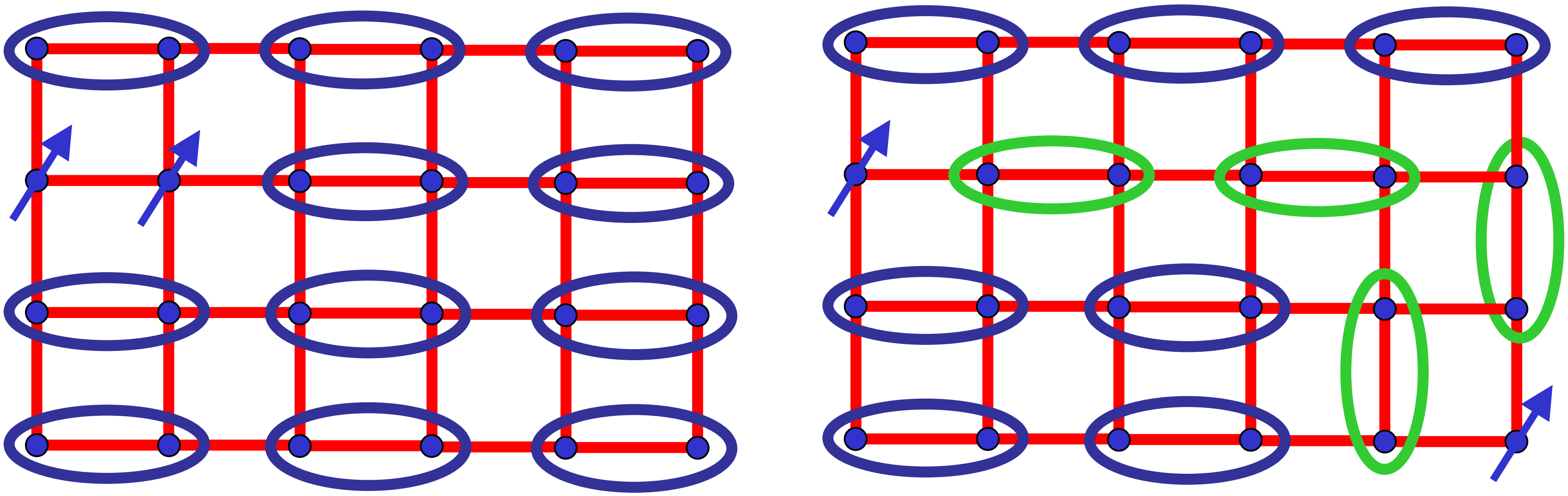
These effects always lead to a broken square lattice symmetry near the transition to the Neel state.

N. Read and S. Sachdev Phys. Rev. B **42**, 4568 (1990).

Excitations

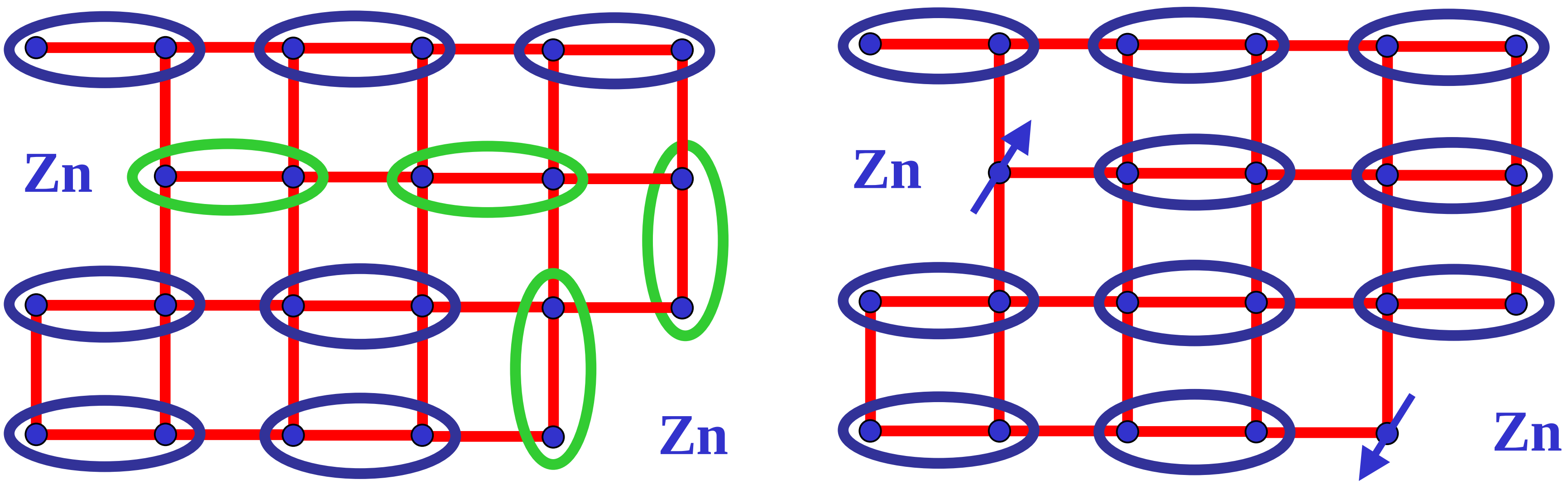
Stable S=1 spin exciton

Energy dispersion $$\varepsilon_k = \Delta + \frac{c_x^2 k_x^2 + c_y^2 k_y^2}{2\Delta}$$ $\Delta \rightarrow$ Spin gap



S=1/2 spinons are linearly confined by the line of "defect" singlet pairs between them

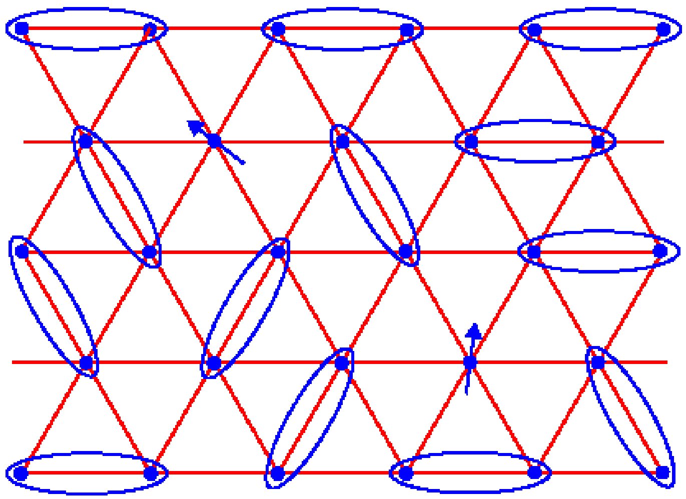
Effect of static non-magnetic impurities (Zn or Li)



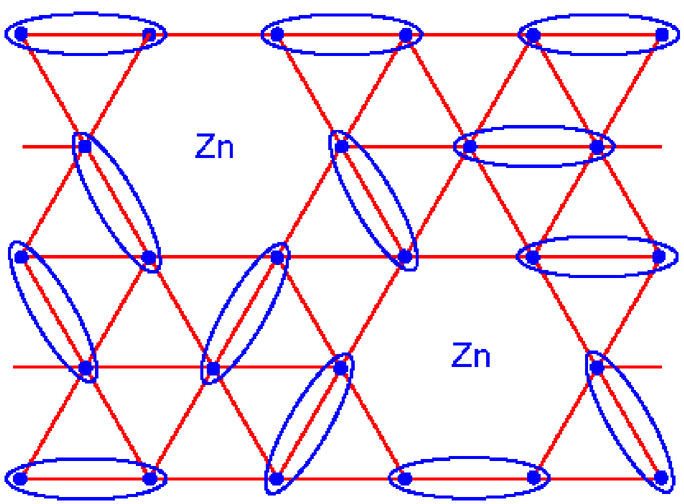
Spinon confinement implies that free S=1/2 moments **must** form near each impurity

