Stringing together the quantum phases of matter

Talk online: sachdev.physics.harvard.edu
The phases of matter:
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Solids  Liquids  Gases
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Solids   Liquids   Gases
Theory of the phases of matter:
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1. Matter is made of atoms

Democritus (4th century B.C.)
Theory of the phases of matter:

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Acharya Kanad (6th century B.C.)
Theory of the phases of matter:

1. Matter is made of atoms

2. The atoms move because of forces acting between them, just like the moon or an apple

Newton (1687)
Theory of the phases of matter:

1. Matter is made of atoms

2. The atoms move because of forces acting between them, just like the moon or an apple

3. The phases of matter are determined by the spatial arrangements of atoms

Boltzmann (1877)
Solids

Ice
Solids

Ice

Salt

Silicon
Solids

Ice
Liquids

Water
Gases

Steam
These solids have very different electrical and magnetic properties.

Copper is a conductor of electricity.
These solids have very different electrical and magnetic properties

Silicon is an insulator
These solids have very different electrical and magnetic properties.

At room temperature, YBCO conducts electricity (but not very well).
These solids have very different electrical and magnetic properties.

When cooled by liquid nitrogen, YBCO conducts electricity *without* resistance.
These solids have very different electrical and magnetic properties. When cooled by liquid nitrogen, YBCO is a SUPERCONDUCTOR!
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When cooled by liquid nitrogen, YBCO is a SUPERCONDUCTOR!
YBCO cables

American Superconductor Corporation
YBCO cables

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Nd-Fe-B magnets, YBaCuO superconductor

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Theory of the electrical phases of matter:
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1. In solids, electrons separate from the atoms and move throughout the entire crystal.
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2. We *cannot* use Newton’s Laws to describe the motion of the electrons.
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Needed:

A theory for the quantum phases of matter.
Quantum superposition and entanglement

Superconductivity

Black Holes and String Theory
Quantum superposition and entanglement

Superconductivity

Black Holes and String Theory

Superconductivity
Quantum superposition and entanglement

Superconductivity

Black Holes and String Theory
Quantum Superposition

The double slit experiment

Interference of water waves
Quantum Superposition

The double slit experiment

Interference of electrons
Quantum Superposition

The double slit experiment

Which slit does an electron pass through?

Interference of electrons
Quantum Superposition

The double slit experiment

Which slit does an electron pass through?

No interference when you watch the electrons

Interference of electrons

Wednesday, June 9, 2010
Quantum Superposition

The double slit experiment

Which slit does an electron pass through?

Each electron passes through both slits!

Interference of electrons

Wednesday, June 9, 2010
Quantum Superposition

The double slit experiment

Let $|L\rangle$ represent the state with the electron in the left slit.
Quantum Superposition

The double slit experiment

Let $|L\rangle$ represent the state with the electron in the left slit.

And $|R\rangle$ represents the state with the electron in the right slit.
Quantum Superposition

The double slit experiment

Let $|L\rangle$ represent the state with the electron in the left slit.

And $|R\rangle$ represents the state with the electron in the right slit.

Actual state of the electron is $|L\rangle + |R\rangle$. 

Wednesday, June 9, 2010
Quantum Entanglement: quantum superposition with more than one particle
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Hydrogen atom:
Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:

\[ \begin{array}{c}
\text{Hydrogen molecule:}
\end{array} \]

\[
\begin{align*}
\frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \\
= \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \\
\end{align*}
\]

Superposition of two electron states leads to non-local correlations between spins
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Quantum Entanglement: quantum superposition with more than one particle

Einstein-Podolsky-Rosen “paradox”: Non-local correlations between observations arbitrarily far apart
Entanglement of chemical bonds

Resonance in benzene leads to a symmetric configuration of chemical bonds

*(F. Kekulé, L. Pauling)*
Resonance in benzene leads to a symmetric configuration of chemical bonds

(F. Kekulé, L. Pauling)
Quantum superposition and entanglement

Superconductivity

Black Holes and String Theory
Quantum superposition and entanglement

Superconductivity

Black Holes and String Theory
Rubidium atoms in a magnetic trap and standing waves of laser light

At very low temperatures and for a weak laser light, the Rubidium atoms obey quantum mechanics and form a **Bose-Einstein condensate**

A Bose-Einstein condensate:
An quantum superposition of all
the atoms in all positions

A liquid which flows without
resistance (a superfluid)
A single atom is superposed between all positions
A single atom is superposed between all positions
A single atom is superposed between all positions
A single atom is superposed between all positions

\[ |G\rangle = \left( |\psi\rangle + |\psi\rangle + |\psi\rangle \right) \]
A single atom is superposed between all positions
A single atom is superposed between all positions

\[ |G\rangle = \left( |\circ| |\rangle + |\circ| |\rangle + |\circ| |\rangle \right) \]
A single atom is superposed between all positions

\[ |G\rangle = (|O| |O\rangle + |O| |O\rangle + |O| |O\rangle) \]
Bose-Einstein condensate: superposition between all atoms

\[ |\text{BEC}\rangle = |G\rangle |G\rangle |G\rangle \]
Bose-Einstein condensate:

superposition between all atoms

\[ |\text{BEC}\rangle = |G\rangle|G\rangle|G\rangle \]

\[ = \left( |\text{o o o o}\rangle + |\text{o o o o}\rangle + |\text{o o o o}\rangle + |\text{o o o o}\rangle + |\text{o o o o}\rangle + |\text{o o o o}\rangle + \cdots \right) \]

