Quantum phase transitions in antiferromagnets and *d*-wave superconductors

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Ying Zhang

Science **286**, 2479 (1999).

Transparencies online at
http://pantheon.yale.edu/~subir

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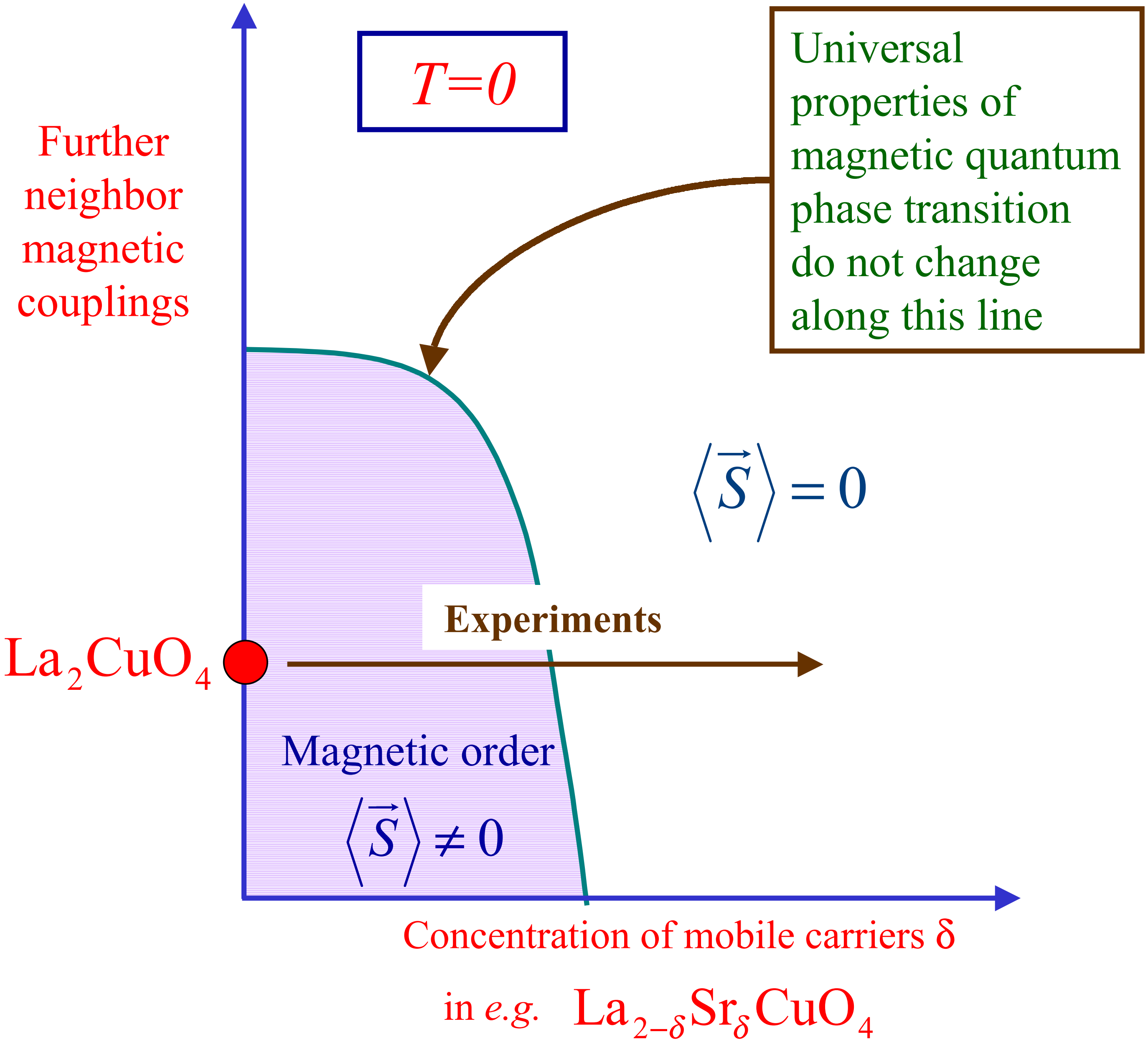

S. Sachdev and J. Ye, Phys. Rev. Lett. **69**, 2411 (1992).
A.V. Chubukov, S. Sachdev, and J. Ye, Phys. Rev. B **49**, 11919 (1994)

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Outline

I. Magnetic ordering transitions

A. Insulators

B. Doped antiferromagnets.

II. Effect of magnetic field on antiferromagnetic order in superconductor.

Comparison of theory with neutron scattering experiments.

III. Effect on Zn/Li impurities on $S=1$ spin exciton.

Comparison of theory with neutron scattering experiments.

IV. Conclusions

V. Damping of nodal quasiparticles.

Non-magnetic quantum transitions in *d*-wave superconductors

I.A Magnetic quantum transition in the insulator (δ=0)

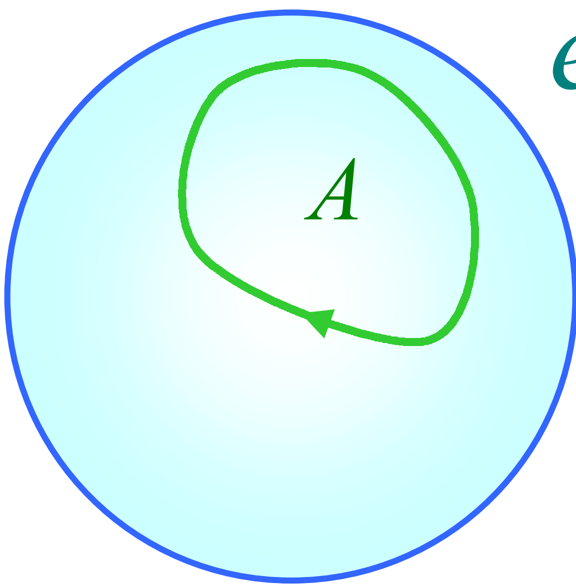
Neel order parameter $\phi_\alpha \quad \alpha=1,2,3$

Action:

$$S_b = \int d^2x d\tau \left[\frac{1}{2}\left(\left(\nabla_x \phi_\alpha\right)^2 + c^2 \left(\partial_\tau \phi_\alpha\right)^2 \right) + V\left(\phi_\alpha^2\right) \right]$$

S. Chakravarty, B.I. Halperin, and D.R. Nelson, Phys. Rev. B **39**, 2344 (1989).

Missing: Spin Berry Phases



$$e^{iSA}$$

Berry phases induce
bond charge order
in quantum "disordered"
phase with $\langle \phi_\alpha \rangle = 0$;
e.g. spin-Peierls or plaquette
order (need not be quasi-1d)
"Dual order parameter"

N. Read and S. Sachdev, Phys. Rev. Lett. **62**, 1694 (1989).

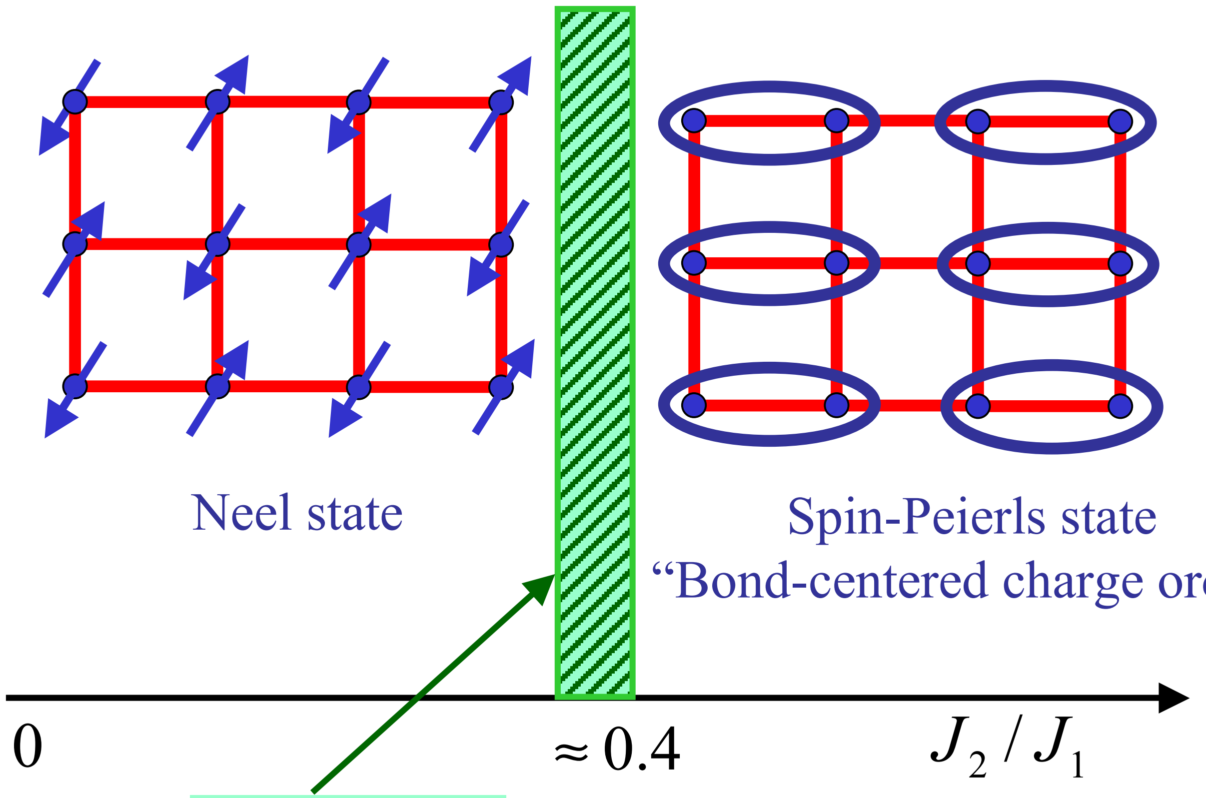
$$H = \sum_{i<j} J_{ij} \; \vec{S}_i \cdot \vec{S}_j$$

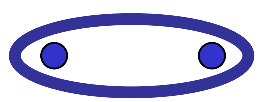
Square lattice with first (J_1) and second (J_2) neighbor exchange interactions

N. Read and S. Sachdev, Phys. Rev. Lett. **62**, 1694 (1989).

O. P. Sushkov, J. Oitmaa, and Z. Weihong, Phys. Rev. B **63**, 104420 (2001).

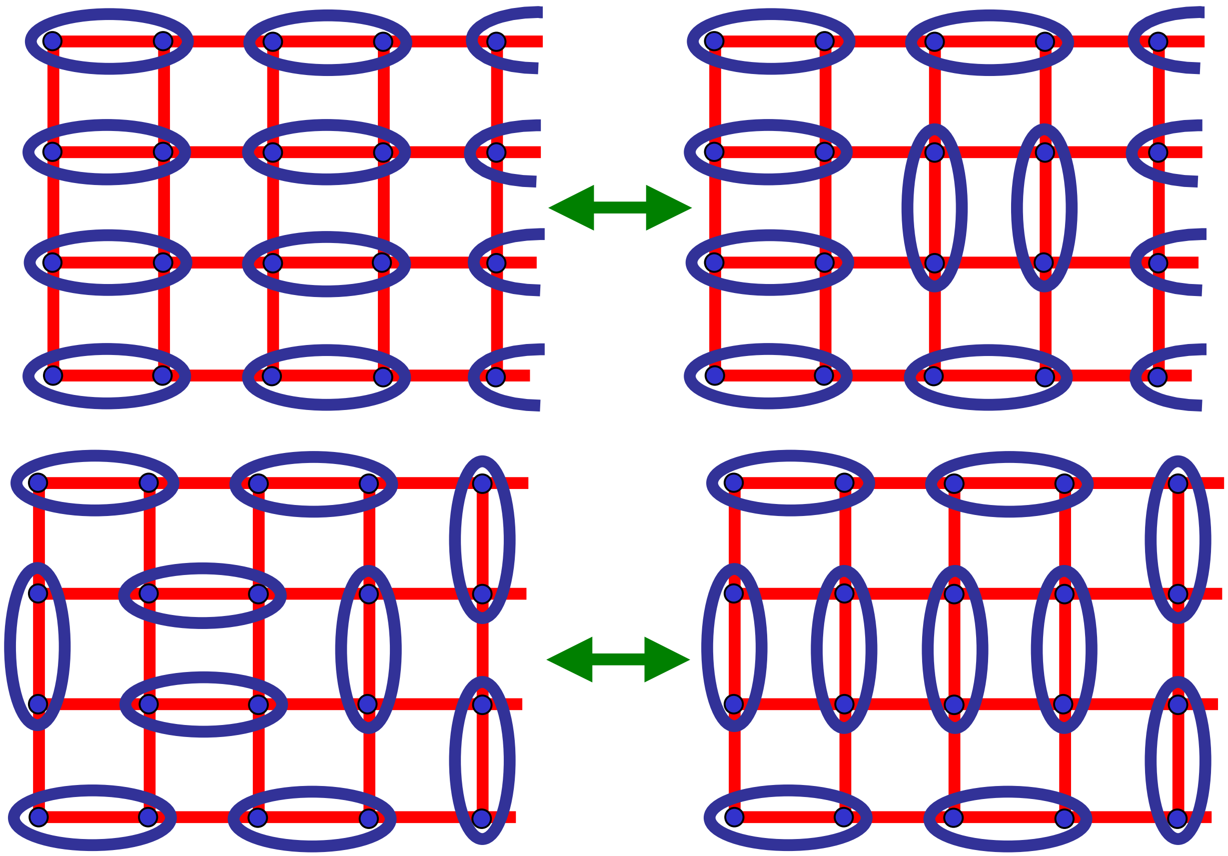
M.S.L. du Croo de Jongh, J.M.J. van Leeuwen, W. van Saarloos, Phys. Rev. B **62**, 14844 (2000).



