A simple model of entangled qubits, describing black holes and superconductors

Trinity Mathematical Society
November 2, 2020

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Talk online: sachdev.physics.harvard.edu
In a landmark study, scientists at Delft University of Technology in the Netherlands reported that they had conducted an experiment that they say proved one of the most fundamental claims of quantum theory — that objects separated by great distance can instantaneously affect each other’s behavior.

Part of the laboratory setup for an experiment at Delft University of Technology, in which two diamonds were set 1.3 kilometers apart, entangled and then shared information.
Quantum entanglement
Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:

\[ \begin{array}{c}
\text{Hydrogen molecule: }
\end{array} \]

\[
\begin{align*}
\text{Hydrogen atom:} & \quad \bullet \quad |\uparrow\rangle \\
\text{Hydrogen molecule:} & \quad \bullet \quad \bullet \quad - \quad \bullet \quad \bullet
\end{align*}
\]

\[
\begin{align*}
= & \quad \bullet \quad \bullet \quad \uparrow\rangle \quad \downarrow\rangle \\
= & \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)
\end{align*}
\]
Quantum Entanglement: quantum superposition with more than one particle
Principles of Quantum Mechanics: II. Quantum Entanglement

Quantum Entanglement: quantum superposition with more than one particle
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Einstein-Podolsky-Rosen “paradox” (1935): Measurement of one particle instantaneously determines the state of the other particle arbitrarily far away
Quantum entanglement
Quantum entanglement

A simple qubit model
A qubit: 2 states $|\uparrow\rangle$, $|\downarrow\rangle$.

Pauli gates:

$X |\uparrow\rangle = |\downarrow\rangle$, $X |\downarrow\rangle = |\uparrow\rangle$

$Y |\uparrow\rangle = i |\downarrow\rangle$, $Y |\downarrow\rangle = -i |\uparrow\rangle$

$Z |\uparrow\rangle = |\uparrow\rangle$, $Z |\downarrow\rangle = - |\downarrow\rangle$
A qubit: 2 states $|\uparrow\rangle$, $|\downarrow\rangle$.

Pauli gates:

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$Z |\uparrow\rangle = |\uparrow\rangle$, $Z |\downarrow\rangle = -|\downarrow\rangle$

A 2-qubit Hamiltonian: $\mathcal{H} = J (X_1 X_2 + Y_1 Y_2 + Z_1 Z_2)$

Ground state: $\frac{1}{\sqrt{2}} (|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2)$

Energy = $-3J$

Excited states: $|\uparrow\rangle_1 |\uparrow\rangle_2$, $|\downarrow\rangle_1 |\downarrow\rangle_2$, $\frac{1}{\sqrt{2}} (|\uparrow\rangle_1 |\downarrow\rangle_2 + |\downarrow\rangle_1 |\uparrow\rangle_2)$

Energy = $J$
The simple model

\[ \mathcal{H} = \sum_{i<j=1}^{N} J_{ij} (X_i X_j + Y_i Y_j + Z_i Z_j) \]

\( J_{ij} \) are random numbers with zero mean, and variance \( \sim J^2 / N \)

(Technical comment: the solvable model has states in the self-conjugate representation of SU(\(M\)), and we take the limit \( N \rightarrow \infty \) followed by the limit \( M \rightarrow \infty \))
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The main result

For $k_B T \ll J$

$$
Z = \text{Tr} \exp \left( - \frac{\mathcal{H}}{k_B T} \right)
$$

$$
= \exp \left( N \frac{S_0}{k_B} \right) \int \mathcal{D} f(\tau) \exp \left( - \frac{1}{\hbar} S_{2D-\text{gravity}} [f(\tau)] \right)
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$S_0$ is the $T \to 0$ entropy of the qubit model.

It maps on to the Bekenstein-Hawking entropy of charged black holes

A. Kitaev (2015)
The main result

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- $f(\tau)$ is the reparameterization of the imaginary time of the qubit model: $\tau$ on a circle of circumference $\hbar/(k_B T)$.
The main result

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- $f(\tau)$ is the reparameterization of the imaginary time of the qubit model: $\tau$ on a circle of circumference $\hbar/(k_B T)$.

- $f(\tau)$ is also the fluctuation of the boundary of a theory of 2D-gravity in 1+1 spacetime dimensions: a ‘boundary graviton’.

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The main result

For $k_B T \ll J$

$$Z = \text{Tr} \exp \left( - \frac{\mathcal{H}}{k_B T} \right)$$

$$= \exp \left( N \frac{S_0}{k_B} \right) \int Df(\tau) \exp \left( -\frac{1}{\hbar} S_{2\text{D-gravity}} [f(\tau)] \right)$$

- $f(\tau)$ is the reparameterization of the imaginary time of the qubit model: $\tau$ on a circle of circumference $\hbar/(k_B T)$.

