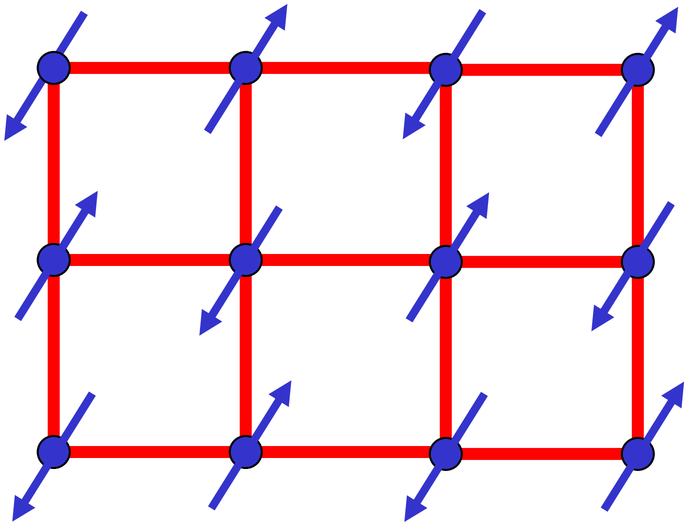
Quantum phase transitions out of the heavy Fermi liquid

Leon Balents (UCSB)
Matthew Fisher (UCSB)
Subir Sachdev (Yale)
T. Senthil (MIT)
Ashvin Vishwanath (Berkeley)
Matthias Vojta (Karlsruhe)

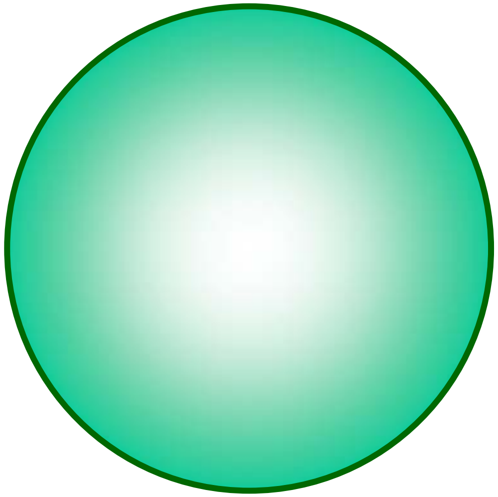
Phys. Rev. Lett. **90**, 216403 (2003).
Science **303**, 1490 (2004).
cond-mat/0409033.

Talk online: Google Sachdev

The Kondo lattice



+



Local moments f_σ

Conduction electrons c_σ

$$H_K = \sum_{i<j} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + J_K \sum_i c^\dagger_{i\sigma} \vec{\tau}_{\sigma\sigma'} c_{i\sigma} \cdot \vec{S}_{fi} + J \sum_{\langle ij \rangle} \vec{S}_{fi} \cdot \vec{S}_{fj}$$

Number of f electrons per unit cell = $n_f = 1$

Number of c electrons per unit cell = n_c

Outline

A. The heavy Fermi liquid (FL)

B. The metallic antiferromagnet

Local moment magnetic metal (LMM),

Spin density wave metal (SDW).

C. FL to SDW transition

D. The route from FL to LMM

The fractionalized Fermi liquid (FL)*

E. Detour: Deconfined criticality in insulators

Landau forbidden quantum transitions

F. Deconfined criticality in the Kondo lattice ?

A. The heavy Fermi liquid (FL)

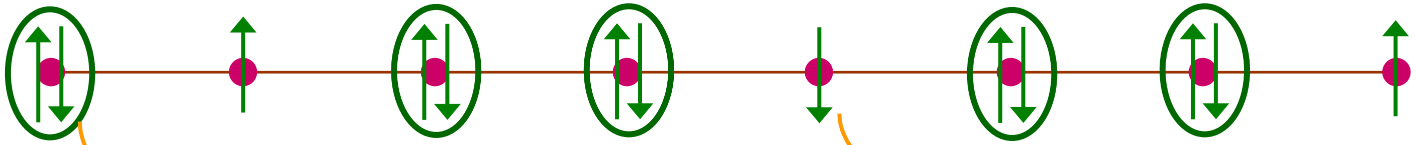
Obtained in the limit of large J_K

The Fermi surface of heavy quasiparticles encloses a volume which counts *all* electrons.

$$\text{Fermi volume} = 1 + n_c$$

Argument for the Fermi surface volume of the FL phase

Single ion Kondo effect implies $J_K \to \infty$ at low energies



$$\boxed{\left(c_{i\uparrow}^{\dagger} f_{i\downarrow}^{\dagger} - c_{i\downarrow}^{\dagger} f_{i\uparrow}^{\dagger}\right)\left|0\right\rangle}$$

$$\boxed{f_{i\downarrow}^{\dagger}\left|0\right\rangle,\ S=1/2 \text{ hole}}$$

Fermi liquid of S=1/2 holes with hard-core repulsion

$$\begin{aligned}\text{Fermi surface volume } &= -\left(\text{density of holes}\right)\text{mod}\,2\\ &= -\left(1-n_c\right) = \left(1+n_c\right)\text{mod}\,2\end{aligned}$$

Operator approach

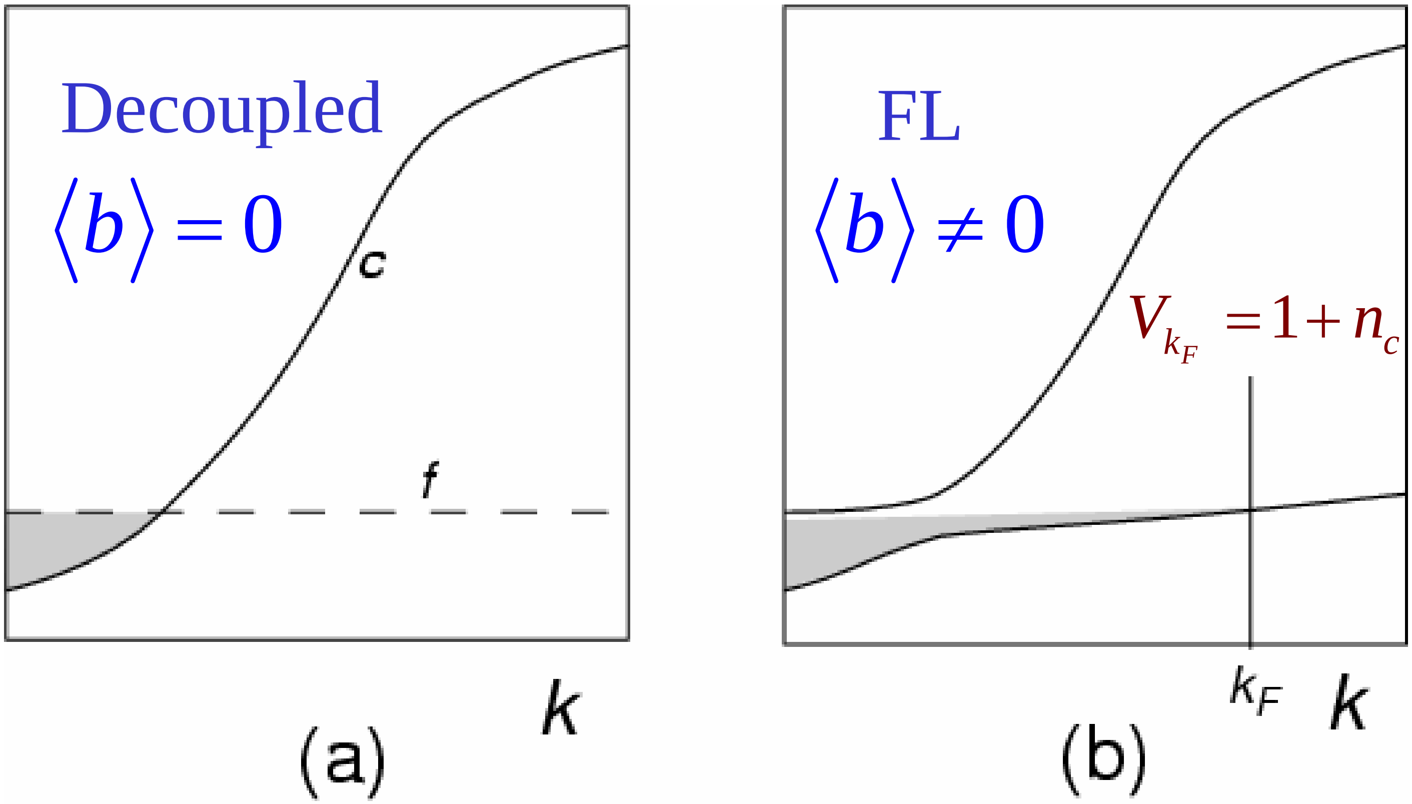
Define a bosonic field which measures the hybridization between the two bands:

$$b_i \sim \sum_{\sigma} c_{i\sigma}^{\dagger} f_{i\sigma}$$

The absence of charge fluctuations on the f sites implies an emergent compact U(1) gauge theory, associated with the gauge transformations

$$f_{i\sigma} \to f_{i\sigma} e^{i\phi_i(\tau)} \;\; ; \;\; b_i \to b_i e^{i\phi_i(\tau)}$$

Operator approach



The FL state is the "Higgs" phase of the U(1) gauge theory. Because of the dispersionless *f* band in the decoupled case, the ground state is always in the Higgs phase.

B. The metallic antiferromagnet

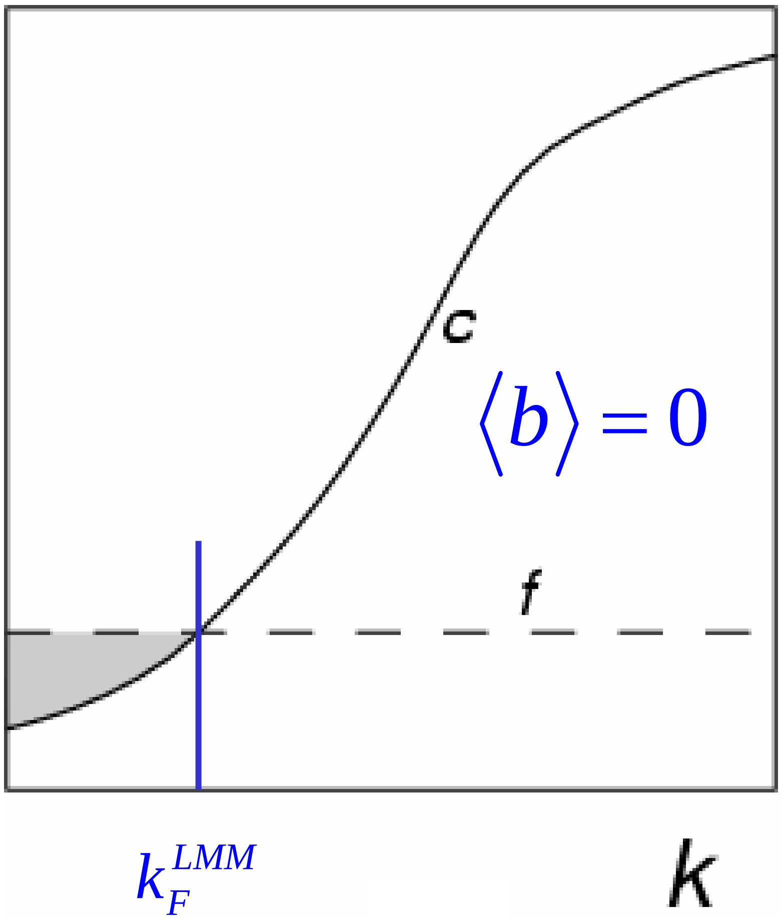
Two possible states:

(A)The local moment magnetic metal (LMM)

(B) Spin density wave metal (SDW)