 $= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$

Quantum dimer model –

D. Rokhsar and S. Kivelson Phys. Rev. Lett. **61**, 2376 (1988)



Quantum “entropic” effects prefer one-dimensional striped structures in which the largest number of singlet pairs can resonate. The state on the upper left has more flippable pairs of singlets than the one on the lower left.

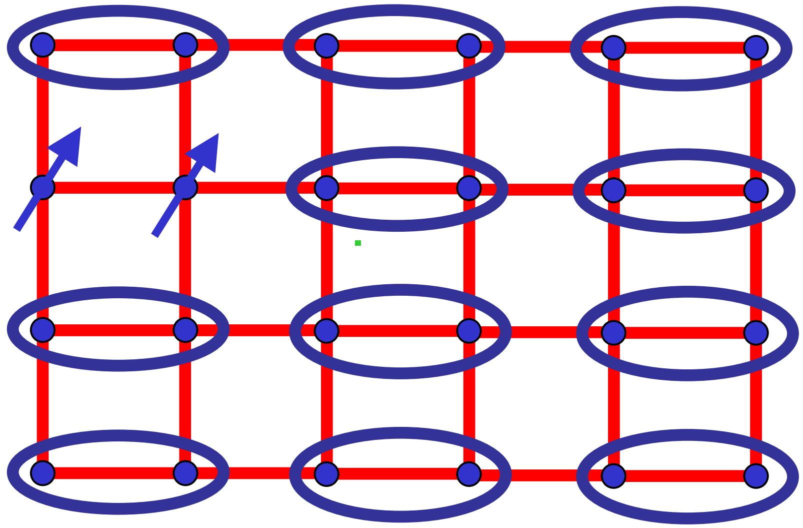
These effects lead to a broken square lattice symmetry near the transition to the Neel state.

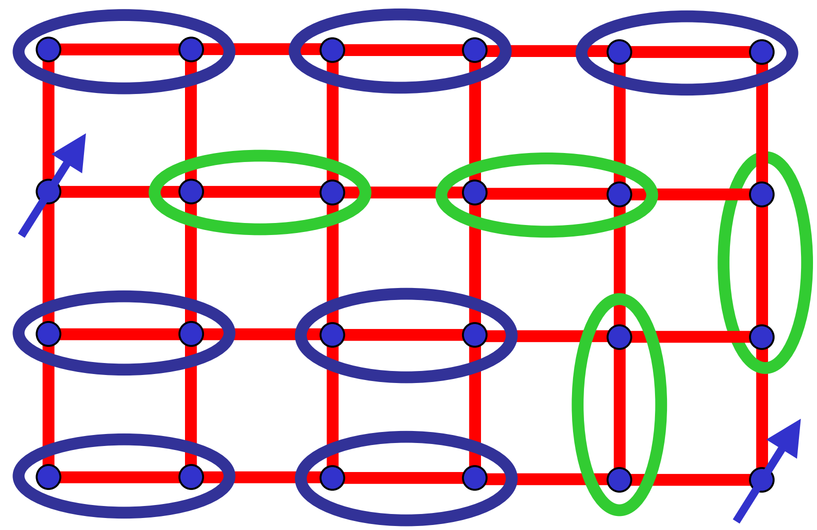
N. Read and S. Sachdev Phys. Rev. B **42**, 4568 (1990).

Properties of paramagnet with bond-charge-order

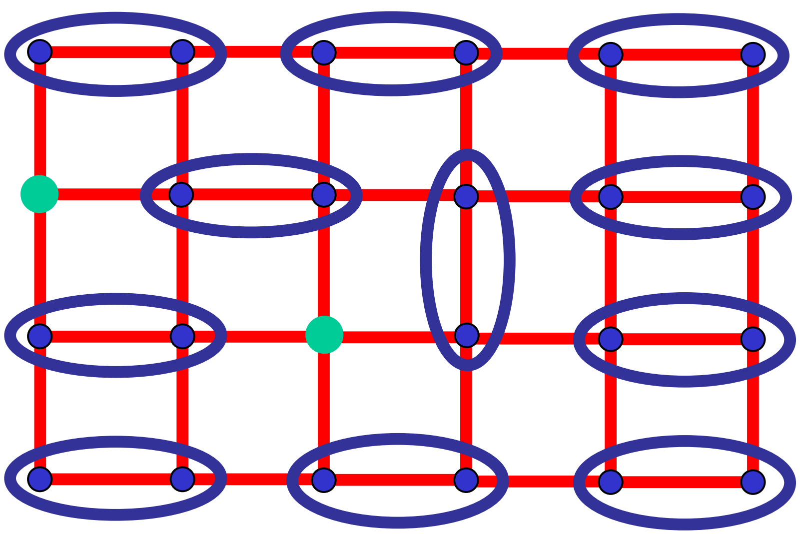
Stable S=1 spin exciton – quanta of 3-component ϕ_α

$$\varepsilon_k = \Delta + \frac{c_x^2 k_x^2 + c_y^2 k_y^2}{2\Delta} \qquad \Delta \rightarrow \text{ Spin gap}$$





S=1/2 spinons are ***confined*** by a linear potential.



S=0 holes are similarly ***confined in pairs***.

E. Fradkin and S. Kivelson, Mod. Phys. Lett B **4**, 225 (1990).
S. Sachdev and N. Read, Int. J. Mod. Phys. B **5**, 219 (1991).

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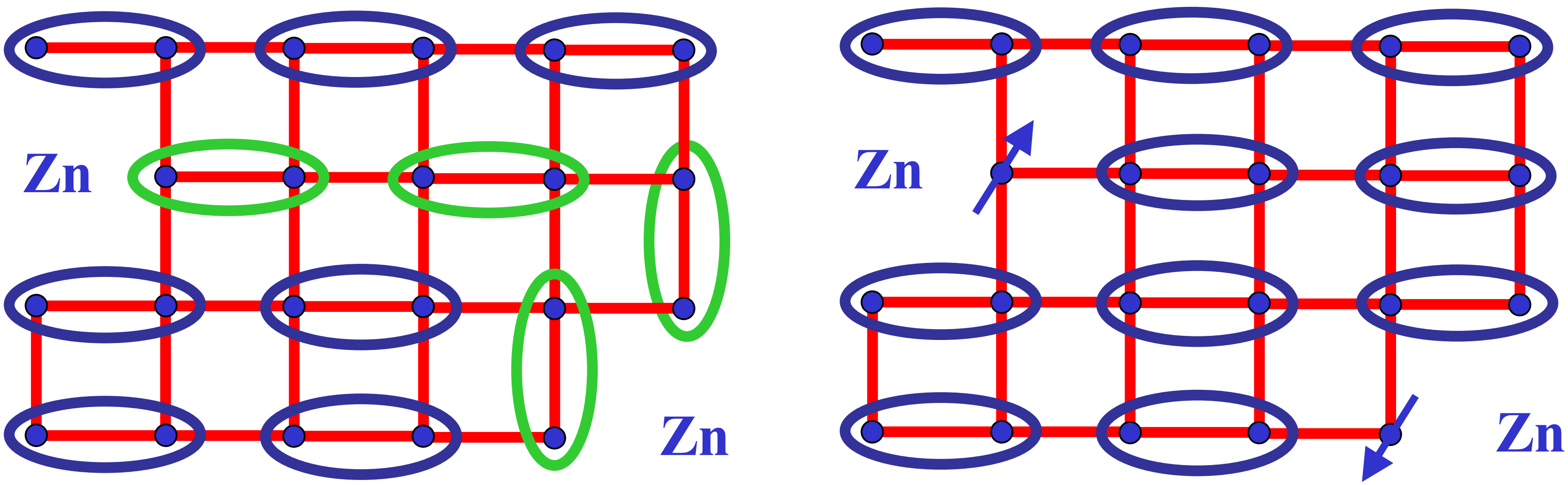
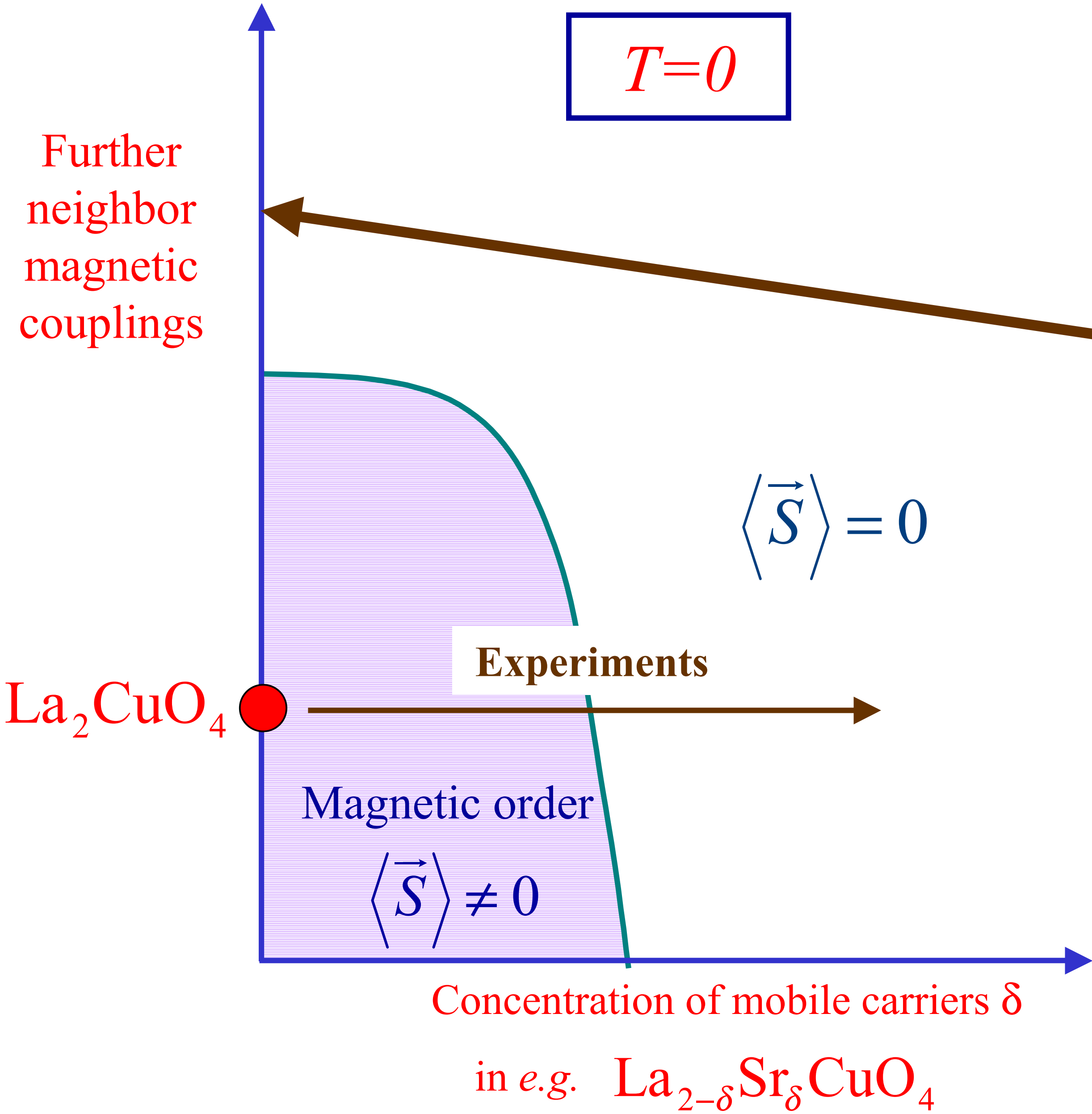
Effect of static non-magnetic impurities (Zn or Li)



Spinon confinement implies that free S=1/2 moments form near each impurity

$$\chi_{\text{impurity}}(T \to 0) = \frac{S(S+1)}{3k_B T}$$

A.M Finkelstein, V.E. Kataev, E.F. Kukovitskii, G.B. Teitel'baum, Physica C **168**, 370 (1990).



