Quantum Entanglement, Strange metals, and black holes

Subir Sachdev, Harvard University and TIFR
Quantum Entanglement, Strange metals, and black holes

Superconductor, levitated by an unseen magnet, in which countless trillions of electrons form a vast interconnected quantum state.

Scientific American, January 2013

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In a landmark study, scientists at Delft University of Technology in the Netherlands reported that they had conducted an experiment that they say proved one of the most fundamental claims of quantum theory — that objects separated by great distance can instantaneously affect each other’s behavior.

Part of the laboratory setup for an experiment at Delft University of Technology, in which two diamonds were set 1.3 kilometers apart, entangled and then shared information.
High temperature superconductors

YBa$_2$Cu$_3$O$_{6+x}$
Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University
Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University
Quantum entanglement
The double slit experiment

Interference of water waves
Principles of Quantum Mechanics: I. Quantum Superposition

The double slit experiment

Interference of water waves
The double slit experiment

Send electrons through the slits
Principles of Quantum Mechanics: I. Quantum Superposition

The double slit experiment

Interference of electrons
The double slit experiment

Is the electron a wave?

Interference of electrons
The double slit experiment

Unlike water waves, electrons arrive one-by-one (so is it like a particle?)

Interference of electrons
The double slit experiment

Unlike water waves, electrons arrive one-by-one (so is it like a particle?)

Interference of electrons
The double slit experiment

But if it is like a particle, which slit does each electron pass through?

Interference of electrons
The double slit experiment

But if it is like a particle, which slit does each electron pass through?

Interference of electrons

No interference when you watch the electrons
The double slit experiment

Principles of Quantum Mechanics: 1. Quantum Superposition

But if it is like a particle, which slit does each electron pass through?

Each electron passes through both slits!

Interference of electrons
The double slit experiment

Let $|L\rangle$ represent the state with the electron in the left slit.
The double slit experiment

Let $|L\rangle$ represent the state with the electron in the left slit.

And $|R\rangle$ represents the state with the electron in the right slit.
Principles of Quantum Mechanics: 1. Quantum Superposition

The double slit experiment

Let $|L\rangle$ represent the state with the electron in the left slit.

And $|R\rangle$ represents the state with the electron in the right slit.

Actual state of each electron is $|L\rangle + |R\rangle$. 

Quantum Entanglement: quantum superposition with more than one particle
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Hydrogen atom:

\[ = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]

Hydrogen molecule:
Quantum Entanglement: quantum superposition with more than one particle
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Quantum Entanglement: quantum superposition with more than one particle

Einstein-Podolsky-Rosen “paradox”: Measurement of one particle instantaneously determines the state of the other particle arbitrarily far away
Quantum entanglement
Quantum entanglement

Black holes

Strange metals
Quantum entanglement

Black holes

Strange metals
High temperature superconductors

$\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$
Strange metal
Antiferromagnet
Superconductor

Figure: K. Fujita and J. C. Seamus Davis
Antiferromagnet

Spins of electrons on Cu sites

Figure: K. Fujita and J. C. Seamus Davis
Square lattice of Cu sites
Square lattice of Cu sites

Remove density $p$ electrons
Square lattice of Cu sites

Electrons entangle in ("Cooper") pairs into chemical bonds

\[ \begin{aligned} &\left| \uparrow \downarrow \right\rangle - \left| \downarrow \uparrow \right\rangle \\
\end{aligned} \]
Square lattice of Cu sites

Cooper pairs form quantum superpositions at different locations: “Bose-Einstein condensation” in which all pairs are “everywhere at the same time”

\[
\begin{align*}
\text{Superconductivity} & \quad \text{Cooper pairs} \\
= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle
\end{align*}
\]
Square lattice of Cu sites

Cooper pairs form quantum superpositions at different locations: “Bose-Einstein condensation” in which all pairs are “everywhere at the same time”

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\[ |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \]
Square lattice of Cu sites

Cooper pairs form quantum superpositions at different locations: “Bose-Einstein condensation” in which all pairs are “everywhere at the same time”
Square lattice of Cu sites

High temperature superconductivity!

Electrons entangle by exchanging partners, and there is long-range quantum entanglement in the strange metal.
Square lattice of Cu sites

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High temperature superconductivity!

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\[ = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \]
Square lattice of Cu sites

High temperature superconductivity!

Electrons entangle by exchanging partners, and there is long-range quantum entanglement in the strange metal.

\[ \begin{align*}
\text{ paired sites } & = | \uparrow \downarrow \rangle - | \downarrow \uparrow \rangle \\
\end{align*} \]
Figure: K. Fujita and J. C. Seamus Davis

Strange metal

Entangled electrons lead to “strange” temperature dependence of resistivity and other properties.
Quantum entanglement

Black holes

Strange metals
Black Holes

Objects so dense that light is gravitationally bound to them.

In Einstein’s theory, the region inside the black hole horizon is disconnected from the rest of the universe.

Horizon radius \( R = \frac{2GM}{c^2} \)
On September 14, 2015, LIGO detected the merger of two black holes, each weighing about 30 solar masses, with radii of about 100 km, 1.3 billion light years away.
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LIGO
September 14, 2015
Around 1974, Bekenstein and Hawking showed that the application of the quantum theory across a black hole horizon led to many astonishing conclusions.
Quantum Entanglement across a black hole horizon
Quantum Entanglement across a black hole horizon
Quantum Entanglement across a black hole horizon

Black hole horizon
Quantum Entanglement across a black hole horizon

There is long-range quantum entanglement between the inside and outside of a black hole.
Quantum Entanglement across a black hole horizon

Hawking used this to show that black hole horizons have an entropy and a temperature.
Quantum Entanglement across a black hole horizon

Hawking used this to show that black hole horizons have an entropy and a temperature (because to an outside observer, the state of the electron inside the black hole is an unknown)
The Hawking temperature influences the radiation from the black hole at the very last stages of the ring-down (not observed so far).
Quantum entanglement

Black holes

Strange metals
Quantum phase transitions

Strange metals

Quantum entanglement

Black holes

Strange metals

A “toy model” which is both a strange metal and a black hole!
The Sachdev-Ye-Kitaev (SYK) model

Pick a set of random positions
The Sachdev-Ye-Kitaev (SYK) model

Place electrons randomly on some sites
The Sachdev-Ye-Kitaev (SYK) model

Entangle electrons pairwise randomly
The Sachdev-Ye-Kitaev (SYK) model

Entangle electrons pairwise randomly
The Sachdev-Ye-Kitaev (SYK) model

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Entangle electrons pairwise randomly
The Sachdev-Ye-Kitaev (SYK) model

Entangle electrons pairwise randomly
The SYK model has “nothing but entanglement”
This describes both a strange metal and a black hole!
SYK and black holes
The SYK model has “dual” description in which an extra spatial dimension, $\zeta$, emerges. The curvature of this “emergent” spacetime is described by Einstein’s theory of general relativity.
An extra spatial dimension emerges from quantum entanglement!
Tensor network of hierarchical entanglement

D-dimensional space

Depth of entanglement
String theory near a “D-brane”

Emergent spatial direction of SYK model or string theory
String theory near a “D-brane”

$\xi$  

Quantum entanglement leads to an emergent spatial dimension

Emergent spatial direction of SYK model or string theory

$D$-dimensional space
Quantum entanglement

Black holes

Strange metals

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