Quantum phase transitions out of the heavy Fermi liquid

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The Kondo lattice

Local moments $f_\sigma$

Conduction electrons $c_\sigma$

$$H_K = \sum_{i<j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + J_K \sum_i c_{i\sigma}^\dagger \vec{\tau}_{\sigma\sigma} c_{i\sigma} \cdot \vec{S}_{fi} + J \sum_{\langle ij \rangle} \vec{S}_{fi} \cdot \vec{S}_{fj}$$

Number of $f$ electrons per unit cell = $n_f = 1$

Number of $c$ electrons per unit cell = $n_c$
Outline

A. The heavy Fermi liquid (FL)

B. The metallic antiferromagnet
   *Local moment magnetic metal (LMM),
   Spin density wave metal (SDW)*

C. FL to SDW transition

D. The route from FL to LMM
   *The fractionalized Fermi liquid (FL*)

E. Detour: Deconfined criticality in insulators
   *Landau forbidden quantum transitions*

F. Deconfined criticality in the Kondo lattice?
A. The heavy Fermi liquid (FL)
Obtained in the limit of large $J_K$

The Fermi surface of heavy quasiparticles encloses a volume which counts all electrons.

Fermi volume $= 1 + n_c$
Argument for the Fermi surface volume of the FL phase

Single ion Kondo effect implies $J_K \to \infty$ at low energies

\[
\left( c_{i\uparrow}^{\dagger} f_{i\downarrow}^{\dagger} - c_{i\downarrow}^{\dagger} f_{i\uparrow}^{\dagger} \right)|0\rangle
\]

\[
f_{i\downarrow}^{\dagger}|0\rangle, \ S=1/2 \ hole
\]

Fermi liquid of $S=1/2$ holes with hard-core repulsion

Fermi surface volume $= - (\text{density of holes}) \mod 2$

$= -(1 - n_c) = (1 + n_c) \mod 2$
Operator approach

Define a bosonic field which measures the hybridization between the two bands:

\[ b_i \sim \sum_{\sigma} c_{i\sigma}^\dagger f_{i\sigma} \]

The absence of charge fluctuations on the \( f \) sites implies an emergent compact U(1) gauge theory, associated with the gauge transformations

\[ f_{i\sigma} \rightarrow f_{i\sigma} e^{i\phi_i(\tau)} \; ; \; b_i \rightarrow b_i e^{i\phi_i(\tau)} \]
The FL state is the “Higgs” phase of the U(1) gauge theory. Because of the dispersionless $f$ band in the decoupled case, the ground state is always in the Higgs phase.
B. The metallic antiferromagnet

Two possible states:

(A) The local moment magnetic metal (LMM)

(B) Spin density wave metal (SDW)
(A) The local moment magnetic metal (LMM)

The local $f$ moments order antiferromagnetically, and this halves the volume of the Brillouin zone.

$$\langle b \rangle = 0$$
(A) The local moment magnetic metal (LMM)

The local $f$ moments order antiferromagnetically, and this halves the volume of the Brillouin zone.

There is an electron-like Fermi surface at $k_F^{LMM}$ with $V_{k_F^{LMM}} = n_c$.

The Luttinger Theorem is obeyed because there are now two $f$ electrons per unit cell. The $f$ electrons are not part of the Fermi sphere.
(B) The spin density wave metal (SDW)

There is incomplete Kondo screening of the local $f$ moments in the FL state, and the static moments order antiferromagnetically. This halves the volume of the Brillouin zone.

$\langle b \rangle \neq 0$
(B) The spin density wave metal (SDW)

There is incomplete Kondo screening of the local $f$ moments in the FL state, and the static moments order antiferromagnetically. This halves the volume of the Brillouin zone.

There is an hole-like Fermi surface at $k_F^{SDW}$ with $V_{k_F^{SDW}} = 1 - n_c$.