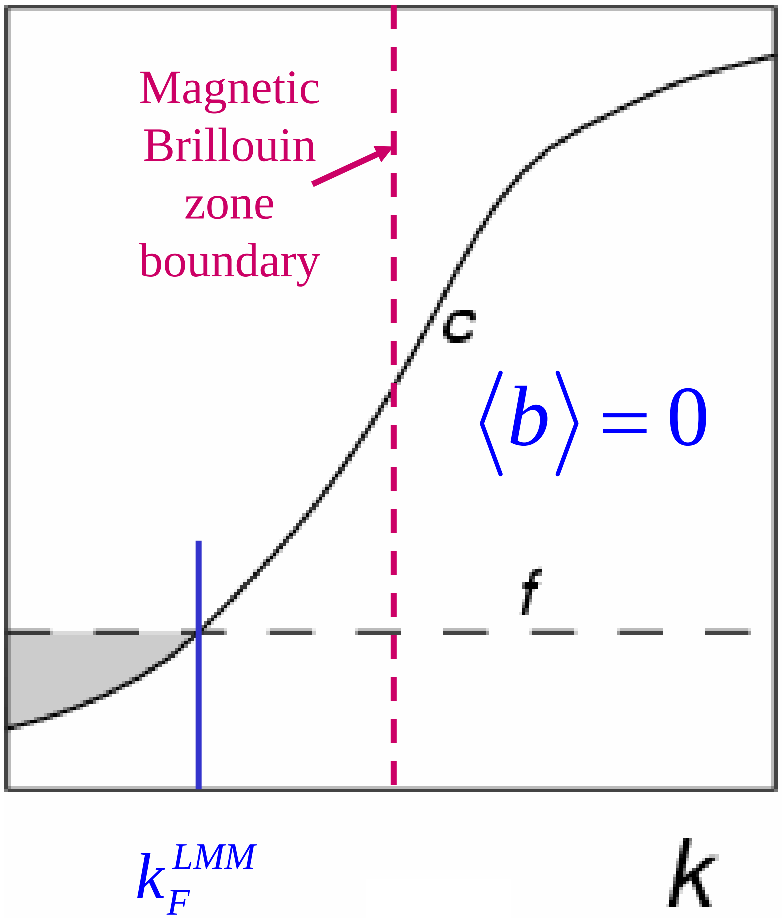
(A) The local moment magnetic metal (LMM)

The local *f* moments order antiferromagnetically, and this halves the volume of the Brillouin zone.



(A) The local moment magnetic metal (LMM)

The local f moments order antiferromagnetically, and this halves the volume of the Brillouin zone.



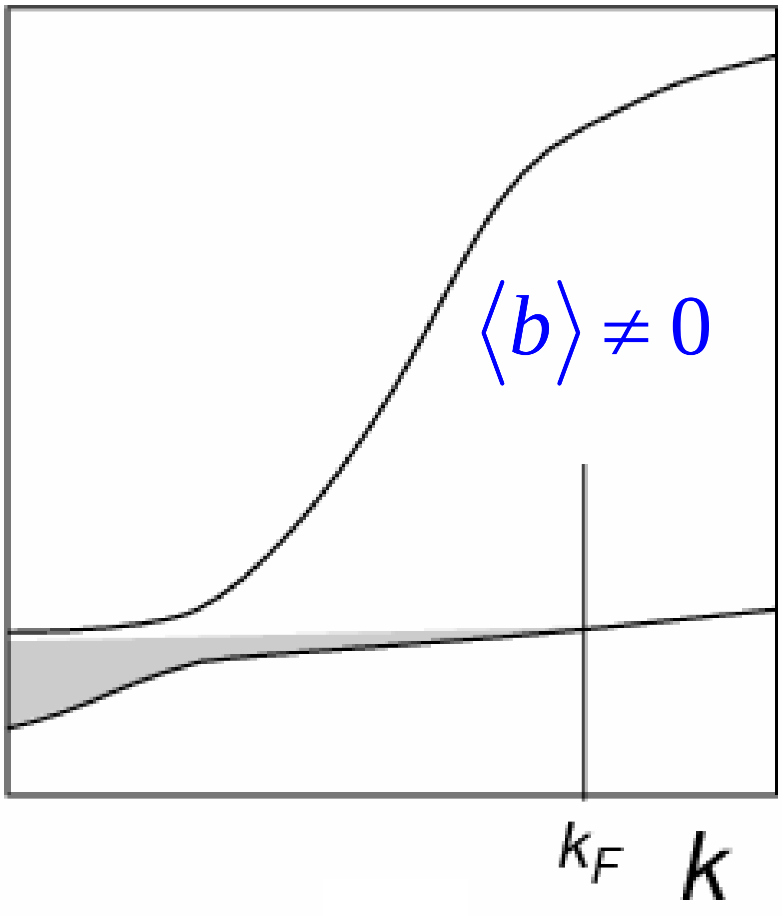
There is an electron-like Fermi surface

at k_F^{LMM} with $V_{k_F^{LMM}} = n_c$.

The Luttinger Theorem is obeyed because there are now two f electrons per unit cell. The f electrons are not part of the Fermi sphere.

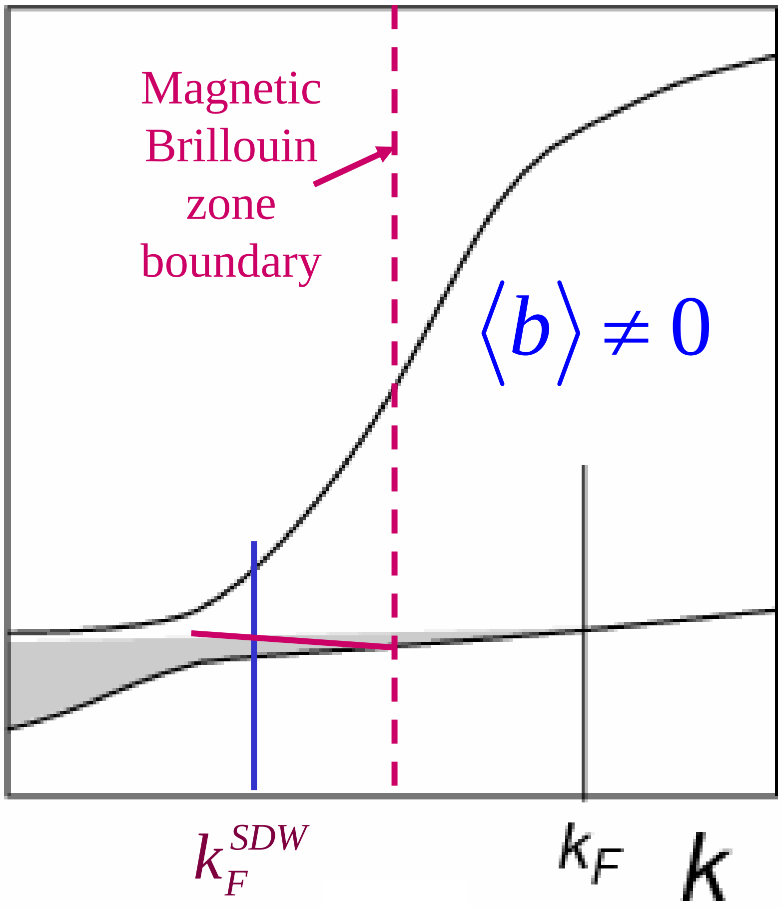
(B) The spin density wave metal (SDW)

There is incomplete Kondo screening of the local *f* moments in the FL state, and the static moments order antiferromagnetically. This halves the volume of the Brillouin zone.



(B) The spin density wave metal (SDW)

There is incomplete Kondo screening of the local *f* moments in the FL state, and the static moments order antiferromagnetically. This halves the volume of the Brillouin zone.



There is an hole-like Fermi surface

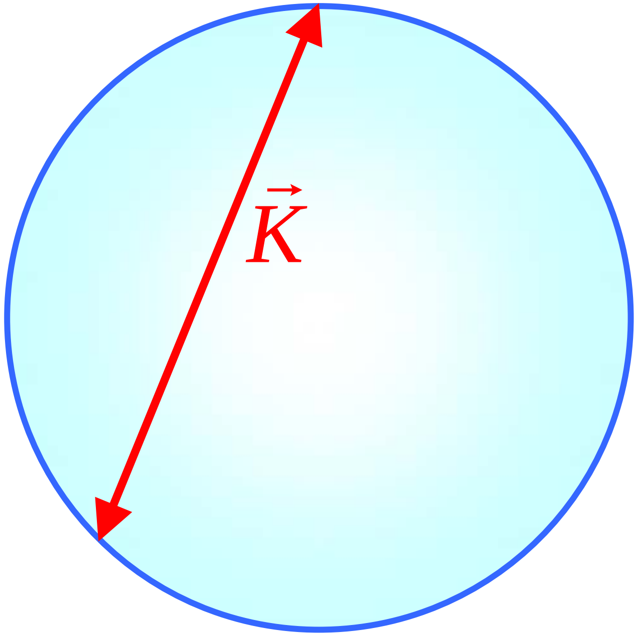
at k_F^{SDW} with $V_{k_F^{SDW}} = 1 - n_c$.

The Luttinger Theorem is again obeyed but the topology of the Fermi surface is different from the LMM metal. The *f* electrons are part of the Fermi sphere.

C. The FL to SDW quantum phase transition

LGW theory for quantum critical point

Write down effective action for SDW order parameter $\vec{\varphi}$



$\vec{\varphi}$ fluctuations are damped by mixing with fermionic quasiparticles near the Fermi surface

$$S_\varphi = \int \frac{d^d q d\omega}{(2\pi)^{d+1}} \left|\vec{\varphi}(q,\omega)\right|^2 \left(q^2 + |\omega| + (J_K - J_{Kc})\right) + \frac{u}{4}\int d^d r d\tau \left(\vec{\varphi}^2\right)^2$$

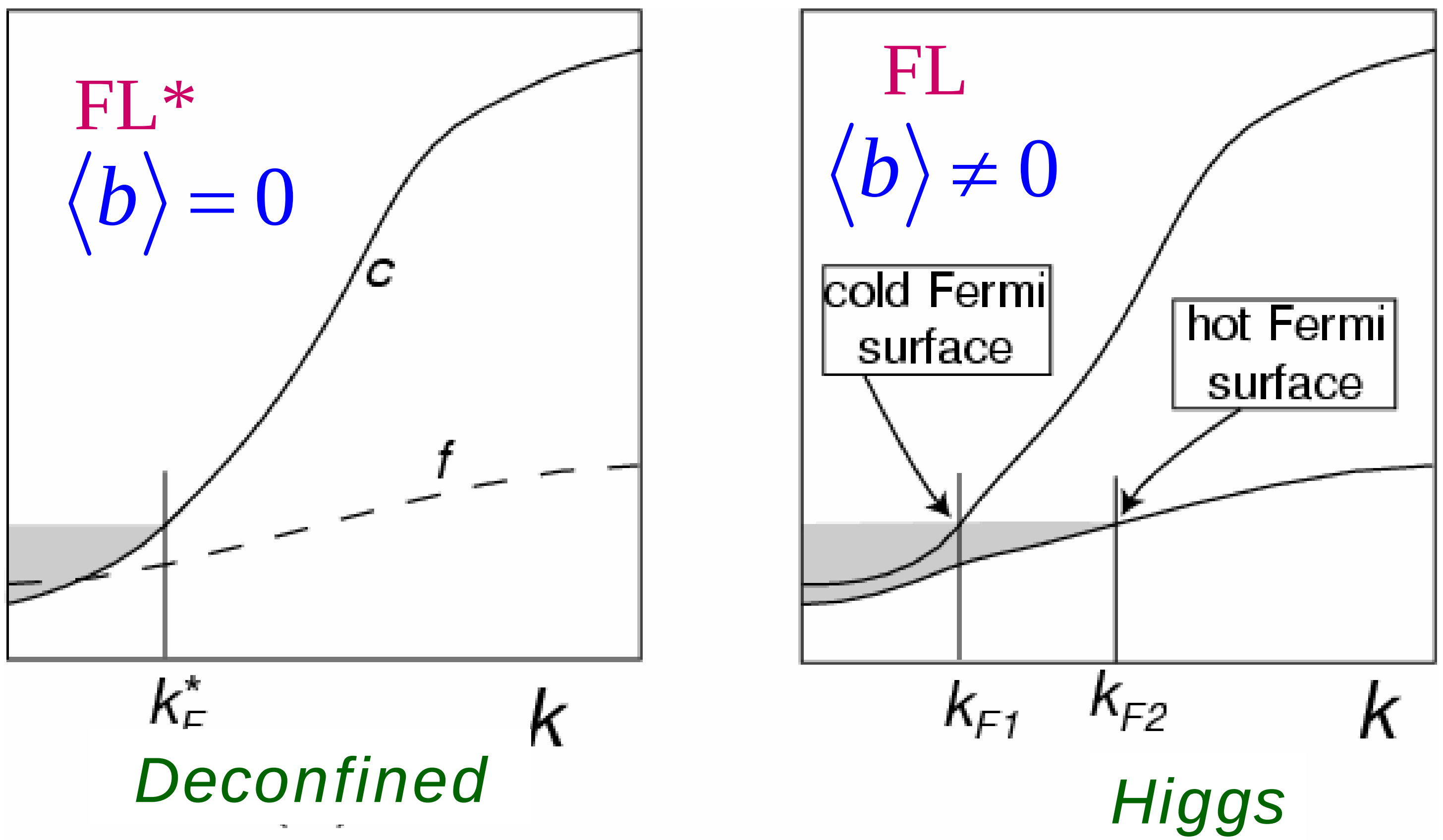
Fluctuations of $\vec{\varphi}$ about $\vec{\varphi} = 0 \Rightarrow$ paramagnons

