Mean field theories of quantum spin glasses

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Classical Sherrington-Kirkpatrick model

$$H = \sum_{i \neq j} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

J_{ij} : a Gaussian random variable with zero mean

$\vec{S}_i$: a unit length n component vector

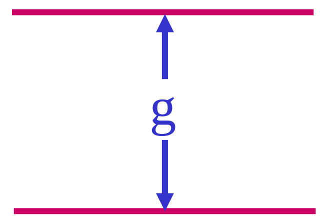
Two routes to quantization

A. Quantum rotor model

$$\text{Action} = \int d\tau \left[\frac{1}{2g} \sum_i \left(\frac{d\vec{S}_i}{d\tau} \right)^2 + H \right]$$

n=1: Ising model is a transverse field g

Spectrum at $J_{ij}=0$

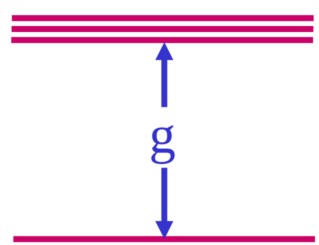


$$|\leftarrow\rangle_j = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_j - |\downarrow\rangle_j \right)$$

$$|\rightarrow\rangle_j = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_j + |\downarrow\rangle_j \right)$$

n=3: randomly coupled spin dimers

Spectrum at $J_{ij}=0$



$$|\uparrow\uparrow\rangle_j, \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle_j + |\downarrow\uparrow\rangle_j \right), |\downarrow\downarrow\rangle_j$$

$$\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle_j - |\downarrow\uparrow\rangle_j \right)$$

Two routes to quantization

B. Heisenberg spins

$$\text{Action} = \int d\tau \left[\sum_j iS\vec{A}\left(\vec{S}_j\right) \bullet \frac{d\vec{S}_j}{d\tau} + H \right]$$

First term is kinematic Berry phase which ensures

$$\left[S_{j\alpha}, S_{k\beta} \right] = i\delta_{jk}\varepsilon_{\alpha\beta\gamma}S_{j\gamma} \text{ and } \vec{S}_j^2 = S(S+1)$$

Spectrum at $J_{ij}=0$ $\quad |\uparrow\rangle_j, |\downarrow\rangle_j$

(2S+1)-fold degeneracy

Generalize model to SU(N) spins and explore phase diagram in N, S plane

Outline

A. Insulating quantum rotors.

B. Insulating Heisenberg spins

C. DMFT of a random *t*-*J* model

D. Metallic spin glasses: DMFT of a random Kondo lattice

A. Insulating quantum rotors

A. Quantum rotor model

$$\text{Action} = \int d\tau \left[\frac{1}{2g} \sum_i \left(\frac{d\vec{S}_i}{d\tau} \right)^2 + \sum_{i \neq j} J_{ij} \vec{S}_i \cdot \vec{S}_j \right]$$

J_{ij} : a Gaussian random variable with zero mean

T=0 phases

Local dynamic spin susceptibility $\chi(\omega_n) = \int_0^{1/T} d\tau \left\langle \vec{S}_j(\tau) \bullet \vec{S}_j(0) \right\rangle e^{i\omega_n \tau}$

$|\uparrow\rangle|\downarrow\rangle|\uparrow\rangle|\uparrow\rangle|\downarrow\rangle|\uparrow\rangle|\downarrow\rangle|\downarrow\rangle|\uparrow\rangle$

Spin glass

$\chi''(\omega) \sim q_{EA}\,\omega\,\delta(\omega) + \omega$

Specific heat $C \sim T$ (?)

$\big(|\uparrow\rangle + |\downarrow\rangle\big)\big(|\uparrow\rangle + |\downarrow\rangle\big)\big(|\uparrow\rangle + |\downarrow\rangle\big)$

Paramagnet

$\chi''(\omega)$ gapped

g

$\chi''(\omega) \sim \omega$

D.A. Huse and J. Miller, *Phys. Rev. Lett.* **70**, 3147 (1993).

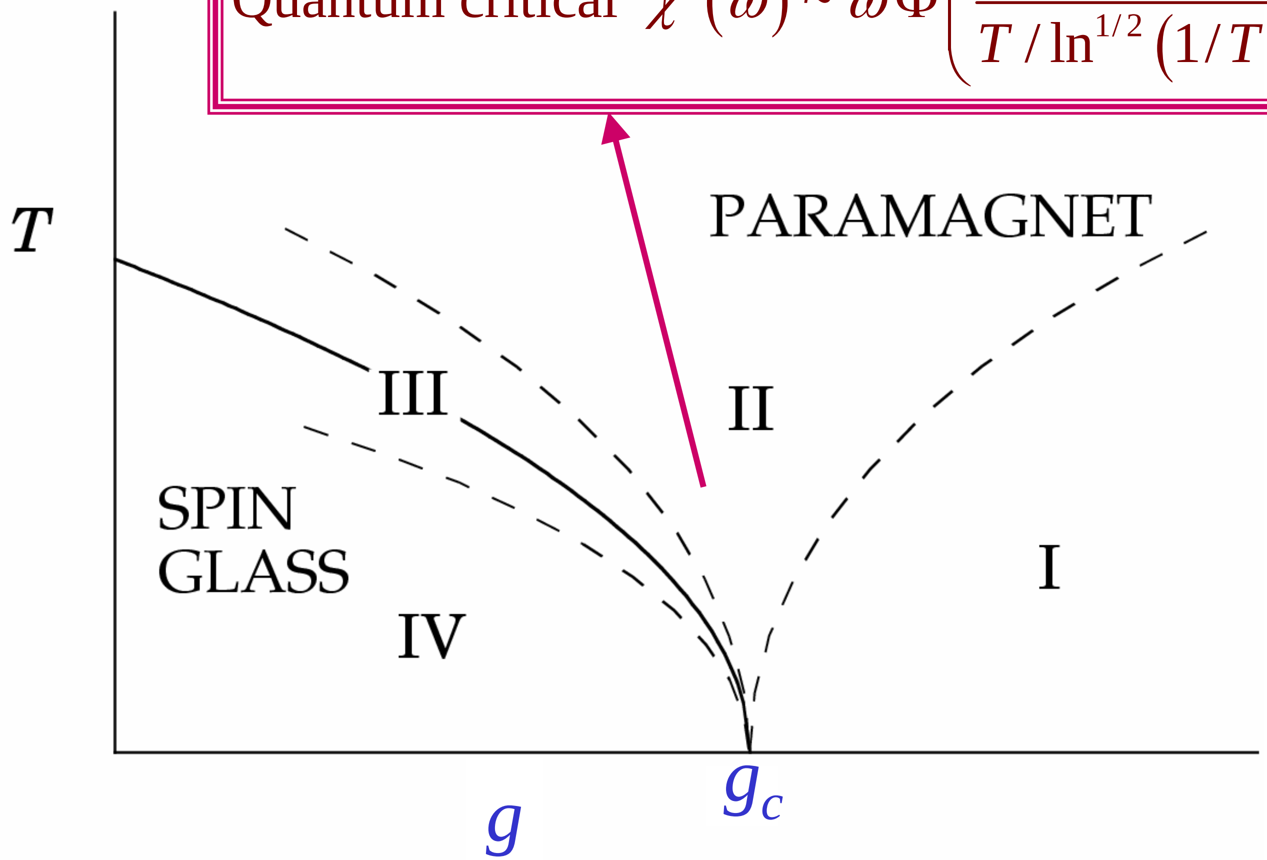
J. Ye, S. Sachdev, and N. Read, *Phys. Rev. Lett.* **70**, 4011 (1993).