$$\chi_{\text{impurity}}(T \to 0) = \frac{S(S+1)}{3k_B T}$$

Paramagnetic ground state with spinon deconfinement



Spinons are deconfined



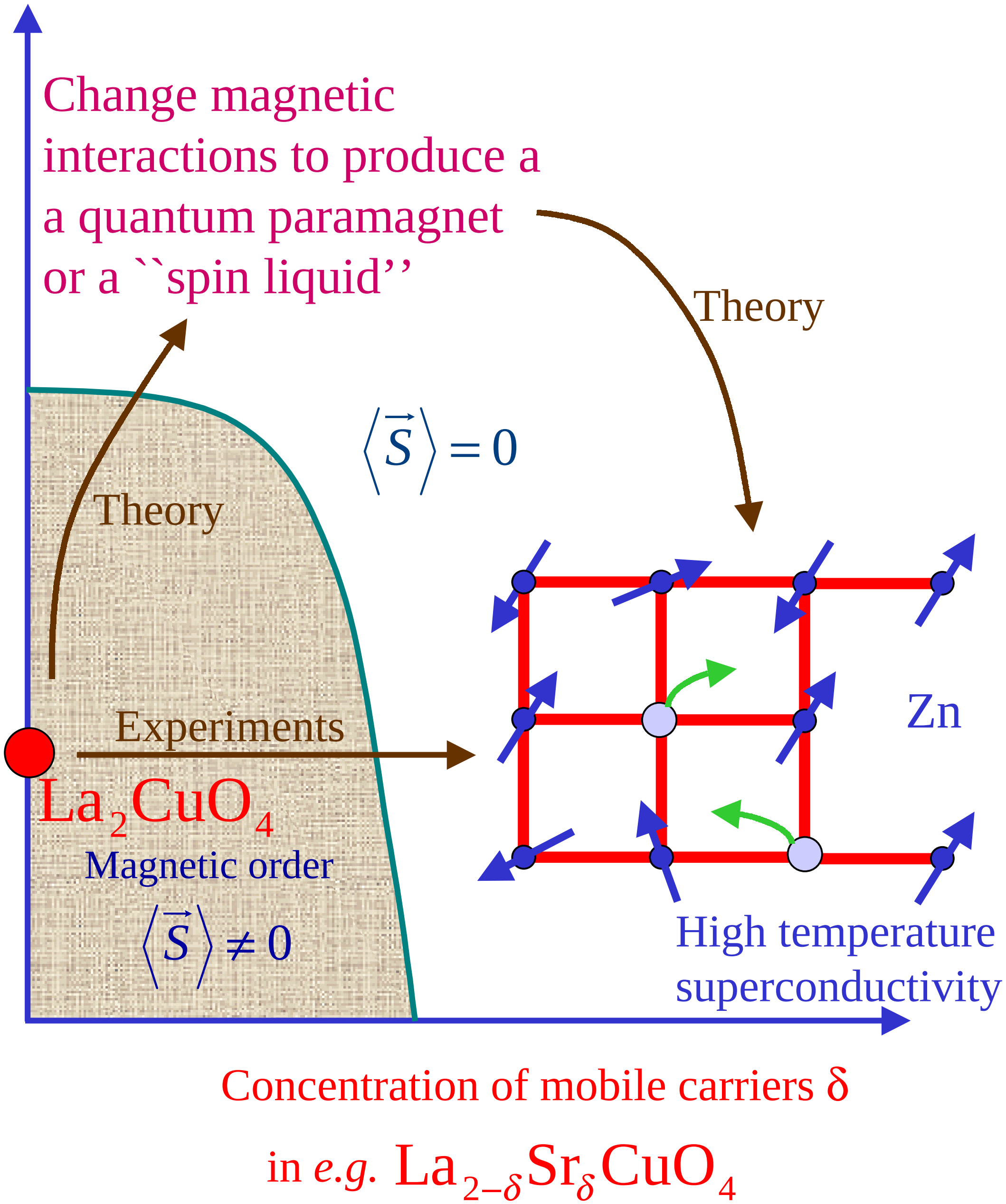
Free S=1/2 moments need not be present near the impurities

Translationally invariant "spin liquid" state obtained by a quantum transition from a magnetically ordered state with **co-planar** spin polarization. Can also appear in frustrated square lattice antiferromagnets – transition to confined states is described by a Z_2 gauge theory

$$\chi_{\text{impurity}}(T \to 0) = 0$$

N. Read and S. Sachdev, Phys. Rev. Lett. **66**, 1773 (1991).
R. Jalabert and S. Sachdev, Phys. Rev. B **44**, 686 (1991).

P. Fazekas and P.W. Anderson, Phil Mag **30**, 23 (1974).
S. Sachdev, Phys. Rev. B **45**, 12377 (1992).
G. Misguich and C. Lhuillier, cond-mat/0002170.
R. Moessner and S.L. Sondhi, Phys. Rev. Lett. **86**, 1881 (2001)



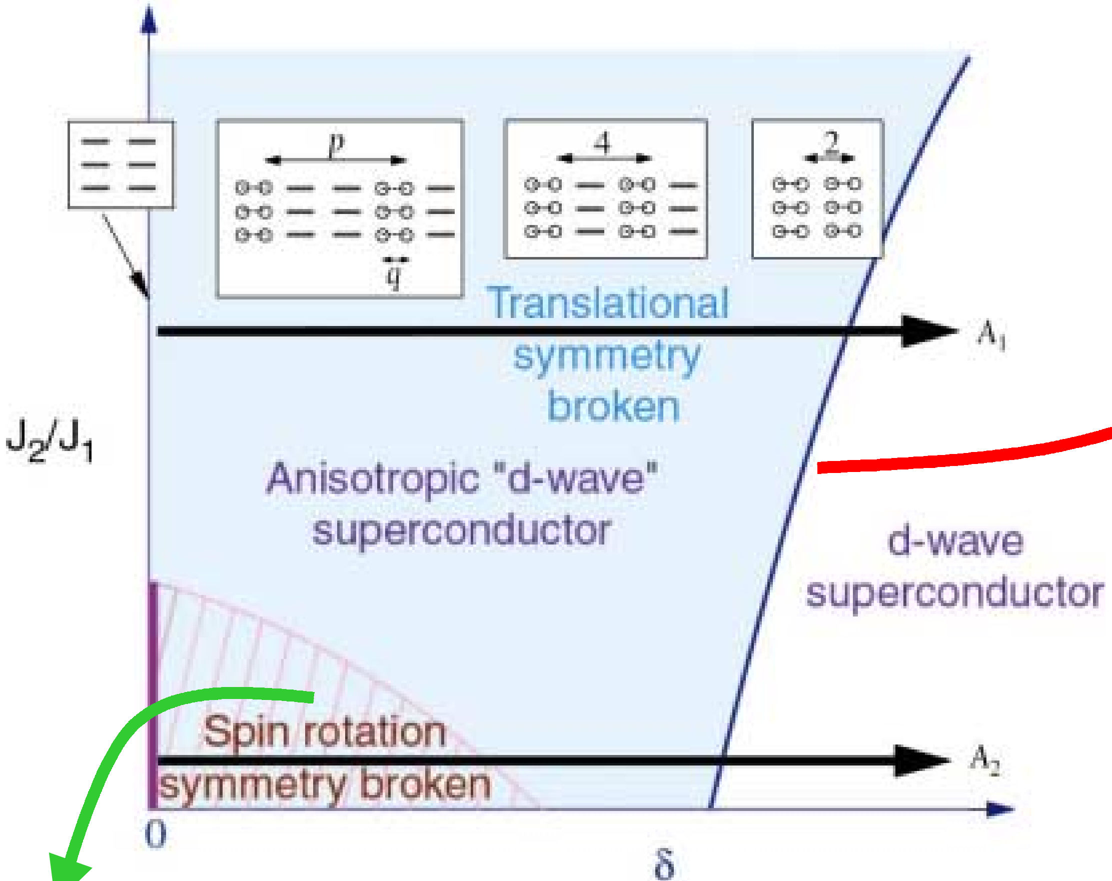
Summary:

Confined, paramagnetic Mott insulator necessarily has

1. Stable *S*=1 spin exciton.
2. Broken translational symmetry:- bond-centered charge stripe order.
3. *S*=1/2 moments near non-magnetic impurities.