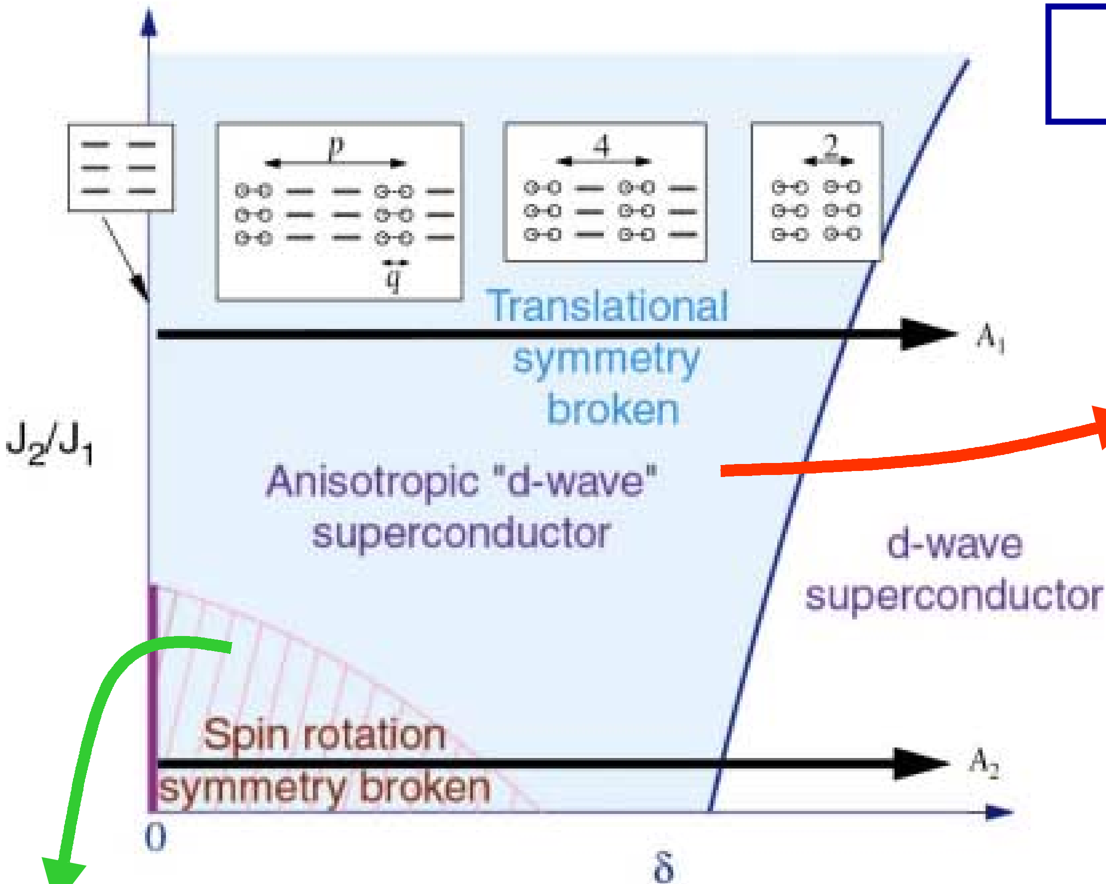
Summary:

Confined, paramagnetic Mott insulator has

1. Stable S=1 spin exciton ϕ_α .
2. Broken translational symmetry:- bond charge order.
3. Pairing of holes.
4. S=1/2 moments near non-magnetic impurities.

These properties survive in the superconductor for a finite range of δ

I.B Phase diagram for doping of confined Mott insulators



T=0

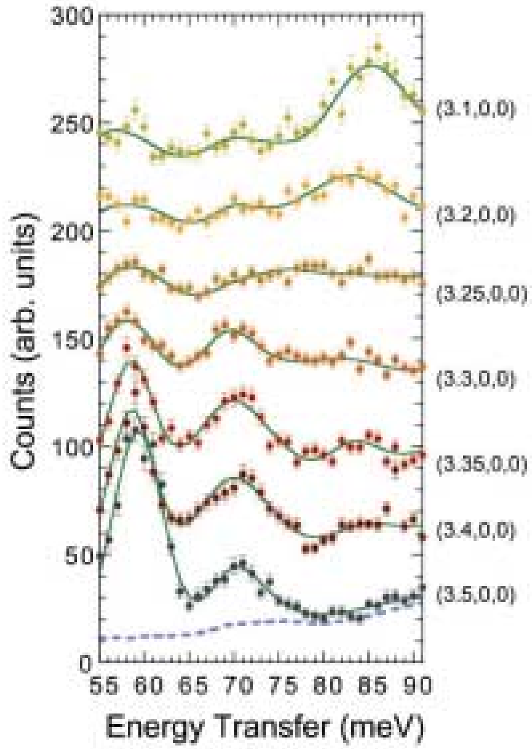
Superconductivity can coexist with *bond* charge order in region without magnetic order

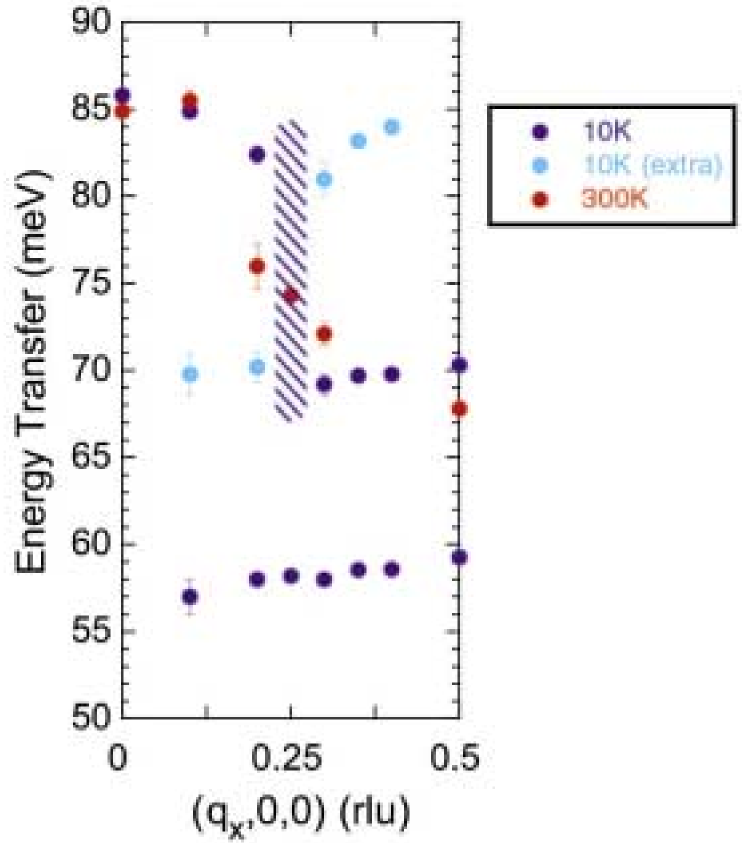
Site charge order likely in regions with magnetic order and weaker superconductivity

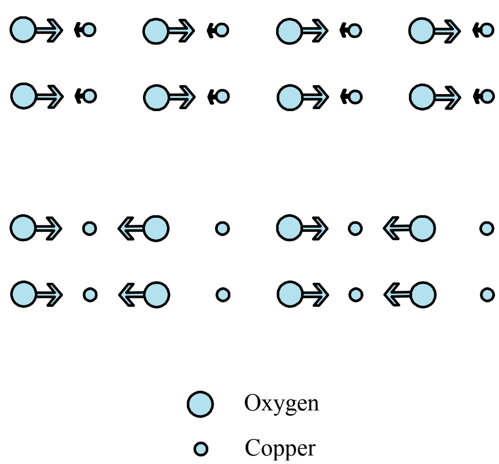
Talks by S. Kivelson, R. Eder, E. Altman, and E. Arrigoni

S. Sachdev and N. Read, Int. J. Mod. Phys. B **5**, 219 (1991).
M. Vojta and S. Sachdev, Phys. Rev. Lett. **83**, 3916 (1999).
M. Vojta, Y. Zhang, and S. Sachdev, Phys. Rev. B **62**, 6721 (2000).
K. Park and S. Sachdev, cond-mat/0104519.
See also J. Zaanen, Physica C **217**, 317 (1999),
S. Kivelson, E. Fradkin and V. Emery, Nature **393**, 550 (1998),
S. White and D. Scalapino, Phys. Rev. Lett. **80**, 1272 (1998).
C. Lannert, M.P.A. Fisher, and T. Senthil, cond-mat/0007002.

Neutron scattering measurements of phonon spectra





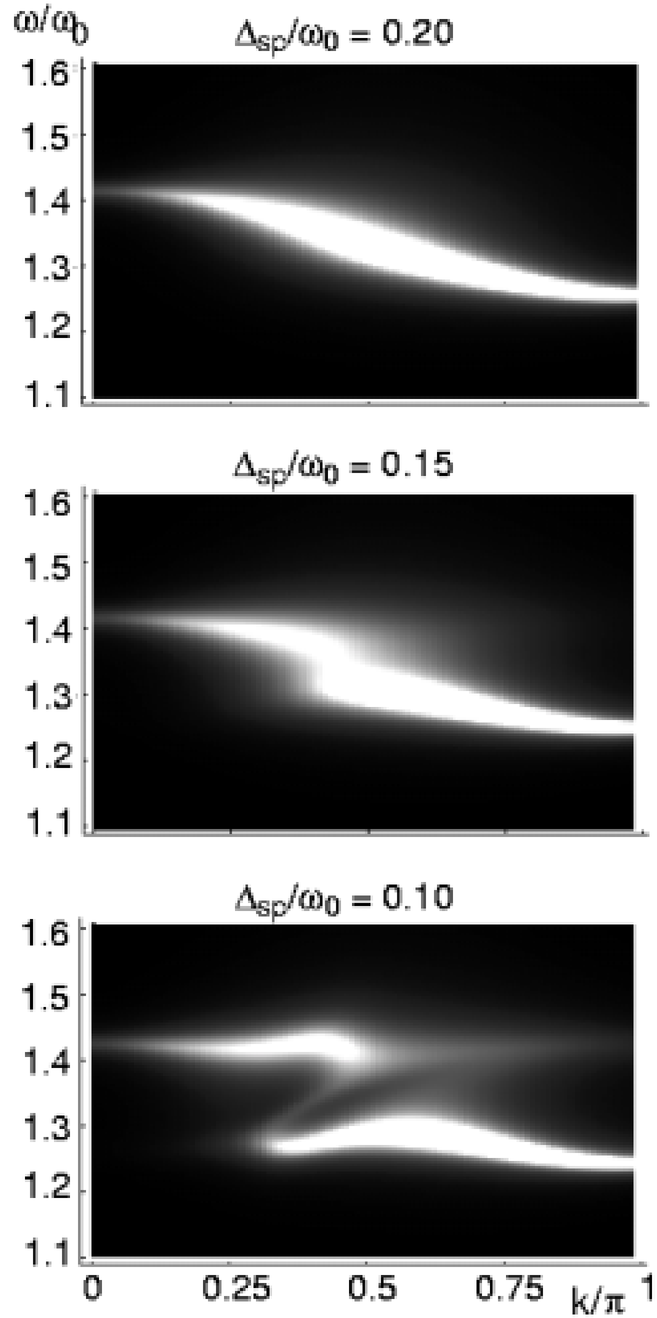


Neutron scattering measurements of phonon spectrum of superconducting $La_{1.85}Sr_{0.15}CuO_4$ by R. J. McQueeney, Y. Petrov, T. Egami, M. Yethiraj, G. Shirane, and Y. Endoh, Phys. Rev. Lett. **82**, 628 (1999).

Also by L. Pintschovius and M. Braden, Phys. Rev. B **60**, R15039 (1999).