N. Read, S. Sachdev, and J. Ye, *Phys. Rev.* B **52**, 384 (1995).

A. Georges, O. Parcollet, and S. Sachdev, *Phys. Rev.* B **63**, 134406 (2001).

$T > 0$ phase diagram

Quantum critical $\chi''(\omega) \sim \omega \Phi\left(\frac{\omega}{T/\ln^{1/2}(1/T)}\right)$



J. Ye, S. Sachdev, and N. Read, *Phys. Rev. Lett.* **70**, 4011 (1993).
N. Read, S. Sachdev, and J. Ye, *Phys. Rev.* B **52**, 384 (1995).

B. Insulating Heisenberg spins

B. Heisenberg spin glass

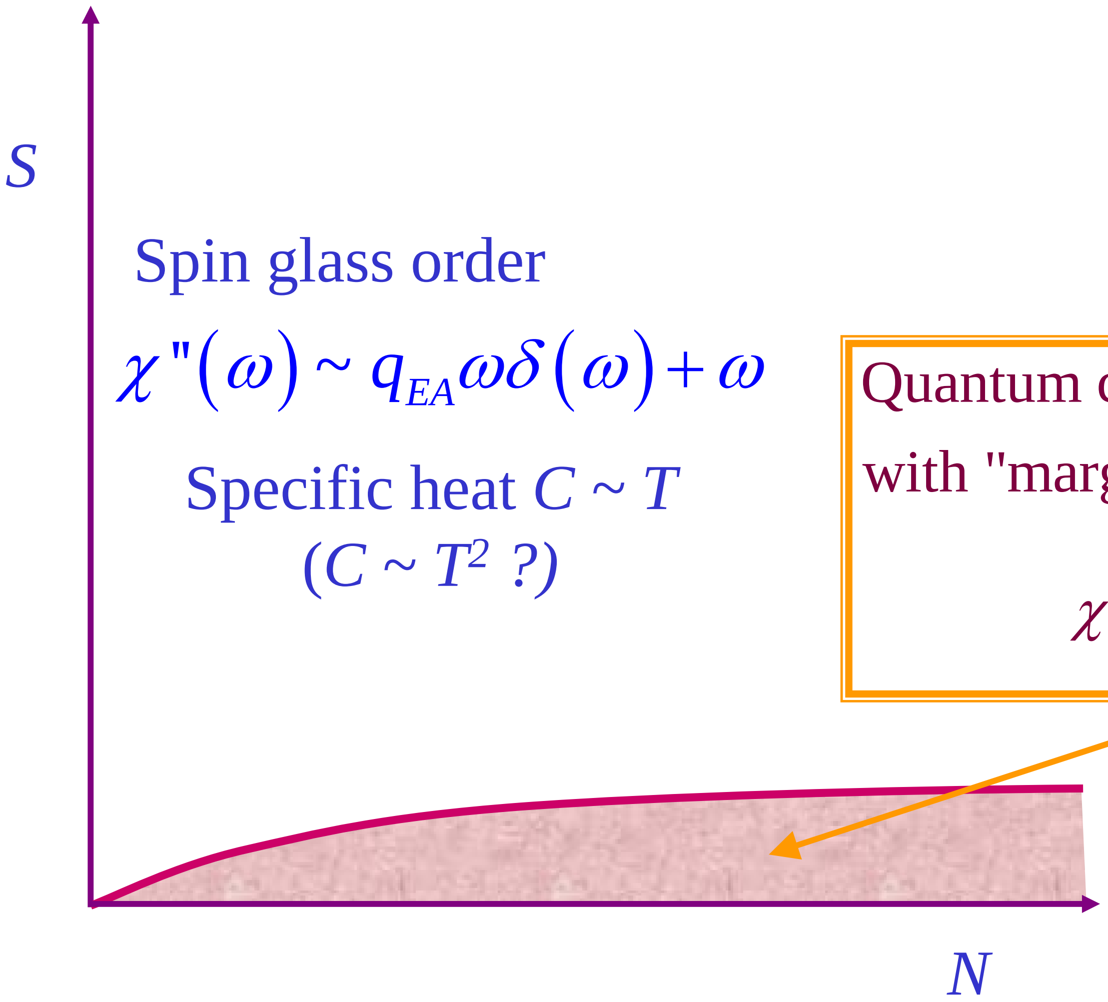
$$\text{Action} = \int d\tau \left[\sum_j iS\vec{A}\left(\vec{S}_j\right) \bullet \frac{d\vec{S}_j}{d\tau} + \sum_{i \neq j} J_{ij} \vec{S}_i \cdot \vec{S}_j \right]$$

J_{ij} : a Gaussian random variable with zero mean

$\vec{S}_j$: a SU(N) spin with $N^2 - 1$ components and "length" S

S. Sachdev and J. Ye, *Phys. Rev. Lett.* **70**, 3339 (1993).

T=0 phase diagram



Spin glass order

$$\chi''(\omega) \sim q_{EA}\,\omega\,\delta(\omega) + \omega$$

Specific heat $C \sim T$

($C \sim T^2$?)