These properties are expected to survive for a finite range of δ in the superconducting states

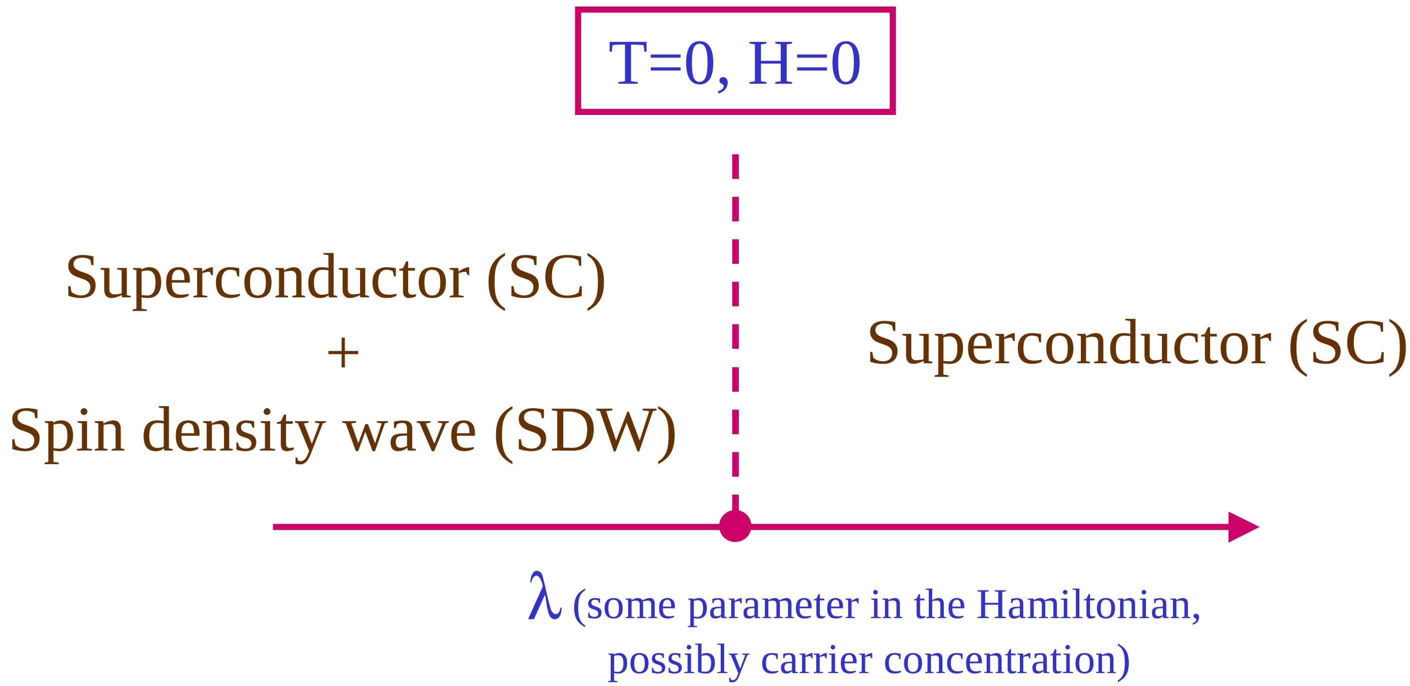
I.B Phase diagram for doping of confined Mott insulators



Superconductivity can coexist with *bond-centered* charge stripe order in region without magnetic order

Site-centered charge stripes likely in regions with magnetic order and weaker superconductivity

S. Sachdev and N. Read, Int. J. Mod. Phys. B **5**, 219 (1991).
M. Vojta and S. Sachdev, Phys. Rev. Lett. **83**, 3916 (1999).
M. Vojta, Y. Zhang, and S. Sachdev, Phys. Rev. B **62**, 6721 (2000).
K. Park and S. Sachdev, cond-mat/0104519.
See also J. Zaanen, Physica C **217**, 317 (1999),
S. Kivelson, E. Fradkin and V. Emery, Nature **393**, 550 (1998),
S. White and D. Scalapino, Phys. Rev. Lett. **80**, 1272 (1998); **81**, 3227 (1998).
C. Lannert, M.P.A. Fisher, and T. Senthil cond-mat/0007002.



Many experimental indications that the cuprate superconductors are not too far from such a quantum phase transition:

G. Aeppli, T.E. Mason, S.M. Hayden, H.A. Mook, J. Kulda, Science **278**, 1432 (1997).
Y. S. Lee, R. J. Birgeneau, M. A. Kastner *et al.*, Phys. Rev. B **60**, 3643 (1999).
S. Katano, M. Sato, K. Yamada, T. Suzuki, and T. Fukase Phys. Rev. B **62**, 14677 (2000).
B. Lake, G. Aeppli *et al.*, Science to appear.
Y. Sidis, C. Ulrich, P. Bourges, *et al.*, cond-mat/0101095.
H. Mook, P. Dai, F. Dogan, cond-mat/0102047.
J.E. Sonier *et al.*, preprint.

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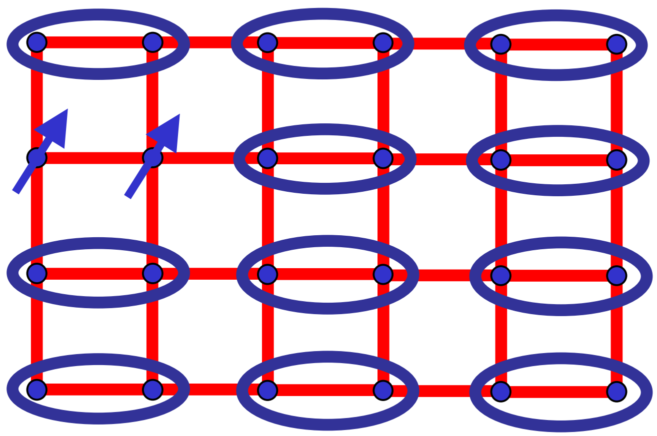
II. Effect of Zn impurities on S=1 spin exciton

S=1 spin exciton mode in YBCO



H.F. Fong, B. Keimer, D. Reznik, D.L. Milius, and I.A. Aksay, Phys. Rev. B **54**, 6708 (1996)

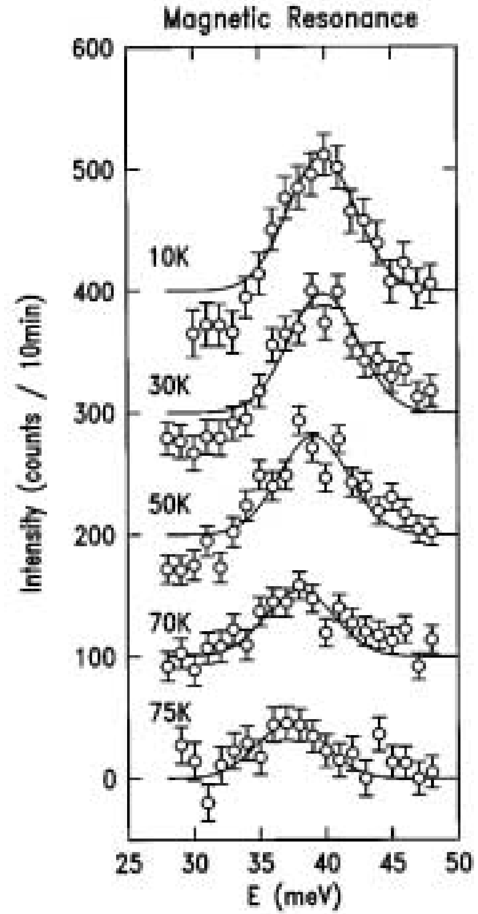


FIG. 8. Unpolarized beam, constant-**Q** data [**Q**=(3/2,1/2,−1.7)] of the 40 meV magnetic resonance obtained by subtracting the signal below T_c from the T=100 K background. The lines are fits to Gaussians, as described in the text. For clarity successive scans are offset by 100.

Spin-1 collective mode in $YBa_2Cu_3O_7$- little observable damping at low T.

