Quantum criticality, the cuprate superconductors, and the AdS/CFT correspondence

Talk online: sachdev.physics.harvard.edu
The cuprate superconductors

Na-CCOC

- Cu
- Ca/Na
- O
- Cl
Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Ground state has long-range Néel order

Order parameter is a single vector field \( \vec{\phi} = \eta_i \vec{S}_i \)

\( \eta_i = \pm 1 \) on two sublattices

\( \langle \vec{\phi} \rangle \neq 0 \) in Néel state.
Central ingredients in cuprate phase diagram: antiferromagnetism, superconductivity, and change in Fermi surface
Antiferromagnetism

Fermi surface

d-wave superconductivity
Crossovers in transport properties of hole-doped cuprates

\[ \frac{\rho(T)}{\rho(0)} = \begin{cases} \frac{1}{T} & \text{or} \frac{1}{T^2} \\ \rho_T & \text{or} \rho_{T^2} \\ \rho_T \rho_{T^2} \end{cases} \]

\( T^* \) upturns in \( \rho(T) \)

S-shaped

AFM

\[ (1 < n < 2) \]

Crossovers in transport properties of hole-doped cuprates

Outline

1. Coupled dimer antiferromagnets
   *Introduction to quantum criticality*

2. Phase diagram of the cuprates
   *Quantum criticality of the competition between antiferromagnetism and superconductivity*

3. Theory of Ising-nematic ordering in a metal
   *Strongly-coupled field theory*

4. The AdS/CFT correspondence
   *Phases of finite density quantum matter at strong coupling*
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Antiferromagnetism

Fermi surface

d-wave superconductivity
An insulator whose spin susceptibility vanishes exponentially as the temperature $T$ tends to zero.
Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Ground state has long-range Néel order

Order parameter is a single vector field \( \vec{\varphi} = \eta_i \vec{S}_i \)

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Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Weaken some bonds to induce spin entanglement in a new quantum phase
Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Ground state is a “quantum paramagnet” with spins locked in valence bond singlets

\[ \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) \]
Pressure in TlCuCl$_3$

\[ = \frac{1}{\sqrt{2}} (\langle \uparrow \downarrow \rangle - \langle \downarrow \uparrow \rangle) \]
Quantum critical point with non-local entanglement in spin wavefunction

\[ \frac{1}{\sqrt{2}} (|\uparrow \downarrow \rangle - |\downarrow \uparrow \rangle) \]
Description using Landau-Ginzburg field theory

\[ S = \int d^2r d\tau \left[ (\partial_\tau \vec{\varphi})^2 + c^2 (\nabla_r \vec{\varphi})^2 + (\lambda - \lambda_c) \vec{\varphi}^2 + u (\vec{\varphi}^2)^2 \right] \]

\( \text{O}(3) \) order parameter \( \vec{\varphi} \)

\( \lambda_c \)
Excitation spectrum in the paramagnetic phase

\[ V(\varphi) = (\lambda - \lambda_c)\varphi^2 + u(\varphi^2)^2 \]

\[ \lambda > \lambda_c \]

Spin \( S = 1 \) “triplon”
Excitation spectrum in the paramagnetic phase

\[ V(\vec{\varphi}) = (\lambda - \lambda_c)\varphi^2 + u (\varphi^2)^2 \]

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Spin \( S = 1 \)

"triplon"
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\[ \lambda > \lambda_c \]
Excitation spectrum in the paramagnetic phase

\[ V(\bar{\phi}) = (\lambda - \lambda_c) \bar{\phi}^2 + u (\bar{\phi}^2)^2 \]

\[ \lambda > \lambda_c \]

Spin \( S = 1 \) “triplon”
TlCuCl$_3$ at ambient pressure

FIG. 1. Measured neutron profiles in the $a^*c^*$ plane of TlCuCl$_3$
for $i=(1.35,0,0)$, $ii=(0,0,3.15)$ [r.l.u.] The spectrum at $T=1.5$ K

N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer
**TlCuCl$_3$ at ambient pressure**

Sharp spin 1 particle excitation above an energy gap (spin gap)