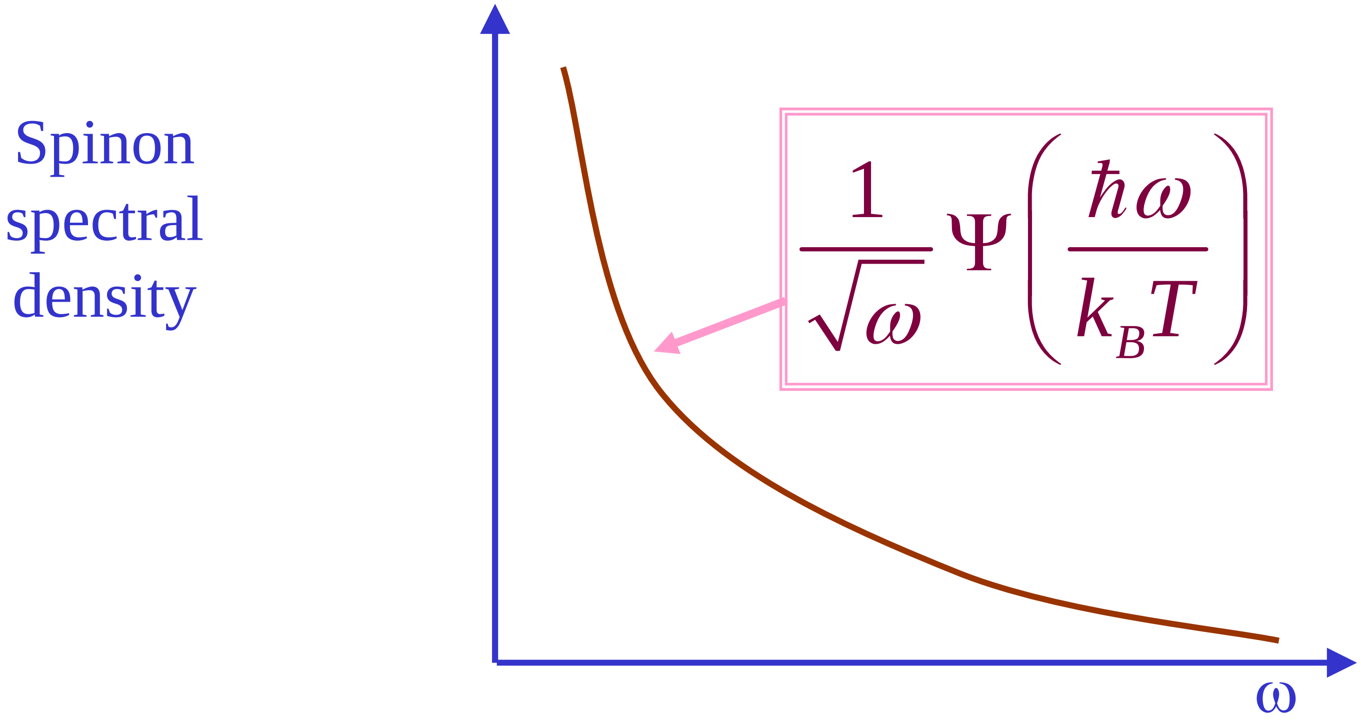
Quantum critical "spin slush" phase with "marginal Fermi liquid" spectrum:

$$\chi''(\omega) \sim \frac{\mathrm{sgn}(\omega)}{J}$$

S. Sachdev and J. Ye, *Phys. Rev. Lett.* **70**, 3339 (1993).
A. Georges, O. Parcollet, and S. Sachdev, *Phys. Rev. Lett.* **85**, 840 (2000).
A. Camjayi and M. J. Rozenberg, *Phys. Rev. Lett.* **90**, 217202 (2003).

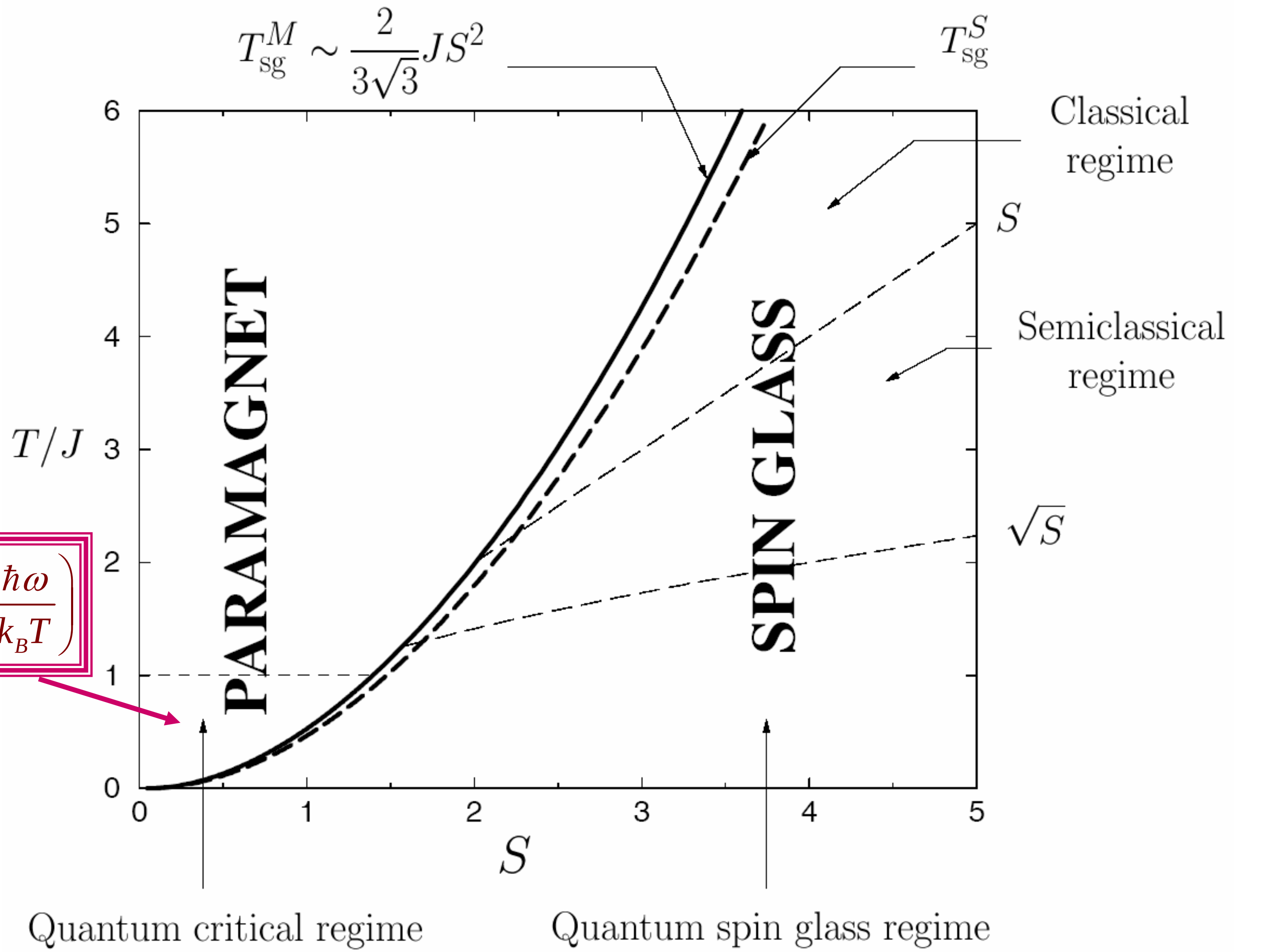
Quantum critical phase is described by fractionalized *S*=1/2 neutral spinon excitations

