Quantum entanglement
and the phases of matter

TIFR, Mumbai
January 17, 2012

sachdev.physics.harvard.edu
Quantum Entanglement: quantum superposition with more than one particle
Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:
Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:

Hydrogen molecule:

\[
\begin{align*}
\text{=} & \quad \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \\
\text{Superposition of two electron states leads to non-local correlations between spins}
\end{align*}
\]
Quantum Entanglement: quantum superposition with more than one particle
Quantum Entanglement: quantum superposition with more than one particle
Quantum Entanglement: quantum superposition with more than one particle
Quantum Entanglement: quantum superposition with more than one particle

Einstein-Podolsky-Rosen “paradox”: Non-local correlations between observations arbitrarily far apart
Outline

1. Quantum critical points and string theory
   Entanglement and emergent dimensions

2. High temperature superconductors and strange metals
   Holography of compressible quantum phases
Outline

1. Quantum critical points and string theory
   *Entanglement and emergent dimensions*

2. High temperature superconductors and strange metals
   *Holography of compressible quantum phases*
Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Examine ground state as a function of \( \lambda \)

S=1/2 spins
At large $\lambda$ ground state is a “quantum paramagnet” with spins locked in valence bond singlets
Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Nearest-neighor spins are “entangled” with each other. Can be separated into an Einstein-Podolsky-Rosen (EPR) pair.
Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

For \( \lambda \approx 1 \), the ground state has antiferromagnetic ("Néel") order, and the spins align in a checkerboard pattern.
Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

For \( \lambda \approx 1 \), the ground state has antiferromagnetic ("Néel") order, and the spins align in a checkerboard pattern.

No EPR pairs
\[ \lambda_c = \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) \]
Pressure in $\text{TlCuCl}_3$

$\lambda = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

An insulator whose spin susceptibility vanishes exponentially as the temperature $T$ tends to zero.
TlCuCl$_3$

Quantum paramagnet at ambient pressure
TlCuCl$_3$

Neel order under pressure

$$\lambda_c = \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$
Excitation spectrum in the paramagnetic phase

Spin $S = 1$ "triplon"
Excitation spectrum in the paramagnetic phase

Spin $S = 1$

"triplon"

$\lambda_c$
Excitation spectrum in the paramagnetic phase

Spin $S = 1$

“triplon”
Excitation spectrum in the paramagnetic phase

Spin $S = 1$

"triplon"
Excitation spectrum in the paramagnetic phase

Spin $S = 1$
“triplon”
Excitation spectrum in the Néel phase

Spin waves
Excitation spectrum in the Néel phase

Spin waves
Excitation spectrum in the Néel phase

Spin waves
Excitations of TlCuCl$_3$ with varying pressure

Excitations of TlCuCl$_3$ with varying pressure

Broken valence bond excitations of the quantum paramagnet

Excitations of TlCuCl$_3$ with varying pressure

Spin waves above the Néel state

Excitations of TlCuCl$_3$ with varying pressure

Longitudinal excitations – similar to the Higgs boson
First observation of the Higgs!

Excitations of TlCuCl$_3$ with varying pressure

"Higgs" particle appears at theoretically predicted energy

Longitudinal excitations—similar to the Higgs boson
First observation of the Higgs!

S. Sachdev, arXiv:0901.4103

\[ \lambda = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]
Quantum critical point with non-local entanglement in spin wavefunction

\[ \lambda = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]
Tensor network representation of entanglement at quantum critical point
• Long-range entanglement
• Long-range entanglement

• Long distance and low energy correlations near the quantum critical point are described by a quantum field theory which is relativistically invariant (where the spin-wave velocity plays the role of the velocity of “light”).
• Long-range entanglement

• Long distance and low energy correlations near the quantum critical point are described by a quantum field theory which is relativistically invariant (where the spin-wave velocity plays the role of the velocity of “light”).

• The quantum field theory is invariant under scale and conformal transformations at the quantum critical point: a CFT3
String theory

- Allows unification of the standard model of particle physics with gravity.

- Low-lying string modes correspond to gauge fields, gravitons, quarks ...
• A $D$-brane is a $D$-dimensional surface on which strings can end.

• The low-energy theory on a $D$-brane is an ordinary quantum field theory with no gravity.
- A $D$-brane is a $D$-dimensional surface on which strings can end.
- The low-energy theory on a $D$-brane is an ordinary quantum field theory with no gravity.
- In $D = 2$, we obtain strongly-interacting CFT$^3$s. These are “dual” to string theory on anti-de Sitter space: AdS$^4$. 
Tensor network representation of entanglement at quantum critical point

D-dimensional space

Depth of entanglement

String theory near a D-brane

Emergent direction of AdS4
Tensor network representation of entanglement at quantum critical point

Emergent direction of AdS4

Brian Swingle, arXiv:0905.1317
\[ \rho_A = \text{Tr}_B \rho = \text{density matrix of region } A \]

**Entanglement entropy** 

\[ S_{EE} = -\text{Tr} (\rho_A \ln \rho_A) \]
Entanglement entropy

A

D-dimensional space

depth of entanglement

Tuesday, January 17, 2012
Entanglement entropy

D-dimensional space

Draw a surface which intersects the minimal number of links

Emergent direction of AdS4
The entanglement entropy of a region A on the boundary equals the minimal area of a surface in the higher-dimensional space whose boundary coincides with that of A.