Excitation spectrum in the Néel phase

\[ V(\vec{\varphi}) = (\lambda - \lambda_c)\varphi^2 + u (\varphi^2)^2 \]

\[ \lambda < \lambda_c \]
Excitation spectrum in the Néel phase

$V(\bar{\phi}) = (\lambda - \lambda_c)\bar{\phi}^2 + u(\bar{\phi}^2)^2$

$\lambda < \lambda_c$

Spin waves
Excitation spectrum in the Néel phase

\[ V(\vec{\phi}) = (\lambda - \lambda_c)\vec{\phi}^2 + u(\vec{\phi}^2)^2 \]

\[ \lambda < \lambda_c \]

Spin waves
Excitation spectrum in the Néel phase

$V(\bar{\varphi}) = (\lambda - \lambda_c)\bar{\varphi}^2 + u(\bar{\varphi}^2)^2$

Field theory yields spin waves ("Goldstone" modes) but also an additional longitudinal "Higgs" particle

$\lambda < \lambda_c$
TlCuCl$_3$ with varying pressure

Observation of 3 $\rightarrow$ 2 low energy modes, emergence of new Higgs particle in the Néel phase, and vanishing of Néel temperature at the quantum critical point

Prediction of quantum field theory

Potential for $\varphi$ fluctuations: $V(\varphi) = (\lambda - \lambda_c)\varphi^2 + u(\varphi^2)^2$

Paramagnetic phase, $\lambda > \lambda_c$

Expand about $\varphi = 0$:

$V(\varphi) \approx (\lambda - \lambda_c)\varphi^2$

Yields 3 particles with energy gap $\sim \sqrt{\lambda - \lambda_c}$
**Prediction of quantum field theory**

Potential for $\bar{\phi}$ fluctuations: $V(\bar{\phi}) = (\lambda - \lambda_c)\bar{\phi}^2 + u (\bar{\phi}^2)^2$

<table>
<thead>
<tr>
<th>Paramagnetic phase, $\lambda &gt; \lambda_c$</th>
</tr>
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<tbody>
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<td>Expand about $\bar{\phi} = 0$:</td>
</tr>
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<table>
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<th>Néel phase, $\lambda &lt; \lambda_c$</th>
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<tr>
<td>Expand $\bar{\phi} = (0, 0, \sqrt{(\lambda_c - \lambda)/(2u)}) + \bar{\phi}_1$:</td>
</tr>
<tr>
<td>$V(\bar{\phi}) \approx 2(\lambda_c - \lambda)\bar{\phi}_{1z}^2$</td>
</tr>
<tr>
<td>Yields 2 gapless spin waves and one Higgs particle with energy gap $\sim \sqrt{2(\lambda_c - \lambda)}$</td>
</tr>
</tbody>
</table>
Prediction of quantum field theory

Energy of Higgs particle

Energy of triplon

$\sqrt{2}$

$V(\phi) = (\lambda - \lambda_c)\phi^2 + u(\phi^2)^2$

S. Sachdev, arXiv:0901.4103
$\lambda$

CFT3

$\frac{1}{\sqrt{2}}(\left\langle \uparrow \downarrow \right\rangle - \left\langle \downarrow \uparrow \right\rangle)$

$\lambda_c$

O(3) order parameter $\vec{\varphi}$

$$S = \int d^2r d\tau \left[ (\partial_\tau \varphi)^2 + c^2 (\nabla_r \vec{\varphi})^2 + s \varphi^2 + u (\vec{\varphi}^2)^2 \right]$$
Classical spin waves

Quantum critical

Dilute triplon gas

Neel order

Pressure in TlCuCl$_3$

Classical spin waves

Quantum critical

CFT3 at $T > 0$

Classical spin waves

Dilute triplon gas

Neel order

Pressure in TlCuCl$_3$

Crossovers in transport properties of hole-doped cuprates
Crossovers in transport properties of hole-doped cuprates

Strange metal: quantum criticality of optimal doping critical point at $x = x_m$?