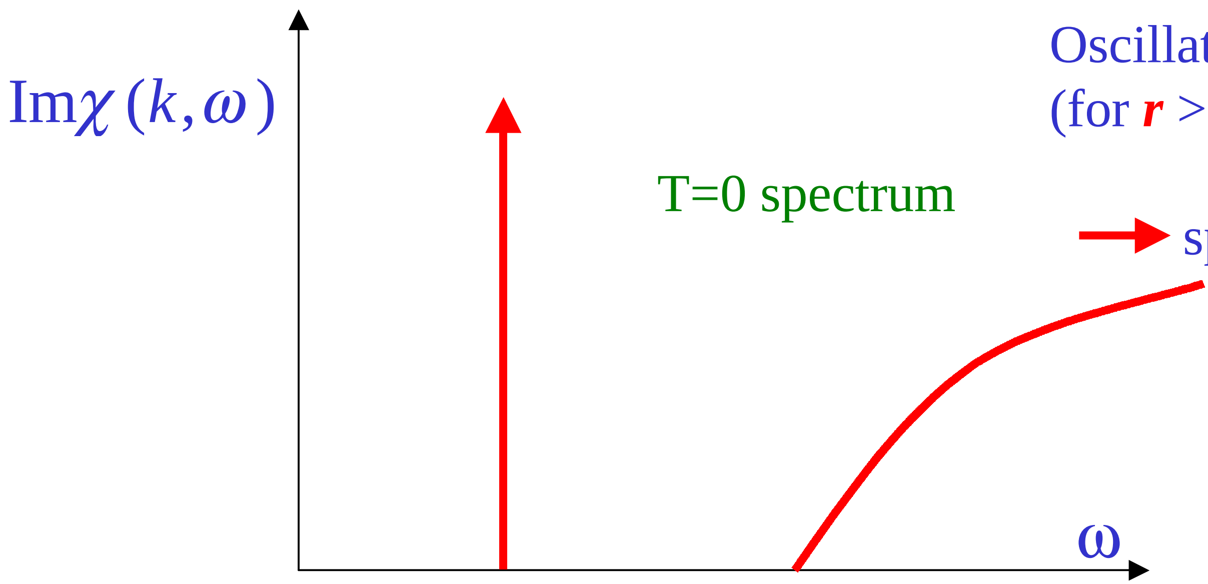
Coupling to superconducting quasiparticles unimportant

Resolution limited width

Quantum field theory for S=1 exciton near SC to SC+SDW quantum phase transition

$$S_b = \int d^2x\, d\tau \left[\frac{1}{2}\left(\left(\nabla_x \phi_\alpha\right)^2 + c^2 \left(\partial_\tau \phi_\alpha\right)^2 + r\phi_\alpha^2 \right) + \frac{g}{4!}\left(\phi_\alpha^2\right)^2 \right]$$

ϕ_α → 3-component antiferromagnetic order parameter

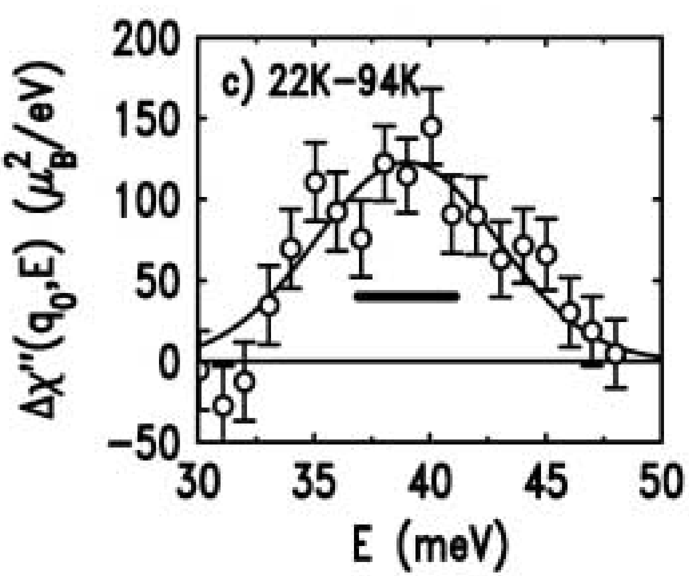


Oscillations of ϕ_α about zero (for ***r*** > 0)

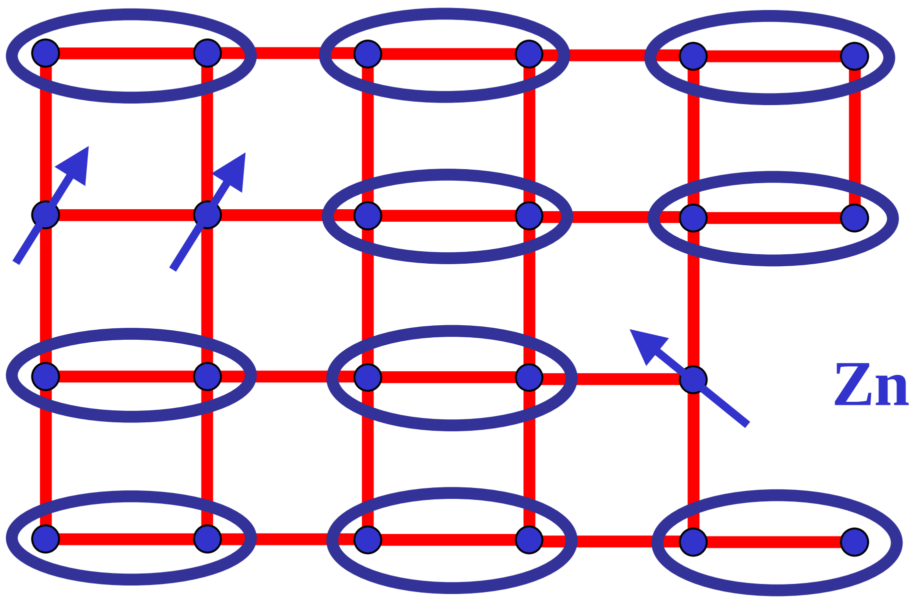
→ spin-1 collective mode

$YBa_2Cu_3O_7 + 0.5\%$ Zn

H. F. Fong, P. Bourges,
Y. Sidis, L. P. Regnault,
J. Bossy, A. Ivanov,
D.L. Milius, I. A. Aksay,
and B. Keimer,
Phys. Rev. Lett. **82**, 1939 (1999)



Zn induced half-width = 4.25 meV



Quantum field theory for S=1 resonance in the presence of a non-magnetic impurity

Orientation of "impurity" spin -- $n_\alpha(\tau)$ (unit vector)

Action of "impurity" spin

$$\mathcal{S}_{\text{imp}} = \int d\tau \left[iSA_\alpha(n) \frac{dn_\alpha}{d\tau} - \gamma S n_\alpha(\tau) \phi_\alpha(x=0,\tau) \right]$$

$A_\alpha(n)$ → Dirac monopole function

Boundary quantum field theory: $\mathcal{S}_b + \mathcal{S}_{\text{imp}}$

Recall -

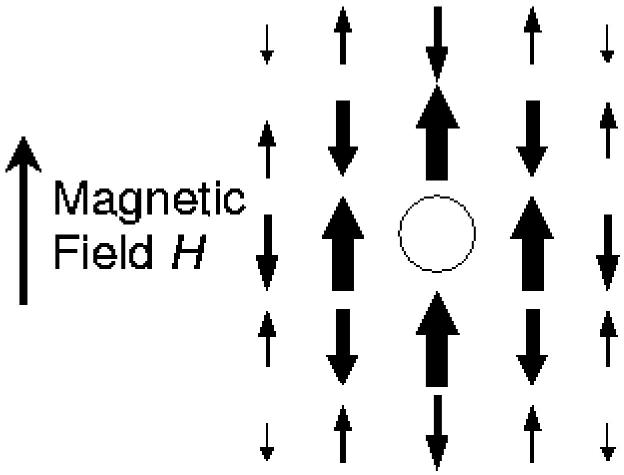
$$\mathcal{S}_b = \int d^2x d\tau \left[\frac{1}{2} \left((\nabla_x \phi_\alpha)^2 + c^2 (\partial_\tau \phi_\alpha)^2 + r\phi_\alpha^2 \right) + \frac{g}{4!} (\phi_\alpha^2)^2 \right]$$

Renormalization group analysis: *g* and γ reach non-zero fixed point values

Zn impurity in $YBa_2Cu_3O_{6.7}$

Moments measured by analysis of Knight shifts

M.-H. Julien, T. Feher, M. Horvatic, C. Berthier, O. N. Bakharev, P. Segransan, G. Collin, and J.-F. Marucco, Phys. Rev. Lett. **84**, 3422 (2000); also earlier work of the group of H. Alloul and the original experiment of A.M Finkelstein, V.E. Kataev, E.F. Kukovitskii, and G.B. Teitel'baum, Physica C **168**, 370 (1990).