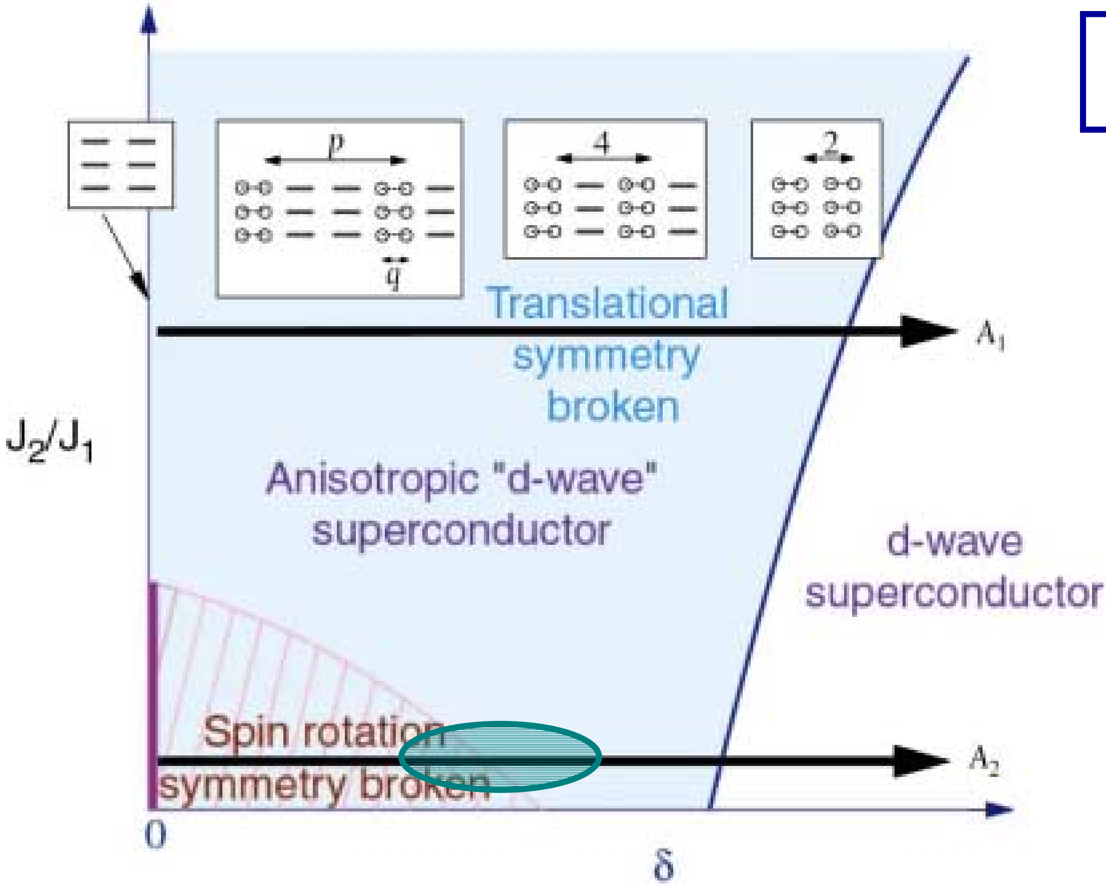
Computation of phonon damping by non-linear coupling to fluctuating spin-Peierls mode.

K. Park and S. Sachdev, cond-mat/0104519

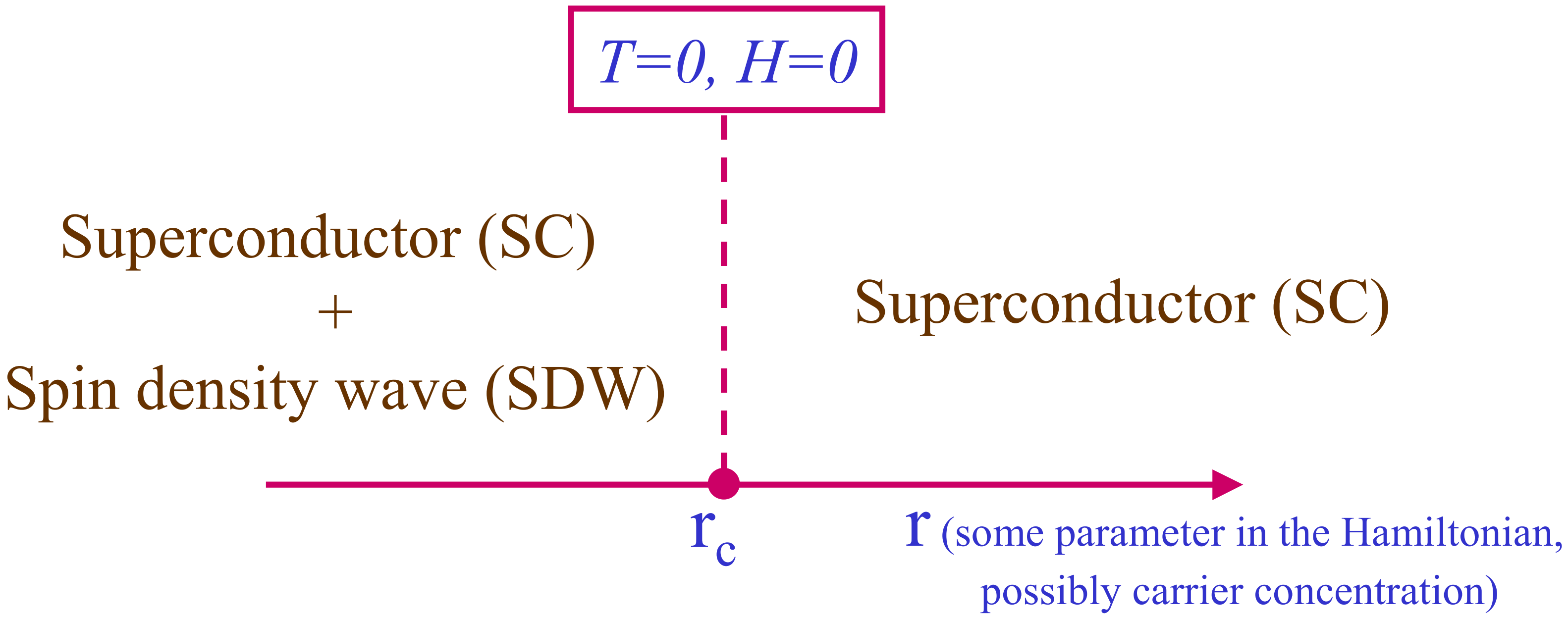


I.B Phase diagram for doping of confined Mott insulators

T=0



II. Effect of magnetic field on SDW order in SC phase



Many experimental indications that the cuprate superconductors are not too far from such a quantum phase transition:

G. Aeppli, T.E. Mason, S.M. Hayden, H.A. Mook, J. Kulda, Science **278**, 1432 (1997).
Y. S. Lee, R. J. Birgeneau, M. A. Kastner *et al.,* Phys. Rev. B **60**, 3643 (1999).
S. Katano, M. Sato, K. Yamada, T. Suzuki, and T. Fukase Phys. Rev. B **62**, 14677 (2000).
B. Lake, G. Aeppli *et al*., Science to appear.
Y. Sidis, C. Ulrich, P. Bourges, *et al.*, cond-mat/0101095.
H. Mook, P. Dai, F. Dogan, cond-mat/0102047.
J.E. Sonier *et al*., preprint.

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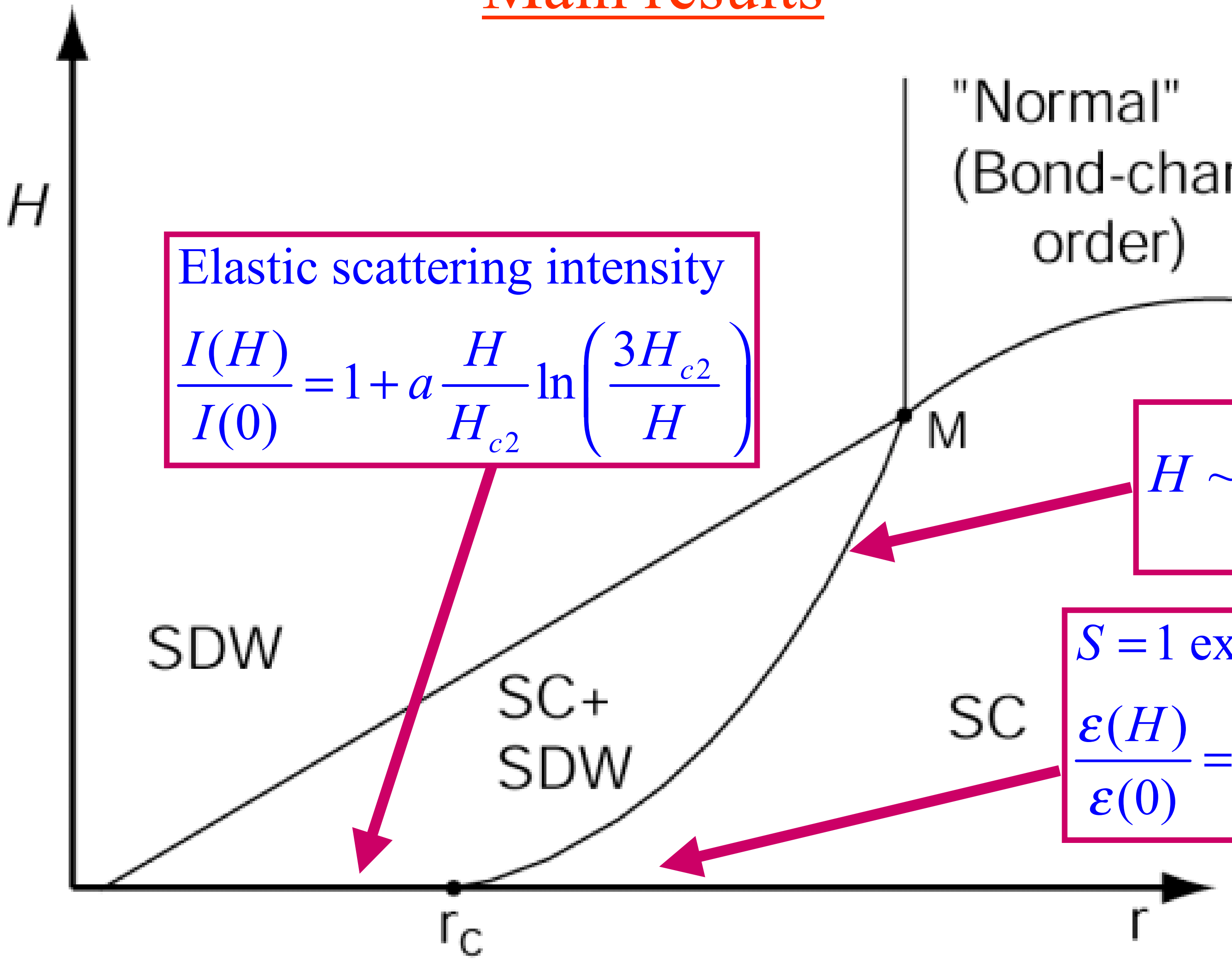
Main results

(extreme Type II superconductivity)

T=0



Elastic scattering intensity

$$\frac{I(H)}{I(0)} = 1 + a\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$$

$$H \sim \frac{(r - r_c)}{\ln\left(1/(r - r_c)\right)}$$

$S = 1$ exciton energy

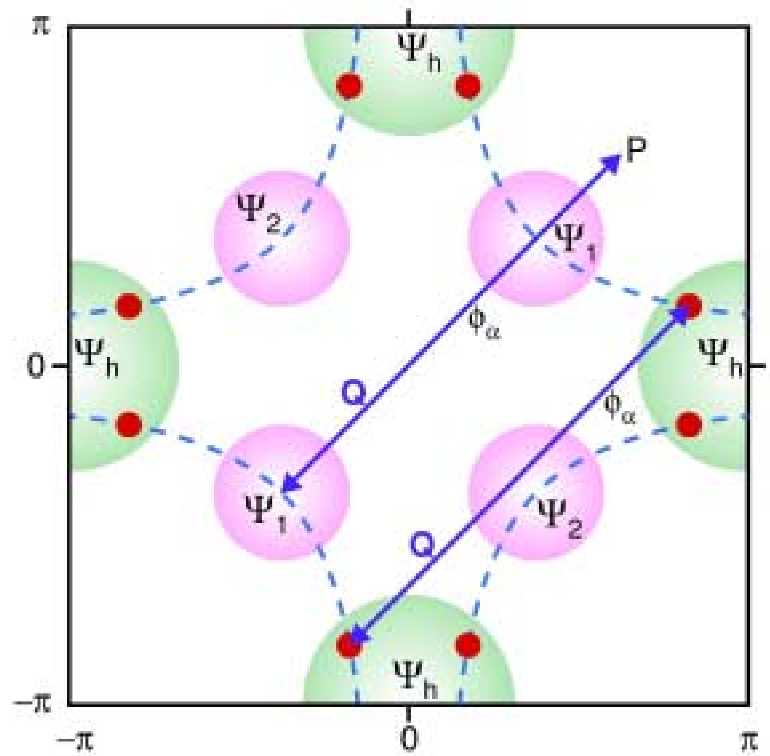
$$\frac{\varepsilon(H)}{\varepsilon(0)} = 1 - b\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$$

- All functional forms are **exact**.
- Similar results apply to other competing orders *e.g.* SC + staggered flux

E. Demler, S. Sachdev, and Y. Zhang, cond-mat/0103192.

Structure of quantum theory

• Charge-order is not critical: can neglect Berry phases.