Only candidate quantum critical point observed at low $T$

Spin density wave order present below a quantum critical point at $x = x_s$ with $x_s \approx 0.12$ in the La series of cuprates
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Antiferromagnetism

Fermi surface

d-wave superconductivity
Antiferromagnetism

Fermi surface

d-wave superconductivity
The electron spin polarization obeys

$$\langle \vec{S}(\mathbf{r}, \tau) \rangle = \varphi(\mathbf{r}, \tau)e^{i\mathbf{K} \cdot \mathbf{r}}$$

where $\mathbf{K}$ is the ordering wavevector.
Hole-doped cuprates

--- Increasing SDW order ---

Hole-doped cuprates

Increasing SDW order

Hole-doped cuprates

Increasing SDW order

Hot spots

Hole-doped cuprates

Fermi surface breaks up at hot spots into electron and hole “pockets”

Hole-doped cuprates

Fermi surface breaks up at hot spots into electron and hole “pockets”

Fermi surface breaks up at hot spots into electron and hole “pockets”

Quantum oscillations

Electron pockets in the Fermi surface of hole-doped high-$T_c$ superconductors


Nature 450, 533 (2007)
Quantum oscillations

Electron pockets in the Fermi surface of hole-doped high-$T_c$ superconductors

Nature 450, 533 (2007)
Evidence for small Fermi pockets

Fermi liquid behaviour in an underdoped high Tc superconductor

Suchitra E. Sebastian, N. Harrison, M. M. Altarawneh, Ruixing Liang, D. A. Bonn, W. N. Hardy, and G. G. Lonzarich

arXiv:0912.3022

FIG. 2: Magnetic quantum oscillations measured in YBa$_2$Cu$_3$O$_{6+x}$ with $x \approx 0.56$ (after background polynomial subtraction). This restricted interval in $B = |B|$ furnishes a dynamic range of $\sim 50$ dB between $T = 1$ and 18 K. The actual $T$ values are provided in Fig. 3.
Theory of quantum criticality in the cuprates

- Fluctuating Fermi pockets
- Strange Metal
- Large Fermi surface

Increasing SDW order

$T^*$

Spin density wave (SDW)

Underlying SDW ordering quantum critical point in **metal** at $x = x_m$
Evidence for connection between linear resistivity and stripe-ordering in a cuprate with a low $T_c$

- Magnetic field of upto 35 T used to suppress superconductivity
- Identifies $x_m \approx 0.24$

Linear temperature dependence of resistivity and change in the Fermi surface at the pseudogap critical point of a high-$T_c$ superconductor

Antiferromagnetism

Fermi surface

d-wave superconductivity
The theory of quantum criticality in the cuprates

Underlying SDW ordering quantum critical point in metal at $x = x_m$
Theory of quantum criticality in the cuprates

Onset of $d$-wave superconductivity hides the critical point $x = x_m$
Theory of quantum criticality in the cuprates

- Small Fermi pockets with pairing fluctuations
- Large Fermi surface
- Strange Metal
- Fluctuating, paired Fermi pockets

\[ T^* \]

**d-wave superconductor**

**Spin density wave (SDW)**

Competition between SDW order and superconductivity moves the actual quantum critical point to \( x = x_s < x_m \).


Theory of quantum criticality in the cuprates

Fluctuating, paired Fermi pockets
Strange Metal
Large Fermi surface
d-wave superconductor

Spin density wave (SDW)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.


Theory of quantum criticality in the cuprates


Spin density wave (SDW)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$. 

Fluctuating, paired Fermi pockets

Strange Metal

Large Fermi surface

$d$-wave superconductor

Thermally fluctuating SDW

Magnetic quantum criticality

Spin gap

$T^*$

$T$

$x$

$x_s$

$x_m$
Theory of quantum criticality in the cuprates

Criticality of the coupled dimer antiferromagnet at $x = x_s$

$\lambda_c$

Classical spin waves

Dilute triplon gas

Quantum critical

Thermally fluctuating SDW

Thermally fluctuating SDW

Magnetic quantum criticality

Spin gap

Neel order

Fluctuating, paired Fermi pockets

Strange Metal

$T^*$

$T$

$\lambda$

$x$

$\lambda_c$

$\chi_s$

$\chi_m$

Spin density wave (SDW)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$. 

Thursday, February 11, 2010
Theory of quantum criticality in the cuprates


Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$. 

Spin density wave (SDW)
Theory of quantum criticality in the cuprates

Fluctuating, paired Fermi pockets

Strange Metal

Large Fermi surface

d-wave superconductor

Magnetic quantum criticality

Spin gap

Thermally fluctuating SDW

Spin density wave (SDW)

Physics of competition: $d$-wave SC and SDW “eat up” same pieces of the large Fermi surface.


