Quantum phase transitions in d-wave superconductors

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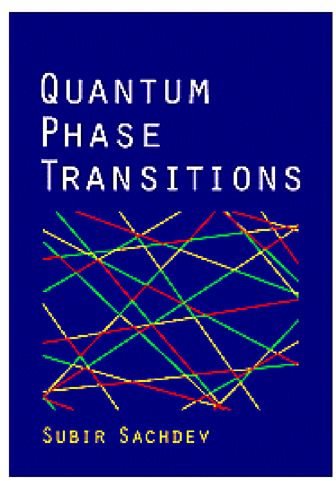


Quantum Phase Transitions
Cambridge University Press

Elementary excitations of a d-wave superconductor

(A) *S=0* Cooper pairs, phase fluctuations

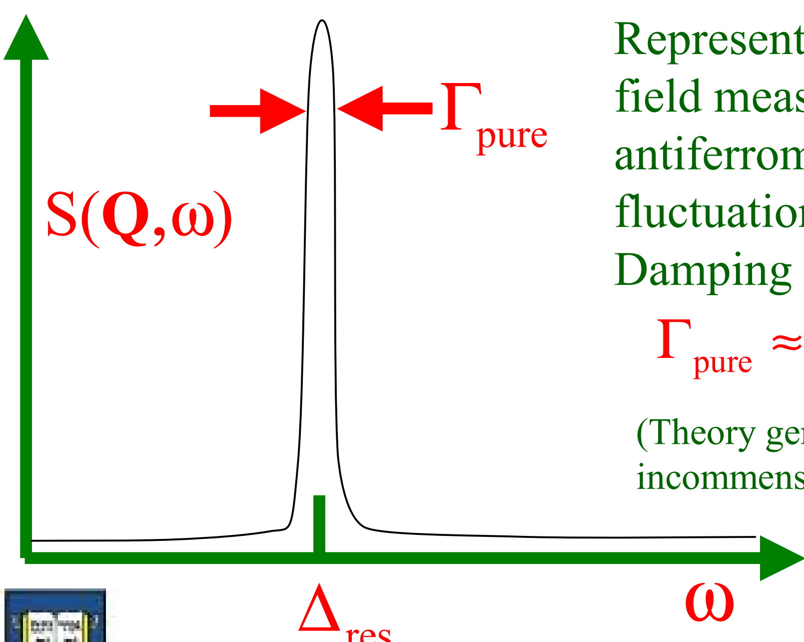
Negligible below T_c except near a T=0 superconductor-insulator transition.

(B) *S=1/2* Fermionic quasiparticles

Ψ_h : strongly paired fermions near $(\pi,0)$, $(0,\pi)$ have an energy gap $\Delta_h \sim$ 30-40 meV

$\Psi_{1,2}$: gapless fermions near the nodes of the superconducting gap at $(\pm K, \pm K)$ with $K = 0.391\pi$

(C) *S=1* Bosonic, resonant collective mode

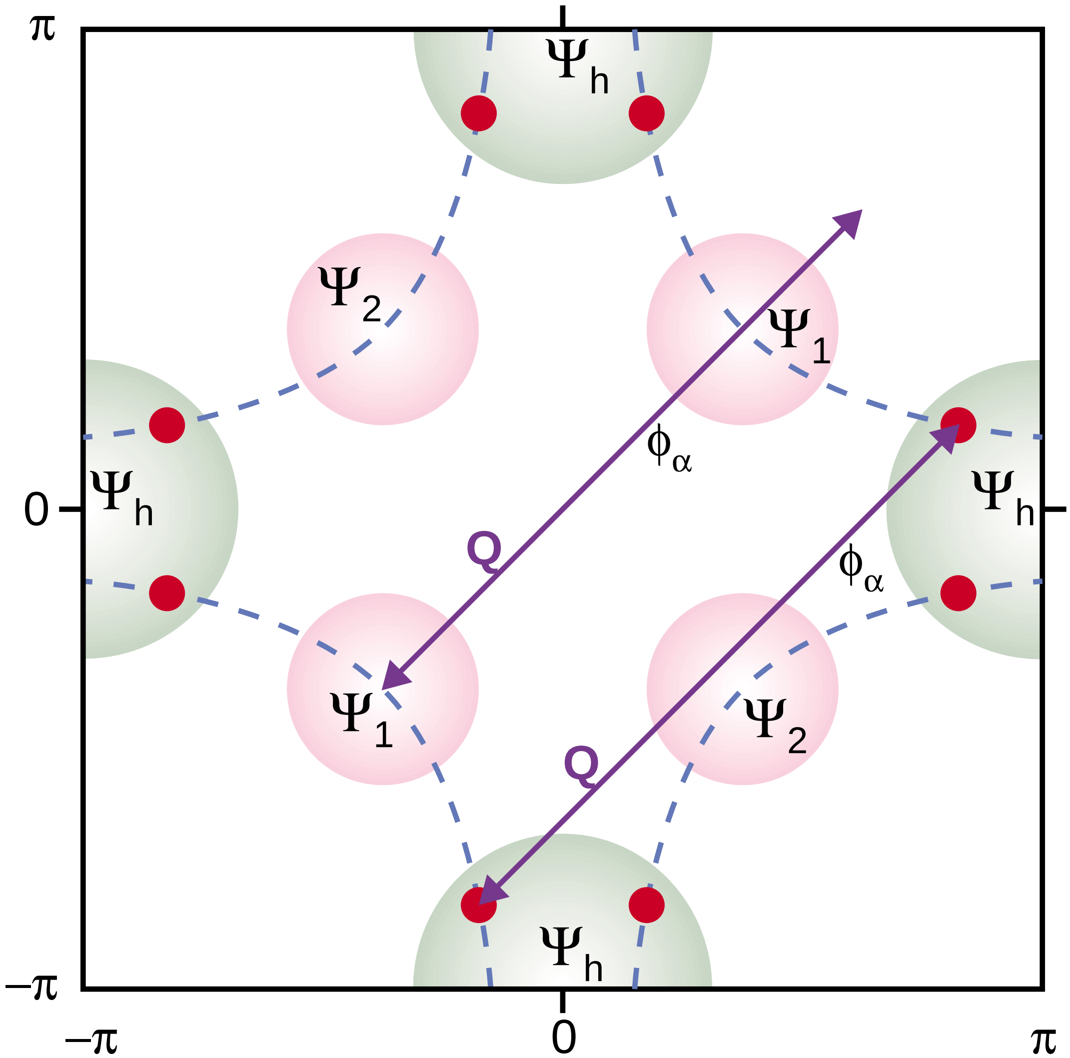


Represented by ϕ_α , a vector field measuring the strength of antiferromagnetic spin fluctuations near $\mathbf{Q} \approx (\pi, \pi)$
Damping is small at *T=0*

$$\Gamma_{pure} \approx 0 \text{ at } T = 0$$

(Theory generalizes to the cases with incommensurate **Q** and $\Gamma_{pure} \neq 0$)

Constraints from momentum conservation



Ψ_h : *strongly coupled* to ϕ_α, but do not damp ϕ_α as long as $\Delta_{res} < 2\,\Delta_h$

$\Psi_{1,2}$: *decoupled* from ϕ_α

I. Zero temperature broadening of resonant collective mode ϕ_α by impurities: comparison with neutron scattering experiments of Fong *et al* Phys. Rev. Lett. **82**, 1939 (1999).
Theory: proximity to a magnetic ordering transition