• Generically, momentum conservation prohibits decay of $S=1$ exciton ϕ_α into $S=1/2$ fermionic excitations at low energies. Virtual pairs of fermions only renormalize parameters in the effective action for ϕ_α .

• Zeeman coupling only leads to corrections at order H^2

• Simple Landau theory couplings between ϕ_α and superconducting order ψ are allowed (S.-C. Zhang, Science **275**, 1089 (1997)), *e.g.*:

$$V\left(\phi_\alpha^2\right) \to V\left(\phi_\alpha^2\right) + \lambda \phi_\alpha^2 \left|\psi\right|^2$$

$$S_b = \int d^2x d\tau \left[\frac{1}{2} \left(\left(\nabla_x \phi_\alpha\right)^2 + c^2 \left(\partial_\tau \phi_\alpha\right)^2 \right) + V\left(\phi_\alpha^2\right) \right]$$

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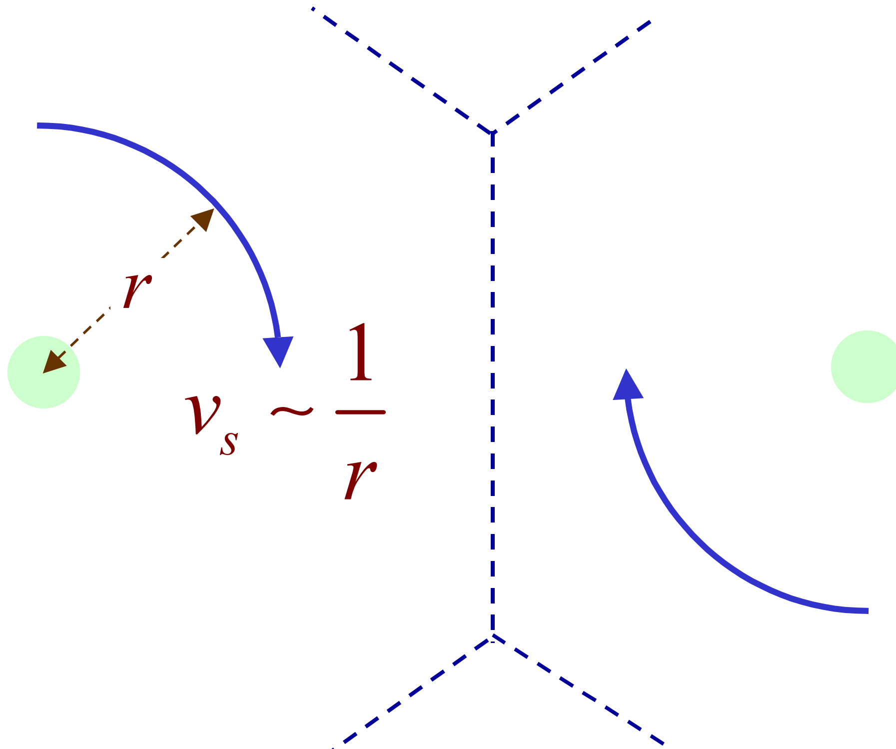
Dominant effect: **uniform** softening of spin excitations by superflow kinetic energy



Spatially averaged superflow kinetic energy

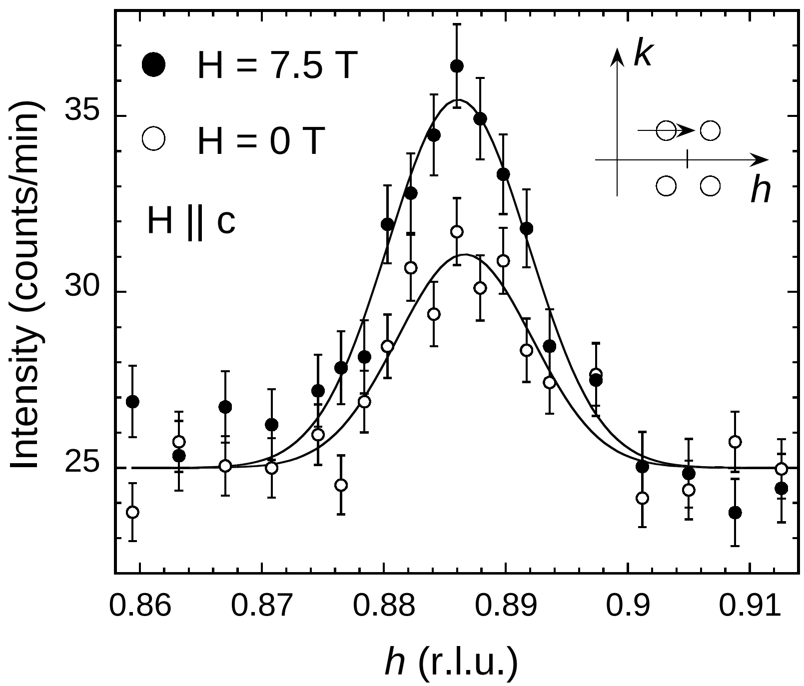
$$\sim \left\langle v_s^2 \right\rangle \sim \frac{H}{H_{c2}} \ln \frac{3H_{c2}}{H}$$

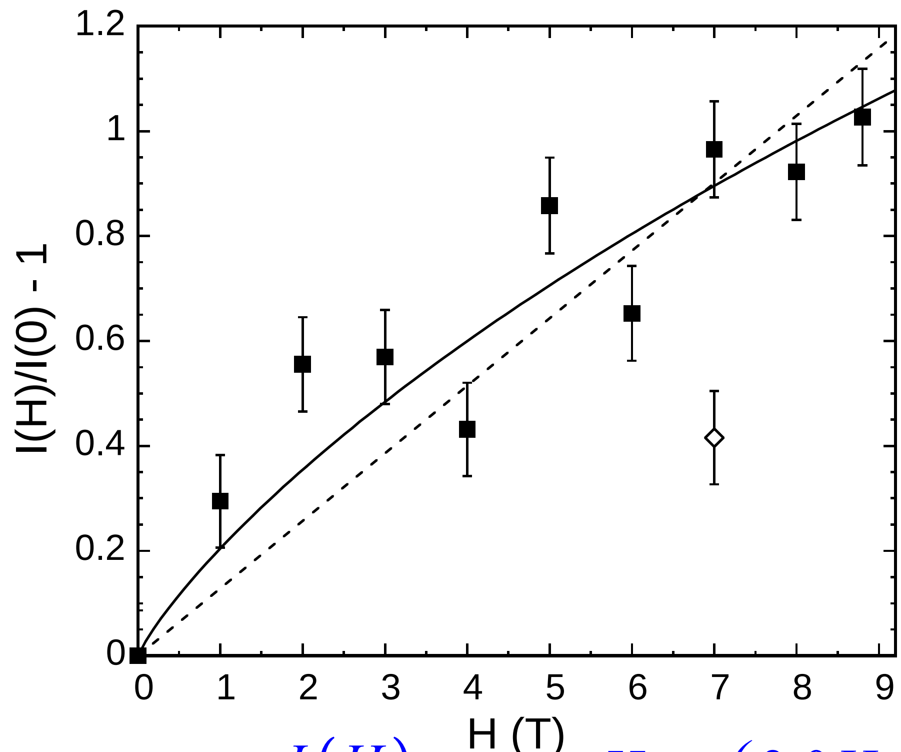
See D. P. Arovas *et al*., Phys. Rev. Lett. **79**, 2871 (1997) for a different viewpoint.

Elastic neutron scattering off La_2CuO_{4+y}

B. Khaykovich, Y. S. Lee, S. Wakimoto, K. J. Thomas,

M. A. Kastner, and R.J. Birgeneau, preprint.





Solid line --- fit to : $\dfrac{I(H)}{I(0)} = 1 + a\dfrac{H}{H_{c2}}\ln\left(\dfrac{3.0H_{c2}}{H}\right)$

a is the only fitting parameter

Best fit value - a = 2.4 with H_{c2} = 60 T

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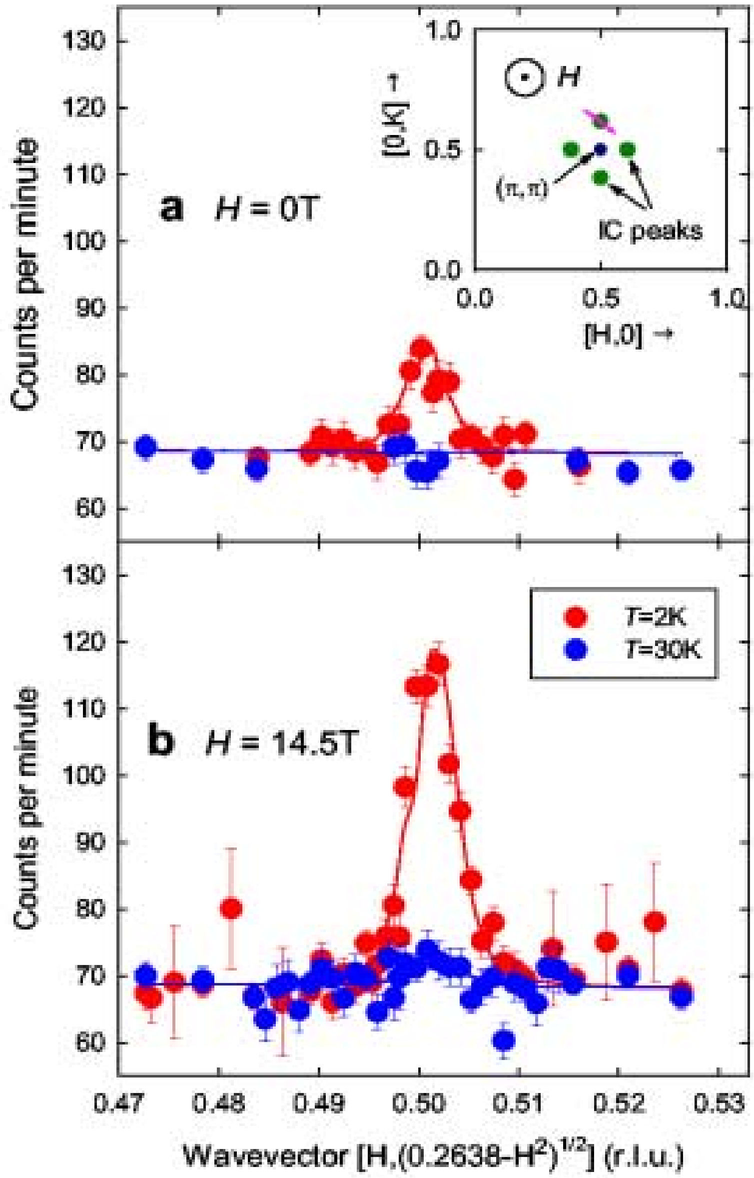
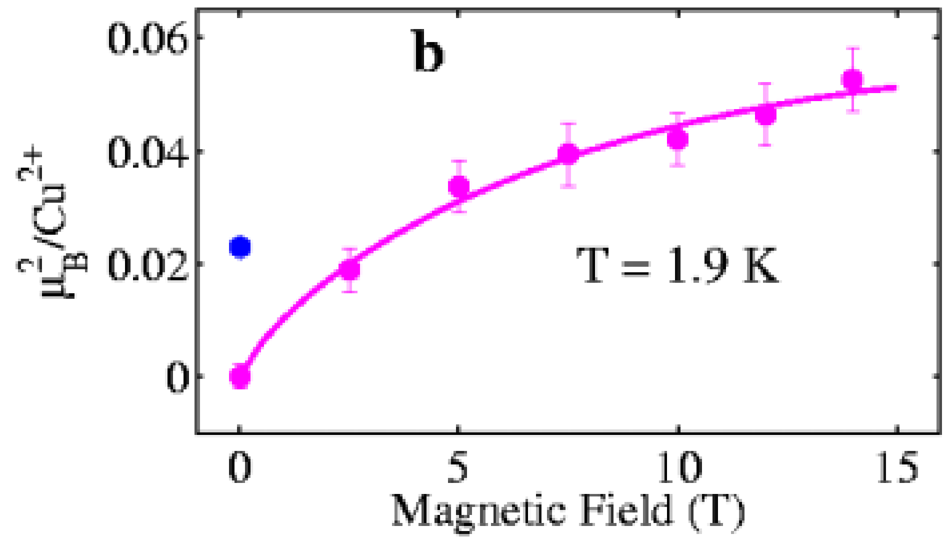
Neutron scattering of $La_{2-x}Sr_xCuO_4$ at x=0.1