Small Fermi
pockets with
pairing fluctuations

Large
Fermi
surface

Strange
Metal

Magnetic
quantum
criticality

Spin gap

Fluctuating,
paired Fermi
pockets

T*
E. Demler, S. Sachdev and Y. Zhang, 
Small Fermi pockets with pairing fluctuations

Large Fermi surface

Strange Metal

Magnetic quantum criticality

Spin gap

Thermally fluctuating SDW

d-wave SC

Fluctuating, paired Fermi pockets


Thursday, February 11, 2010
Small Fermi pockets with pairing fluctuations

Large Fermi surface

Strange Metal

Magnetic quantum criticality

Spin gap

d-wave SC

Thermally fluctuating SDW

Quantum oscillations

"Normal" (Large Fermi surface)
Nd$_{2-x}$Ce$_x$CuO$_4$

Quantum oscillations

Phase diagram in a magnetic field agrees with numerous neutron scattering and $\mu$SR experiments.
Fluctuating, paired Fermi pockets

Large Fermi surface

Strange Metal

Small Fermi pockets

Pairing fluctuations

d-wave SC

$T_{sdw}$

$T^*$

$T_{m}$

$T_{s}$

SC+ SDW

SDW (Small Fermi pockets)

"Normal" (Large Fermi surface)
Similar phase diagram for CeRhIn$_5$

Similar phase diagram for the pnictides

Fluctuating, paired Fermi pockets

Large Fermi surface

Strange Metal

SDW (Small Fermi pockets)

SDW (Large Fermi surface)

"Normal"
Onset of superconductivity disrupts SDW order, but valence bond solid (VBS) and/or Ising-nematic ordering can survive.

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Quantum criticality of Pomeranchuk instability

Fermi surface with full square lattice symmetry
Quantum criticality of Pomeranchuk instability

Spontaneous elongation along $x$ direction:

Quantum criticality of Pomeranchuk instability

Spontaneous elongation along $y$ direction:

Ising-nematic order parameter

\[ \phi \sim \int d^2 k \left( \cos k_x - \cos k_y \right) c_{k\sigma}^\dagger c_{k\sigma} \]

Measures spontaneous breaking of square lattice point-group symmetry of underlying Hamiltonian
Quantum criticality of Pomeranchuk instability

Spontaneous elongation along $x$ direction:
Ising order parameter $\phi > 0$.

Quantum criticality of Pomeranchuk instability

Spontaneous elongation along $y$ direction:
Ising order parameter $\phi < 0$.

Quantum criticality of Pomeranchuk instability

Quantum criticality of Pomeranchuk instability

Phase diagram as a function of $T$ and $r$

$T_l-n$

$\langle \phi \rangle \neq 0$

$\langle \phi \rangle = 0$

$T$

$\phi$

$T_c$

$T_l-n$

$\langle \phi \rangle \neq 0$

$\langle \phi \rangle = 0$

Phase diagram as a function of $T$ and $r$
Quantum criticality of Pomeranchuk instability

Phase diagram as a function of $T$ and $r$
Nematic order in YBCO

Broken rotational symmetry in the pseudogap phase of a high-Tc superconductor

R. Daou, J. Chang, David LeBoeuf, Olivier Cyr-Choiniere, Francis Laliberte, Nicolas Doiron-Leyraud, B. J. Ramshaw, Ruixing Liang, D. A. Bonn, W. N. Hardy, and Louis Taillefer

A $\phi$ fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm \vec{k}_0$.

Expand fermion kinetic energy at wavevectors about $\vec{k}_0$. 

Thursday, February 11, 2010
• Critical point is described by an infinite set of 2+1 dimensional field theories, one for each direction $\hat{q}$.
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Contrast with “Fermi surface bosonization” methods where there are an infinite set of 1+1 dimensional field theories, one for each direction $\hat{q}$. 

![Diagram of a 2+1 dimensional field theory](image)
• Critical point is described by an infinite set of 2+1 dimensional field theories, one for each direction $\hat{q}$.