II. Intrinsic inelastic lifetime of nodal quasiparticles $\Psi_{1,2}$ (Valla *et al* Science **285**, 2110 (1999) and Corson *et al* cond-mat/0003243)
Theory: proximity to a quantum phase transition with a spin-singlet fermion bilinear order parameter

Independent low energy quantum field theories for the ϕ_α and the $\Psi_{1,2}$

I. Zero temperature broadening of resonant collective mode by impurities

Analogy with deformation of quantum coherence by a dilute concentration of impurities n_{imp}

Magnetic impurities in a Fermi liquid

Quasiparticle scattering rate

$$\Gamma_{\text{imp}}(\varepsilon) \sim \begin{cases} n_{\text{imp}} J^2 a^{2d} \rho(E_F) & \varepsilon >> T_K \\ \dfrac{n_{\text{imp}}}{\rho(\mathrm{E_F})} & \varepsilon << \mathrm{T_K} \end{cases}$$

Main result for collective spin resonant mode in two dimensions

Effect of *arbitrary localized deformations* ("impurities") of density n_{imp}

Each impurity is characterized by an integer/half-odd-integer S

As $\Delta_{\text{res}} \to 0$

$$\frac{\Gamma_{\text{imp}}}{\Delta_{\text{res}}} = n_{\text{imp}} \left(\frac{\hbar c}{\Delta_{\text{res}}} \right)^2 \left[C_S + O\left(\frac{\Delta_{\text{res}}}{J} \right) \right]$$

Correlation length ξ

$C_S \to$ Universal numbers dependent only on S

$$C_0 = 0 \ ; \ C_{1/2} \approx 1$$

Zn impurities in YBCO have S=1/2

"Swiss-cheese" model of quantum impurities (Uemura):

Inverse Q of resonance ~ fractional volume of holes in Swiss cheese.

As $\Delta_{res} \rightarrow 0$ there is a quantum phase transition to a magnetically ordered state

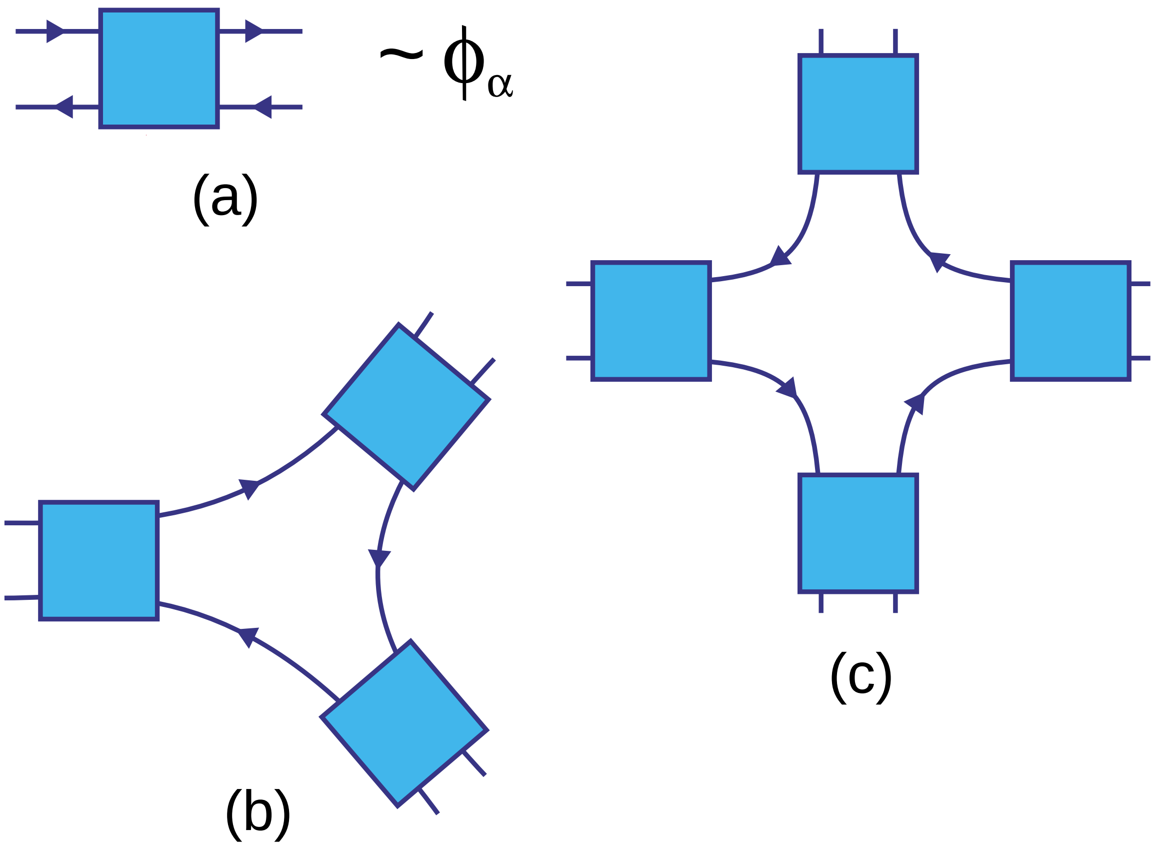
(A) Insulating Neel state (or collinear SDW at wavevector **Q**) $\Leftrightarrow$ insulating quantum paramagnet

(B) *d*-wave superconductor with collinear SDW at wavevector **Q** $\Leftrightarrow$ *d*-wave superconductor (paramagnet)

Transition (B) is in the same universality class as (A) provided Ψ_h fermions remain gapped at quantum-critical point.

Why appeal to proximity to a quantum phase transition ?

$\phi_\alpha \sim S=1$ bound state in particle-hole channel at the antiferromagnetic wavevector



Quantum field theory of critical point allows systematic treatment of the *strongly relevant* multi-point interactions in (b) and (c).