B. Lake, G. Aeppli, *et al.*

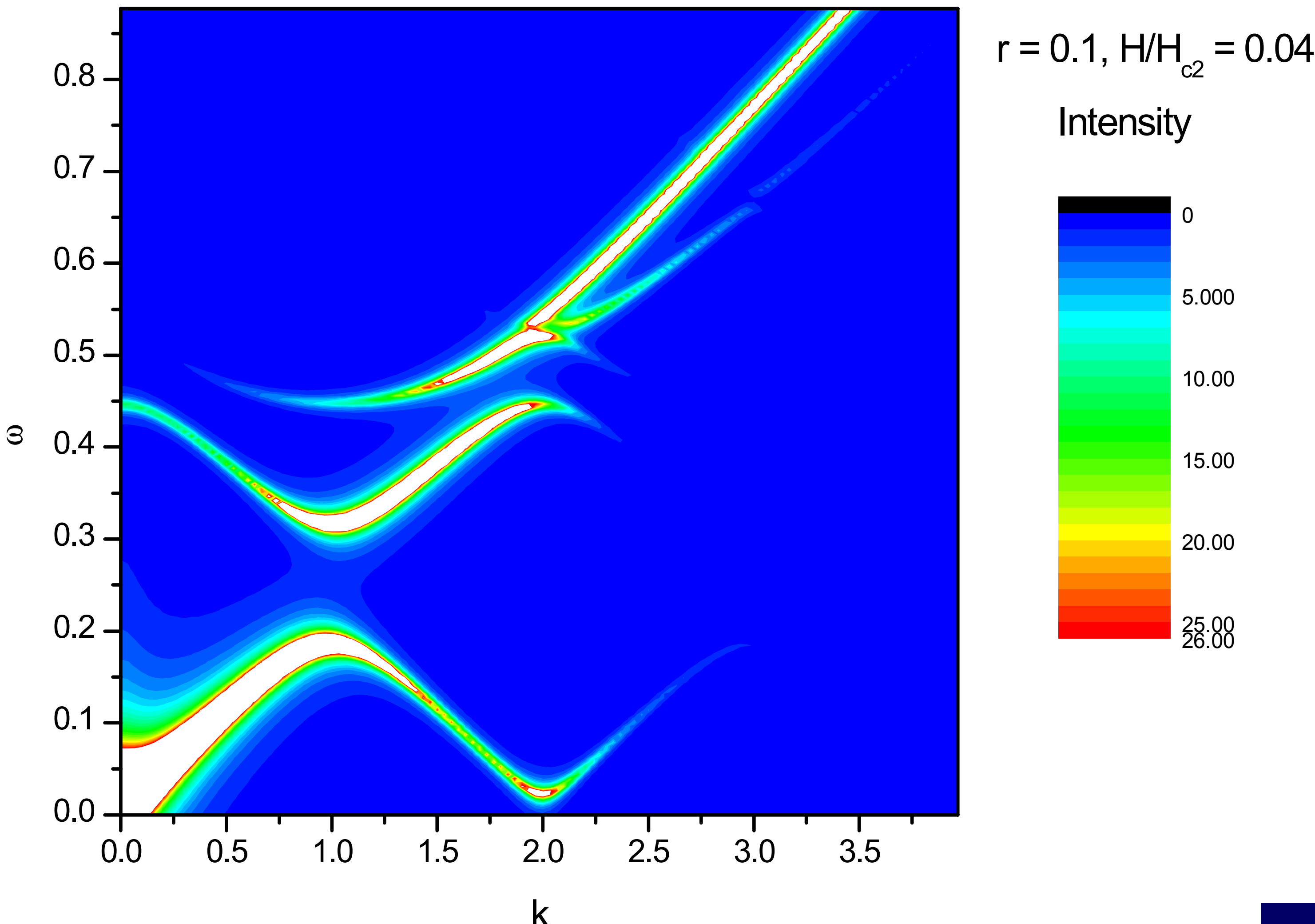




Solid line - fit to : $I(H) = a\frac{H}{H_{c2}}\ln\left(\frac{H_{c2}}{H}\right)$

Presence of vortex lattice leads to supermodulation in the spin exciton spectrum

Computation of spin susceptibility $\chi''(k,\omega)$ in self-consistent large N theory of ϕ_α fluctuations in a vortex lattice



2π / (vortex lattice spacing)

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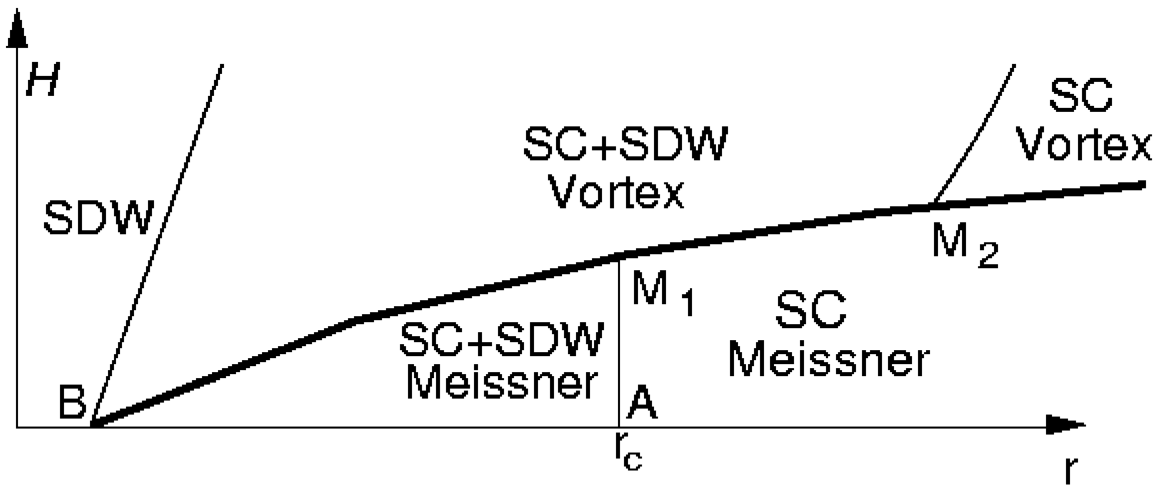
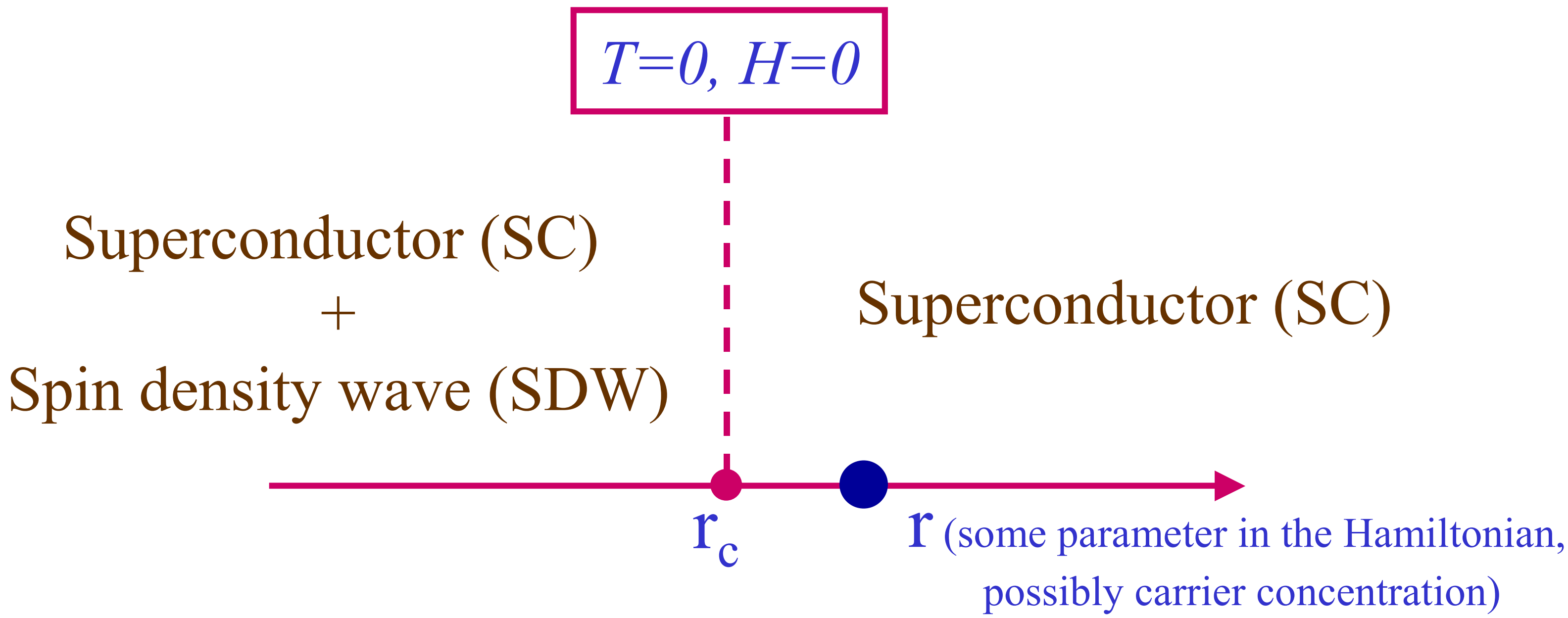
Consequences of a finite London penetration depth (finite κ)

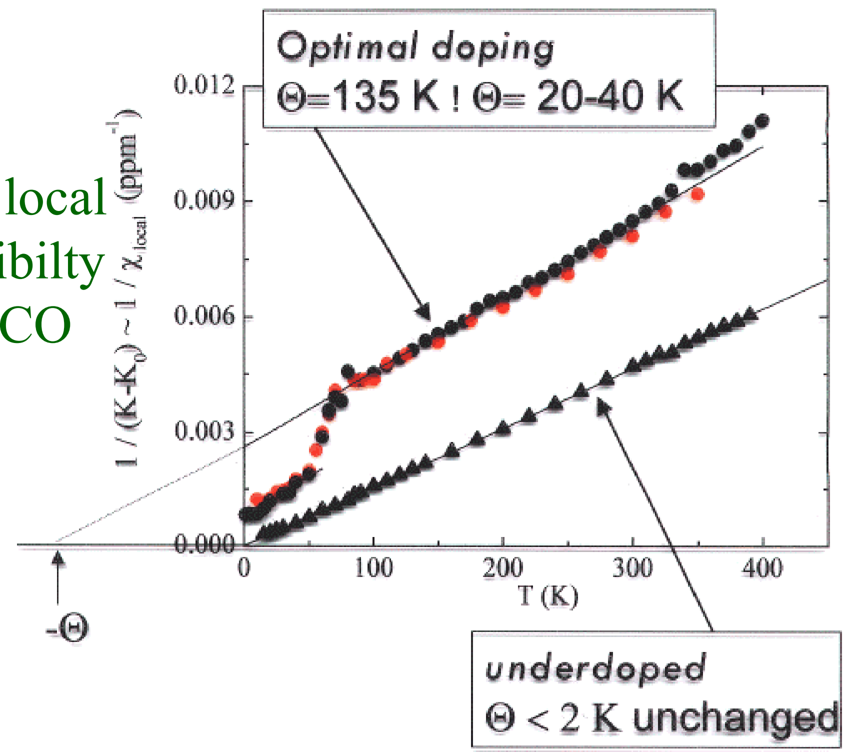


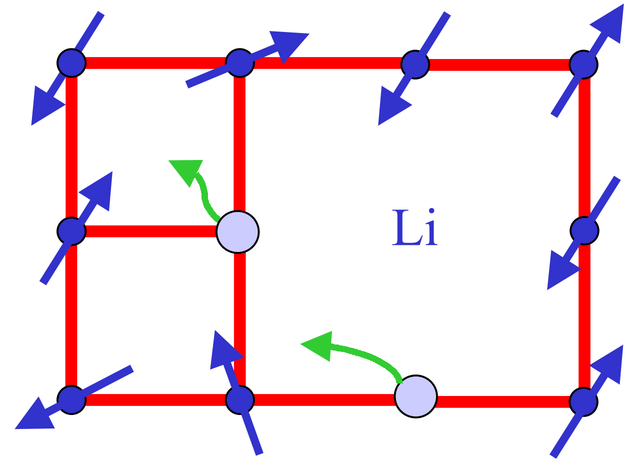
III. Effect on Zn/Li impurities on *S=1* spin exciton



Measurement of spin susceptibility near non-magnetic (Zn/Li) impurities

Inverse local susceptibilty in YBCO





^{7}Li NMR below T_c

J. Bobroff, H. Alloul, W.A. MacFarlane, P. Mendels, N. Blanchard, G. Collin, and J.-F. Marucco, Phys. Rev. Lett. **86**, 4116 (2001)

See also D. L. Sisson, S. G. Doettinger, A. Kapitulnik, R. Liang, D. A. Bonn, and W. N. Hardy, Phys. Rev. B **61**, 3604 (2000).