$$\vec{S} \sim f_\alpha^\dagger \vec{\sigma}_{\alpha\beta} f_\beta$$



S. Sachdev and J. Ye, *Phys. Rev. Lett.* **70**, 3339 (1993).

$T > 0$ phase diagram



$$\chi''(\omega) \sim \mathrm{sgn}(\omega)\Phi\left(\frac{\hbar\omega}{k_B T}\right)$$

S. Sachdev and J. Ye, *Phys. Rev. Lett.* **70**, 3339 (1993).
A. Georges, O. Parcollet, and S. Sachdev, *Phys. Rev. Lett.* **85**, 840 (2000).
A. Camjayi and M. J. Rozenberg, *Phys. Rev. Lett.* **90**, 217202 (2003).

C. Doping the quantum critical spin liquid

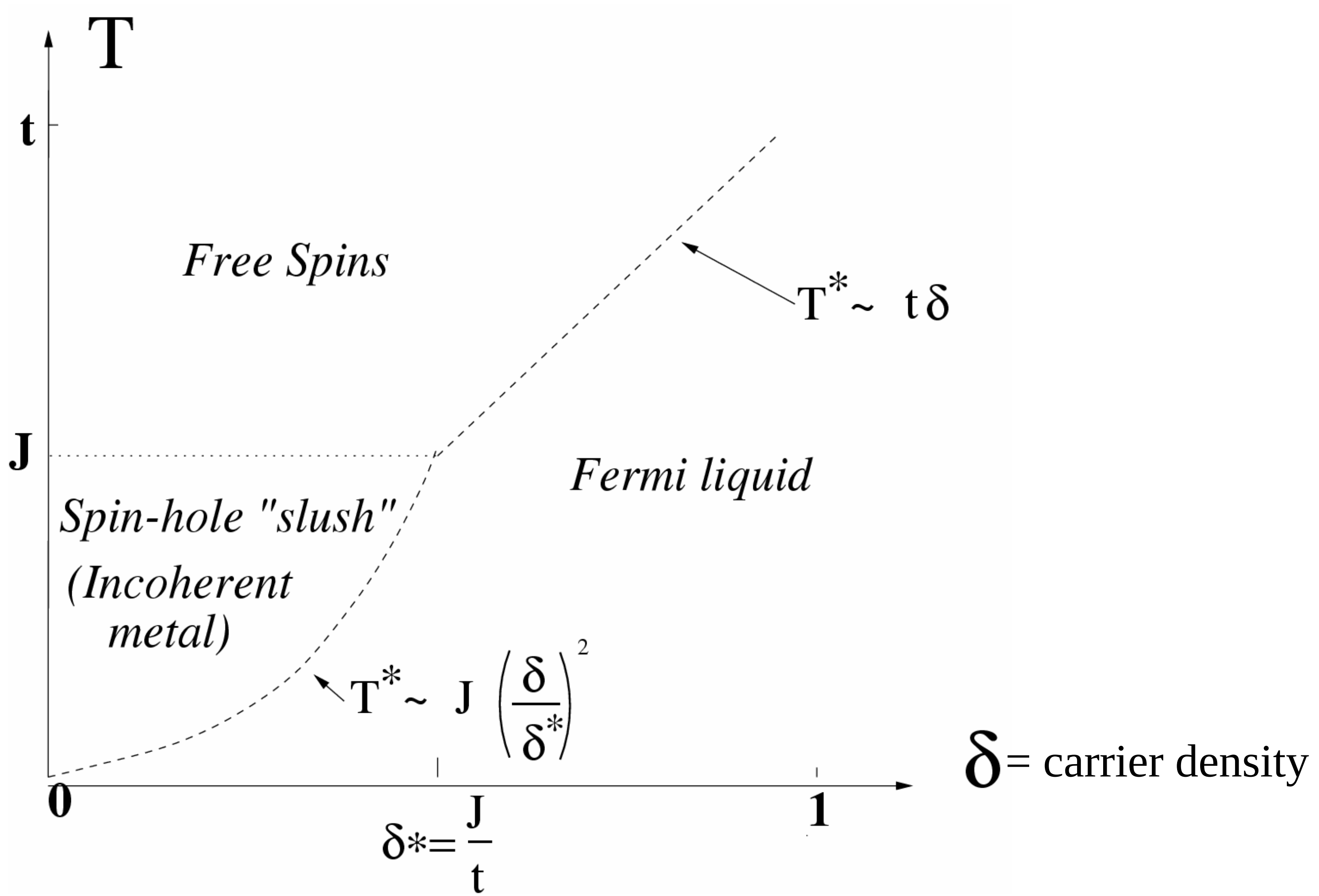
C. DMFT of a random *t-J* model

$$\text{Hamiltonian} = \sum_{\langle ij \rangle} t_{ij} P c_{i\alpha}^{\dagger} c_{j\alpha} P + \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$\vec{S}_i \equiv \frac{1}{2} c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}$$