Large fluctuations in number of atoms in each site – superfluidity (atoms can “flow” without dissipation)
Bose-Einstein condensate: superposition between all atoms

\[ |\text{BEC}\rangle = |\text{G}\rangle |\text{G}\rangle |\text{G}\rangle = \left( |\circ\circ\circ\circ\rangle + |\circ\circ\circ\circ\rangle + |\circ\circ\circ\rangle + |\circ\circ\rangle + |\circ\rangle + \ldots \text{27 terms} \right) \]

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\[
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\[ + |\text{o}\text{\text{o}}\text{\text{o}}\rangle + |\text{o}\text{o}\text{o}\text{o}\rangle + |\text{o}\text{o}\text{o}\text{o}\rangle + \ldots \text{27 terms} \]

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\[ |\text{BEC}\rangle = |G\rangle |G\rangle |G\rangle \]

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Large fluctuations in number of atoms in each site – superfluidity (atoms can “flow” without dissipation)
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Bose-Einstein condensate: superposition between all atoms

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Large fluctuations in number of atoms in each site – *superfluidity* (atoms can “flow” without dissipation)
At very low temperatures and for a weak laser light, the Rubidium atoms form a Bose-Einstein condensate

Bose-Einstein condensate: superposition between all atoms

\[ |\text{BEC}\rangle = |G\rangle |G\rangle |G\rangle \]

(Strictly speaking: this is not entanglement between the atoms because the BEC is a product of simple “wave” states of the atoms)
A superconductor: a Bose condensate of pairs of electrons in a “chemical bond” in a metal

\[ |\text{BEC}\rangle = |G\rangle|G\rangle|G\rangle \]

\[ |G\rangle \equiv |\uparrow\downarrow - \downarrow\uparrow\rangle \]

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High temperature superconductors

\[ Ca_{1.90}Na_{0.10}CuO_2Cl_2 \]

\[ Bi_{2.2}Sr_{1.8}Ca_{0.8}Dy_{0.2}Cu_2O_y \]

\[ a_0 = 3.9\AA \]

\[ a_0 = 5.4\AA \]
High temperature superconductors

$\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$
Square lattice of Cu sites
Square lattice of Cu sites

I. Remove some electrons
Square lattice of Cu sites

1. Remove some electrons
2. Electrons entangle into chemical bonds

\[ |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \]
Square lattice of Cu sites

1. Remove some electrons
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\[ |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \]
Square lattice of Cu sites

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|↑↓ − |↓↑

Equation: \[ |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \]
Square lattice of Cu sites

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\[ |↑↓⟩ - |↓↑⟩ \]
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Nd-Fe-B magnets, YBaCuO superconductor

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Quantum superposition and entanglement

Superconductivity

Black Holes and String Theory
Objects so massive that light is gravitationally bound to them.
Objects so massive that light is gravitationally bound to them.

In Einstein’s theory, the region inside the black hole horizon is disconnected from the rest of the universe.

Horizon radius \( R = \frac{2GM}{c^2} \)
Around 1974, Bekenstein and Hawking showed that the application of the quantum theory across a black hole horizon led to many astonishing conclusions.
Quantum Entanglement across a black hole horizon
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Black hole horizon

Black hole horizon
Quantum Entanglement across a black hole horizon

Wednesday, June 9, 2010
Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole.
Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole.
Quantum Entanglement across a black hole horizon

There is a non-local quantum entanglement between the inside and outside of a black hole.

This entanglement leads to a black hole temperature (the Hawking temperature) and a black hole entropy (the Bekenstein entropy).
Quantum superposition and entanglement

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Wednesday, June 9, 2010
Superconducting Black Holes

Add electrical charge to a black hole in a curved spacetime: initially the charges fall past the horizon into the black hole
Superconducting Black Holes

However, eventually there is a balance between the gravitational forces pulling the charges into the black hole, and the repulsive electrical forces which push them out, and the resulting state is a superconductor!
More generally, string theory shows that there is a deep correspondence between the states of a black hole, and the quantum phases of matter (AdS/CFT correspondence)
More generally, string theory shows that there is a correspondence between the states of a black hole, and the quantum phases of matter (AdS/CFT correspondence).

This has helped enrich our understanding of the physics of black holes, and also of the possible quantum phases of electrons in crystals.
Quantum phases we do not understand yet:
Quantum phases we do not understand yet:

The phases around the high temperature superconductor YBCO as we vary the density of electrons
High temperature superconductors

YBa$_2$Cu$_3$O$_{6+x}$
Phase diagram of YBCO

Electron density vs. Temperature

Superconductor region

AFM phase
Phase diagram of YBCO

Electron density

Superconductor

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Phase diagram of YBCO

Phases with different forms of electrical resistance

Electron density

Superconductor
Hole doping

\( x \)

- \( d^-\text{wave SC} \)

Phase diagram of YBCO

Phases with different forms of electrical resistance

Electron density

Superconductor
Phase diagram of YBCO

Phases with different forms of electrical resistance

"Strange" metal

Superconductor

Electron density

T (K)
Phase diagram of YBCO

- "Strange" metal
- Superconductor
- "Quantum critical point?"
A “quantum critical point” is a special point between quantum phases where quantum entanglement is truly long-range.
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Long-range quantum entanglement is also found in string theories of black holes.
A “quantum critical point” is a special point between quantum phases where quantum entanglement is truly long-range.

Can string theory improve our understanding of quantum critical points, and of high temperature superconductors like YBCO?