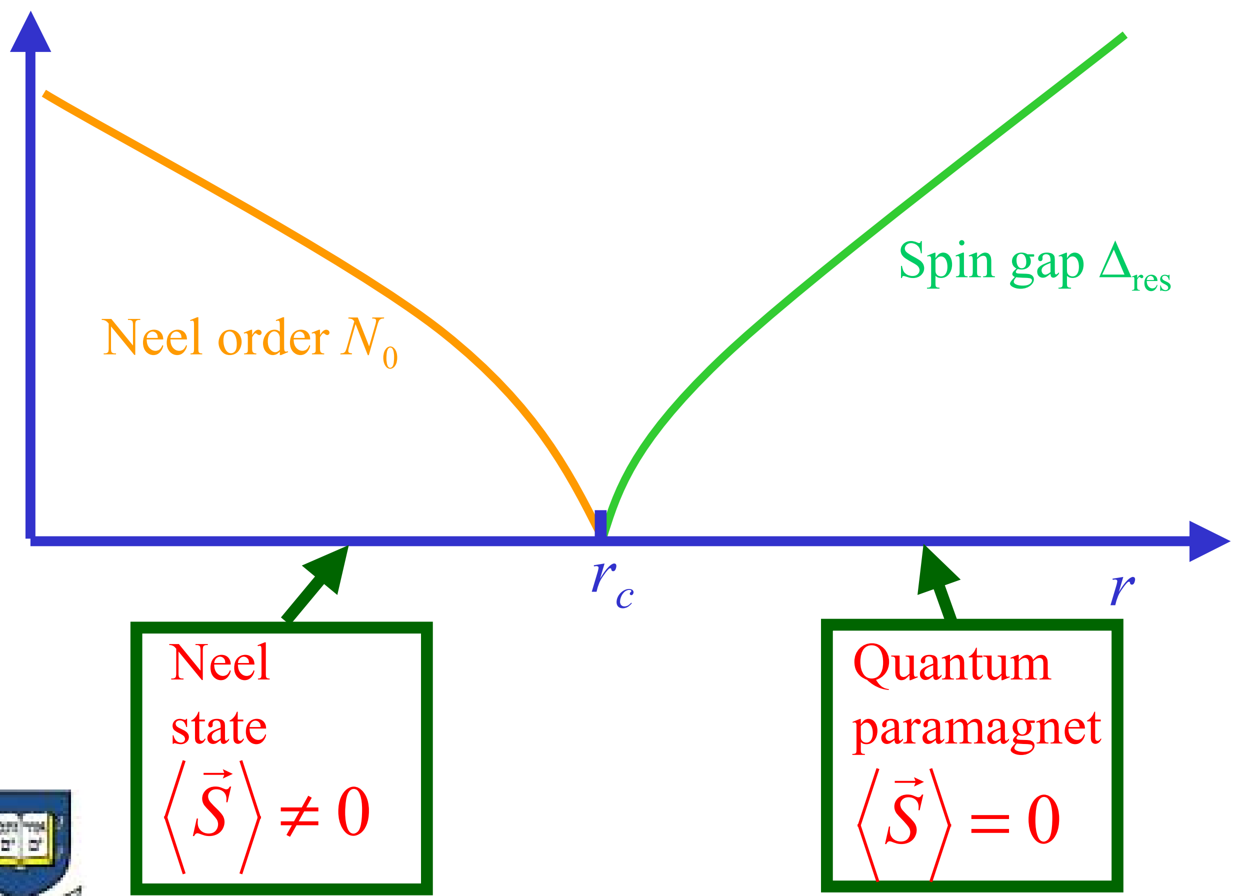
Nearly-critical paramagnets

Quantum field theory:

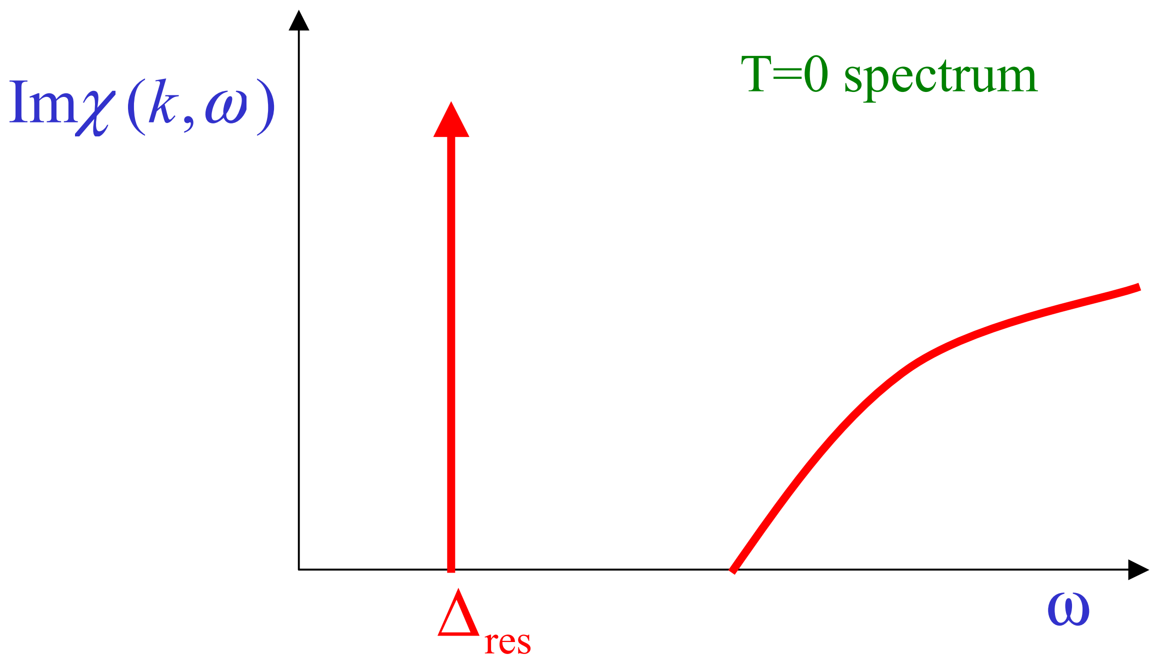
$$S_b = \int d^d x d\tau \left[\frac{1}{2}\left((\nabla_x \phi_\alpha)^2 + c^2(\partial_\tau \phi_\alpha)^2 + r\phi_\alpha^2\right) + \frac{g}{4!}(\phi_\alpha^2)^2\right]$$

ϕ_α → 3-component antiferromagnetic order parameter

No Berry phase terms because of almost perfect cancellation of the two sublattice contributions



Oscillations of ϕ_α about zero (for $\boldsymbol{r} > 0$)
→ spin-1 collective mode



Coupling *g* approaches fixed-point value under renormalization group flow: beta function ($\varepsilon = 3\text{-}d$) :

$$\beta(g) = -\epsilon g + \frac{11g^2}{6} - \frac{23g^3}{12} + \mathcal{O}(g^4)$$

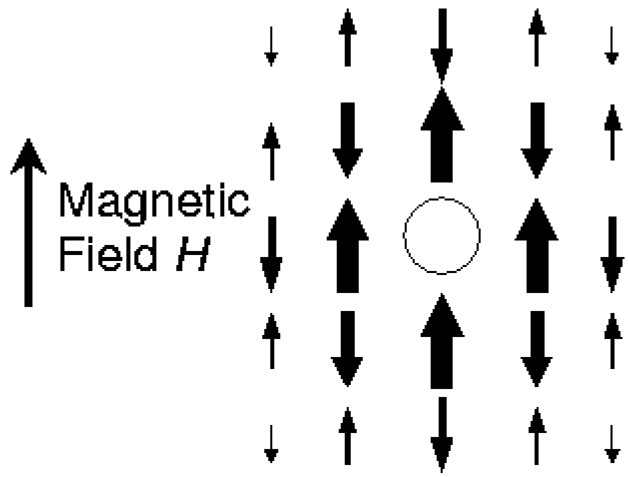
Only relevant perturbation – ***r***
strength is measured by the spin gap Δ_{res}

Δ_{res} and *c* completely determine entire spectrum of quasi-particle peak and multiparticle continua, the S matrices for scattering between the excitations, and $T > 0$ modifications.