The Luttinger Theorem is again obeyed but the topology of the Fermi surface is different from the LMM metal. The $f$ electrons are part of the Fermi sphere.
C. The FL to SDW quantum phase transition
Write down effective action for SDW order parameter $\bar{\phi}$

$$S_\phi = \int \frac{d^d q d\omega}{(2\pi)^{d+1}} |\bar{\phi}(q, \omega)|^2 \left( q^2 + |\omega| + (J_K - J_{Kc}) \right) + \frac{u}{4} \int d^d r \, d\tau \left( \bar{\phi}^2 \right)^2$$

$\bar{\phi}$ fluctuations are damped by mixing with fermionic quasiparticles near the Fermi surface

Fluctuations of $\bar{\phi}$ about $\bar{\phi} = 0 \Rightarrow$ paramagnons

D. The route from FL to LMM:

*the fractionalized Fermi liquid (FL*)
Because of direct exchange $J$ between local moments, allow $f$ band to disperse in the decoupled limit.

For large $J/J_K$, the gauge theory can enter its deconfined phase, and the fractionalized Fermi liquid (FL*) is obtained.
Work in the regime with small $J_K$, and consider destruction of magnetic order by frustrating (RKKY) exchange interactions between $f$ moments.

Ground state has Neel order with $\vec{\phi} \neq 0$.
Work in the regime with small $J_K$, and consider destruction of magnetic order by frustrating (RKKY) exchange interactions between $f$ moments

Destroy SDW order by perturbations which preserve full square lattice symmetry e.g. second-neighbor or ring exchange.
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A \textit{spin liquid} ground state with $\langle \bar{\phi} \rangle = 0$ and no broken lattice symmetries. Such a state has emergent excitations described by a $Z_2$ or U(1) gauge theory.

Influence of conduction electrons

Local moments $f_\sigma$

\[ H = \sum_{i<j} t_{ij} c_i^{\dagger} c_j + \sum_i (J_K c_i^{\dagger} \vec{\tau}_{i\sigma} c_i \cdot \vec{S}_i) + \sum_{i<j} J_H (i, j) \vec{S}_i \cdot \vec{S}_j \]

Determine the ground state of the quantum antiferromagnet defined by $J_H$, and then couple to conduction electrons by $J_K$

Choose $J_H$ so that ground state of antiferromagnet is a $Z_2$ or U(1) spin liquid
Influence of conduction electrons

Local moments $f_{\sigma}$

At $J_K = 0$ the conduction electrons form a Fermi surface on their own with volume determined by $n_c$.

Perturbation theory in $J_K$ is regular, and so this state will be stable for finite $J_K$.

So volume of Fermi surface is determined by

$$(n_c + n_f - 1) = n_c \text{ (mod 2)},$$

and does not equal the Luttinger value.

The $(U(1)$ or $Z_2)$ FL* state
A new phase: FL*

This phase preserves spin rotation invariance, and has a Fermi surface of *sharp* electron-like quasiparticles.

The state has “*topological order*” and associated neutral excitations. The topological order can be detected by the violation of Luttinger’s Fermi surface volume. It can only appear in dimensions $d > 1$

$$2 \times \frac{V_0}{(2\pi)^d} \left( \text{Volume enclosed by Fermi surface} \right)$$

$$= \left( n_f + n_c - 1 \right) \left( \text{mod } 2 \right)$$

Phase diagram

\( b = 0 \), Deconfined

\( J_Kc \)

\( b \neq 0 \), Higgs

\( J_K \)

\( U(1) FL^* \)

\( FL \)
Fractionalized Fermi liquid with moments paired in a spin liquid. Fermi surface volume does not include moments and is unequal to the Luttinger value.

\[ \langle b \rangle = 0, \text{ Deconfined} \quad J_{Kc} \quad \langle b \rangle \neq 0, \text{ Higgs} \quad J_K \]
Fractionalized Fermi liquid with moments paired in a spin liquid. Fermi surface volume does not include moments and is unequal to the Luttinger value.

“Heavy” Fermi liquid with moments Kondo screened by conduction electrons. Fermi surface volume equals the Luttinger value.
Phase diagram

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“Heavy” Fermi liquid with moments Kondo screened by conduction electrons. Fermi surface volume equals the Luttinger value.

U(1) FL* \[ \langle b \rangle = 0, \text{ Deconfined} \]

FL \[ \langle b \rangle \neq 0, \text{ Higgs} \]

Sharp transition at \( T=0 \) in compact U(1) gauge theory; compactness “irrelevant” at critical point
No transition for $T>0$ in compact U(1) gauge theory; compactness essential for this feature.