This can be seen both the string and tensor-network pictures

Brian Swingle, arXiv:0905.1317
Emergent holographic direction

Quantum matter with long-range entanglement

J. McGreevy, arXiv0909.0518
$\text{AdS}_{d+2}$

$\mathbb{R}^{d,1}$

Minkowski

Emergent holographic direction

$CFT_{d+1}$

Quantum matter with long-range entanglement

$\rightarrow$

Emergent holographic direction
$\text{AdS}_{d+2}$

$\text{Minkowski}$

$\mathbb{R}^{d,1}$

Emergent holographic direction

Area measures entanglement entropy

$CFT_{d+1}$

Quantum matter with long-range entanglement

$r$
Quantum critical point with non-local entanglement in spin wavefunction

\[ \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]
Classical spin waves

Dilute triplon gas

Quantum critical

Neel order

Pressure in TlCuCl$_3$

Classical spin waves

Dilute triplon gas

Quantum critical

Neel order

Short-range entanglement

Pressure in TlCuCl$_3$

Classical spin waves

Dilute triplon gas

Quantum critical


Excitations of a ground state with long-range entanglement

Neel order

Pressure in $\text{TlCuCl}_3$

$T$

$\lambda$

$\lambda_c$
AdS/CFT correspondence at non-zero temperatures

AdS$_4$-Schwarzschild black-brane

A 2+1 dimensional system at its quantum critical point

Tuesday, January 17, 2012
AdS/CFT correspondence at non-zero temperatures

AdS$_4$-Schwarzschild black-brane

Black-brane at temperature of 2+1 dimensional quantum critical system

A 2+1 dimensional system at its quantum critical point

Tuesday, January 17, 2012
AdS/CFT correspondence at non-zero temperatures

AdS$_4$-Schwarzschild black-brane

Black-brane at temperature of 2+1 dimensional quantum critical system

Friction of quantum criticality = waves falling into black brane

A 2+1 dimensional system at its quantum critical point
AdS/CFT correspondence at non-zero temperatures

AdS$_4$-Schwarzschild black-brane

A 2+1 dimensional system at its quantum critical point

Black-brane at temperature of 2+1 dimensional quantum critical system

Provides successful description of many properties of quantum critical points at non-zero temperatures
Outline

1. Quantum critical points and string theory
   *Entanglement and emergent dimensions*

2. High temperature superconductors
   and strange metals
   *Holography of compressible quantum phases*
Outline

1. Quantum critical points and string theory
   *Entanglement and emergent dimensions*

2. High temperature superconductors and strange metals
   *Holography of compressible quantum phases*
The cuprate superconductors
Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Ground state has long-range Néel order
Hole-doped Superconductor

Bose condensate of pairs of electrons

Short-range entanglement
Electron-doped cuprate superconductors

Hole-doped

La$_{2-x}$Sr$_x$CuO$_4$

$T^*$

$T_c$

$T_N$

$\sim 300K$

$\sim 30K$

Electron-doped

RE$_{2-x}$Ce$_x$CuO$_4$

$T^*$

$T_N$

$T_c$

Hole doping / Sr content (x)

Electron doping / Ce content (x)
Electron-doped cuprate superconductors

Figure prepared by K. Jin and R. L. Greene based on N. P. Fournier, P. Armitage, and R. L. Greene, Rev. Mod. Phys. 82, 2421 (2010).

Resistivity $\sim \rho_0 + AT^n$
Electron-doped cuprate superconductors

Figure prepared by K. Jin and R. L. Greene based on N. P. Fournier, P. Armitage, and R. L. Greene, Rev. Mod. Phys. 82, 2421 (2010).

Electron doping / Ce content (x)

Resistivity

∼ ρ₀ + AT^n

Ordinary metal (Fermi liquid)
Short-range entanglement

Tuesday, January 17, 2012
Electron-doped cuprate superconductors

Figure prepared by K. Jin and R. L. Greene based on N. P. Fournier, P. Armitage, and R. L. Greene, Rev. Mod. Phys. 82, 2421 (2010).

Resistivity \( \sim \rho_0 + AT^n \)
Electron-doped cuprate superconductors

Figure prepared by K. Jin and R. L. Greene based on N. P. Fournier, P. Armitage, and R. L. Greene, Rev. Mod. Phys. 82, 2421 (2010).
Electron-doped cuprate superconductors

Figure prepared by K. Jin and R. L. Greene based on N. P. Fournier, P. Armitage, and R. L. Greene, Rev. Mod. Phys. 82, 2421 (2010).