J. Mathon, *Proc. R. Soc. London* A, **306**, 355 (1968); T.V. Ramakrishnan, *Phys. Rev.* B **10**, 4014 (1974);
M. T. Beal-Monod and K. Maki, *Phys. Rev. Lett.* **34**, 1461 (1975); J.A. Hertz, *Phys. Rev.* B **14**, 1165 (1976).
T. Moriya, *Spin Fluctuations in Itinerant Electron Magnetism*, Springer-Verlag, Berlin (1985);
G. G. Lonzarich and L. Taillefer, *J. Phys.* C **18**, 4339 (1985); A.J. Millis, *Phys. Rev.* B **48**, 7183 (1993).

D. The route from FL to LMM:

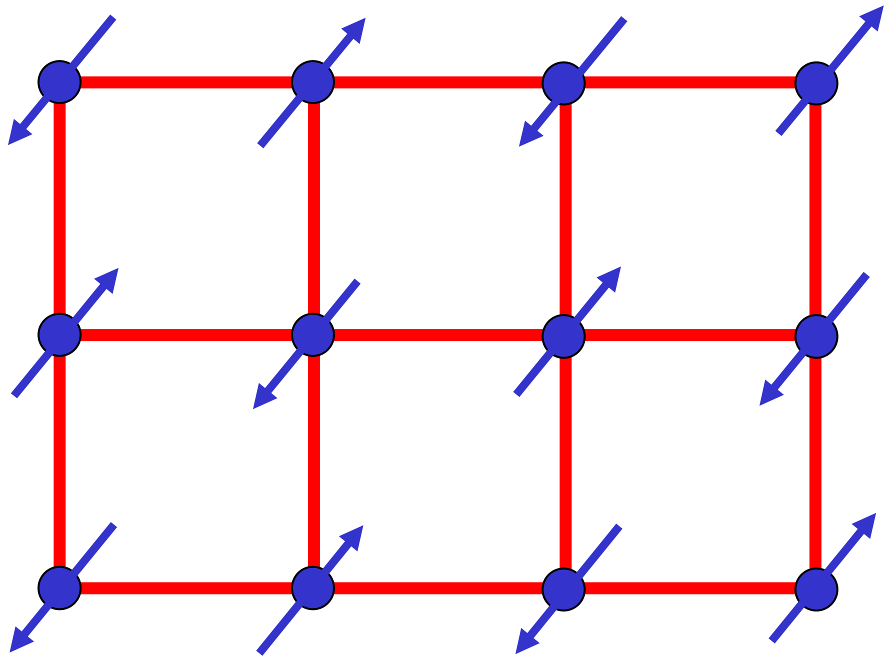
the fractionalized Fermi liquid (FL)*

Because of direct exchange J between local moments, allow f band to disperse in the decoupled limit.



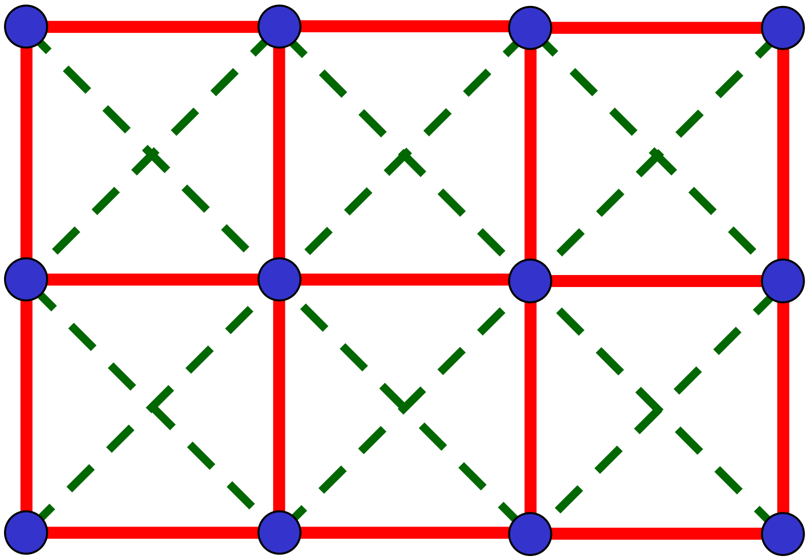
For large J/J_K, the gauge theory can enters its deconfined phase, and the fractionalized Fermi liquid (FL*) is obtained.

Work in the regime with small J_K, and consider destruction of magnetic order by frustrating (RKKY) exchange interactions between f moments



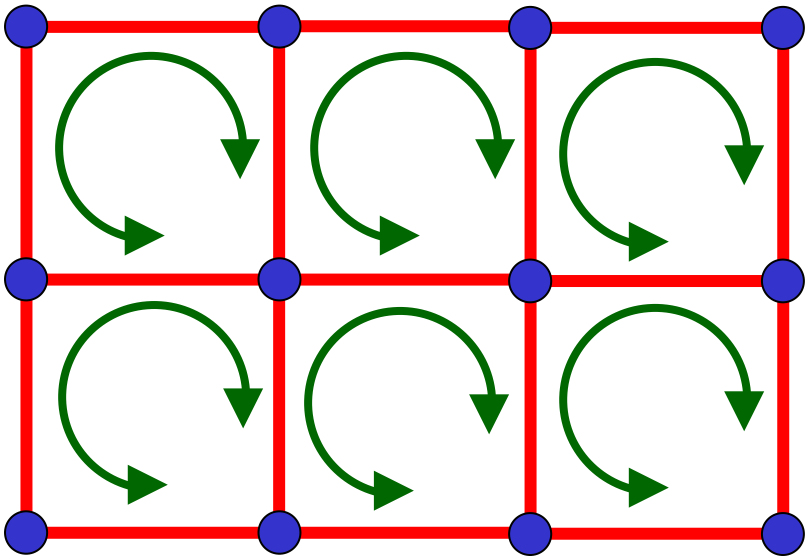
Ground state has Neel order with $\vec{\varphi} \neq 0$

Work in the regime with small J_K, and consider destruction of magnetic order by frustrating (RKKY) exchange interactions between *f* moments



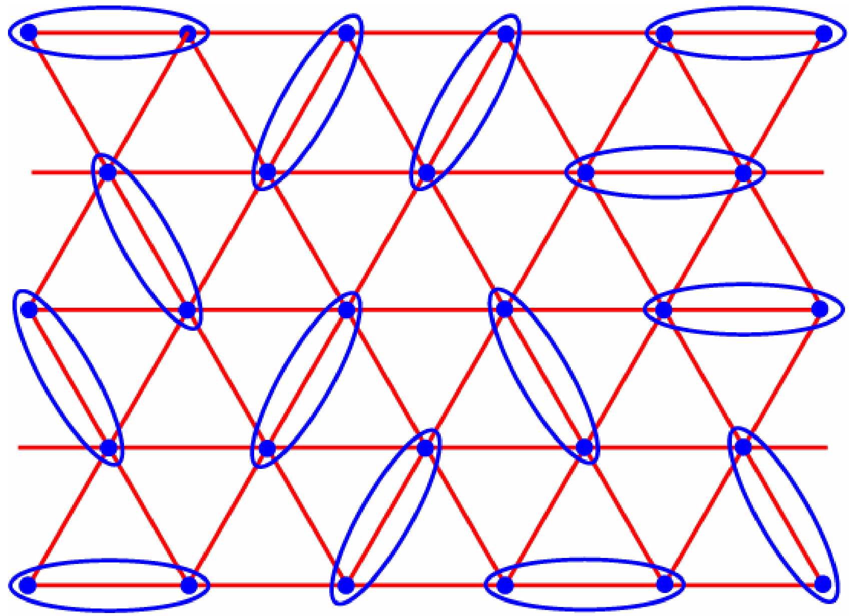
Destroy SDW order by perturbations which preserve full square lattice symmetry *e.g.* second-neighbor or ring exchange.

Work in the regime with small J_K, and consider destruction of magnetic order by frustrating (RKKY) exchange interactions between *f* moments



Destroy SDW order by perturbations which preserve full square lattice symmetry *e.g.* second-neighbor or ring exchange.

Work in the regime with small J_K, and consider destruction of magnetic order by frustrating (RKKY) exchange interactions between *f* moments

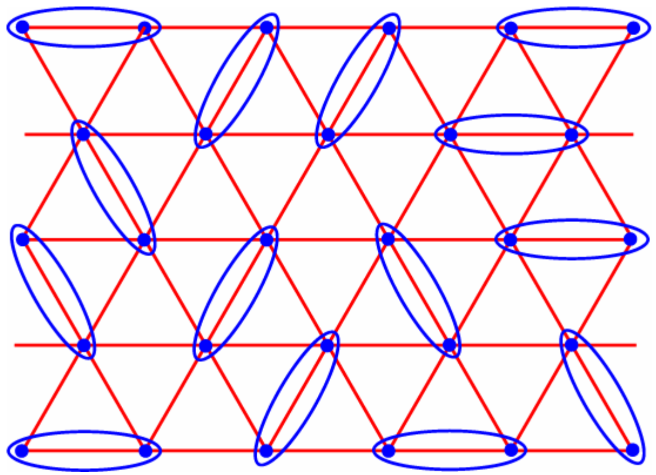


A *spin liquid* ground state with $\langle \vec{\varphi} \rangle = 0$ and no broken lattice symmetries.

Such a state has emergent excitations described by a Z_2 or U(1) gauge theory

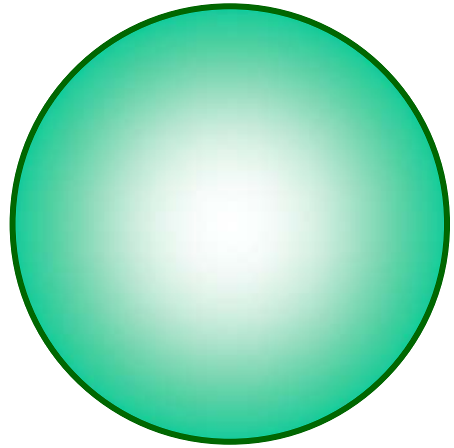
P. Fazekas and P.W. Anderson, *Phil Mag* **30**, 23 (1974).
N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991);
X. G. Wen, *Phys. Rev*. B **44**, 2664 (1991).