J_{ij} : a Gaussian random variable with zero mean

O. Parcollet and A. Georges, *Phys. Rev*. B **59**, 5341 (1999).



Quantum critical "incoherent" physics with universal $\hbar\omega/k_B T$ scaling above

a coherence scale $\varepsilon_F^* \sim k_B T^* \sim \frac{(\delta t)^2}{J}$

O. Parcollet and A. Georges, *Phys. Rev.* B **59**, 5341 (1999).

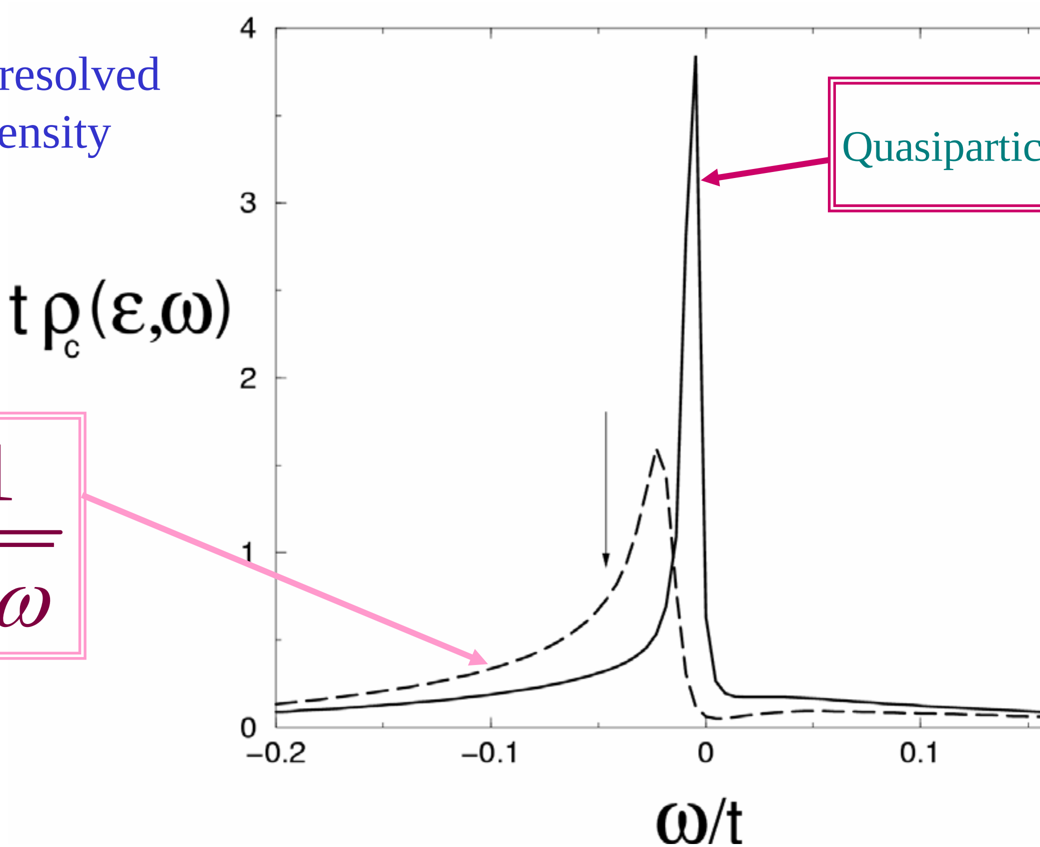
Physical consequences of quantum criticality

1. Electron spectral function (photoemission)

Momentum integrated electron spectral density at $T = 0$: $\rho(\omega) = \frac{1}{t}\phi\left(\frac{\omega}{\varepsilon_F^*}\right)$

$$\phi(\varpi) \to \frac{1}{\pi} \text{ as } \varpi \to 0 \text{ and } \phi(\varpi) \to \frac{1}{\sqrt{\varpi}} \text{ as } \varpi \to \infty$$