Zn impurity in $YBa_2Cu_3O_{6.7}$

Moments measured by analysis of Knight shifts

M.-H. Julien, T. Feher, M. Horvatic, C. Berthier, O. N. Bakharev, P. Segransan, G. Collin, and J.-F. Marucco, Phys. Rev. Lett. **84**, 3422 (2000); also earlier work of the group of H. Alloul



Berry phases of precessing spins do not cancel between the sublattices in the vicinity of the impurity: net uncancelled phase of S=1/2

Orientation of "impurity" spin -- $n_\alpha(\tau)$ (unit vector)

Action of "impurity" spin

$$S_{\rm imp} = \int d\tau \left[i S A_\alpha(n) \frac{dn_\alpha}{d\tau} - \gamma S n_\alpha(\tau) \phi_\alpha(x=0,\tau) \right]$$

$A_\alpha(n)$ → Dirac monopole function

Boundary quantum field theory: $S_b + S_{\rm imp}$

Recall -

$$S_b = \int d^d x d\tau \left[\frac{1}{2} \left((\nabla_x \phi_\alpha)^2 + c^2 (\partial_\tau \phi_\alpha)^2 + r \phi_\alpha^2 \right) + \frac{g}{4!} (\phi_\alpha^2)^2 \right]$$

Coupling γ approaches *also* approaches a fixed-point value under the renormalization group flow

(Sengupta, 97
Sachdev+Ye, 93
Smith+Si 99)

Beta function:

$$\beta(\gamma) = -\frac{\epsilon\gamma}{2} + \gamma^3 - \gamma^5 + \frac{5g^2\gamma}{144} + \frac{\pi^2}{3}\left(S(S+1) - \frac{1}{3}\right) g\gamma^3 + \mathcal{O}\left((\gamma, \sqrt{g})^7\right)$$

No new relevant perturbations on the boundary;
All other boundary perturbations are irrelevant –

e.g. $\lambda \int d\tau \phi_\alpha^2(x=0,\tau)$

(This is the simplest allowed boundary perturbation for S=0 – its irrelevance implies $C_0 = 0$)

Δ_{res} and c completely determine spin dynamics near an impurity –

No new parameters are necessary !

Finite density of impurities n_{imp}

Relevant perturbation – strength determined by only energy scale that is linear in n_{imp} and contains only bulk parameters

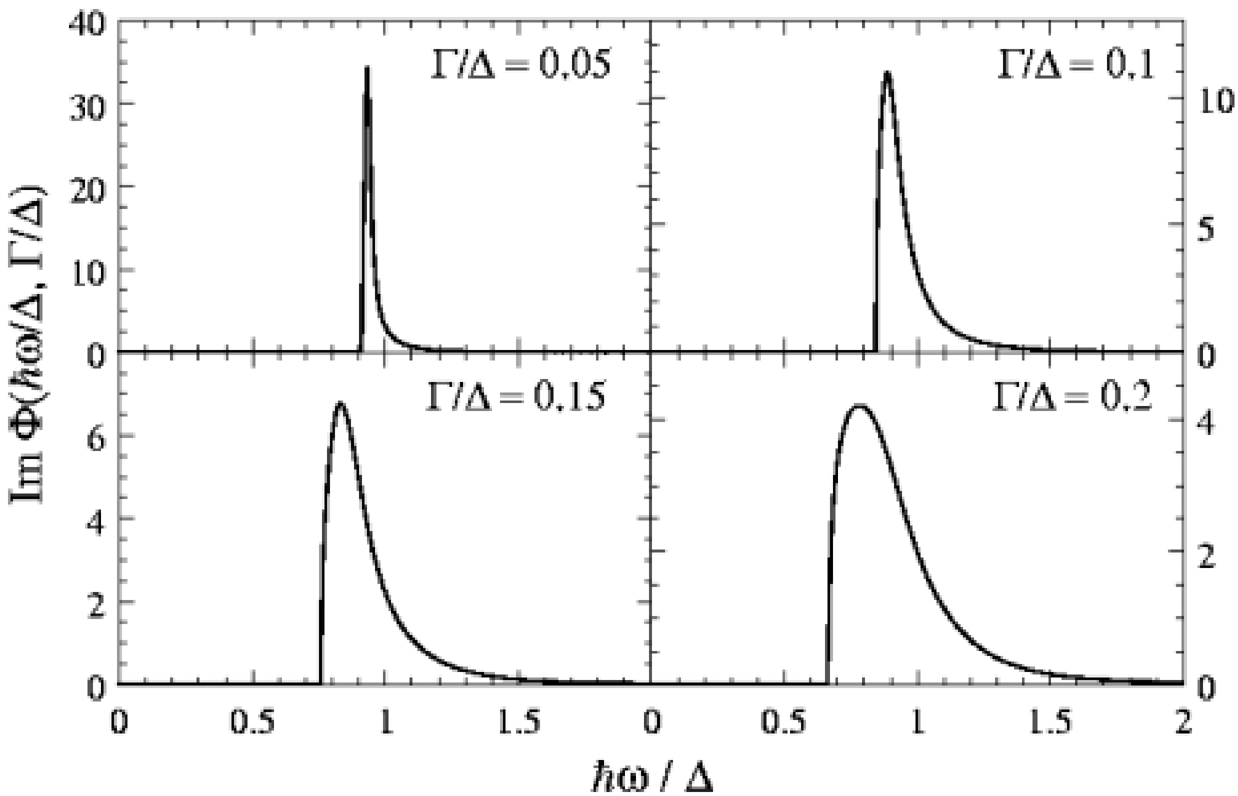
$$\Gamma \equiv \frac{n_{\text{imp}}(\hbar c)^2}{\Delta_{\text{res}}}$$

Fate of collective mode peak

Without impurities $$\chi(G,\omega) = \frac{A}{\Delta_{\mathrm{res}}^2 - \omega^2}$$

With impurities $$\chi(G,\omega) = \frac{A}{\Delta_{\mathrm{res}}^2}\Phi\left(\frac{\hbar\omega}{\Delta_{\mathrm{res}}}, \frac{\Gamma}{\Delta_{\mathrm{res}}}\right)$$

Φ → *Universal* scaling function. We computed it in a "self-consistent, non-crossing" approximation



Predictions: Half-width of line $\approx \Gamma$

Universal asymmetric lineshape

Coupling of impurity to fermionic quasiparticles $\Psi_{1,2}$

$$\sum_r J_K(r) S\, n_\alpha \Psi^\dagger(r)\sigma^\alpha\Psi(r) + U\Psi^\dagger(0)\Psi(0)$$

Kondo couplings

Potential scattering

(Many works (*e.g.* Pepin and Lee, Salkola, Balatasky and Scalapino) have ignored impurity spin and treated and effective potential scattering model with $U \to \infty$; we take U finite and include Kondo resonance effects)

Because density of states vanishes linearly at the Fermi level, there is no Kondo screening for any finite J_K (below a finite J_K) with (without) particle-hole symmetry

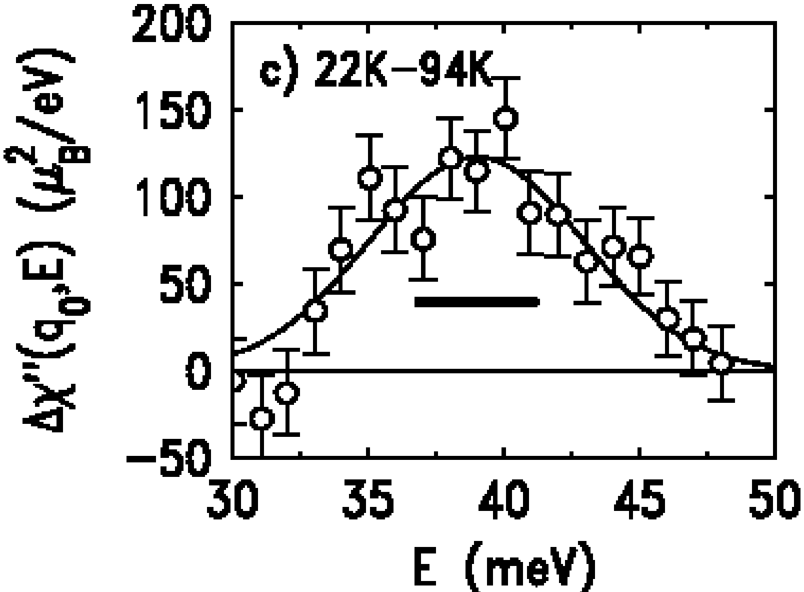
(Withoff+Fradkin, Chen+Jayaprakash, Buxton+Ingersent)