Berry phases of precessing spins do not cancel between the sublattices in the vicinity of the impurity: net uncancelled phase of S=1/2

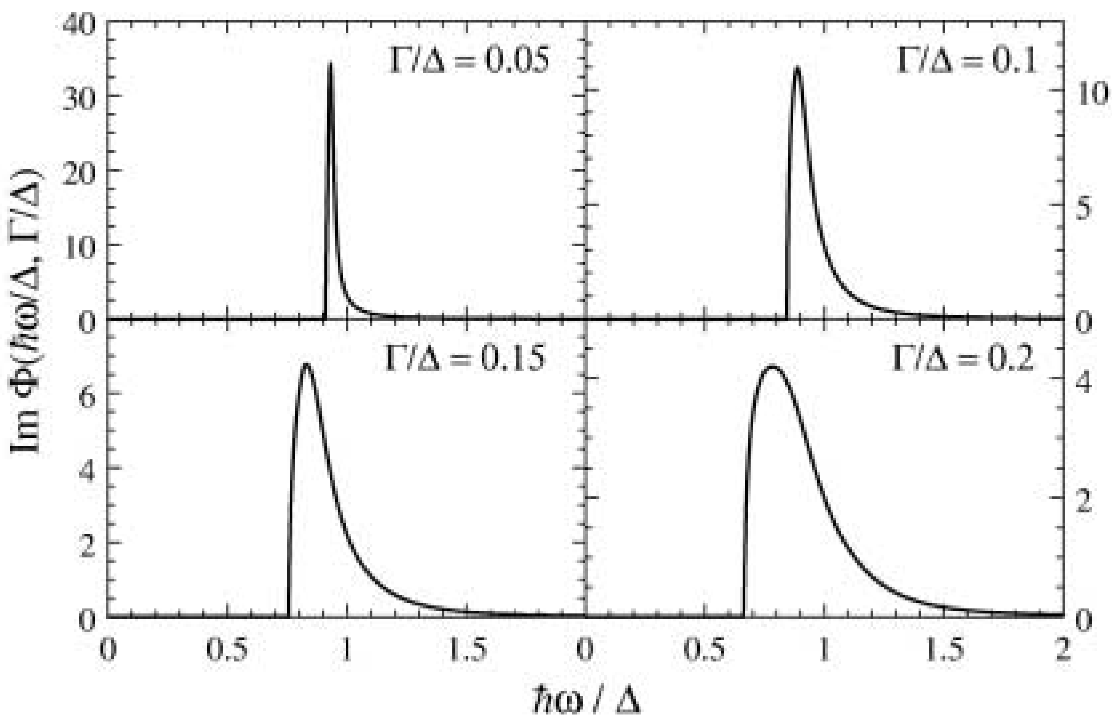
Predictions of quantum field theory

Without impurities $$\chi(G,\omega) = \frac{A}{\Delta^2 - \omega^2}$$

With impurities $$\chi(G,\omega) = \frac{A}{\Delta^2}\Phi\left(\frac{\hbar\omega}{\Delta}, \frac{\Gamma}{\Delta}\right)$$

$$\Gamma \equiv \frac{n_{\text{imp}}(\hbar c)^2}{\Delta}$$

$\Phi \rightarrow$ *Universal* scaling function. We computed it in a "self-consistent, non-crossing" approximation



Predictions:

Half-width of line $\approx \Gamma$

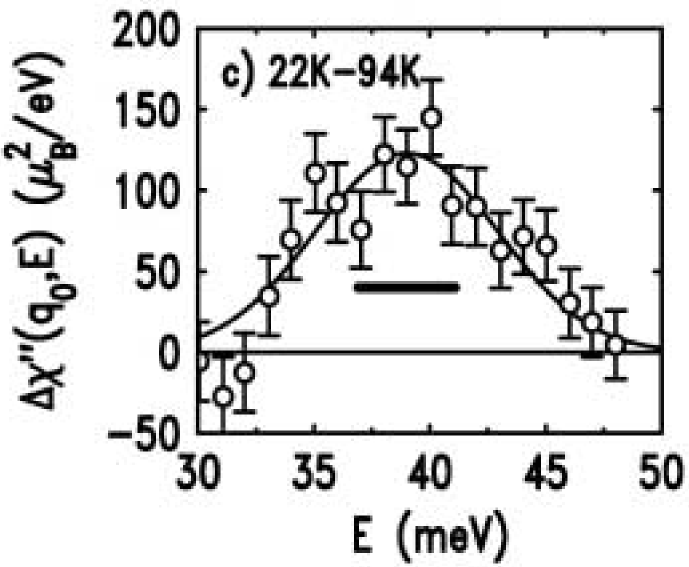
Universal asymmetric lineshape

S. Sachdev, C. Buragohain, M. Vojta, Science **286**, 2479 (1999).

M. Vojta, C. Buragohain, and S. Sachdev, Phys. Rev. B **61**, 15152 (2000).

$YBa_2Cu_3O_7 + 0.5\%$ Zn

H. F. Fong, P. Bourges, Y. Sidis, L. P. Regnault, J. Bossy, A. Ivanov, D.L. Milius, I. A. Aksay, and B. Keimer, Phys. Rev. Lett. **82**, 1939 (1999)