Momentum resolved spectral density



$\frac{1}{\sqrt{\omega}}$

Quasiparticle peak with residue $Z \sim \frac{\delta^2}{\delta^*}$

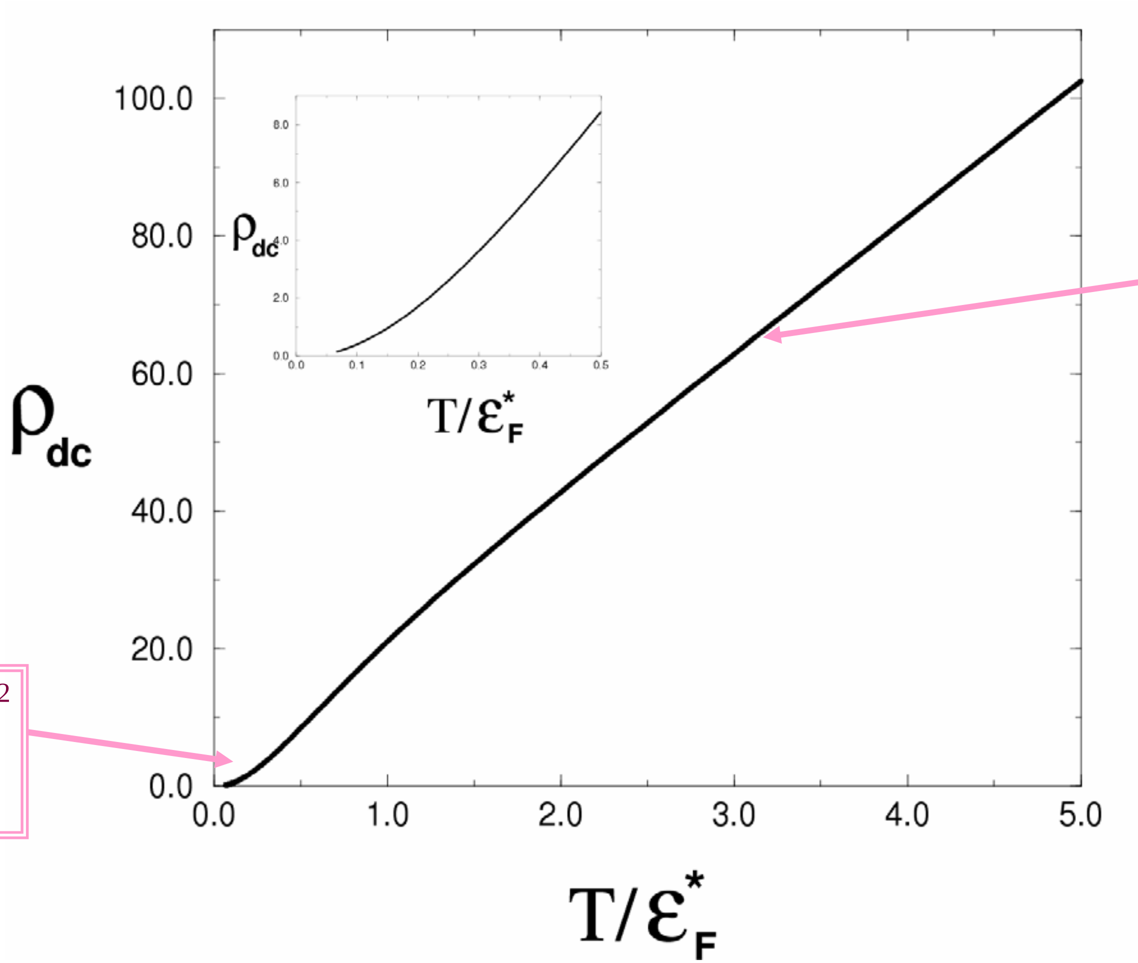
O. Parcollet and A. Georges, *Phys. Rev*. B **59**, 5341 (1999).

Physical consequences of quantum criticality

2. d.c Resistivity

$$\rho_{\mathrm{dc}}\left(T\right)=\frac{h}{e^2}\psi\left(\frac{T}{\varepsilon_F^*}\right)$$

$$\left(\frac{T}{\varepsilon_F^*}\right)^2$$



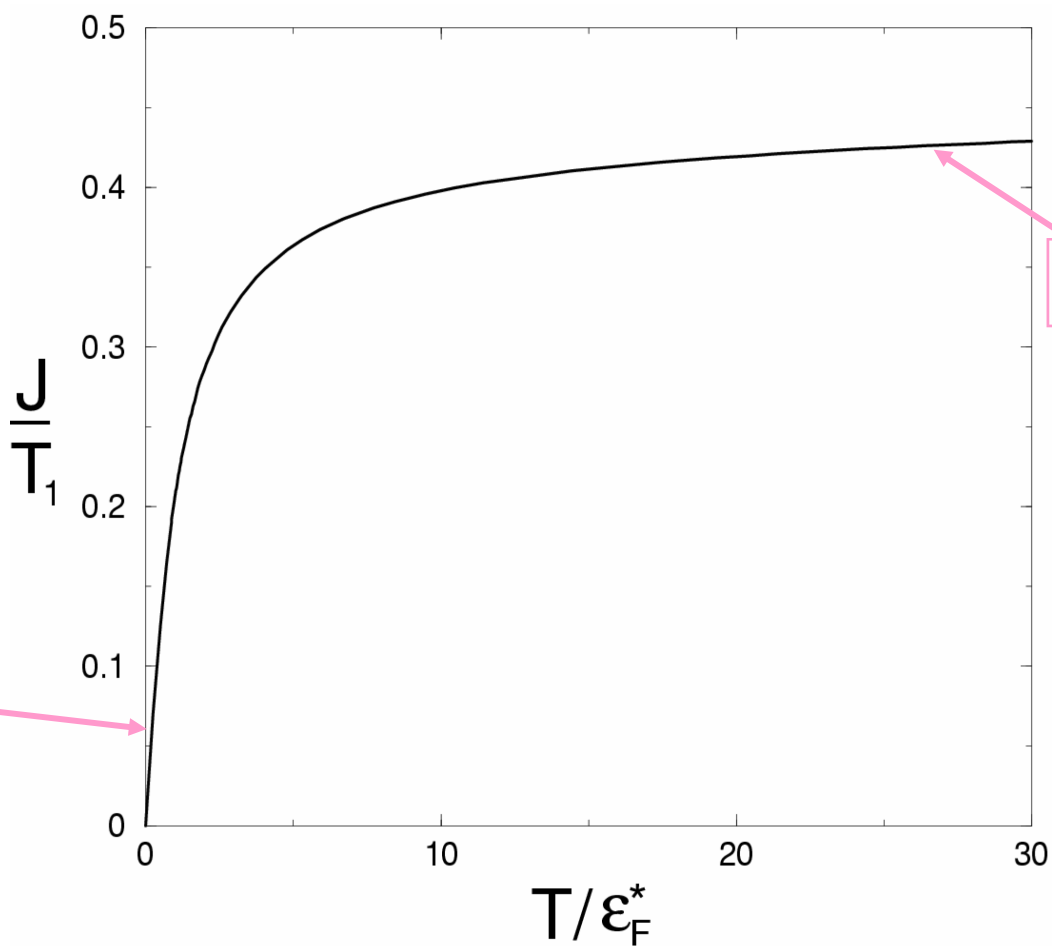
$$\frac{T}{\varepsilon_F^*} \;!$$

O. Parcollet and A. Georges, *Phys. Rev.* B **59**, 5341 (1999).

Physical consequences of quantum criticality

3. NMR $1/T_1$ relaxation rate

$$\frac{1}{T_1} = \frac{1}{J}\psi\left(\frac{T}{\varepsilon_F^*}\right)$$



constant (MFL)