Our theory applies for $\Delta_{res} > T_K$

Implications of impurity spin for STM experiments: A. Polkovnikov, S. Sachdev and M. Vojta, to appear

$YBa_2Cu_3O_7 + 0.5\%$ Zn

H. F. Fong, P. Bourges, Y. Sidis, L. P. Regnault, J. Bossy, A. Ivanov, D.L. Milius, I. A. Aksay, and B. Keimer, Phys. Rev. Lett. **82**, 1939 (1999)



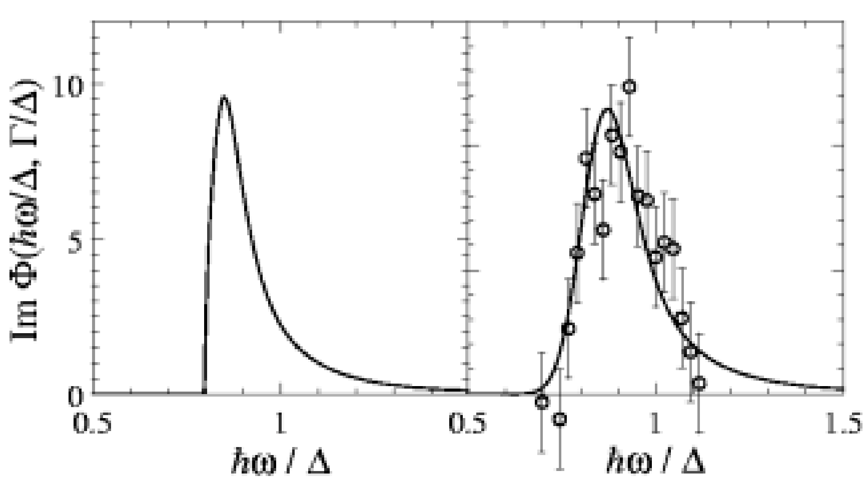
$$n_{\text{imp}} = 0.005$$

$$\Delta_{\text{res}} = 40 \text{ meV}$$

$$\hbar c = 0.2 \text{ eV}$$

$$\Rightarrow \quad \Gamma = 5 \text{ meV}, \ \Gamma/\Delta_{\text{res}} = 0.125$$

Quoted half-width = 4.25 meV



Conclusions: Part I

1. Universal *T=0* damping of S=1 collective mode by non-magnetic impurities.

   Linewidth: $$\Gamma \equiv \frac{n_{\text{imp}} (\hbar c)^2}{\Delta_{\text{res}}}$$

   independent of impurity parameters.

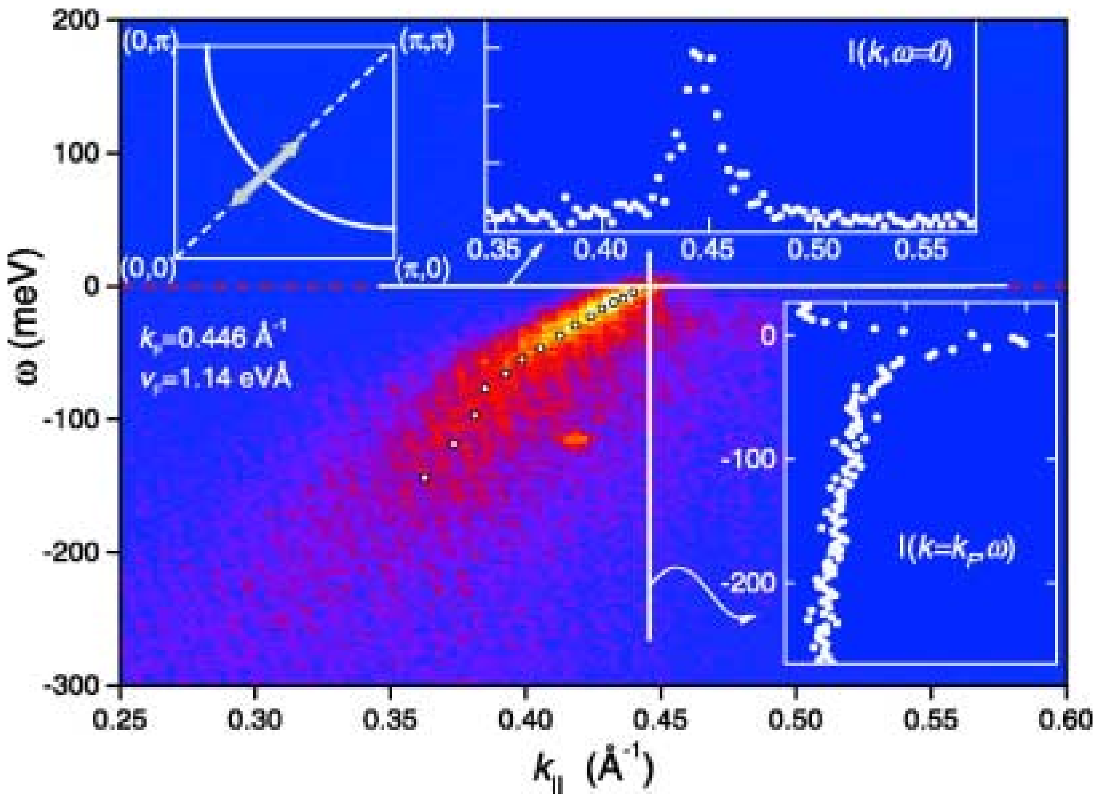
2. New interacting boundary conformal field theory in 2+1 dimensions

3. Universal irrational spin near the impurity at the critical point.

II. Intrinsic inelastic lifetime of nodal quasiparticles $\Psi_{1,2}$

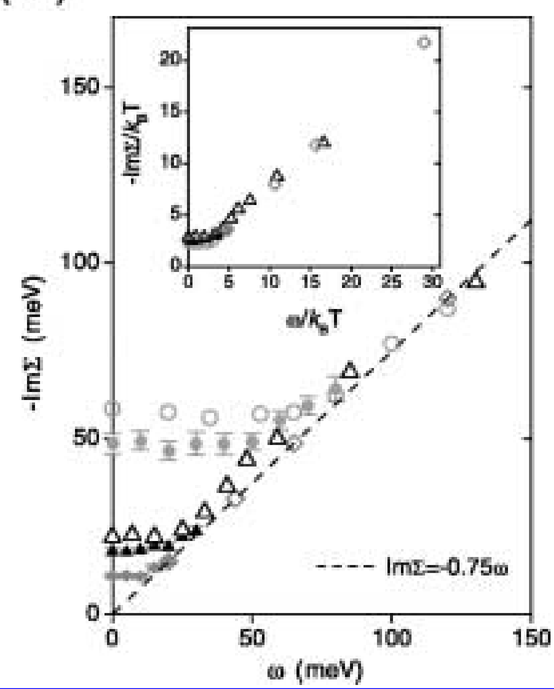
Photoemission on BSSCO

(Valla et al Science **285**, 2110 (1999))