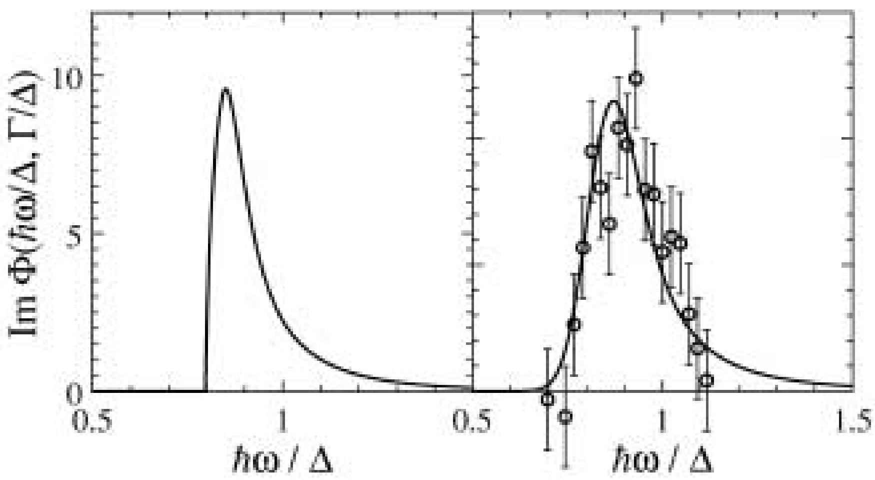
$$n_{\text{imp}} = 0.005$$

$$\Delta = 40\ \text{meV}$$

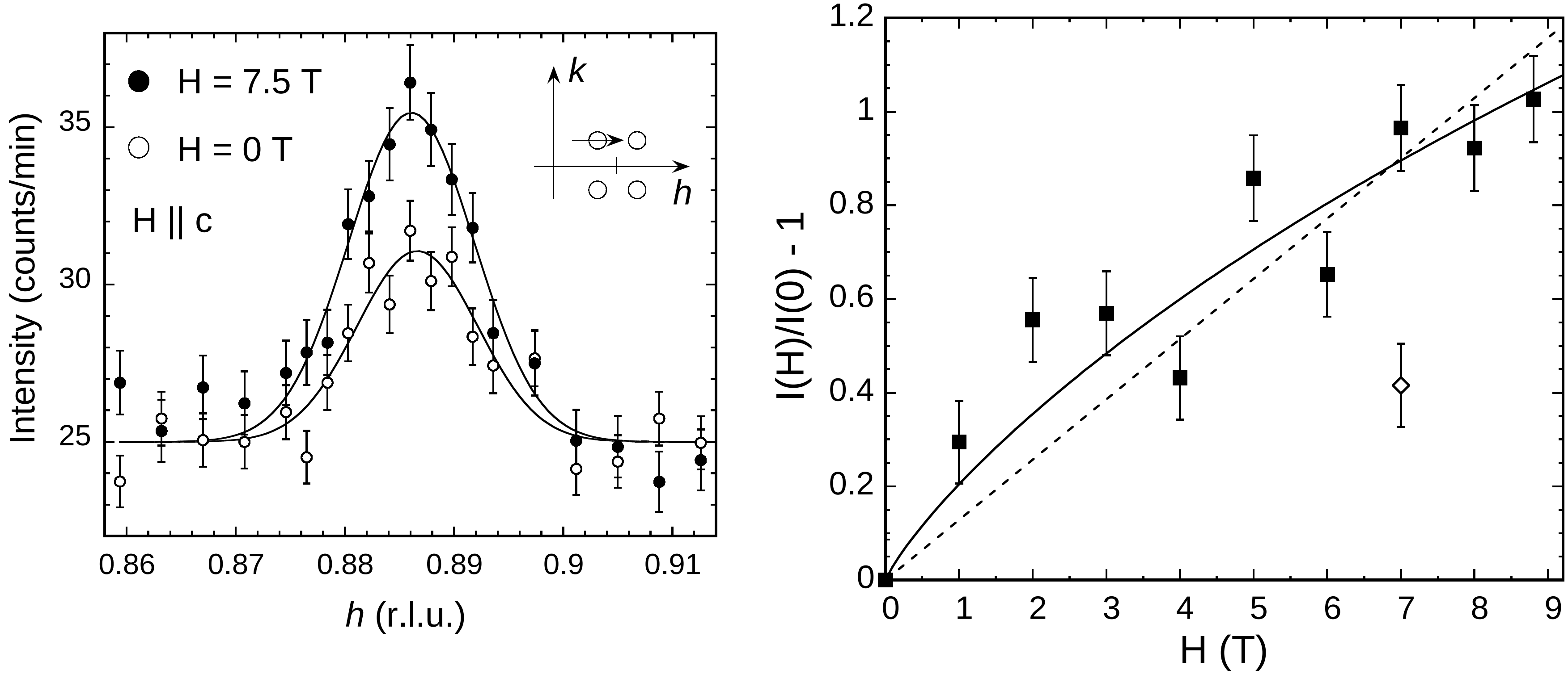
$$\hbar c = 0.2\ \text{eV}$$

$$\Rightarrow\quad \Gamma = 5\ \text{meV},\ \Gamma/\Delta = 0.125$$

Quoted half-width = 4.25 meV



III. Effect of magnetic field on SDW order in SC phase



Elastic neutron scattering off La_2CuO_{4+y}

B. Khaykovich, Y. S. Lee, S. Wakimoto, K. J. Thomas, M. A. Kastner, and R.J. Birgeneau, preprint.

- Theory should account for quantum spin fluctuations
- All effects are ~ H^2 except those associated with H induced superflow.
- Can treat SC order in a static Ginzburg-Landau theory

Action $\quad F_{GL}/T + S_b + S_c$

$$F_{GL} = \int d^2x \left[-|\psi|^2 + \frac{|\psi|^4}{2} + \left|(\nabla_x - iA)\psi\right|^2 \right]$$

$$S_c = \int d^2x d\tau \left[\frac{v}{2} \phi_\alpha^2 |\psi|^2 \right]$$

See also S.C. Zhang, Science, **275**, 1089 (1997); D. P. Arovas *et al*., Phys. Rev. Lett. **79**, 2871 (1997).

$$S_b = \int d^2x \int_0^{1/T} d\tau \left[\frac{1}{2}\left((\nabla_x \phi_\alpha)^2 + c^2 (\partial_\tau \phi_\alpha)^2 + r\phi_\alpha^2 \right) + \frac{g}{4!}(\phi_\alpha^2)^2 \right]$$

Self-consistent Hartree theory of quantum spin fluctuations (large N limit)

$$\chi(x,x',\omega_n)\delta_{\alpha\beta} = \langle \phi_\alpha(x,\omega_n)\phi_\beta(x',-\omega_n)\rangle$$

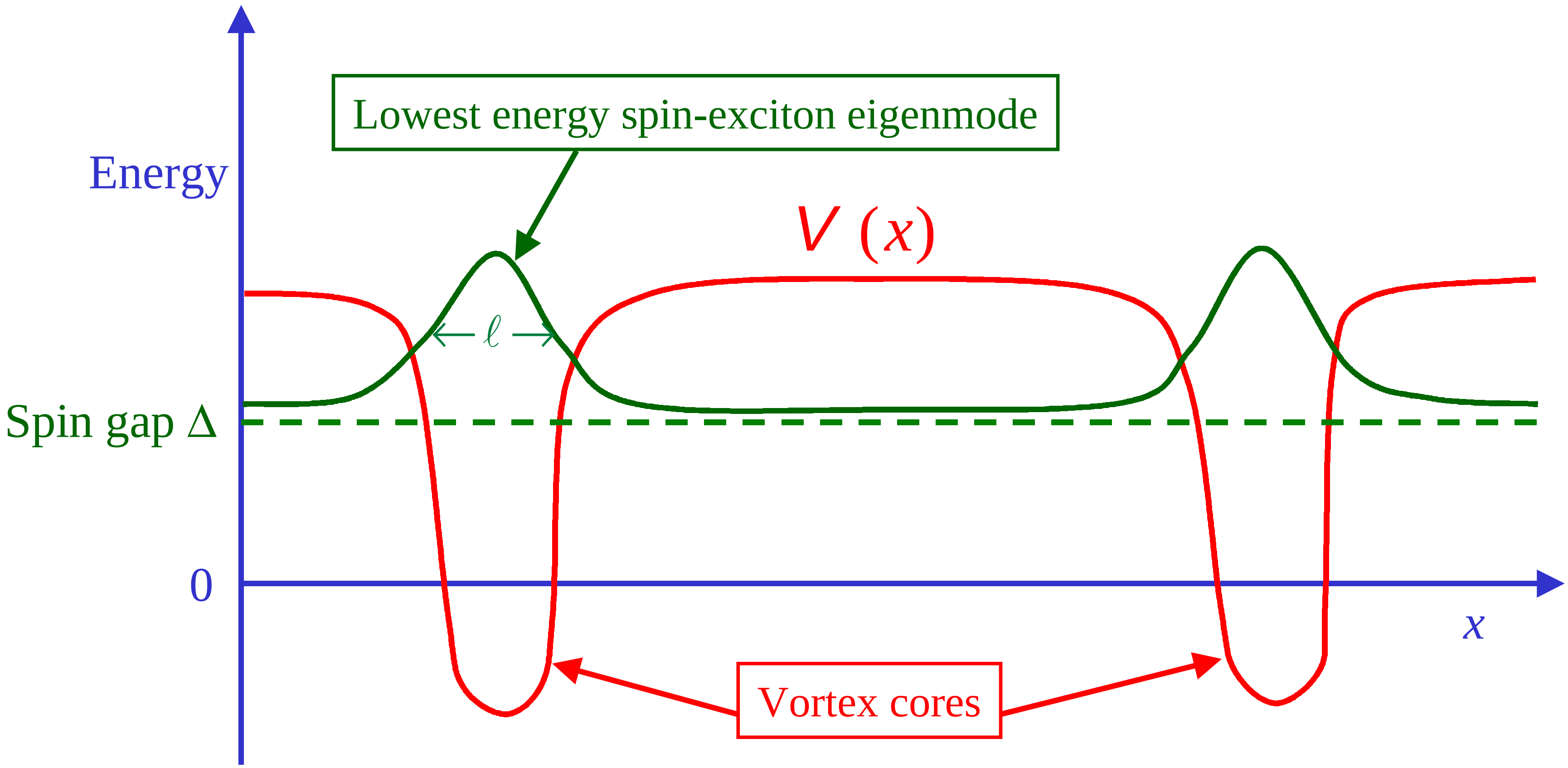
$$\left(\omega_n^{\ 2} - c^2\nabla_x^2 + V(x)\right)\chi(x,x',\omega_n) = \delta(x-x')$$

$$V(x) = r + v\left|\psi(x)\right|^2 + (NgT/6)\sum_{\omega_n}\chi(x,x,\omega_n)$$

$$\left[-1+\left|\psi(x)\right|^2 - \left(\nabla_x - i\vec{A}\right)^2\right]\psi(x) + (NvT/2)\sum_{\omega_n}\chi(x,x,\omega_n)\psi(x) = 0$$

$V(x) \rightarrow$ local classical energy of spin fluctuations; can become negative in vortex cores for $v > 0$.

However, spin gap remains finite because of quantum fluctuations



As $\Delta \rightarrow 0$, $\ell \rightarrow \infty$, because of self interaction, g, of spin excitations.

A.J. Bray and M.A. Moore, J. Phys. C **15**, L765 (1982).

J.A. Hertz, A. Fleishman, and P.W. Anderson, Phys. Rev. Lett. **43**, 942 (1979).