Korringa $\left(\frac{T}{\varepsilon_F^*}\right)$

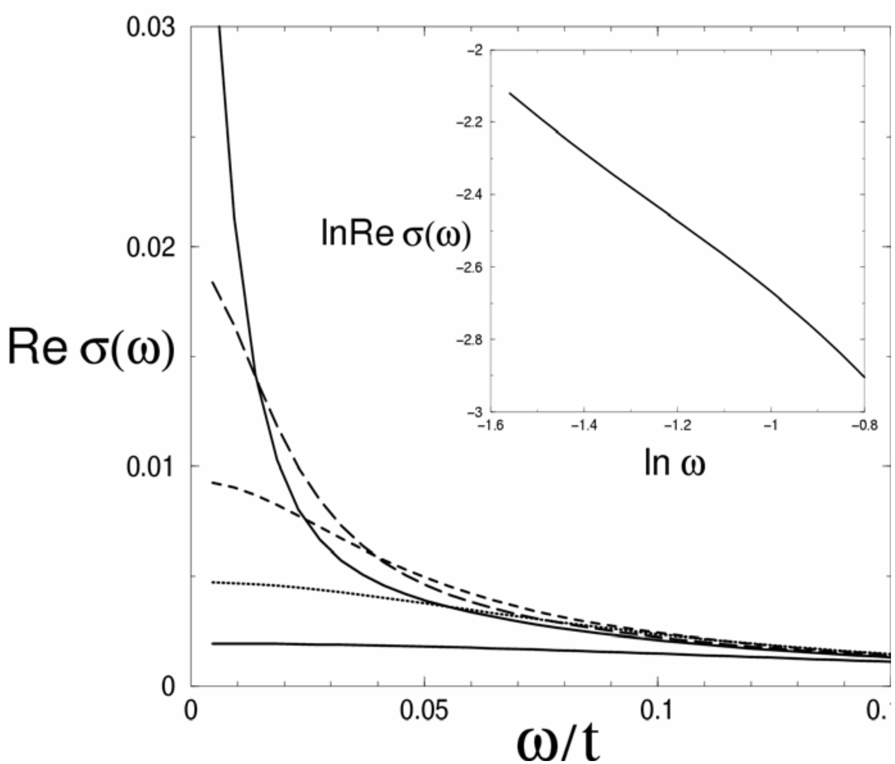
O. Parcollet and A. Georges, *Phys. Rev*. B **59**, 5341 (1999).

Physical consequences of quantum criticality

4. Optical conductivity

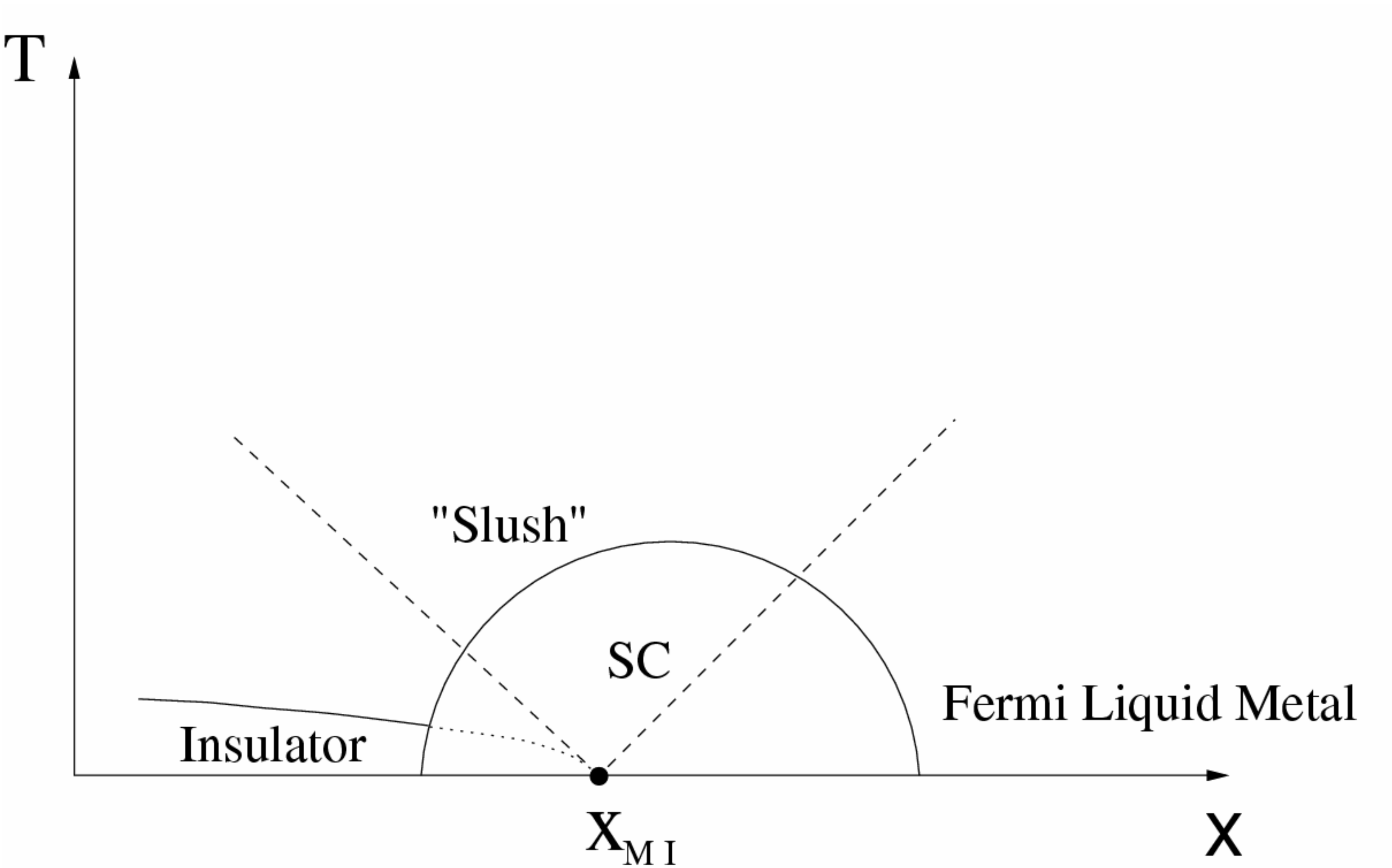
In quantum critical regime, with $\varepsilon_F^* < T < J$, $\mathrm{Re}\,\sigma(\omega) = \frac{\varepsilon_F^*}{\omega}\vartheta\left(\frac{\hbar\omega}{k_B T}\right)$

$$\text{with } \mathrm{Re}\,\sigma(\omega) = \begin{cases} \frac{\varepsilon_F^*}{\omega} & \text{for } T < \omega < J \\ \frac{\varepsilon_F^*}{T} & \text{for } \omega < T \end{cases}$$