Influence of conduction electrons



Local moments f_σ

+



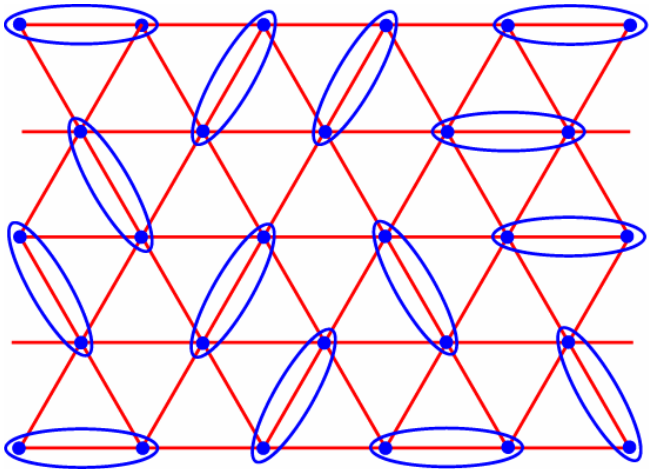
Conduction electrons c_σ

$$H = \sum_{i<j} t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + \sum_{i} \left(J_K c_{i\sigma}^{\dagger} \vec{\tau}_{\sigma\sigma'} c_{i\sigma} \cdot \vec{S}_{fi} \right) + \sum_{i<j} J_H (i, j) \vec{S}_{fi} \cdot \vec{S}_{fj}$$

Determine the ground state of the quantum antiferromagnet defined by J_H, and then couple to conduction electrons by J_K

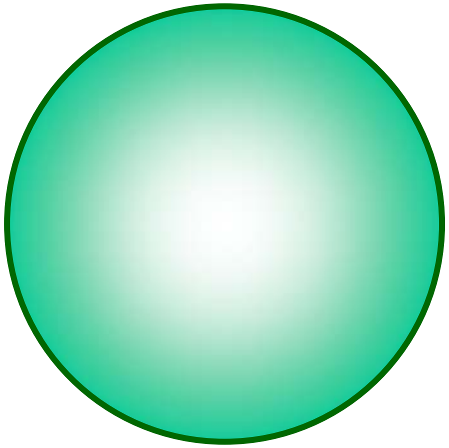
Choose J_H so that ground state of antiferromagnet is a Z_2 or U(1) spin liquid

Influence of conduction electrons



Local moments f_σ

+



Conduction electrons c_σ

At J_K= 0 the conduction electrons form a Fermi surface on their own with volume determined by n_c.

Perturbation theory in J_K is regular, and so this state will be stable for finite J_K.

So volume of Fermi surface is determined by **$(n_c+n_f\,-1)= n_c$(mod 2),** and does not equal the Luttinger value.

The (U(1) or Z_2) FL* state

A new phase: FL*

This phase preserves spin rotation invariance, and has a Fermi surface of *sharp* electron-like quasiparticles.

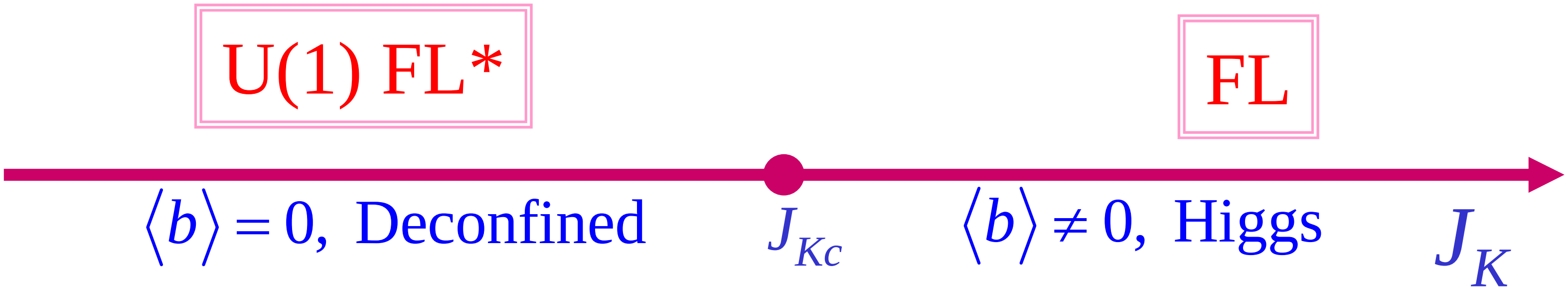
The state has "*topological order*" and associated neutral excitations. The topological order can be detected by the violation of Luttinger's Fermi surface volume. It can only appear in dimensions $d > 1$

$$2 \times \frac{v_0}{(2\pi)^d} \left(\text{Volume enclosed by Fermi surface} \right)$$

$$= \left(n_f + n_c - 1 \right) \left(\text{mod } 2 \right)$$

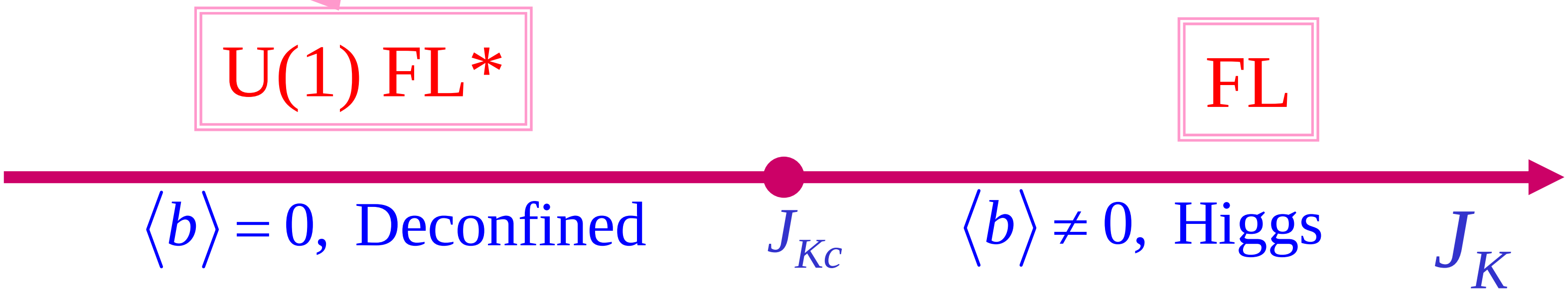
Precursors: N. Andrei and P. Coleman, *Phys. Rev. Lett*. **62**, 595 (1989).
Yu. Kagan, K. A. Kikoin, and N. V. Prokof'ev, *Physica* B **182**, 201 (1992).
Q. Si, S. Rabello, K. Ingersent, and L. Smith, *Nature* **413**, 804 (2001).
S. Burdin, D. R. Grempel, and A. Georges, *Phys. Rev*. B 66, 045111 (2002).
L. Balents and M. P. A. Fisher and C. Nayak, *Phys. Rev*. B **60**, 1654, (1999);
T. Senthil and M.P.A. Fisher, *Phys. Rev*. B **62**, 7850 (2000).
F. H. L. Essler and A. M. Tsvelik, *Phys. Rev*. B **65**, 115117 (2002).

Phase diagram



Phase diagram

Fractionalized Fermi liquid with moments paired in a spin liquid. Fermi surface volume does not include moments and is unequal to the Luttinger value.



Phase diagram

Fractionalized Fermi liquid with moments paired in a spin liquid. Fermi surface volume does not include moments and is unequal to the Luttinger value.

U(1) FL*

"Heavy" Fermi liquid with moments Kondo screened by conduction electrons. Fermi surface volume equals the Luttinger value.

FL

$\langle b \rangle = 0$, Deconfined — J_{Kc} — $\langle b \rangle \neq 0$, Higgs — J_K

Phase diagram

Fractionalized Fermi liquid with moments paired in a spin liquid. Fermi surface volume does not include moments and is unequal to the Luttinger value.

"Heavy" Fermi liquid with moments Kondo screened by conduction electrons. Fermi surface volume equals the Luttinger value.