Influence of $\psi(x)$ on extended spin eigenmodes:

$$|\psi(x)| = 1 - \frac{1}{2x^2}$$ outside each vortex core because of superflow kinetic energy

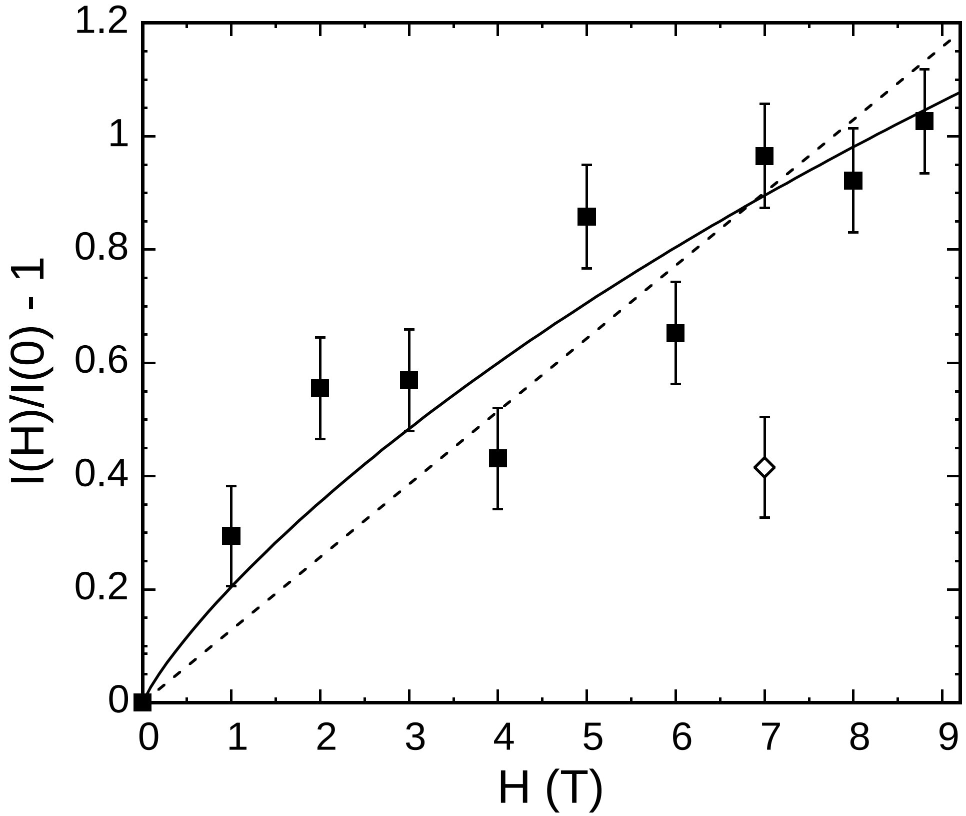
$$\left\langle |\psi(x)|^2 \right\rangle = 1 - \frac{H}{2H_{c2}} \ln\left(\frac{3.0H_{c2}}{H}\right)$$

In SC phase, spin gap obeys:

$$\Delta(H) = \Delta(0) - \frac{24\pi c^2 v}{Ng\left(1 - \frac{3v^2}{g}\right)} \frac{H}{2H_{c2}} \ln\left(\frac{3.0H_{c2}}{H}\right)$$

In SC+SDW phase, intensity of elastic scattering obeys:

$$I(H) = I(0) + \frac{6v}{g\left(1 - \frac{3v^2}{g}\right)} \frac{H}{2H_{c2}} \ln\left(\frac{3.0H_{c2}}{H}\right)$$



Elastic neutron scattering off La_2CuO_{4+y}

B. Khaykovich, Y. S. Lee, S. Wakimoto, K. J. Thomas, M. A. Kastner, and R.J. Birgeneau, preprint.

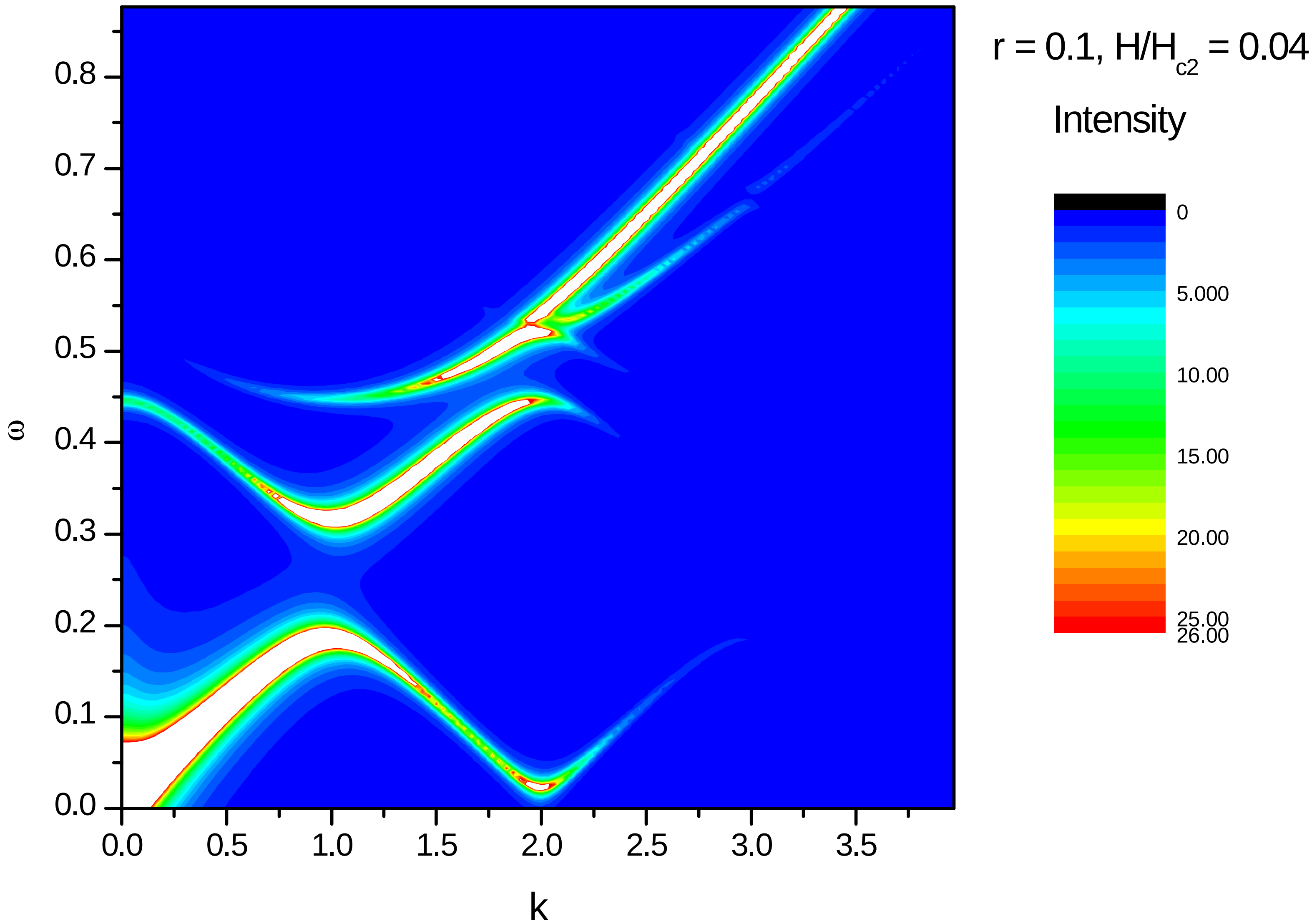
Solid line --- fit to :

$$\frac{I(H)}{I(0)} = 1 + a\frac{H}{H_{c2}}\ln\left(\frac{3.0H_{c2}}{H}\right)$$

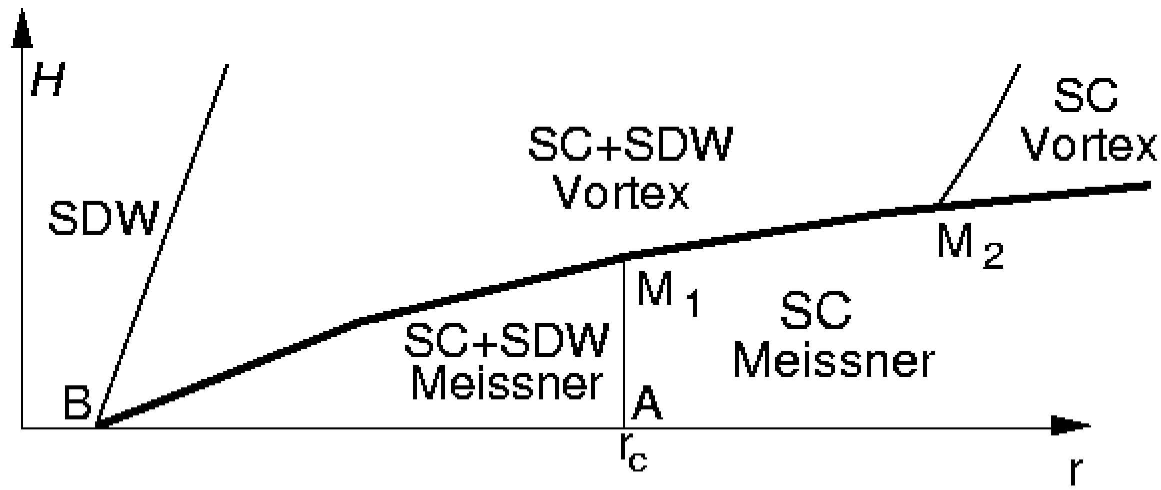
a is the only fitting parameter

Best fit value - a = 2.4 with H_{c2} = 60 T

Dynamic spin susceptibility $\chi''(k,\omega)$ in vortex lattice phase



Consequences of a finite London penetration depth (finite κ)



Conclusions

1. Strong experimental evidence for *S=1/2* moment near Zn and Li impurities in the underdoped high temperature superconductor.