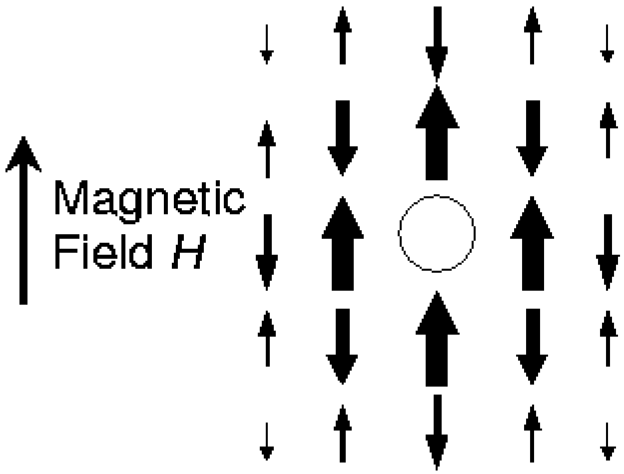
Measured $\chi_{\text{impurity}}(T \to 0) = \dfrac{S(S+1)}{3k_B T}$ with $S = 1/2$ in underdoped sample.

Not expected from BCS theory, which predicts $\chi_{\text{impurity}}(T \to 0) \neq \infty$ for a non-magnetic impurity with strong potential scattering.

Zn impurity in $YBa_2Cu_3O_{6.7}$

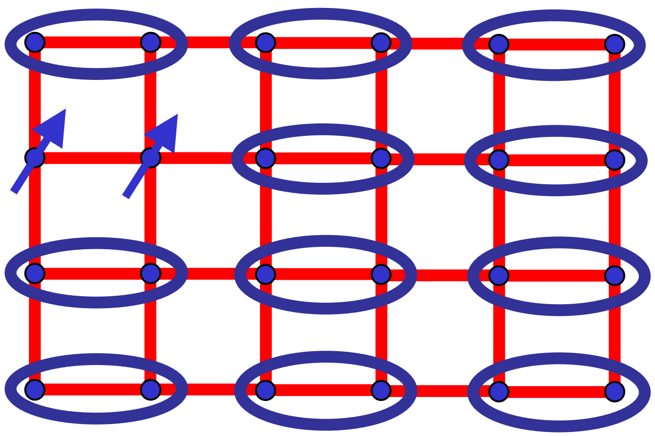
Moments measured by analysis of Knight shifts

M.-H. Julien, T. Feher, M. Horvatic, C. Berthier, O. N. Bakharev, P. Segransan, G. Collin, and J.-F. Marucco, Phys. Rev. Lett. **84**, 3422 (2000); also earlier work of the group of H. Alloul and the original experiment of A.M Finkelstein, V.E. Kataev, E.F. Kukovitskii, and G.B. Teitel'baum, Physica C **168**, 370 (1990).



Berry phases of precessing spins do not cancel between the sublattices in the vicinity of the impurity: net uncancelled phase of S=1/2

<u>*S=1* spin exciton mode in YBCO</u>



H.F. Fong, B. Keimer, D. Reznik, D.L. Milius, and I.A. Aksay, Phys. Rev. B **54**, 6708 (1996)

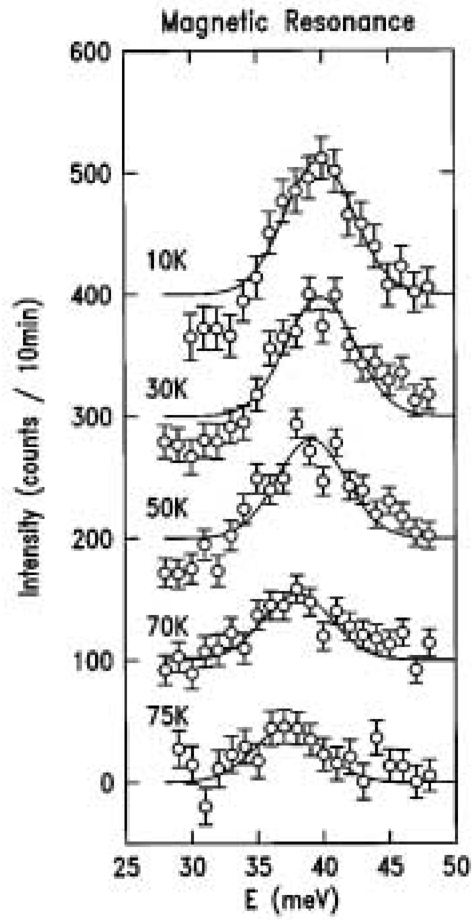


FIG. 8. Unpolarized beam, constant-**Q** data [**Q**=(3/2,1/2,−1.7)] of the 40 meV magnetic resonance obtained by subtracting the signal below T_c from the T=100 K background. The lines are fits to Gaussians, as described in the text. For clarity successive scans are offset by 100.

Spin-1 collective mode in $YBa_2Cu_3O_7$- little observable damping at low T.

Coupling to superconducting quasiparticles unimportant

Resolution limited width

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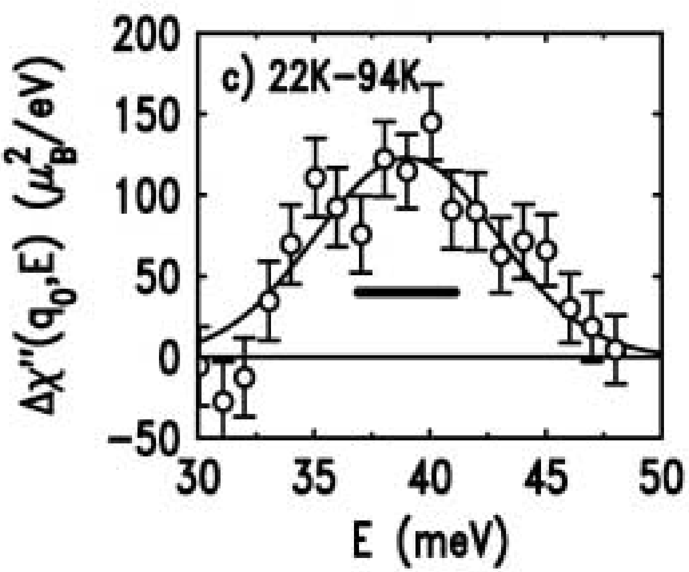
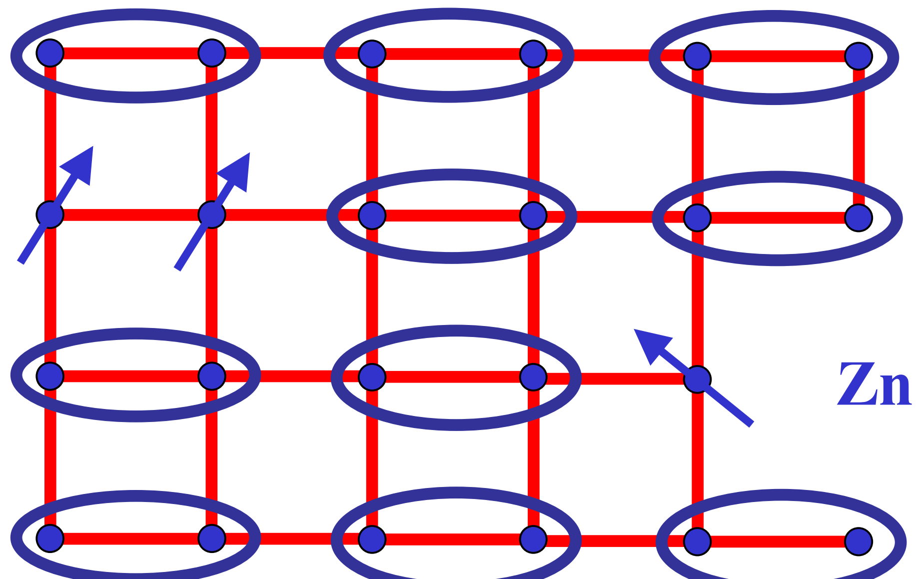
$YBa_2Cu_3O_7 + 0.5\%$ Zn

H. F. Fong, P. Bourges, Y. Sidis, L. P. Regnault, J. Bossy, A. Ivanov, D.L. Milius, I. A. Aksay, and B. Keimer, Phys. Rev. Lett. **82**, 1939 (1999)



Zn induced half-width = 4.25 meV



Quantum field theory for S=1 resonance in the presence of a non-magnetic impurity

Orientation of "impurity" spin -- $n_\alpha(\tau)$ (unit vector)

Action of "impurity" spin

$$S_{\text{imp}} = \int d\tau \left[iSA_\alpha(n)\frac{dn_\alpha}{d\tau} - \gamma S n_\alpha(\tau)\phi_\alpha(x=0,\tau) \right]$$

$A_\alpha(n)$ → Dirac monopole function

Boundary quantum field theory: $\mathcal{S}_b + \mathcal{S}_{\text{imp}}$

Recall -

$$\mathcal{S}_b = \int d^2x d\tau \left[\frac{1}{2}\left((\nabla_x \phi_\alpha)^2 + c^2(\partial_\tau \phi_\alpha)^2 + r\phi_\alpha^2 \right) + \frac{g}{4!}\left(\phi_\alpha^2\right)^2 \right]$$

Renormalization group analysis: g and γ reach non-zero fixed point values

"Swiss cheese" model

Inverse Q of resonance = $C\ n_{\text{imp}} \xi^2$; Linewidth $\Gamma = C\ n_{\text{imp}} \dfrac{(\hbar c)^2}{\Delta}$

$C \rightarrow$ universal number

$\xi \rightarrow$ spin correlation length which diverges at the onset of SDW order