U(1) FL*

FL

$\langle b \rangle = 0$, Deconfined

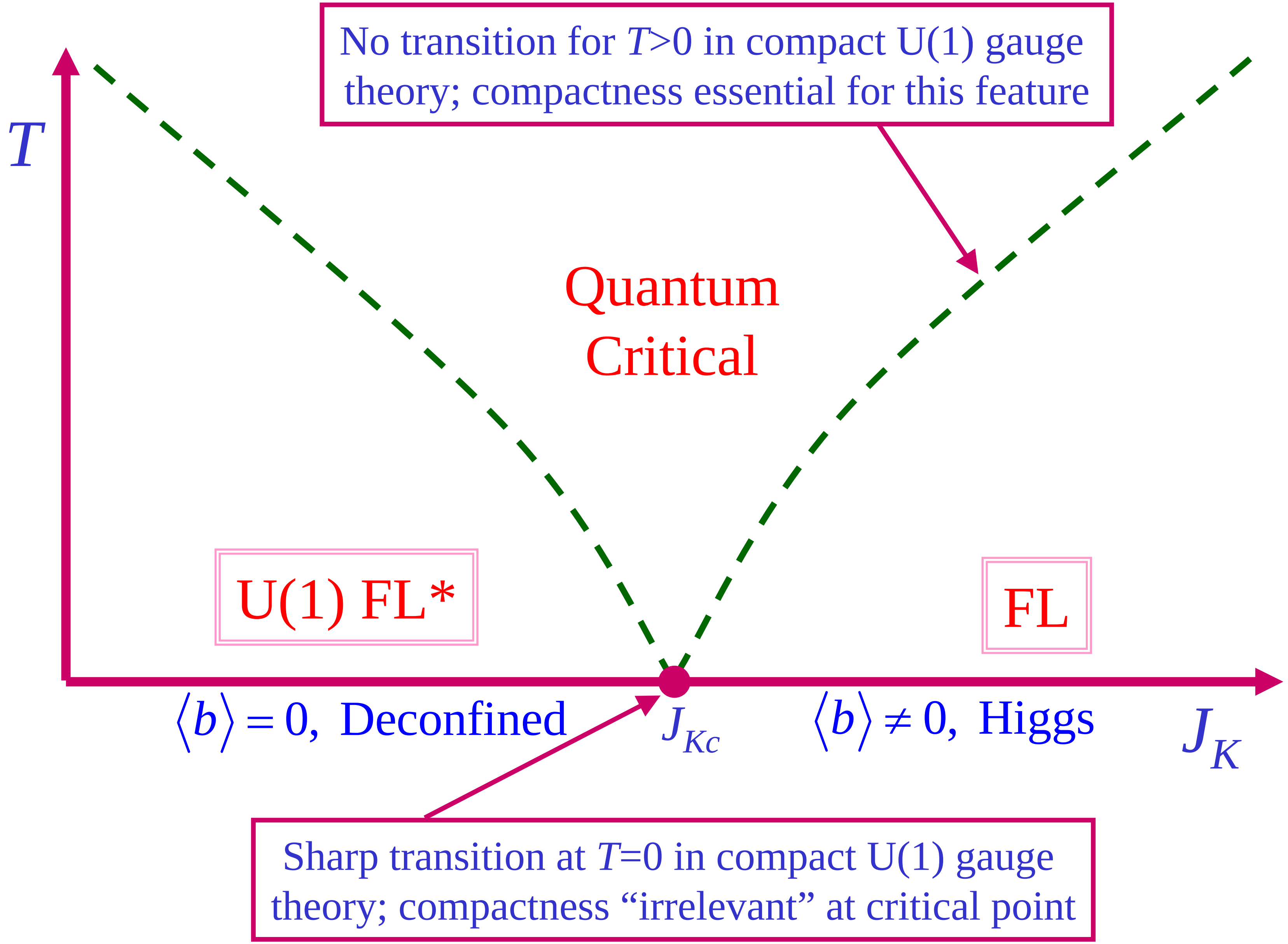
J_{Kc}

$\langle b \rangle \neq 0$, Higgs

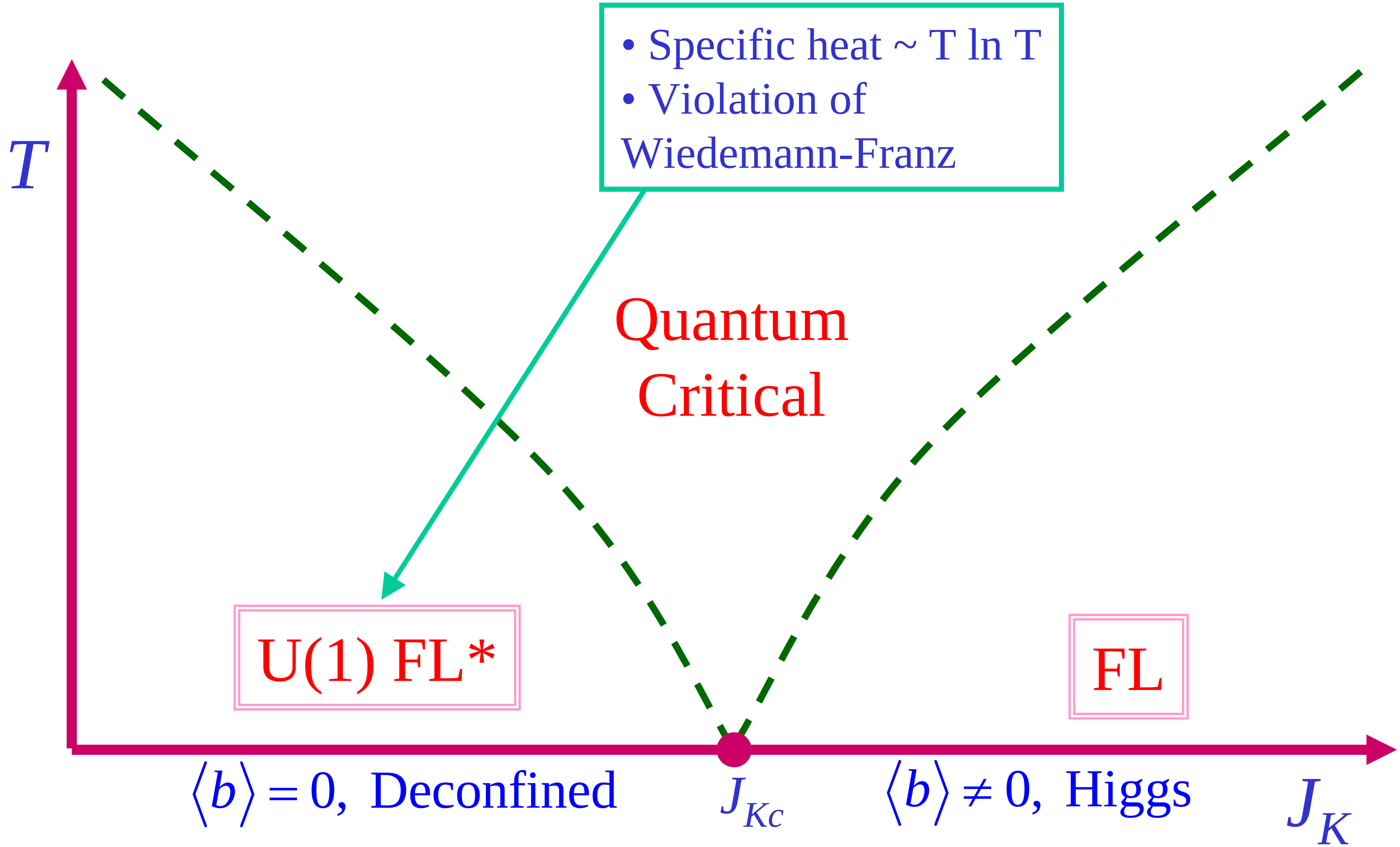
J_K

Sharp transition at T=0 in compact U(1) gauge theory; compactness "irrelevant" at critical point

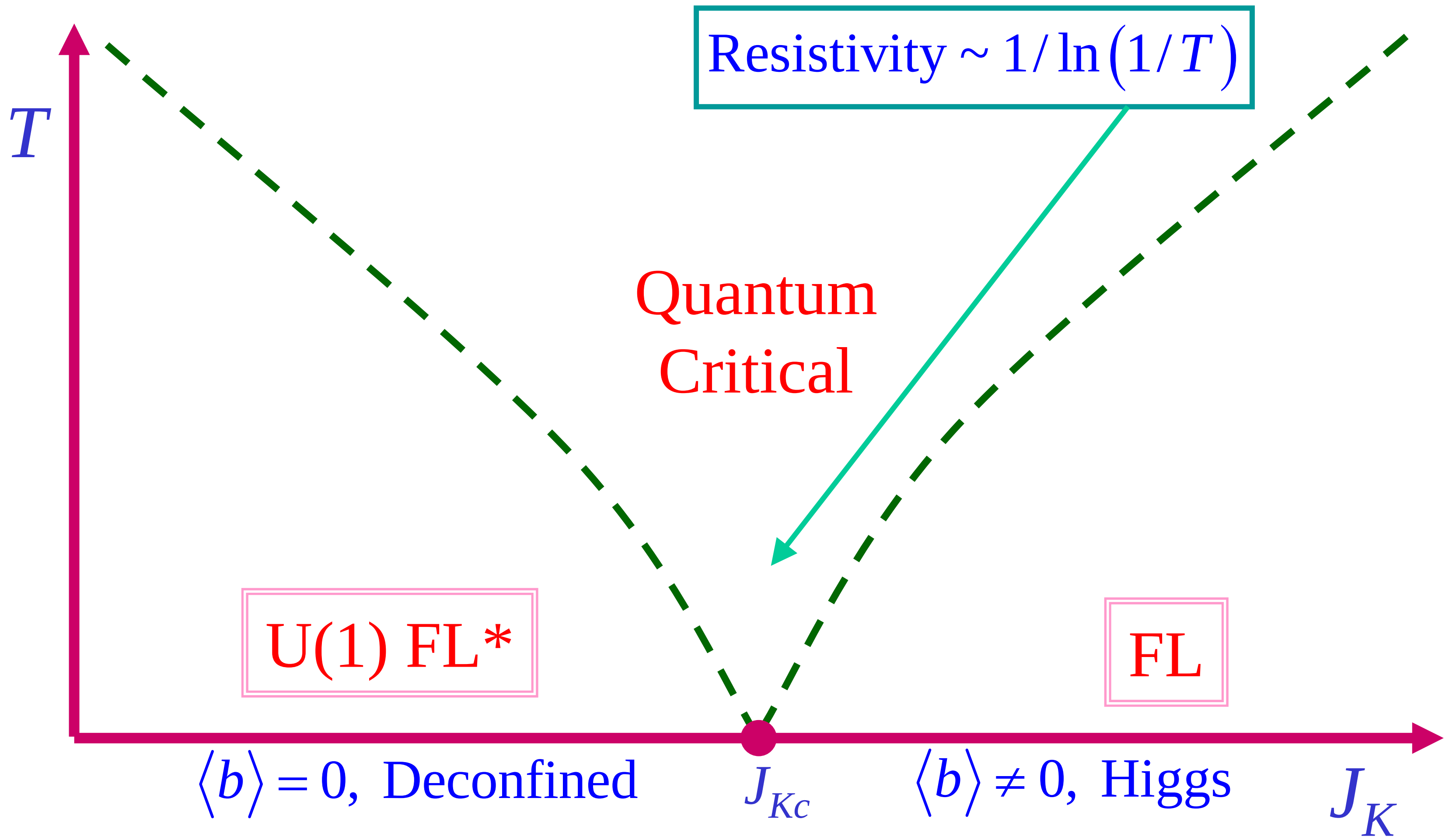
Phase diagram



Phase diagram



Phase diagram

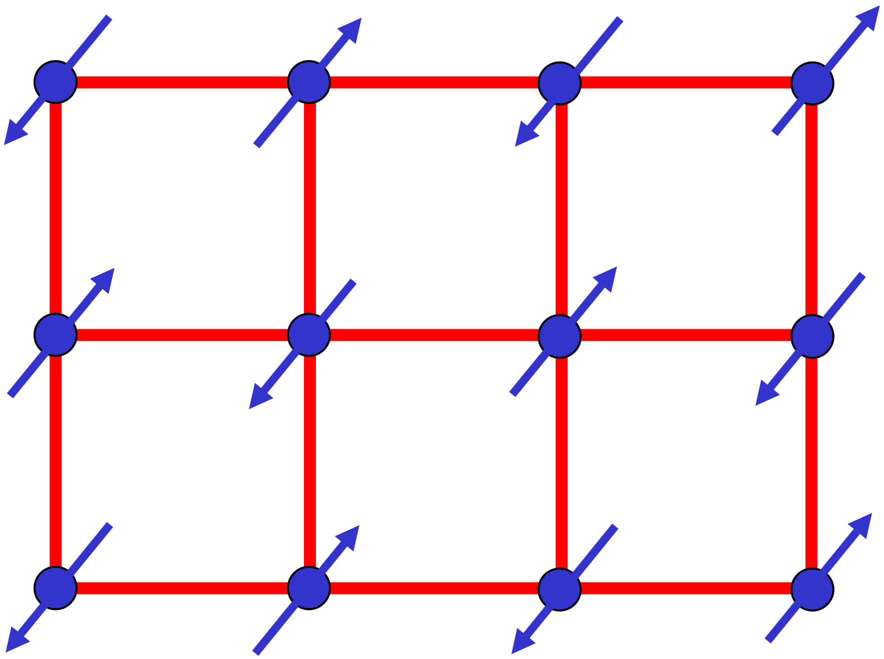


Is the U(1) FL* phase unstable to the LMM metal at the lowest energy scales ?

E. Detour: Deconfined criticality in insulating antiferromagnets

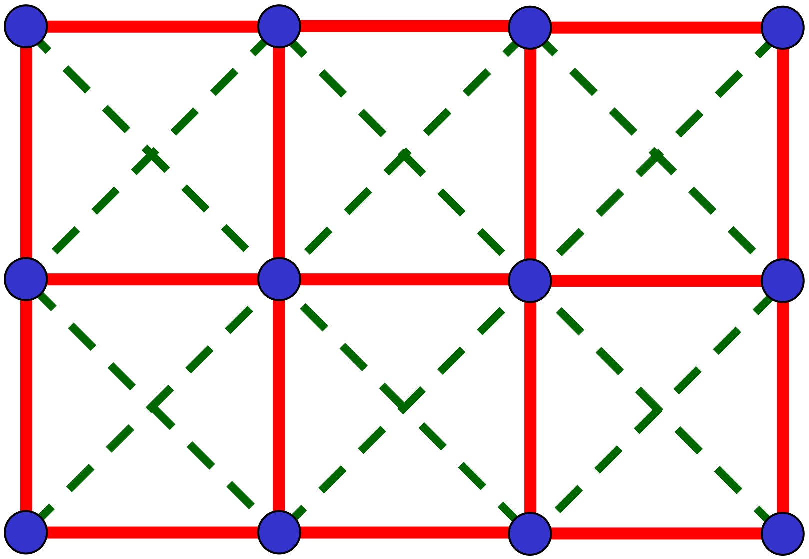
Landau forbidden quantum transitions

Reconsider destruction of magnetic order by frustrating (RKKY) exchange interactions between *f* moments in an insulator.