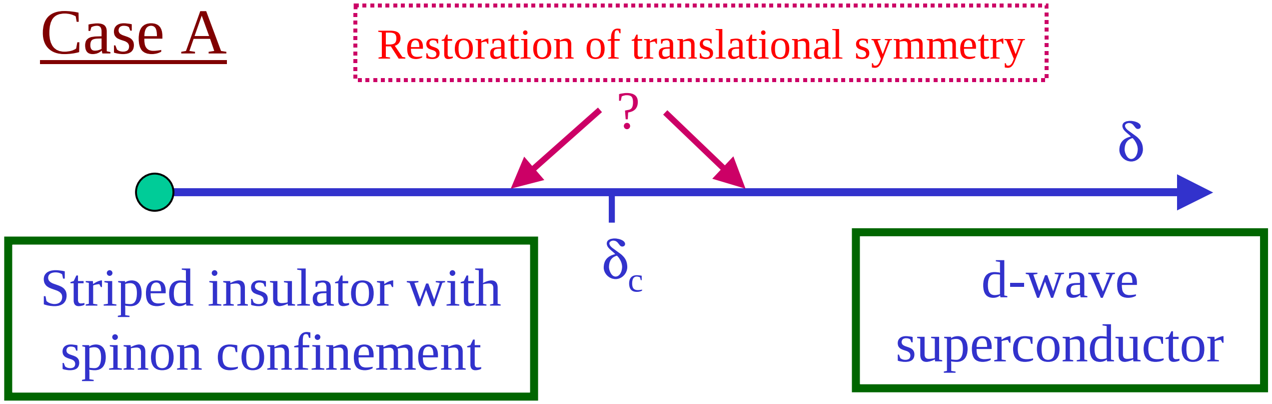
2. New boundary conformal quantum field theory in 2+1 dimensions describes scattering of spin resonance mode off "non-magnetic" impurities.

3. This, and other properties of the high temperature superconductors (existence of *S=1* spin resonance mode, possible bond-centered charge stripe (spin-Peierls) order) are naturally understood by a theory of doping Mott insulators with confinement.

4. Much to be learned from interplay of SDW and superconductivity by applying an external magnetic field.

II.B Zn or Li impurities in doped Mott insulators

Case A

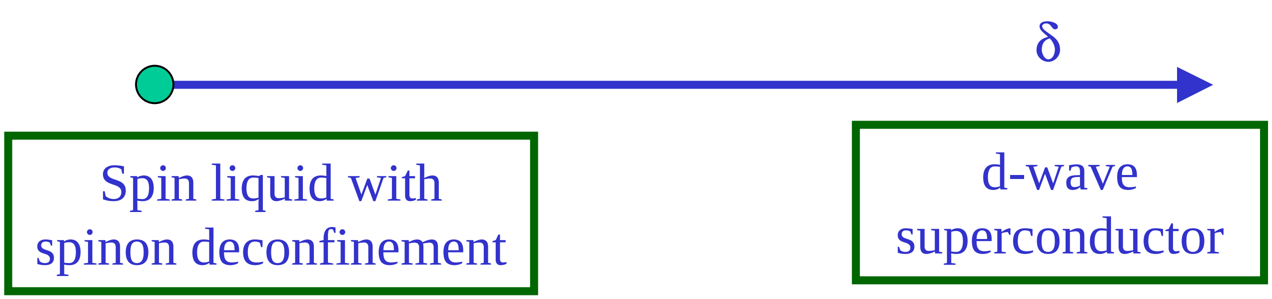


Moments form near each Zn or Li.

This moment is quenched by fermionic Bogoliubov quasiparticles at a quantum phase transition at $\delta=\delta_c$.

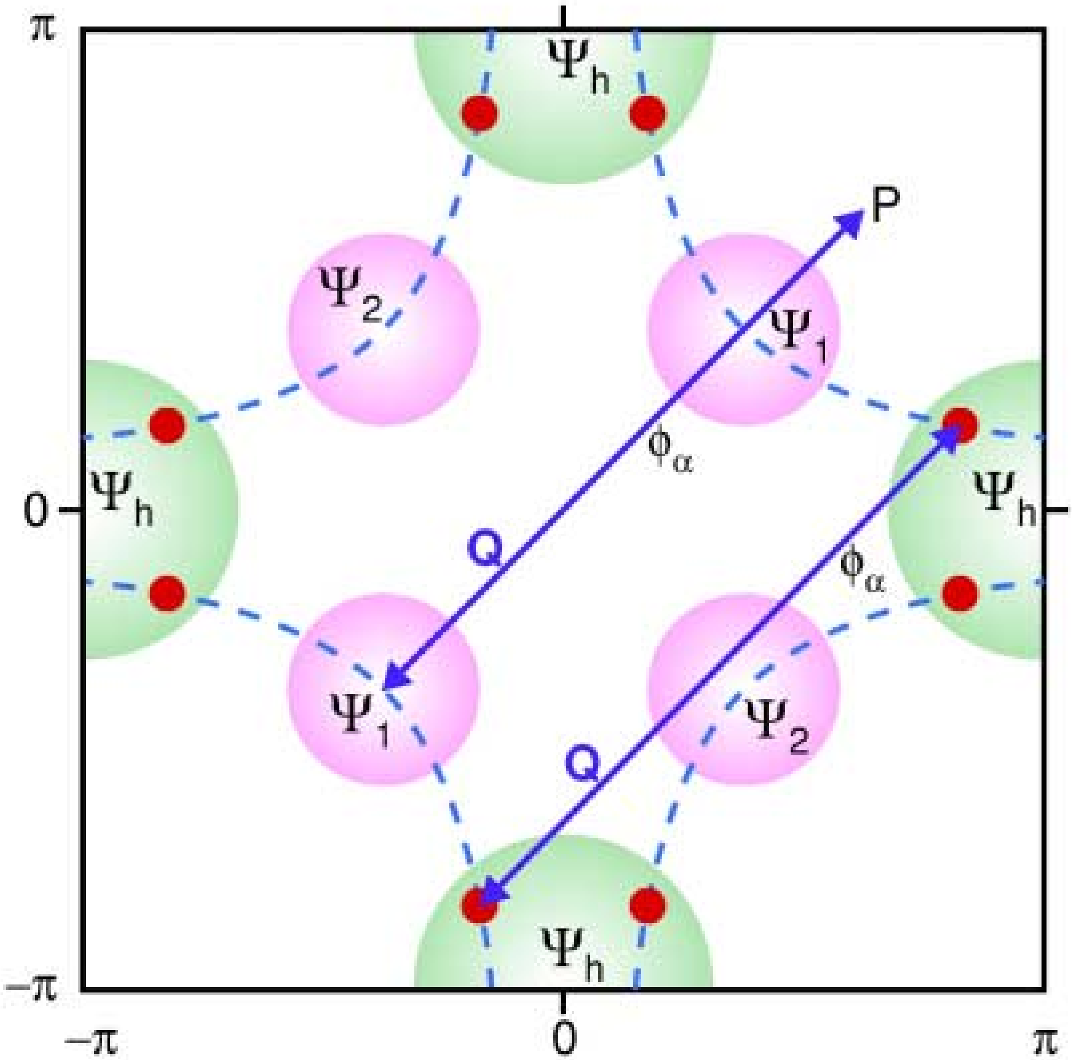
D. Withoff and E. Fradkin, Phys. Rev. Lett. **64**, 1835 (1990).
C. Gonzalez-Buxton and K. Ingersent, Phys. Rev. B **57**, 14254 (1998).

Case B



No moments form near Zn or Li ions substituted for Cu and impurity response evolves smoothly

Constraints from momentum conservation



Ψ_h : <u>*strongly coupled*</u> to ϕ_α, but do not damp ϕ_α as long as $\Delta < 2\,\Delta_h$

$\Psi_{1,2}$: <u>*decoupled*</u> from ϕ_α