• Contrast with “Fermi surface bosonization” methods where there are an infinite set of 1+1 dimensional field theories, one for each direction $\hat{q}$.

• Our approach leads to a redundant description of underlying degrees of freedom. A “Galilean symmetry” ensures consistency of redundant description.
• Critical point is described by an infinite set of 2+1 dimensional field theories, one for each direction $\hat{q}$.

• Contrast with “Fermi surface bosonization” methods where there are an infinite set of 1+1 dimensional field theories, one for each direction $\hat{q}$.

• Our approach leads to a redundant description of underlying degrees of freedom. A “Galilean symmetry” ensures consistency of redundant description.

• Infinite set of 2+1 dimensional field theories: implies an emergent dimension of spacetime, and suggests a string-theoretic description and application of the AdS/CFT correspondence.
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Conformal field theory in 2+1 dimensions at $T = 0$

Einstein gravity on AdS$_4$

e.g. Graphene at zero bias
Conformal field theory in 2+1 dimensions at $T > 0$

Einstein gravity on AdS$_4$ with a Schwarzschild black hole

e.g. Graphene at zero bias
Conformal field theory in 2+1 dimensions at $T > 0$, with a non-zero chemical potential, $\mu$ and applied magnetic field, $B$.

Einstein gravity on AdS$_4$ with a Reissner-Nordstrom black hole carrying electric and magnetic charges.

e.g. Graphene at non-zero bias.
AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions.

Maldacena, Gubser, Klebanov, Polyakov, Witten
AdS/CFT correspondence
The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions.

3+1 dimensional AdS space

A 2+1 dimensional system at its quantum critical point

Maldacena, Gubser, Klebanov, Polyakov, Witten
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3+1 dimensional AdS space

Black hole entropy = entropy of quantum criticality

Quantum criticality in 2+1 dimensions
AdS/CFT correspondence
The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions.
AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions.
Magneto-thermo-electric dynamics of graphene obtained using insights from the dynamics of a Reissner-Nordstom black hole in AdS$_4$. 


\[ T(K) \]

-1 \[ \sim \] \[ \sqrt{n} \]

1 \[ \sim \] \[ (1 + \lambda \ln \Lambda \sqrt{n}) \]

-1 \[ \mu \leq 0 \]

0 \[ \mu > 0 \]

100 \[ \frac{n}{10^{12}/m^2} \]

Hole

Fermi liquid

Electron

Fermi liquid

Quantum critical

Dirac liquid
At low $T$ graphene is a Fermi liquid
At low $T$ graphene is a Fermi liquid.

At low $T$ graphene is a Fermi liquid
At low $T$ graphene is a Fermi liquid

What is the dual of the black hole theory at low $T$?

Examine free energy and Green’s function of a probe particle

Short time behavior depends upon conformal AdS$_4$ geometry near boundary

Long time behavior depends upon near-horizon geometry of black hole

Radial direction of gravity theory is measure of energy scale in CFT

Figure 1: The extra (‘radial’) dimension of the bulk is the resolution scale of the field theory. The left figure indicates a series of block spin transformations labelled by a parameter $z$. The right figure is a cartoon of AdS space, which organizes the field theory information in the same way. In this sense, the bulk picture is a hologram: excitations with different wavelengths get put in different places in the bulk image.
Infrared physics of Fermi surface is linked to the near horizon $\text{AdS}_2$ geometry of Reissner-Nordstrom black hole

Geometric interpretation of RG flow

Geometric interpretation of RG flow

Green’s function of a fermion

\[ G(k, \omega) \approx \frac{1}{\omega - v_F(k - k_F) - i\omega^{\theta(k)}} \]

Green’s function of a fermion

\[ G(k, \omega) \approx \frac{1}{\omega - v_F(k - k_F) - i\omega\theta(k)} \]

Similar to our theory of the singular Fermi surface near the Ising-nematic quantum critical point

Conclusions

Identified quantum criticality in cuprate superconductors with a critical point at optimal doping associated with onset of spin density wave order in a metal.

Elusive optimal doping quantum critical point has been “hiding in plain sight”.

It is shifted to lower doping by the onset of superconductivity.
Conclusions

Theories for the onset of spin density wave and Ising-nematic order in metals are strongly coupled in two dimensions.