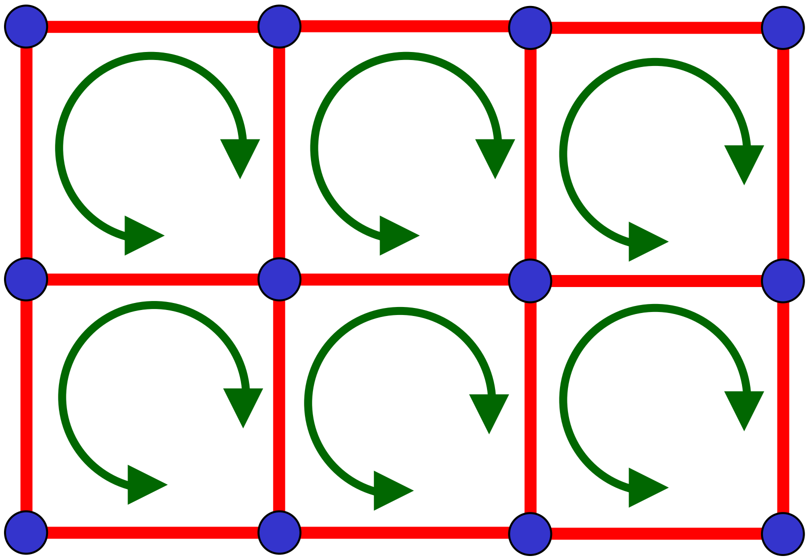
Ground state has Neel order with $\vec{\varphi} \neq 0$

Reconsider destruction of magnetic order by frustrating (RKKY) exchange interactions between *f* moments in an insulator.



Destroy SDW order by perturbations which preserve full square lattice symmetry *e.g.* second-neighbor or ring exchange.

Reconsider destruction of magnetic order by frustrating (RKKY) exchange interactions between *f* moments in an insulator.



Destroy SDW order by perturbations which preserve full square lattice symmetry *e.g.* second-neighbor or ring exchange.

Attempted theory for the destruction of Néel order

Express Néel order $\vec{\varphi}$ in terms of $S = 1/2$ bosonic spinons z_α by

$$\vec{\varphi} \sim z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta.$$

This introduces a U(1) gauge invariance under $z_\alpha \to z_\alpha e^{i\phi(x,\tau)}$.
Field theory for the z_α spinons:

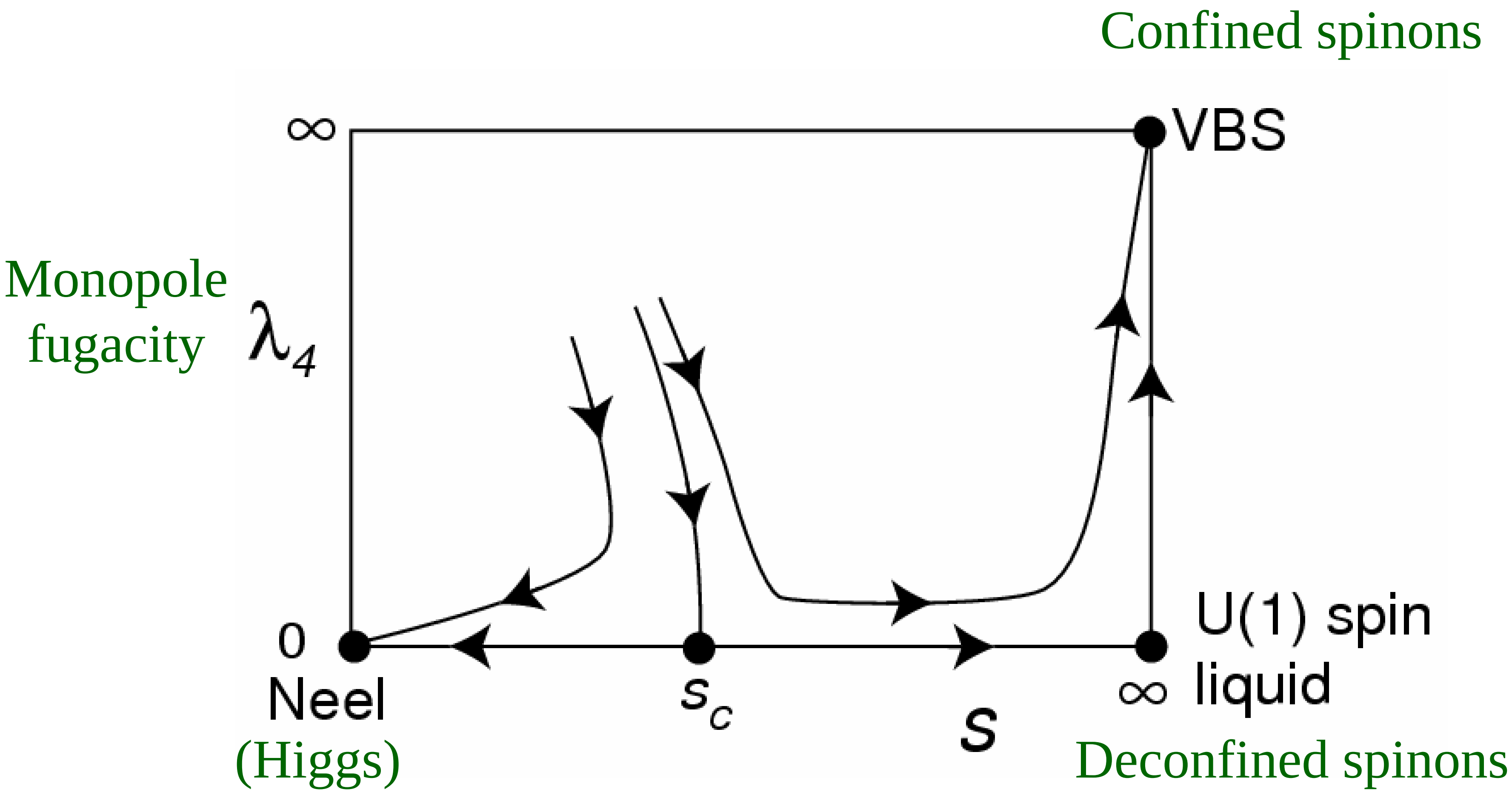
$$\begin{aligned}\mathcal{S}_{\text{critical}} &= \int d^2x d\tau \Big[\left|(\partial_\mu - iA_\mu) z_\alpha\right|^2 + s\left|z_\alpha\right|^2 + \frac{u}{2}\left(\left|z_\alpha\right|^2\right)^2 \\ &\qquad + \frac{1}{4e^2}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 \Big]\end{aligned}$$

where A_μ is a U(1) gauge field.

Phases of theory

$$s < s_c \;\Rightarrow\; \text{Néel (Higgs) phase with } \langle z_\alpha \rangle \neq 0$$

$$s > s_c \;\Rightarrow\; \text{Deconfined U(1) spin liquid with } \langle z_\alpha \rangle = 0$$

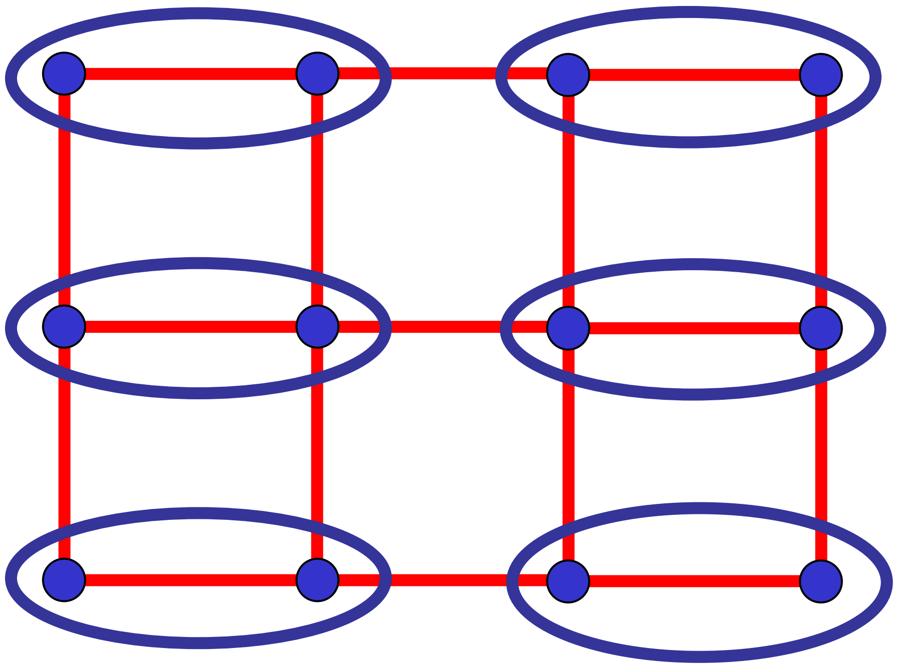


N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).

A. V. Chubukov, S. Sachdev, and J. Ye, *Phys. Rev. B* **49**, 11919 (1994).

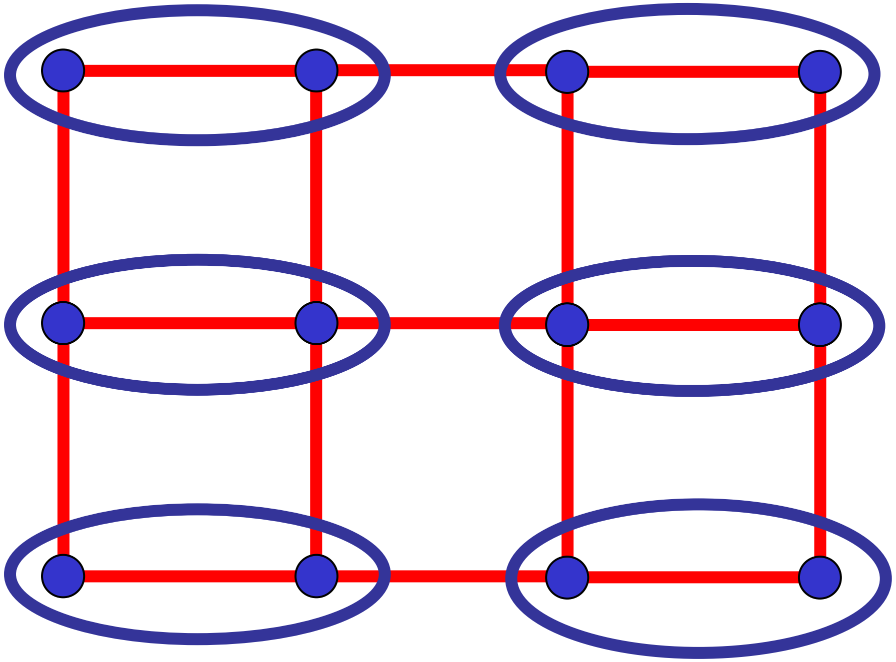
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

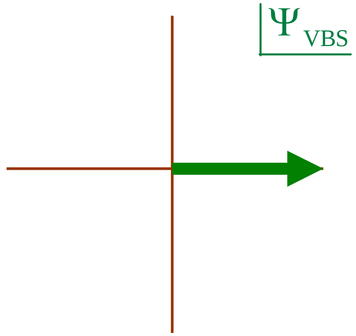
Valence bond solid order



Possible paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Valence bond solid order

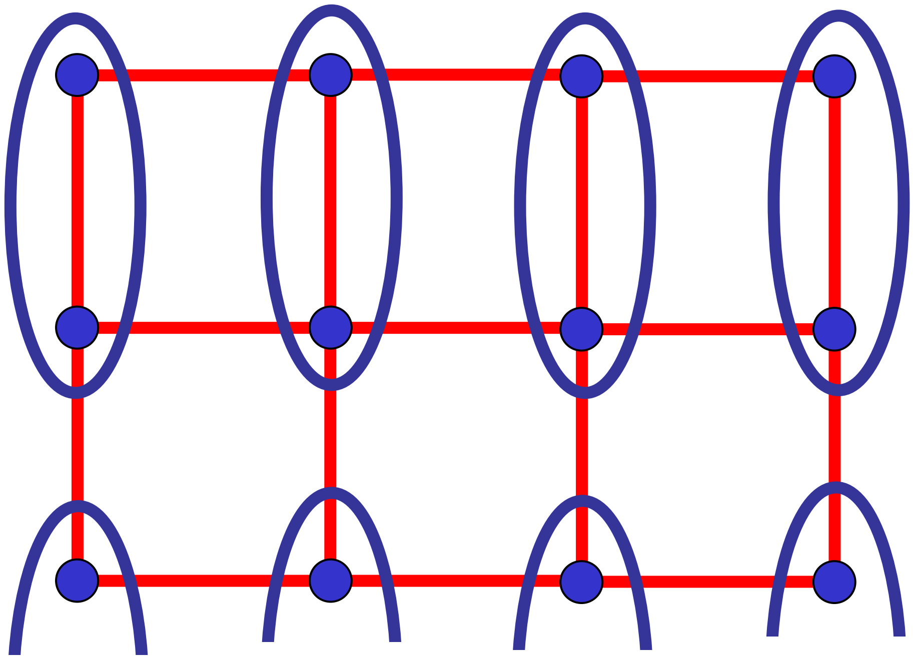


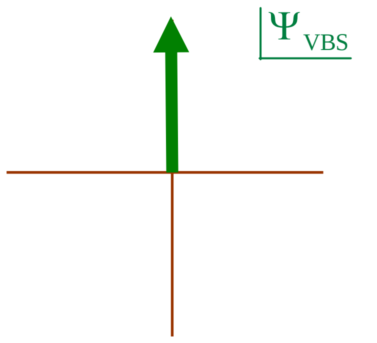


Possible paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Such a state breaks lattice symmetry and has $\langle \Psi_{VBS} \rangle \neq 0$,
where Ψ_{VBS} is the ***valence bond solid (VBS) order parameter***