Sharp transition at $T=0$ in compact U(1) gauge theory; compactness “irrelevant” at critical point.
Phase diagram

- Specific heat \( \sim T \ln T \)
- Violation of Wiedemann-Franz

Quantum Critical

\[ \langle b \rangle = 0, \text{ Deconfined} \]
\[ J_{Kc} \]
\[ \langle b \rangle \neq 0, \text{ Higgs} \]
Is the U(1) FL* phase unstable to the LMM metal at the lowest energy scales?
E. Detour: Deconfined criticality in insulating antiferromagnets

*Landau forbidden quantum transitions*
Reconsider destruction of magnetic order by frustrating (RKKY) exchange interactions between \( f \) moments in an insulator.

Ground state has Neel order with \( \tilde{\varphi} \neq 0 \).
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Destroy SDW order by perturbations which preserve full square lattice symmetry e.g. second-neighbor or ring exchange.
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Destroy SDW order by perturbations which preserve full square lattice symmetry \( e.g. \) second-neighbor or ring exchange.
Attempted theory for the destruction of Néel order

Express Néel order $\varphi$ in terms of $S = 1/2$ bosonic spinons $z_\alpha$ by

$$\varphi \sim z_\alpha^* \tilde{\sigma}_\alpha \tilde{z}_\beta.$$ 

This introduces a U(1) gauge invariance under $z_\alpha \rightarrow z_\alpha e^{i\phi(x,\tau)}$.

Field theory for the $z_\alpha$ spinons:

$$S_{\text{critical}} = \int d^2 x d\tau \left[ |(\partial_\mu - i A_\mu) z_\alpha|^2 + s |z_\alpha|^2 + \frac{u}{2} (|z_\alpha|^2)^2 + \frac{1}{4e^2} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \right]$$

where $A_\mu$ is a U(1) gauge field.

**Phases of theory**

$s < s_c \Rightarrow$ Néel (Higgs) phase with $\langle z_\alpha \rangle \neq 0$

$s > s_c \Rightarrow$ Deconfined U(1) spin liquid with $\langle z_\alpha \rangle = 0$
Confined spinons


Possible paramagnetic ground state with $\langle \bar{\phi} \rangle = 0$
Valence bond solid order

Possible paramagnetic ground state with $\langle \bar{\phi} \rangle = 0$

Such a state breaks lattice symmetry and has $\langle \Psi_{\text{VBS}} \rangle \neq 0$, where $\Psi_{\text{VBS}}$ is the *valence bond solid (VBS) order parameter*.
Valence bond solid order

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Valence bond solid order

Possible paramagnetic ground state with $\langle \bar{\psi} \rangle = 0$

Such a state breaks lattice symmetry and has $\langle \Psi_{_{\text{VBS}}} \rangle \neq 0$, where $\Psi_{_{\text{VBS}}}$ is the \textit{valence bond solid (VBS) order parameter}. 
Phase diagram of S=1/2 square lattice antiferromagnet

Neel order
\[ \bar{\phi} \sim z^*_\alpha \tilde{\sigma}_{\alpha\beta} z_\beta \neq 0 \] (Higgs)

VBS order
\[ \Psi_{\text{VBS}} \neq 0, \]
\[ S = 1/2 \text{ spinons } z_\alpha \text{ confined}, \]
\[ S = 1 \text{ triplon excitations} \]

Deconfined critical point described by a theory of spinons

\[ S_{\text{critical}} = \int d^2 x d\tau \left[ |(\partial_\mu - i A_\mu) z_\alpha|^2 + s |z_\alpha|^2 + \frac{u}{2} (|z_\alpha|^2)^2 + \frac{1}{4 e^2} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \right] \]

Landau-forbidden transition between phases which break "unrelated" symmetries
F. Deconfined criticality in the Kondo lattice?
Is the U(1) FL* phase unstable to the LMM metal at the lowest energy scales?
U(1) FL* phase generates magnetism at energies much lower than the critical energy of the FL to FL* transition.
Local moments choose some static spin arrangement. The “hot” Fermi surface of the FL phase disappears at the quantum critical point.

“Heavy” Fermi liquid with moments Kondo screened by conduction electrons. Fermi surface volume equals the Luttinger value.