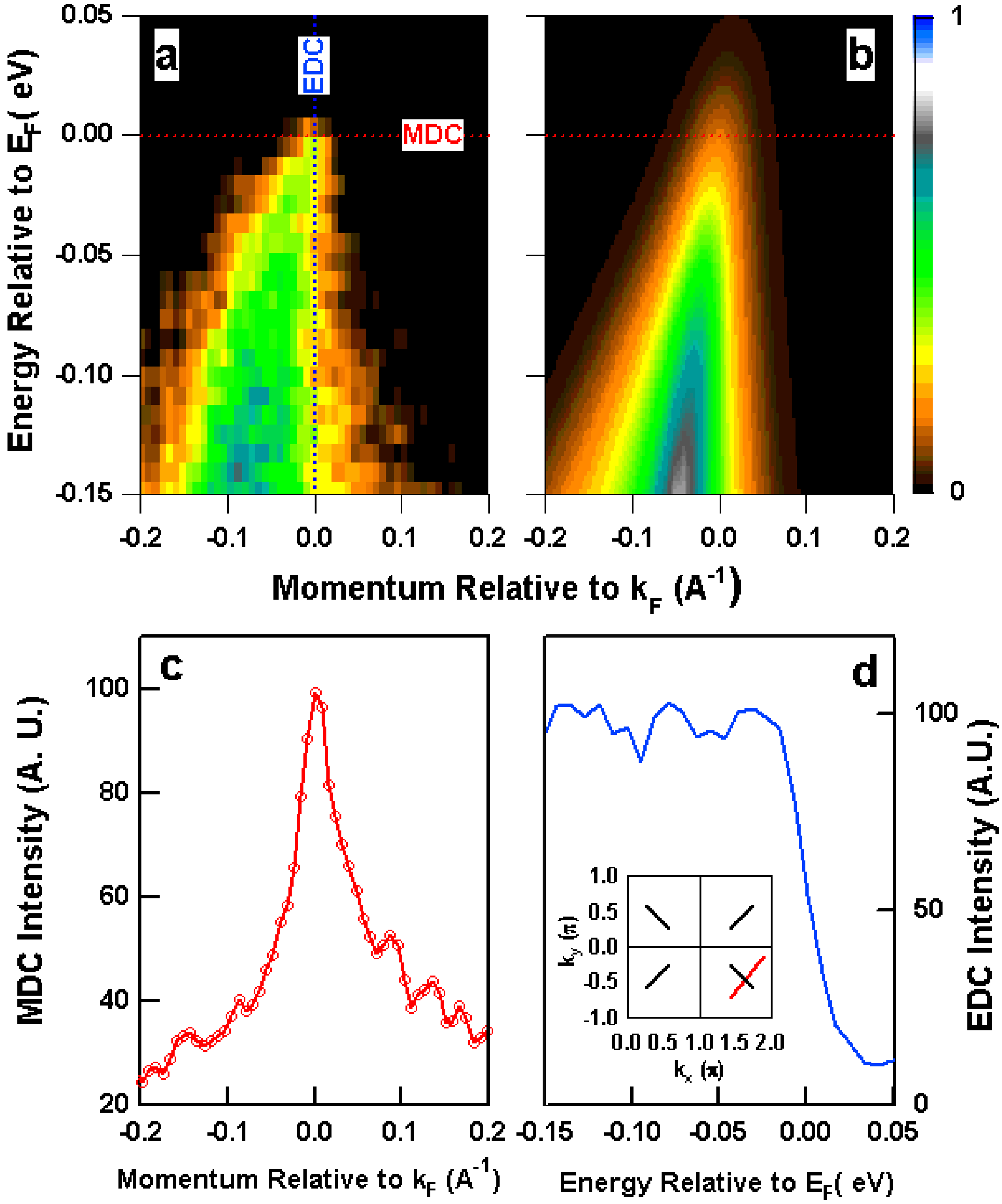


Quantum-critical damping of quasi-particles along (1,1)

Quasi-particles sharp along (1,0)



D. Orgad *et al*, cond-mat/0005457 :
Photoemission on LNSCO

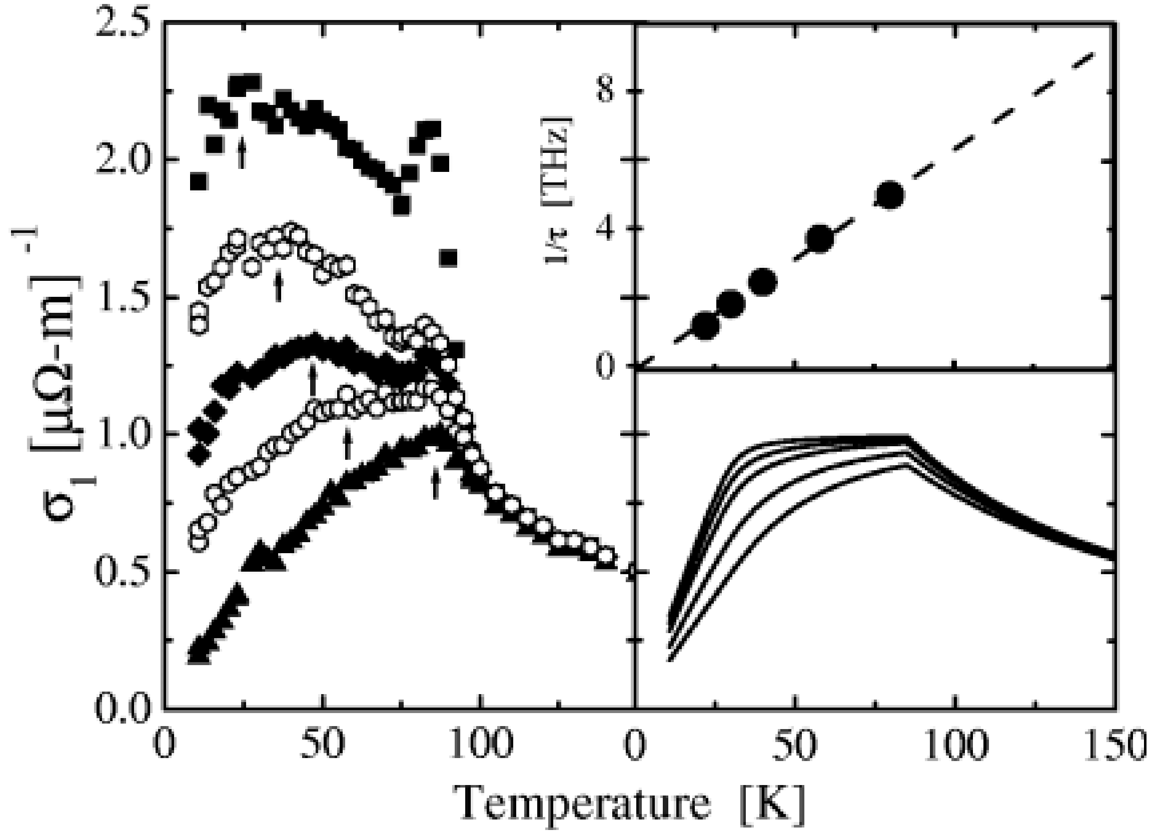


Large ω tail in the fermion spectral function

$$G(k,\omega) \sim \frac{1}{(v_F k - \omega)^{1-\eta_F}}$$

THz conductivity of BSCCO

(Corson et al cond-mat/0003243)



Quantum-critical damping of nodal quasi-particles

Origin of inelastic scattering ?