Result also holds near SDW transitions in Mott insulators

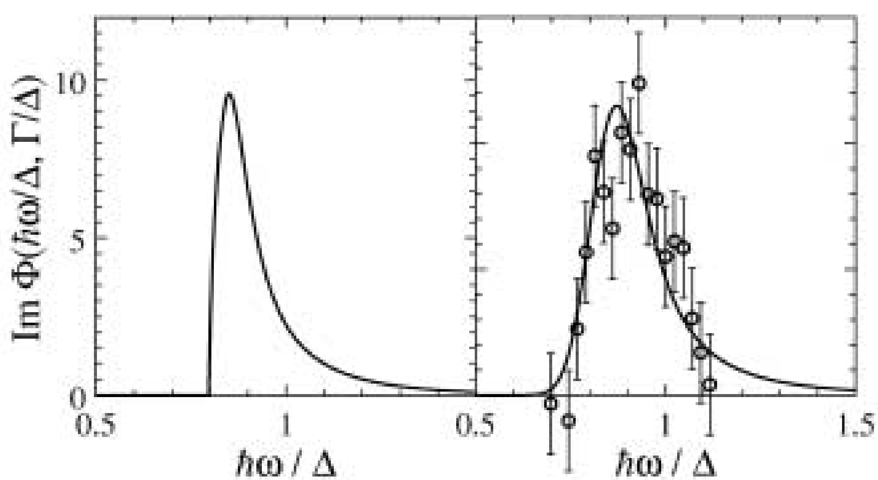
$n_{\text{imp}} = 0.005$

$\Delta = 40$ meV

$\hbar c = 0.2$ eV

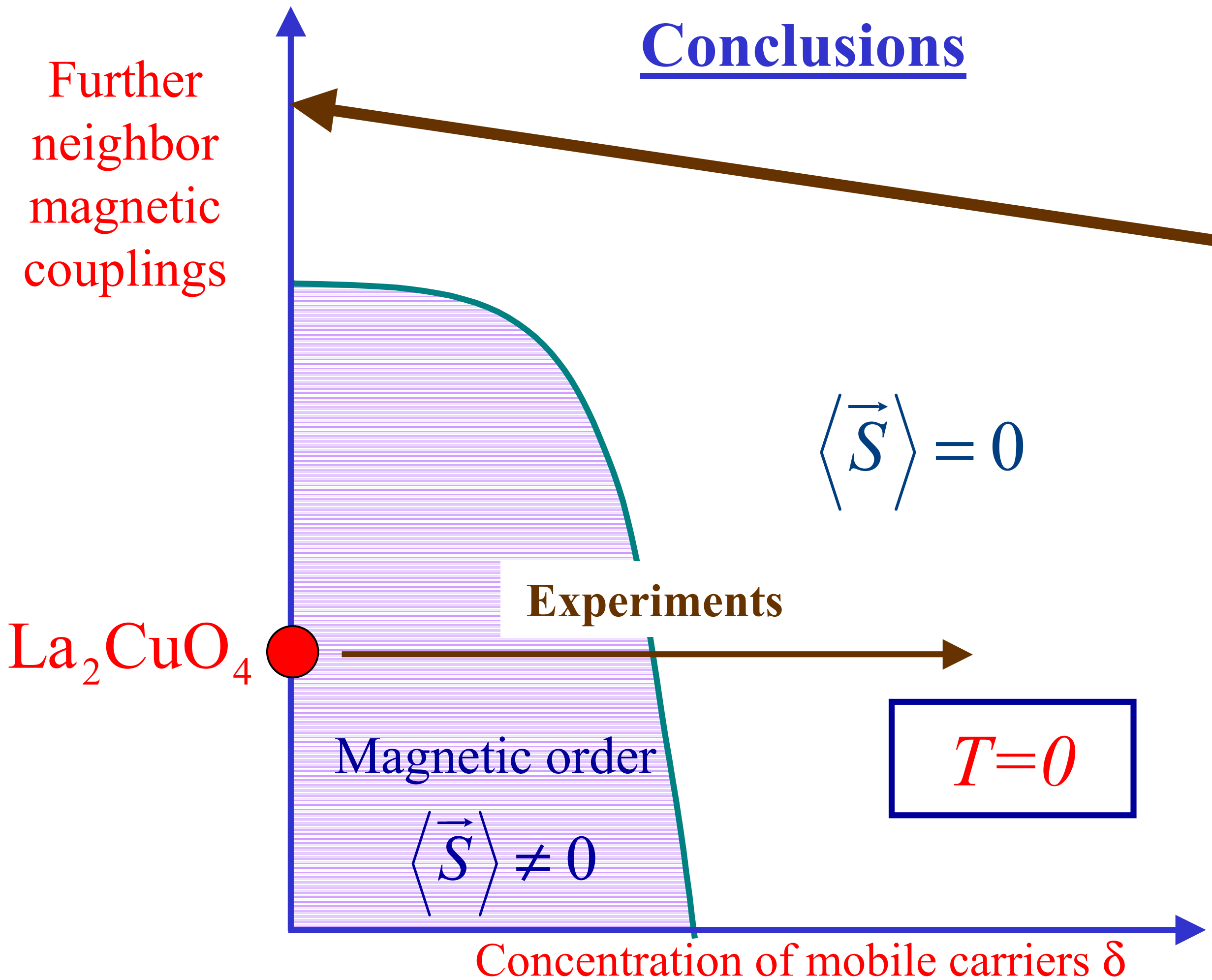
$\Rightarrow \quad \Gamma = 5$ meV, $\Gamma/\Delta = 0.125$

Quoted half-width = 4.25 meV



M. Vojta, C. Buragohain, and S. Sachdev, Phys. Rev. B **61**, 15152 (2000)

Conclusions



Confined, paramagnetic Mott insulator has

1. Stable S=1 spin exciton ϕ_α .
2. Broken translational symmetry:- bond-centered charge order.
3. Pairing of holes.
4. S=1/2 moments near non-magnetic impurities

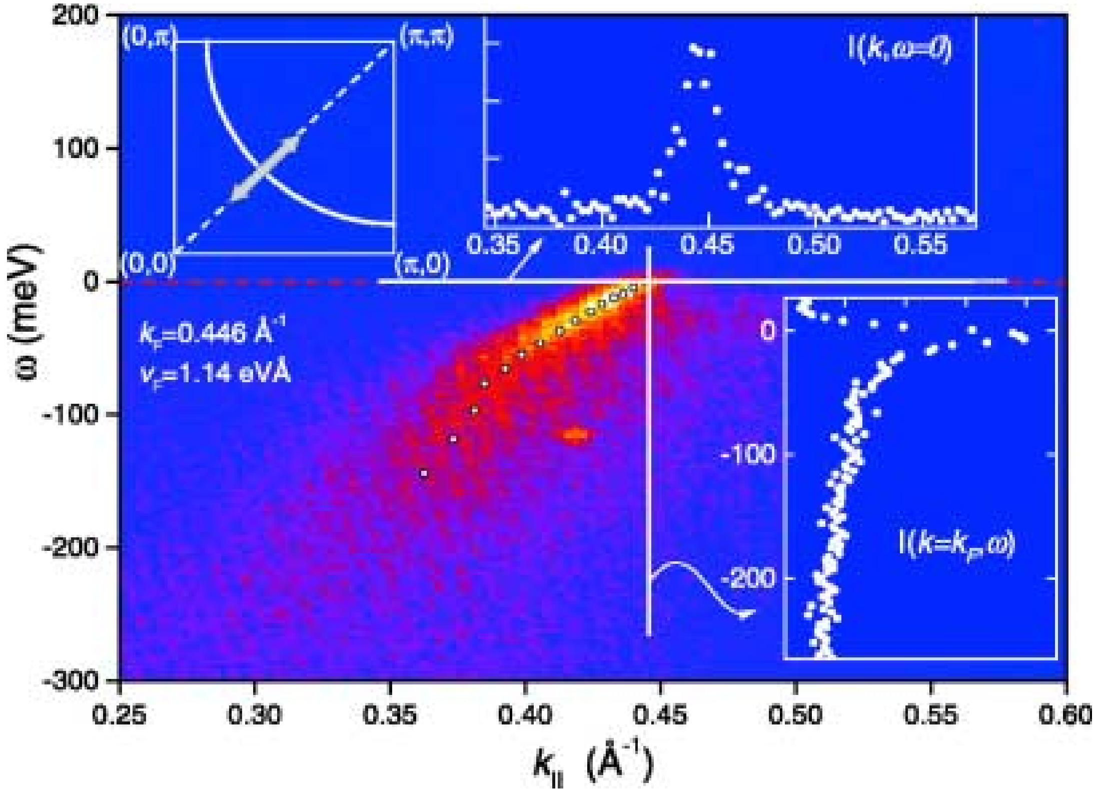
Theory of magnetic ordering quantum transitions in antiferromagnets and superconductors leads to quantitative theories for

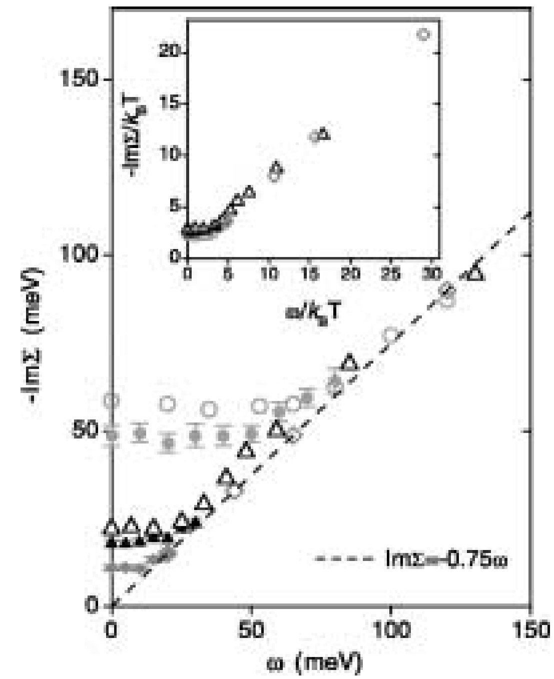
- Spin correlations in a magnetic field
- Effect of Zn/Li impurities on collective spin excitations

V. Damping of Nodal Quasiparticles

Photoemission on BSSCO

(Valla et al Science **285**, 2110 (1999))





Goal: Classify theories in which, with minimal fine tuning, a *d*-wave superconductor has a fermionic quasiparticle momentum distribution curve (MDC), at the nodal points, with a width proportional to $k_B T$

In a Fermi liquid, MDC width $\sim T^2$

In a BCS d-wave superconductor, MDC width $\sim T^3$

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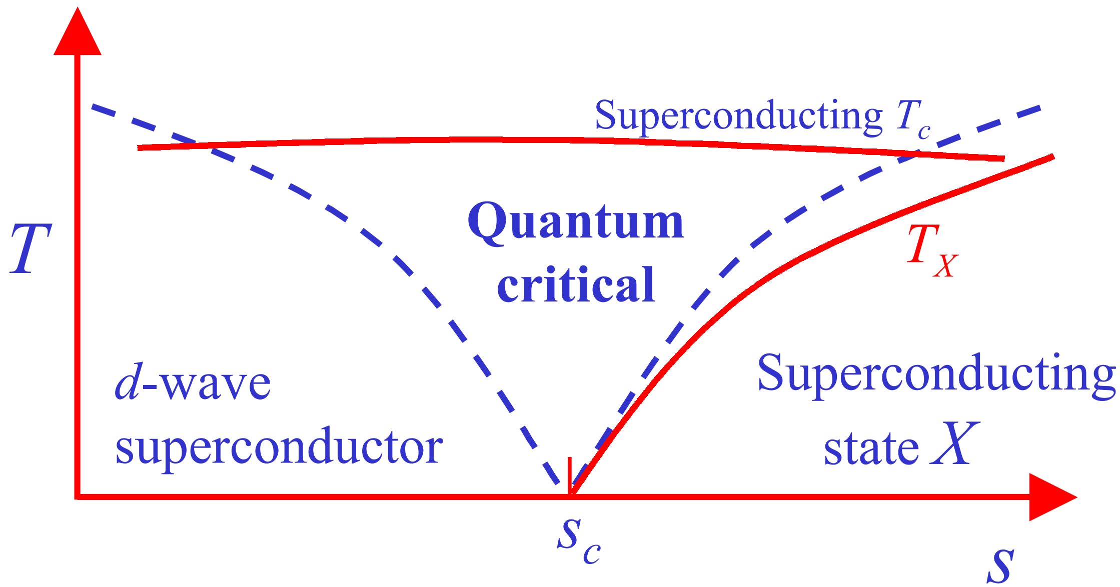
Proximity to a quantum-critical point



doping ⟷ ?

S. Sachdev and J. Ye, Phys. Rev. Lett. **69**, 2411 (1992).
M. Vojta, Y. Zhang, and S. Sachdev, Phys. Rev. Lett. **85**, 4940 (2000).

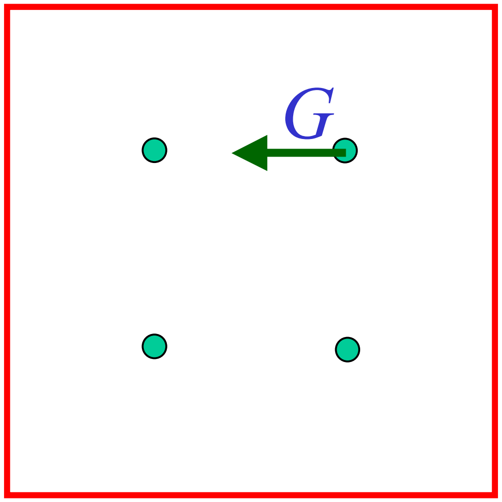
Necessary conditions

1. Quantum-critical point should be below its upper-critical dimension and obey hyperscaling.
2. Nodal quasi-particles should be part of the critical-field theory.
3. Critical field theory should not be free – required to obtain damping in the scaling limit.

A spin-singlet, fermion bilinear, zero momentum order parameter for *X* is preferred.

An order parameter with momentum *G*:
Charge (or spin) density-wave order

$$\delta\rho \sim \mathrm{Re}\left[\Phi_x e^{iGx} + \Phi_y e^{iGy}\right]$$



If *G* does not connect two nodal points, fermions are not part of the critical theory

Order parameter for X should be a component of

$$\Delta_k = \langle c_{k\uparrow} c_{-k\downarrow} \rangle \text{ (fermion pairing)}$$

or

$$A_k = \langle c^{\dagger}_{k\alpha} c_{k\alpha} \rangle \text{ (excitonic order)}$$

Complete group-theoretic classification

X has $d_{x^2-y^2}$ pairing plus

(A) is pairing → fermion spectrum fully gapped

(B) id_{xy} pairing → fermion spectrum fully gapped

(C) ig pairing

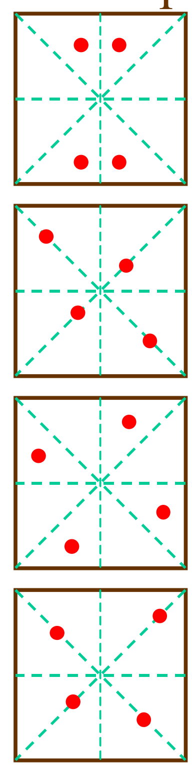
Nodal points

(D) s pairing → superconducting nematics

(E) d_{xy} excitons → superconducting nematics

(F) d_{xy} pairing

(G) p excitons



Main results

Only cases

(A) $d_{x^2-y^2} \Leftrightarrow d_{x^2-y^2} + is$ pairing and

(B) $d_{x^2-y^2} \Leftrightarrow d_{x^2-y^2} + id_{xy}$ pairing

have renormalization group fixed points with a non-zero interaction strength between the bosonic order parameter mode and the nodal fermions.

Only cases (A) and (B) satisfy conditions 1,2,3

Transition to d_{xy} pairing is expected with increasing J_2

M. Vojta, Y. Zhang, and S. Sachdev, Phys. Rev. Lett. **85**, 4940 (2000).