- $f(\tau)$ is also the fluctuation of the boundary of a theory of 2D-gravity in 1+1 spacetime dimensions: a ‘boundary graviton’.

- The action of 2D-gravity, $S_{2\text{D-gravity}}$, is constrained by an emergent time reparameterization symmetry which is broken down to a conformal symmetry ($\text{SL}(2,\mathbb{R})$).

A. Kitaev (2015)
Maria Tikhanovskaya

Haoyu Guo

Grigory Tarnopolsky

arXiv:2010.09742
Consequences of 2D-gravity for the dynamic spin susceptibility

\[ \chi_L(\omega) = \sum_n |\langle 0 | X_i | n \rangle|^2 \delta(\hbar\omega - E_n + E_0), \text{ (at } T = 0) \]

\[ \chi_L(\omega) \sim \tanh \left( \frac{\hbar\omega}{2k_B T} \right) \left[ 1 - C\gamma\omega \tanh \left( \frac{\hbar\omega}{2k_B T} \right) - \ldots \right] \]
Consequences of 2D-gravity for the dynamic spin susceptibility

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Conformally (SL(2,R)) invariant result with characteristic dissipative time \( \sim \hbar / (k_B T) \)

A. Georges and O. Parcollet
PRB 59, 5341 (1999)
Consequences of 2D-gravity for the dynamic spin susceptibility

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**Correction from the boundary graviton**
Quantum entanglement

A simple qubit model

Black holes
Black Holes

Objects so dense that light is gravitationally bound to them.

In Einstein’s theory, the region inside the black hole horizon is disconnected from the rest of the universe.

Horizon radius \( R = \frac{2GM}{c^2} \)

\( G \) Newton’s constant, \( c \) velocity of light, \( M \) mass of black hole
Quantum Entanglement across a black hole horizon
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Quantum Entanglement across a black hole horizon

There is quantum entanglement between the inside and outside of a black hole.
Quantum Entanglement across a black hole horizon

Hawking used this to show that black hole horizons have an entropy and a temperature (because to an outside observer, the state of the electron inside the black hole is an unknown).
- Black holes have an entropy and a temperature, $T_H$

- The entropy is proportional to their surface area.

$S \propto \frac{A}{k_B}$

$T_H = \frac{\hbar c^3}{8 \pi GM}$

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J. D. Bekenstein, PRD 7, 2333 (1973)
On September 14, 2015, LIGO detected the merger of two black holes, each weighing about 30 solar masses, with radii of about 100 km, 1.3 billion light years away 0.1 seconds later!
LIGO
September 14, 2015

- The ring-down time $\frac{8\pi GM}{c^3} \sim 8$ milliseconds. Curiously, for essentially all types of black holes, the ring-down time equals $\frac{\hbar}{k_B T_H}$, where $\hbar$ Planck’s constant, $k_B$ Boltzmann’s constant.
Quantum Black holes

- Black holes have an entropy and a temperature, $T_H$
- The entropy is proportional to their surface area.
- They relax to thermal equilibrium in a Planckian time $\sim \hbar/(k_B T_H)$. 
Maxwell’s electromagnetism and Einstein’s general relativity allow black hole solutions with a net charge.
Maxwell’s electromagnetism and Einstein’s general relativity allow black hole solutions with a net charge. Zooming into the near-horizon region of a charged black hole at low temperature, yields a gravitational theory in one space ($\zeta$) and one time dimension.
Maxwell’s electromagnetism and Einstein’s general relativity allow black hole solutions with a net charge. This 2D-gravity theory is precisely that appearing in the low T limit of the Sachdev-Ye-Kitaev (SYK) models (including the qubit model)!
Charged black holes

\[ I_{EM} = \int d^{d+2}x \sqrt{g} \left[ -\frac{1}{2\kappa^2} \left( \mathcal{R}_{d+2} + \frac{d(d+1)}{L^2} \right) + \frac{1}{4g_F^2} F^2 \right] \]

Black hole horizon of radius \( r_0 \)

Solutions of \( I_{EM} \) have metric and gauge field \((F = dA)\)

\[ ds^2 = V(r) d\tau^2 + r^2 d\Omega^2_d + \frac{dr^2}{V(r)} , \quad i\mu \left( 1 - \frac{r_0^{d-1}}{r^{d-1}} \right) d\tau \]

\[ V(r) = 1 + \frac{r^2}{L^2} + \frac{\Theta^2}{r^{2d-2}} - \frac{M}{r^{d-1}}. \]

where \( d\Omega^2_d \) is the metric of the \( d \)-sphere. All parameters of the solution are determined in terms of the chemical potential \( \mu \), and the Hawking temperature of horizon, \( T \).