In a Fermi liquid $\mathrm{Im}\Sigma \sim T^2$

In a BCS d-wave superconductor $\mathrm{Im}\Sigma \sim T^3$

Classify theories in which a *d*-wave superconductor at $T \ll T_c$ has, with minimal fine-tuning:

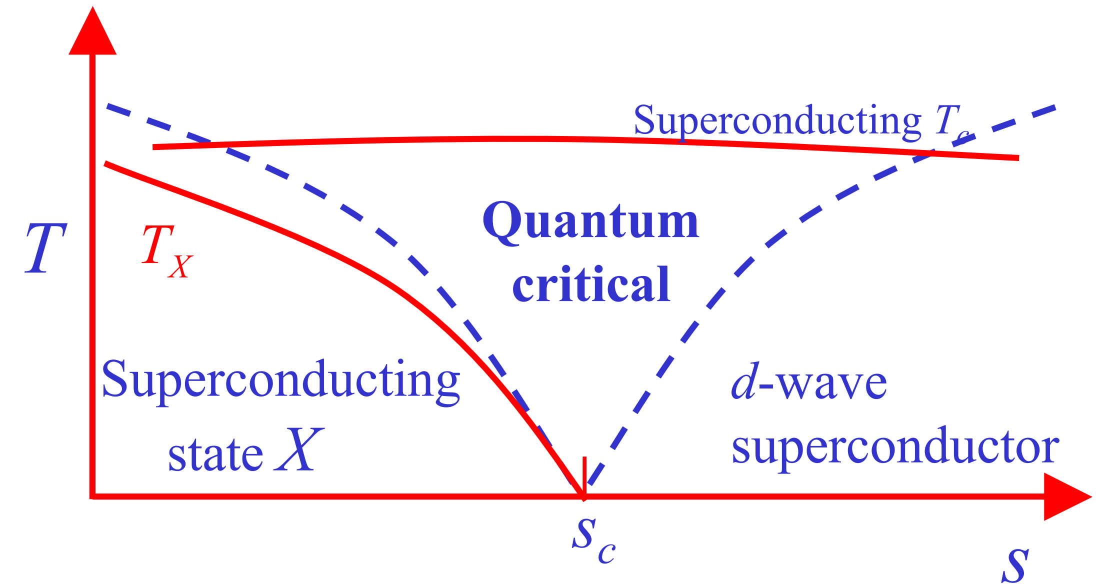
(a) nodal quasiparticle lifetime $\sim \hbar / k_B T$

and possibly

(b) negligible scattering of quasiparticles along (1,0), (0,1) directions

We will find that theories which obey (a) also have a large ω tail in nodal quasiparticle spectral function

Proximity to a quantum-critical point



(Crossovers analogous to those near quantum phase transitions in boson models Weichmann *et al* 1986, Chakravarty *et al* 1989)

Relaxational dynamics in quantum critical region
(Sachdev+Ye, 1992)

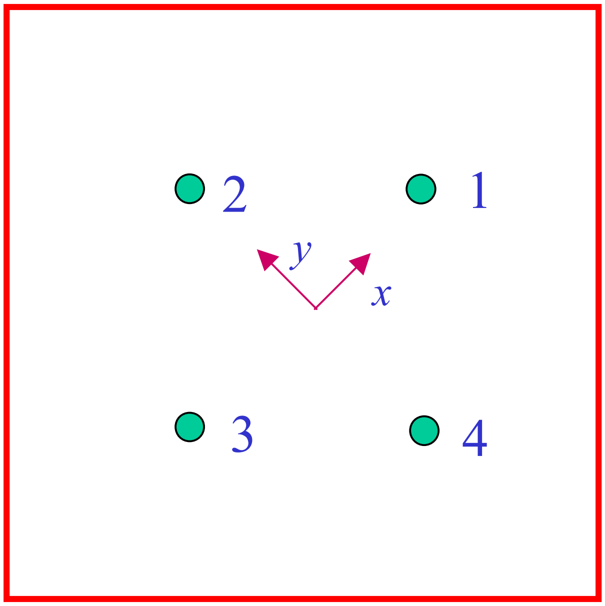
$$G_F(k,\omega) = \frac{\Lambda^{-\eta_F}}{(k_B T)^{1-\eta_F}} \Phi\left(\frac{ck}{k_B T}, \frac{\hbar\omega}{k_B T}\right)$$

Nodal quasiparticle Green's function
k → wavevector separation from node

Necessary conditions

1. Quantum-critical point should be below its upper-critical dimension and obey hyperscaling.
2. Critical field theory should not be free – required to obtain damping in the scaling limit. Combined with (1) this implies that characteristic relaxation times ~ $\hbar / k_B T$, so satisfying (a)
3. Nodal quasi-particles should be part of the critical-field theory.
4. Quasi-particles along (1,0), (0,1) should not couple to critical degrees of freedom to satisfy (b)

Low energy fermionic excitations of a *d*-wave superconductor



Gapless Fermi Points in a *d*-wave superconductor at wavevectors $(\pm K, \pm K)$

$K = 0.391\pi$

$$\Psi_1 = \begin{pmatrix} f_{1\uparrow} \\ f_{3\downarrow}^* \\ f_{1\downarrow} \\ -f_{3\uparrow}^* \end{pmatrix} \qquad \Psi_2 = \begin{pmatrix} f_{2\uparrow} \\ f_{4\downarrow}^* \\ f_{2\downarrow} \\ -f_{4\uparrow}^* \end{pmatrix}$$

$$\begin{aligned} S_\Psi \;=\; & \int \frac{d^2k}{(2\pi)^2} T \sum_{\omega_n} \Psi_1^\dagger \left(-i\omega_n + v_F k_x \tau^z + v_\Delta k_y \tau^x\right) \Psi_1 \\ & + \int \frac{d^2k}{(2\pi)^2} T \sum_{\omega_n} \Psi_2^\dagger \left(-i\omega_n + v_F k_y \tau^z + v_\Delta k_x \tau^x\right) \Psi_2 . \end{aligned}$$

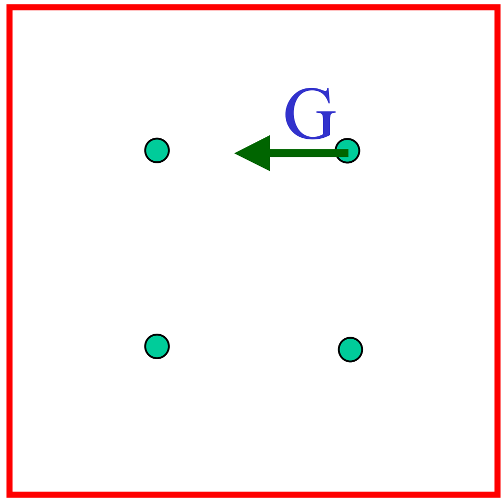
τ^x, τ^z are Pauli matrices in Nambu space

Order parameter for X should be a spin-singlet fermion bilinear at zero total momentum

e.g. Charge stripe order

$$\delta\rho \sim \mathrm{Re}\left[\Phi_x e^{iGx} + \Phi_y e^{iGy}\right]$$

If $G \neq 2K$ fermions do not couple efficiently to the order parameter and are not part of the critical theory



Action for quantum fluctuations of order parameter

$$\begin{aligned} S_\Phi &= \int d^2x d\tau \Big[|\partial_\tau \Phi_x|^2 + |\partial_\tau \Phi_y|^2 + |\nabla \Phi_x|^2 + |\nabla \Phi_y|^2 \\ &\quad + s\left(|\Phi_x|^2 + |\Phi_y|^2\right) + \frac{u_0}{2}\left(|\Phi_x|^4 + |\Phi_y|^4\right) \\ &\quad + v_0 |\Phi_x|^2 |\Phi_y|^2 \Big] \end{aligned}$$