O. Parcollet and A. Georges, *Phys. Rev*. B **59**, 5341 (1999).

Phenomenological phase diagram for cuprates



O. Parcollet and A. Georges, *Phys. Rev.* B **59**, 5341 (1999).

D. Metallic spin glasses

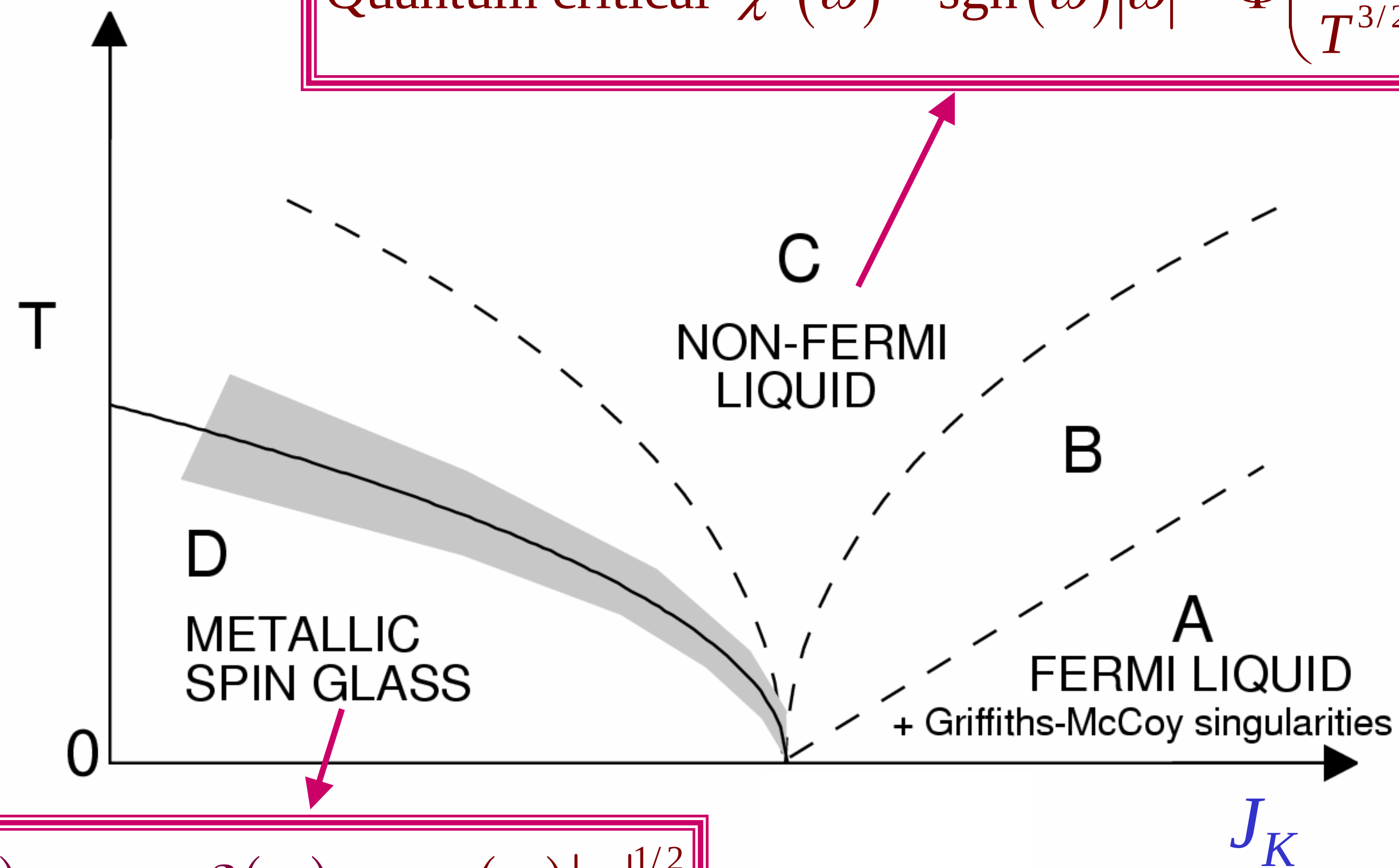
C. DMFT of a random Kondo lattice model

$$\text{Hamiltonian} = \sum_{\langle ij \rangle} t_{ij} c_{i\alpha}^{\dagger} c_{j\alpha} + \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j + \frac{J_K}{2} \sum_i \vec{S}_i \cdot c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}$$

J_{ij} : a Gaussian random variable with zero mean

S. Sachdev, N. Read, and R. Oppermann, *Phys. Rev.* B **52**, 10286 (1995).
A. M. Sengupta and A. Georges, *Phys. Rev.* B **52**, 10295 (1995).

Quantum critical $\chi''(\omega) \sim \mathrm{sgn}(\omega)|\omega|^{1/2}\, \Phi\left(\frac{\omega}{T^{3/2}}\right)$



$$\chi''(\omega) \sim q_{EA}\,\omega\,\delta(\omega) + \mathrm{sgn}(\omega)|\omega|^{1/2}$$

S. Sachdev, N. Read, and R. Oppermann, *Phys. Rev*. B **52**, 10286 (1995).
A. M. Sengupta and A. Georges, *Phys. Rev*. B **52**, 10295 (1995).

Outlook

• Spin glass order is an attractive candidate for a quantum critical point in the cuprates, on both theoretical and experimental grounds.
(Impurities break the translational symmetry associated with charge-ordered states, and the Imry-Ma argument then prohibits a quantum critical point associated with charge order in the presence of randomness in two dimensions)

• A simple mean-field theory of a doped Heisenberg spin glass naturally reproduces all the "marginal" phenomenology.

• Needed: better theory of fluctuations in low dimensions