A. Chamblin, R. Emparan, C.V. Johnson, and R.C. Myers, PRD 60, 064018 (1999)
Charged black holes

In the $T \to 0$ limit, at fixed $\mu$, we obtain a charged black hole solution with radius $r_0(T \to 0, \mu) = R_h$. All properties of this black hole can be expressed in terms of $R_h$

- In the near-horizon region, we change co-ordinates from $r$ to $\zeta$ so that

$$r - R_h = \frac{R_2^2}{\zeta} , \quad R_2 = \frac{LR_h}{\sqrt{d(d+1)R_h^2 + (d-1)^2L^2}}.$$  

Then the near-horizon metric becomes $\text{AdS}_2 \times S_d$, with

$$ds^2 = R_2^2 \left[ -dt^2 + d\zeta^2 \zeta^2 \right] + R_h^2 d\Omega_d^2 , \quad A = \frac{\mathcal{E}}{\zeta} dt.$$  

where the dimensionless electric field $\mathcal{E}$ is

$$\mathcal{E} = \frac{g_F R_h \sqrt{2d[(d+1)R_h^2 + (d-1)L^2]}}{2 \left[ d(d+1)R_h^2 + (d-1)^2L^2 \right]}.$$  

Charged black holes

Black hole horizon of radius $R_h$ and entropy $s_0$

\[ ds^2 = \frac{(d\zeta^2 - dt^2)}{\zeta^2} + d\vec{x}^2 \]

Gauge field: $A = (\mathcal{E}/\zeta)dt$

- The entropy $S_0$, the charge $Q$, and the dimensionless electric field $\mathcal{E}$ obey the same thermodynamic relation as the SYK model

\[ \frac{dS_0}{dQ} = 2\pi\mathcal{E} \]

2D gravity and black holes

- Reparameterization invariance is a defining property of Einstein’s theory of gravity

- In imaginary time, AdS$_2$ is the homogeneous hyperbolic space: two-dimensional surface of constant negative curvature. Its metric is invariant under $\text{SL}(2,\mathbb{R})$

\[ ds^2 = (d\tau^2 + d\zeta^2)/\zeta^2 \] is invariant under

\[ \tau' + i\zeta' = \frac{a(\tau + i\zeta) + b}{c(\tau + i\zeta) + d} \] with $ad - bc = 1$. 

A. Kitaev, 2015

2D gravity and black holes

- Reparameterization invariance is a defining property of Einstein’s theory of gravity
- In imaginary time, AdS$_2$ is the homogeneous hyperbolic space: two-dimensional surface of constant negative curvature. Its metric is invariant under SL(2,R)

Low $T$ quantum fluctuations about the Einstein-Maxwell theory of charged black holes in $d \geq 2$ spatial dimensions leads to the same 2D gravity theory as the SYK models.

P. Chaturvedi, Yingfei Gu, Wei Song, Boyang Yu, arXiv:1808.08062
SYK model and charged black holes

Horizon

\[ ds^2 = R_2^2 \frac{(d\zeta^2 - dt^2)}{\zeta^2} + R_h^2 d\Omega_2^2 \]

Gauge field: \[ A = \frac{\mathcal{E}}{\zeta} dt \]
Quantum entanglement

A simple qubit model

Black holes

Copper-based superconductors
Cuprate superconductors

\[ \text{YBa}_2\text{Cu}_3\text{O}_{6+x} \]
Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University
High temperature superconductors

\[ \text{YBa}_2\text{Cu}_3\text{O}_{6+x} \]

CuO\(_2\) plane
Insulating antiferromagnet
Antiferromagnet doped with hole density $p$
We consider the hole-doped case, with no double occupancy.
$p$ mobile holes in a background of fluctuating spins
$p$ mobile holes in a background of fluctuating spins
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$p$ mobile holes in a background of fluctuating spins
Real-space view

$p$ mobile holes in a background of fluctuating spins
\( p \) mobile holes in a background of fluctuating spins
Real-space view

$p$ mobile holes in a background of fluctuating spins
$p$ mobile holes in a background of fluctuating spins
$p$ mobile holes in a background of fluctuating spins
Momentum-space view