Valence bond solid order

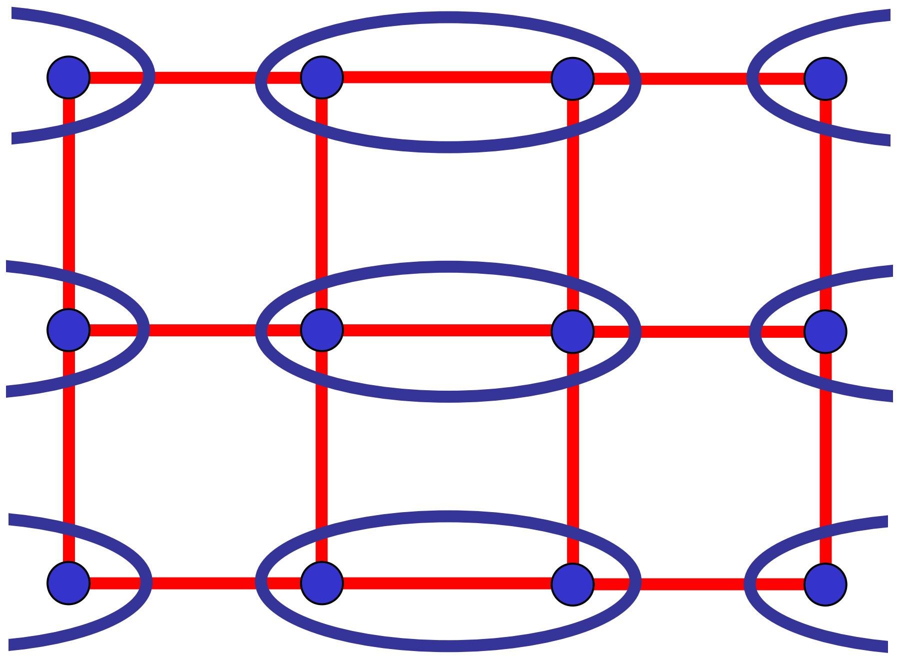


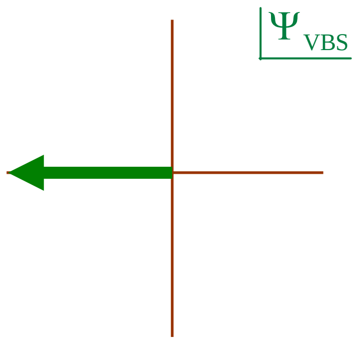


Possible paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Such a state breaks lattice symmetry and has $\langle \Psi_{\text{VBS}} \rangle \neq 0$,
where Ψ_{VBS} is the ***valence bond solid (VBS) order parameter***

Valence bond solid order

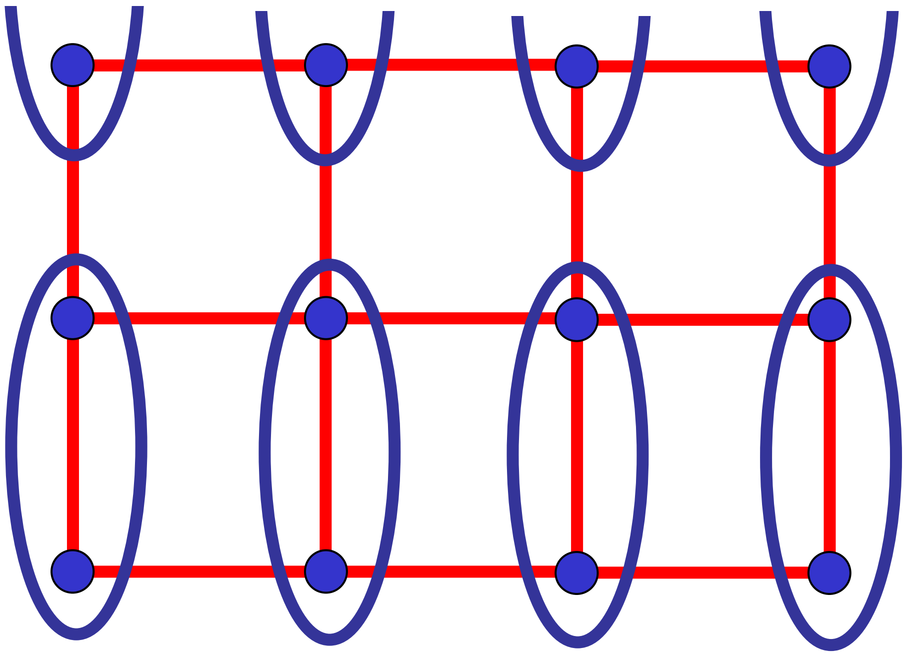


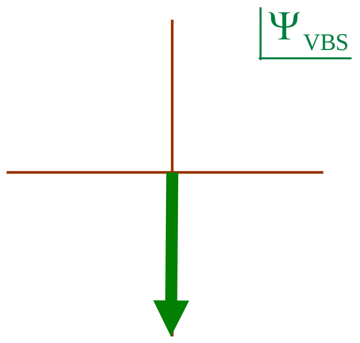


Possible paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Such a state breaks lattice symmetry and has $\langle \Psi_{\text{VBS}} \rangle \neq 0$,
where Ψ_{VBS} is the ***valence bond solid (VBS) order parameter***

Valence bond solid order

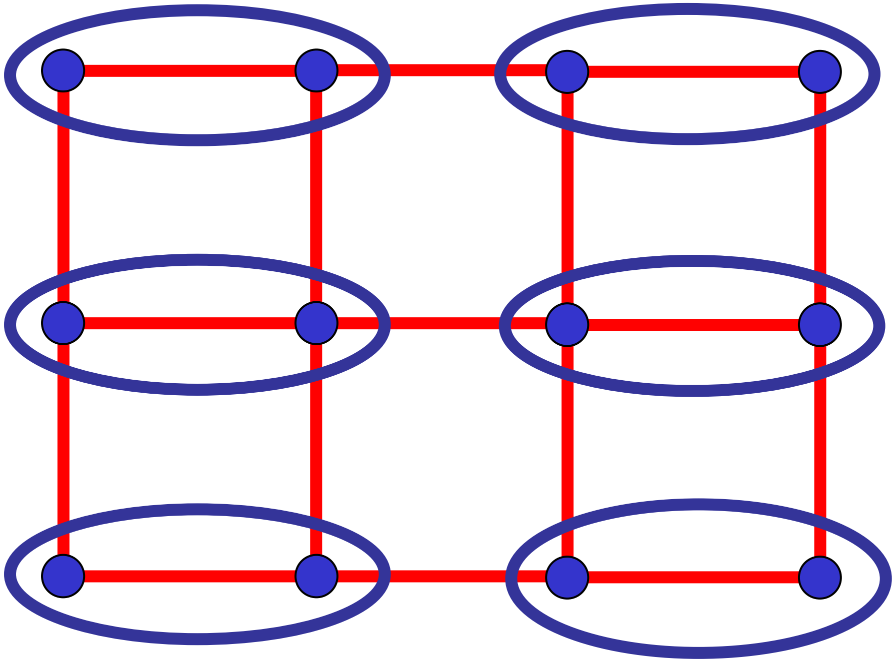


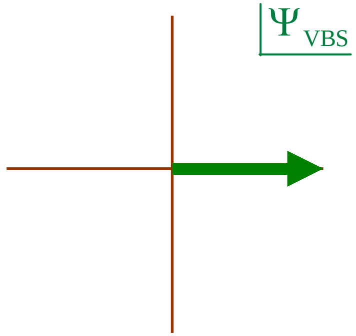


Possible paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Such a state breaks lattice symmetry and has $\langle \Psi_{\text{VBS}} \rangle \neq 0$,
where Ψ_{VBS} is the ***valence bond solid (VBS) order parameter***

Valence bond solid order

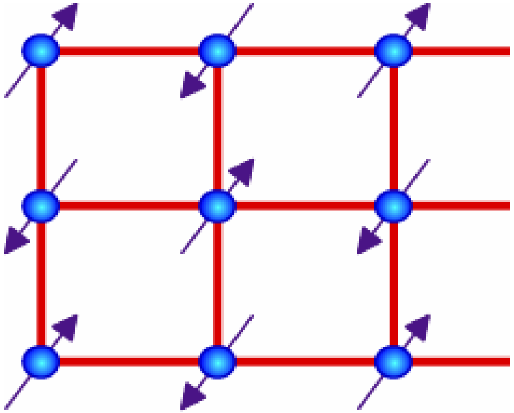




Possible paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Such a state breaks lattice symmetry and has $\langle \Psi_{VBS} \rangle \neq 0$,
where Ψ_{VBS} is the ***valence bond solid (VBS) order parameter***

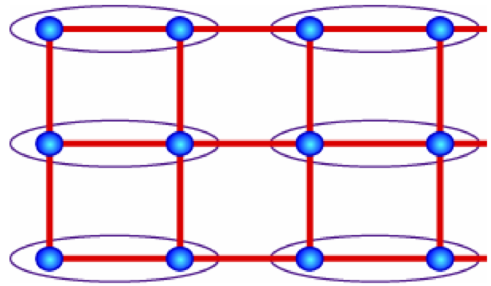
Phase diagram of S=1/2 square lattice antiferromagnet