Coupling to fermions $\sim \lambda \int d^d x d\tau \left|\Phi_a\right|^2 \Psi^\dagger \tau^z \Psi$

and λ is irrelevant at the critical point

$$\mathrm{Im}\Sigma \sim T^{2d+1-2/\nu}$$

$$\sim T^{(\text{between 2 and 3})} \text{ for } 2/3 < \nu < 1$$

Similarly exclude staggered flux state, which has G=(π,π) and a gradient coupling to fermions

Order parameter for X should be a component of

$$\Delta_k = \langle c_{k\uparrow} c_{-k\downarrow} \rangle \text{ (fermion pairing)}$$

or

$$A_k = \langle c^\dagger_{k\alpha} c_{k\alpha} \rangle \text{ (excitonic order)}$$

Complete group-theoretic classification

X has $d_{x^2-y^2}$ pairing plus

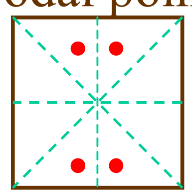
(A) is pairing → fermion spectrum fully gapped

(B) id_{xy} pairing → fermion spectrum fully gapped

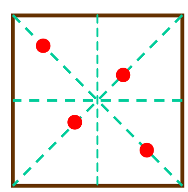
(C) ig pairing

Nodal points

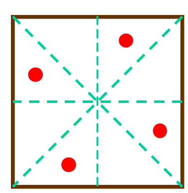
(D) s pairing → superconducting nematics



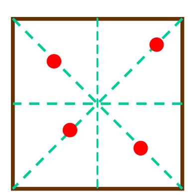
(E) d_{xy} excitons → superconducting nematics



(F) d_{xy} pairing



(G) p excitons



Quantum field theory for critical point

Ising order parameter ϕ (except for case (G))

$$S_\phi = \int d^2x d\tau \left[\frac{1}{2}(\partial_\tau \phi)^2 + \frac{c^2}{2}(\nabla \phi)^2 + \frac{s}{2}\phi^2 + \frac{u}{24}\phi^4 \right]$$

Coupling to nodal fermions

$$S_{\Psi\phi} = \int d^2x d\tau \left[\lambda\phi \left(\Psi_1^\dagger M_1 \Psi_1 + \Psi_2^\dagger M_2 \Psi_2 \right) \right].$$

(A) $M_1 = \tau^y;\ M_2 = \tau^y$

(B) $M_1 = \tau^y;\ M_2 = -\tau^y$

(C) λ=0, so (a) is not obeyed

(D) $M_1 = \tau^x;\ M_2 = \tau^x$

(E) $M_1 = \tau^z;\ M_2 = -\tau^z$

(F) $M_1 = \tau^x;\ M_2 = -\tau^x$

(G) $M_1 = 1;\ M_2 = 1$ but ϕ has 2 components

Main results

Only cases

(A) $d_{x^2-y^2} \Leftrightarrow d_{x^2-y^2} + is$ and

(B) $d_{x^2-y^2} \Leftrightarrow d_{x^2-y^2} + id_{xy}$

have renormalization group fixed points with

$\lambda = \lambda^* \neq 0$ and $u = u^* \neq 0$

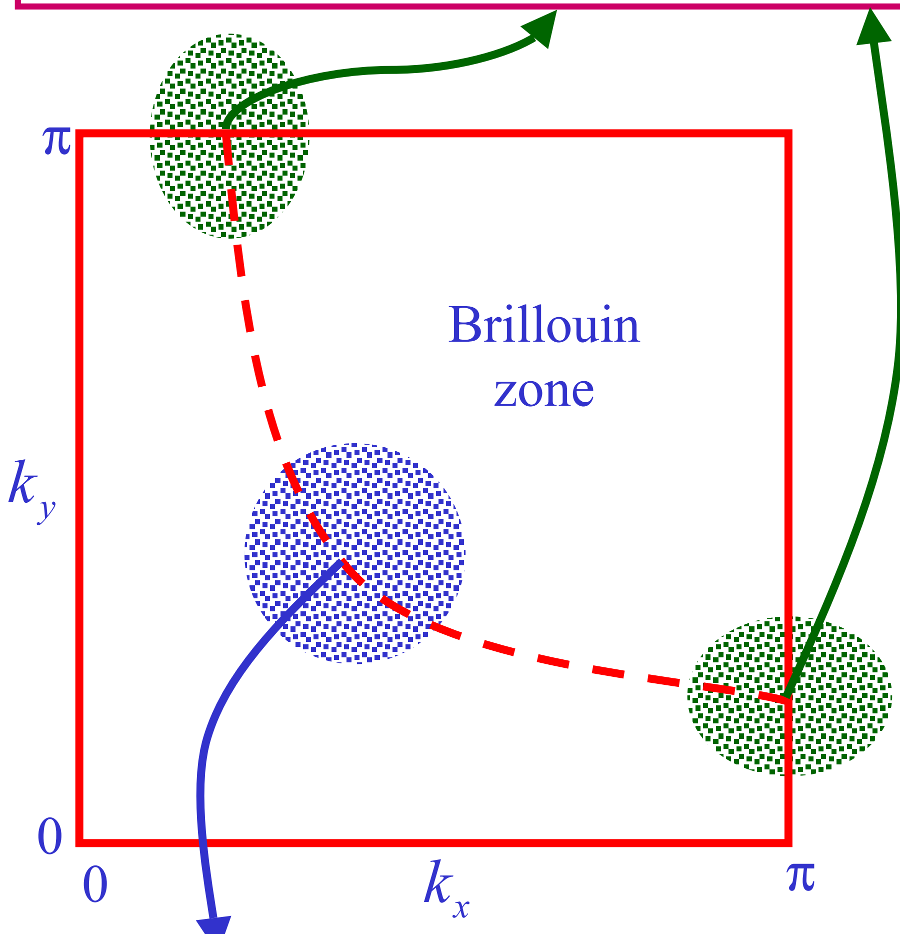
Only cases (A) and (B) satisfy condition (a)

d_{xy} order vanishes along the (1,0),(0,1) directions, and so only case (B) satisfies condition (b)

Gapped quasiparticles:

Below T_c : negligible damping

Above T_c: damping from strong coupling to superconducting phase and SDW fluctuations.



Nodal quasiparticles:

Below T_c : damping from fluctuations to $d_{x^2-y^2} + id_{xy}$ order

Above T_c: same mechanism applies as long as quantum-critical length < superconducting phase coherence length. Quasiparticles do not couple to phase or SDW fluctuations.

Conclusions: Part II

Classification of quantum-critical points leading to critical damping of quasiparticles in superconductor

Most attractive possibility: $\mathcal{T}$ breaking transition from a $d_{x^2-y^2}$ superconductor to a $d_{x^2-y^2} + id_{xy}$ superconductor

Leads to quantum-critical damping along (1,1), and no damping along (1,0), with no unnatural fine-tuning.

Note: stable ground state of cuprates can always be a $d_{x^2-y^2}$ superconductor; only need thermal/quantum fluctuations to $d_{x^2-y^2} + id_{xy}$ order in quantum-critical region.

Experimental update: Tafuri+Kirtley (cond-mat/0003106) claim signals of $\mathcal{T}$ breaking near non-magnetic impurities in YBCO films