Filled Band
$1-p$ mobile electrons = $1+p$ mobile holes in a filled band
In slightly more overdoped samples, these measurements which found 1.30 and 1.24 itinerant holes, respectively, appear to be approaching a topological transition from hole-pocket centered at energy across the gap, which, however, still correspond to about 24% for equivalent trimerized with respect to the BZ diagonal, taking an average was downfolded to the reduced zone scheme and symmetrized with the BZ. For the ARPES spectra normalized at high temperature dependent experiments on a less overdoped Tl2201-OD30 sample could not be followed via the shift of the leading edge midpoint (LEM) across Tl2201-OD30 along the nodal and antinodal symmetrized spectra [15], in particular, the ARPES and AMRO results, better agreement would require the inclusion in the calculations of correlation effect. The FS of Tl2201-OD30 [Fig. 1(d)] consists of a large nodal and antinodal regions of the BZ, as indicated by the arrows in Fig. 1(a). Dispersive features are clearly observable, with a behavior which is ubiquitous among the cuprates. As for the detailed shape of the Fermi function and convoluted with the instrumental energy, one obtains an estimate for the normal-state FS [Fig. 1(d); the dashed line] and 63% (blue, solid line) of the BZ. (b),(c) ARPES spectra along different arrows in Fig. 1(a). Close to the nodal direction the QP peak exhibits a pronounced dispersion that can be followed over the TlO band that in LDA calculations crosses over the CuO band. In the ARPES spectra Fig. 1(b) and 1(c), on the other hand, the TlO band (as well as 0.109 from the CuO band), already produces both the FS shape [Fig. 1(d)] and the QP energy consistent with a SC gap. This, however, is no surprise even within the independent-exciton approximation [1]. The ARPES spectra along different k directions [signature of a FS crossing; bold line in Fig. 2(a)] shows clearly that the ARPES spectra at (0,0) and near (π,π) on Tl2201-OD30/[Figs. 2(f) and 2(g)].

Momentum-space view at small $p$

Ca$_{2-x}$Na$_x$CuO$_2$Cl$_2$

at $x = 0.10$

“Fermi arcs”

Hard antinodal gap revealed by quantum oscillations in the pseudogap regime of underdoped high-$T_c$ superconductors

Máté Hartstein, Yu-Te Hsu, Kimberly A. Modic, Juan Porras, Toshinao Loew, Matthieu Le Tacon, Huakun Zuo, Jinhua Wang, Zengwei Zhu, Mun K. Chan, Ross D. McDonald, Gilbert G. Lonzarich, Bernhard Keimer, Suchitra E. Sebastian and Neil Harrison

“Fermi arcs” reconstructed by charge density wave order at high magnetic fields in YBa$_2$Cu$_3$O$_{6.55}$
Fermi surface transformation at the pseudogap critical point of a cuprate superconductor  

We use angle-dependent magnetoresistance (ADMR) to measure the Fermi surface of the cuprate La$_{1.6-x}$Nd$_{0.4}$Sr$_x$CuO$_4$. Above the critical doping $p^*$—outside of the pseudogap phase—we find a Fermi surface that is in quantitative agreement with angle-resolved photoemission. Below $p^*$, however, the ADMR is qualitatively different, revealing a clear change in Fermi surface topology. We find that our data is most consistent with a Fermi surface that has been reconstructed by a $Q = (\pi, \pi)$ wavevector. While static $Q = (\pi, \pi)$ antiferromagnetism is not found at these dopings, our results suggest that this wavevector is a fundamental organizing principle of the pseudogap phase.

$p > p_c$ Large Fermi surface

$p < p_c$ Reconstructed Fermi surface
An Isotropic, $T$-linear Scattering Rate

\[
\frac{1}{\tau} = \frac{1}{\tau_{\text{iso}}(\vec{k})} + \frac{\alpha}{\hbar} k_B T
\]

G. Grissonanche, Y. Fang et al., 2020
**t-J model**

\[
H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j
\]

We consider the hole-doped case, with no double occupancy.

\[
\alpha = \uparrow, \downarrow, \quad \{c_{i\alpha}, c_{j\beta}^{\dagger}\} = \delta_{ij} \delta_{\alpha\beta}, \quad \{c_{i\alpha}, c_{j\beta}\} = 0
\]

\[
\vec{S}_i = \frac{1}{2} c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \sum_{\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} \leq 1, \quad \frac{1}{N} \sum_{i\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} = 1 - p
\]
Random $t$-$J$ model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

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$$J_{ij} \text{ random, } \overline{J_{ij}} = 0, \quad \overline{J_{ij}^2} = J^2$$
$$t_{ij} \text{ random, } \overline{t_{ij}} = 0, \quad \overline{t_{ij}^2} = t^2$$
Random $t$-$J$ model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

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**Random $t$-$J$ model**

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} c^\dagger_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.
Evidence for a quantum critical point at $p = p_c \approx 0.3$ with SYK criticality. Spin glass order for $p < p_c$. 
Numerical exact diagonalization and SYK theory

Numerics matches many other observations, including the breakdown of the Luttinger-volume Fermi surface for $p < p_c$, and Planckian dissipation at scale $\hbar/(k_B T)$. 
Quantum entanglement

A simple qubit model

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Copper-based superconductors
Quantum entanglement

2D-gravity theory of charged black holes

A simple qubit model

Copper-based superconductors
Quantum entanglement

A simple qubit model

SYK criticality near $p = p_c$

Black holes

Copper-based superconductors