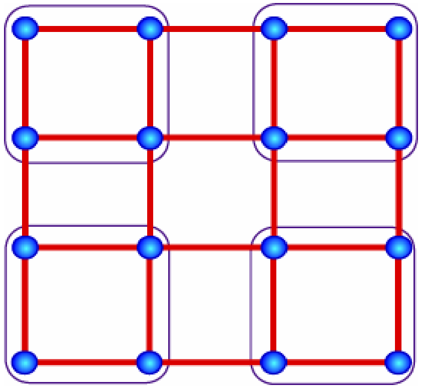
Neel order

$\vec{\varphi} \sim z^*_\alpha \vec{\sigma}_{\alpha\beta} z_\beta \neq 0$

(Higgs)



or



VBS order $\Psi_{\mathrm{VBS}} \neq 0$,

$S = 1/2$ spinons z_α confined,

$S = 1$ triplon excitations

s

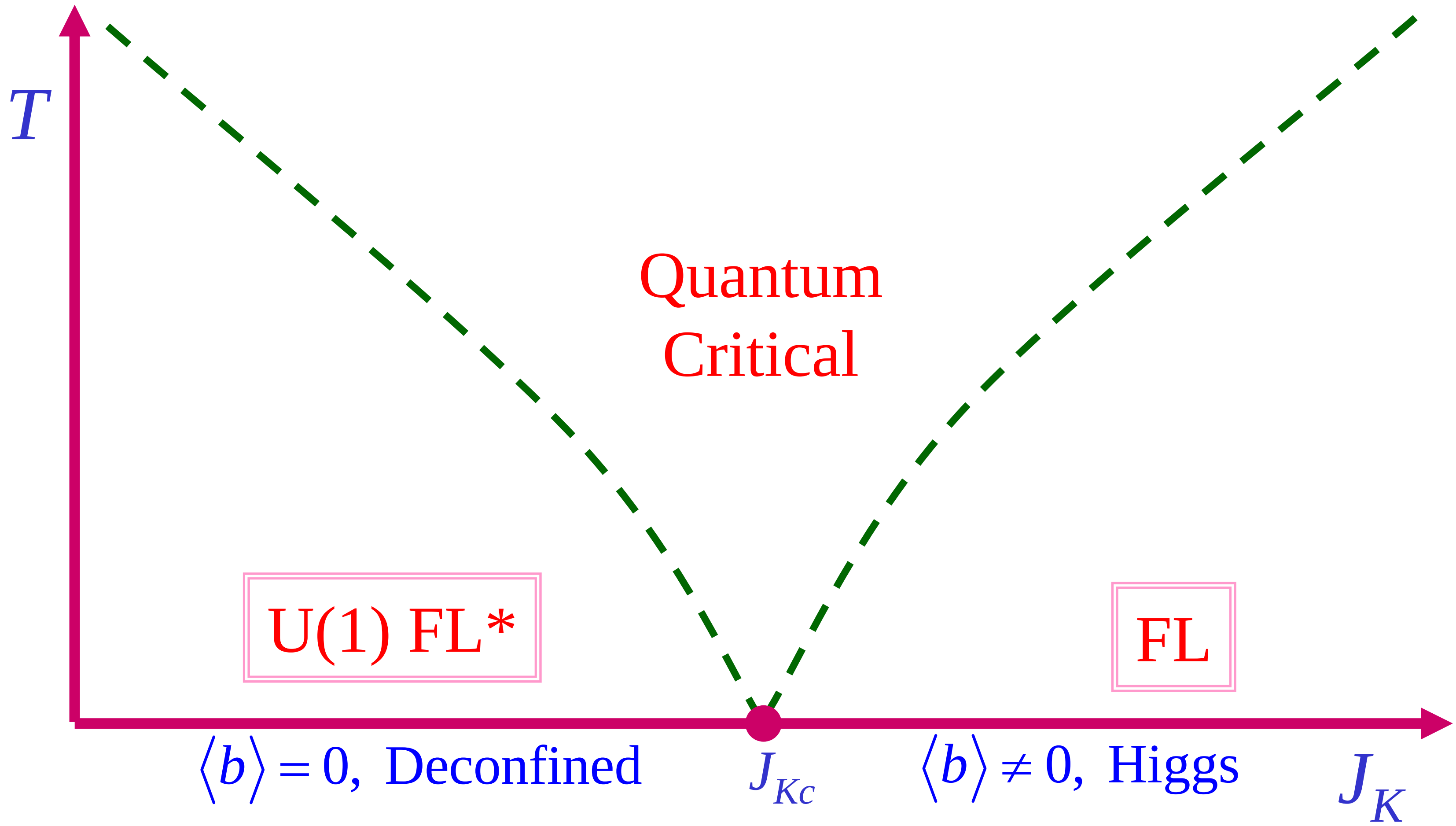
Deconfined critical point described by a theory of spinons

$$\mathcal{S}_{\text{critical}} = \int d^2x d\tau \left[\left|(\partial_\mu - iA_\mu) z_\alpha\right|^2 + s\left|z_\alpha\right|^2 + \frac{u}{2}\left(\left|z_\alpha\right|^2\right)^2 + \frac{1}{4e^2}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 \right]$$

Landau-forbidden transition between phases which break "unrelated" symmetries

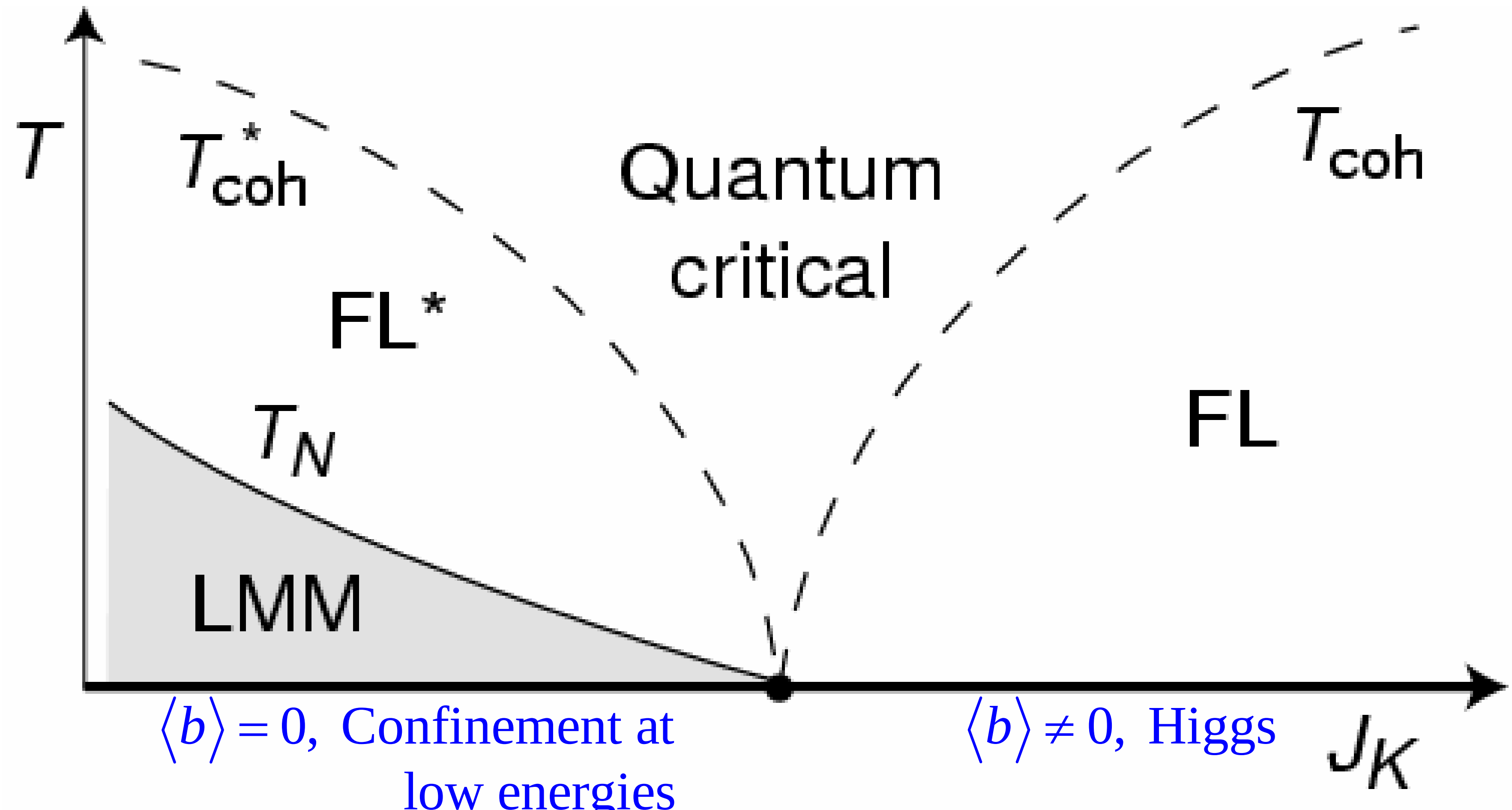
F. Deconfined criticality in the Kondo lattice ?

Phase diagram



Is the U(1) FL* phase unstable to the LMM metal at the lowest energy scales ?

Phase diagram ?

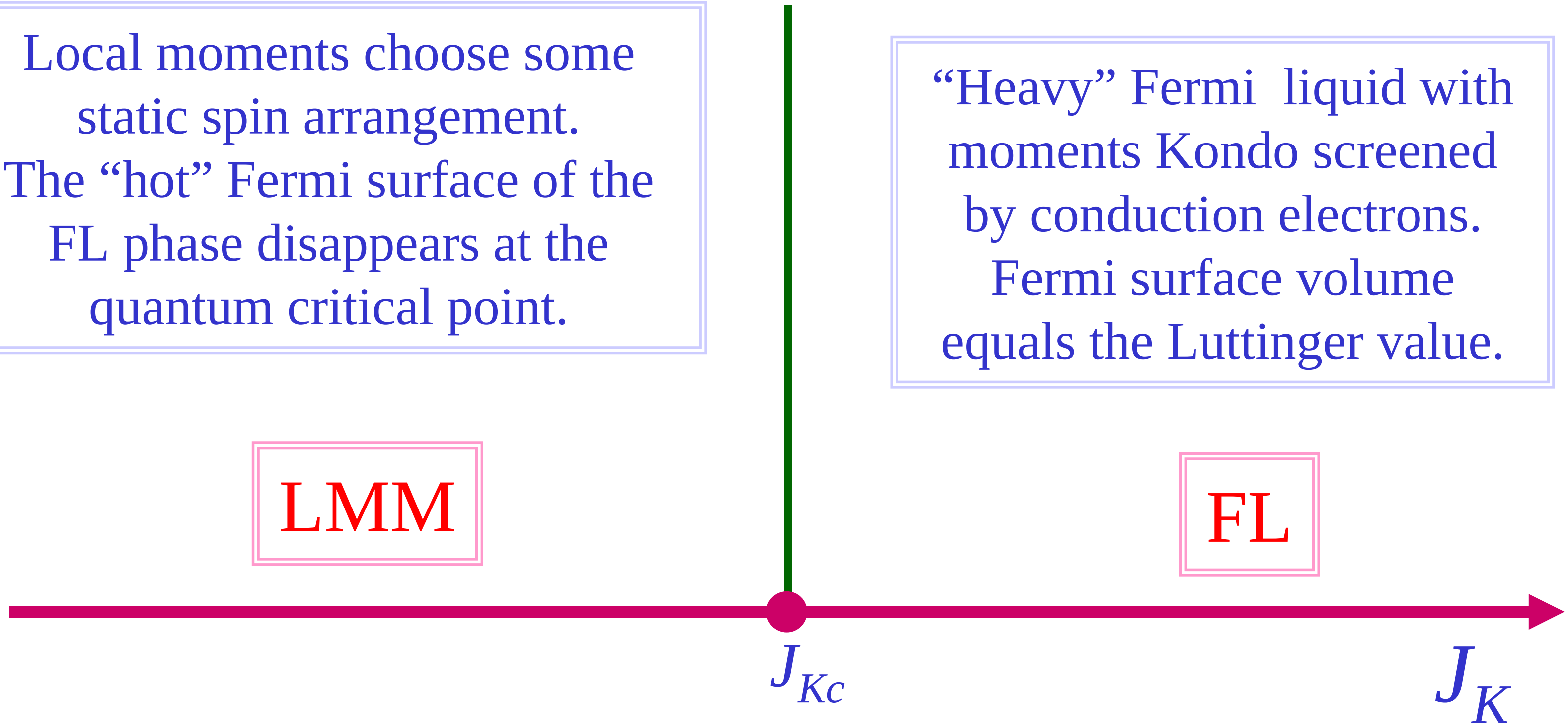


U(1) FL* phase generates magnetism at energies much lower than the critical energy of the FL to FL* transition

Phase diagram for the Kondo lattice ?

Local moments choose some static spin arrangement. The "hot" Fermi surface of the FL phase disappears at the quantum critical point.

"Heavy" Fermi liquid with moments Kondo screened by conduction electrons. Fermi surface volume equals the Luttinger value.



See also Q. Si, S. Rabello, K. Ingersent, and J. L. Smith, *Nature* 413, 804 (2001);
S. Paschen, T. Luehmann, C. Langhammer, O. Trovarelli, S. Wirth, C. Geibel, F. Steglich, *Acta Physica Polonica* B **34**, 359 (2003).