Resistivity

\[ \sim \rho_0 + AT^n \]
Electron-doped cuprate superconductors

Strange Metal

Excitations of a ground state with long-range entanglement

Resistivity $\sim \rho_0 + AT^n$

Figure prepared by K. Jin and R. L. Greene based on N. P. Fournier, P. Armitage, and R. L. Greene, Rev. Mod. Phys. 82, 2421 (2010).
Iron pnictides: 

a new class of high temperature superconductors


Temperature-doping phase diagram of the iron pnictides:

Temperature-doping phase diagram of the iron pnictides:

\[ \text{BaFe}_2(\text{As}_{1-x}\text{P}_x)_2 \]

Ordinary metal (Fermi liquid)

Short-range entanglement

\[ \text{Resistivity} \sim \rho_0 + A T^\alpha \]
Temperature-doping phase diagram of the iron pnictides:

\[ \text{Resistivity } \sim \rho_0 + AT^\alpha \]


Temperature-doping phase diagram of the iron pnictides:

\[ T_{\text{SDW}} \]

BaFe\(_2\)(As\(_{1-x}\)P\(_x\))\(_2\)

Strange Metal

Resistivity \( \sim \rho_0 + AT^\alpha \)


Temperature-doping phase diagram of the iron pnictides:

\[ \text{Resistivity} \sim \rho_0 + AT^\alpha \]


Temperature-doping phase diagram of the iron pnictides:

Excitations of a ground state with long-range entanglement

(As$_{1-x}$P$_x$)$_2$

Strange Metal

AF

Superconductivity

Resistivity $\sim \rho_0 + AT^\alpha$


Temperature-pressure phase diagram of an organic superconductor

Temperature-pressure phase diagram of an heavy-fermion superconductor

Ordinary metals (Fermi liquids)

Metal with “large” Fermi surface

Momenta with electron states occupied

Momenta with electron states empty
The electron spin polarization obeys

\[ \langle \vec{S}(\vec{r}, \tau) \rangle = \varphi(\vec{r}, \tau) e^{i\vec{K} \cdot \vec{r}} \]

where \( \vec{K} \) is the ordering wavevector.
Fermi surface + antiferromagnetism

Metal with electron and hole pockets

Metal with “large” Fermi surface

Increasing interaction

Fermi surface reconstruction and onset of antiferromagnetism

AF \langle \phi \rangle \neq 0

\langle \phi \rangle = 0
Fermi surface + antiferromagnetism

Metal with "large" Fermi surface

Increasing interaction
Fermi surface reconstruction and onset of antiferromagnetism

AF $\langle \varphi \rangle \neq 0$

Metal with electron and hole pockets

$\langle \varphi \rangle = 0$
Fermi surface + antiferromagnetism

Metal with electron and hole pockets

Metal with “large” Fermi surface

Increasing interaction

Fermi surface reconstruction and onset of antiferromagnetism

Quantum critical point realizes a strongly-coupled “strange metal” with long-range entanglement!
AF metal with “small” Fermi pockets

Fermi liquid with “large” Fermi surface
AF metal with “small” Fermi pockets

High temperature Superconductor ✓

Fermi liquid with “large” Fermi surface

AF metal with “small” Fermi pockets

Fermi liquid with “large” Fermi surface

High temperature Superconductor

Strange Metal with long-range entanglement

AF metal with “small” Fermi pockets

High temperature Superconductor ✓

Fermi liquid with “large” Fermi surface

Tuesday, January 17, 2012
Challenge to string theory:

Describe quantum critical points and phases of metallic systems
Challenge to string theory:

Describe quantum critical points and phases of metallic systems

Can we obtain holographic theories of superconductors and ordinary metals (Fermi liquids)?
Challenge to string theory:

Describe quantum critical points and phases of metallic systems

Can we obtain holographic theories of superconductors and ordinary metals (Fermi liquids)?

Yes
Challenge to string theory:

Describe quantum critical points and phases of metallic systems

Does holography yield metals other than ordinary metals?
Challenge to string theory:

Describe quantum critical points and phases of metallic systems

Does holography yield metals other than ordinary metals?

Yes, lots of them, with many “strange” properties!
Challenge to string theory:

Describe quantum critical points and phases of metallic systems

Do any of the holographic “strange metals” have the correct type of long-range entanglement?
Challenge to string theory:

Describe quantum critical points and phases of metallic systems

Do any of the holographic “strange metals” have the correct type of long-range entanglement?

Yes, a very select subset has the proper logarithmic violation of the area law of entanglement!!
These are now being studied intensively.........

Conclusions

Phases of matter with long-range quantum entanglement are prominent in numerous modern materials.
Conclusions

Simplest examples of long-range entanglement are at quantum-critical points of insulating antiferromagnets
Conclusions

More complex examples in metallic states are experimentally ubiquitous, but pose difficult strong-coupling problems to conventional methods of field theory.
Conclusions

String theory and holography offer a remarkable new approach to describing states with long-range quantum entanglement.
Conclusions

String theory and holography offer a remarkable new approach to describing states with long-range quantum entanglement.

Much recent progress offers hope of a holographic description of “strange metals”