Competing orders in the underdoped cuprates

Talk online: sachdev.physics.harvard.edu
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Competing orders in the underdoped cuprates, Eun Gook Moon and S. Sachdev, *to appear*
Outline

1. Survey of experiments and theory
   (a) Quantum oscillations
   (b) Competing orders
   (c) Nodal-anti-nodal dichotomy

2. Spin-fluctuation exchange mechanism of d-wave superconductivity
   Successful at large doping, but cannot account for competing orders, the nodal-anti-nodal dichotomy (and other phenomena) at low doping

3. Superconductivity of electron and hole pockets in a background of fluctuating antiferromagnetism
   Pairing by gauge forces the unusual d-wave superconductivity of the underdoped cuprates.
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   Pairing by gauge forces the unusual d-wave superconductivity of the underdoped cuprates.
The cuprate superconductors
Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Ground state has long-range spin density wave (SDW) order

Order parameter is a single vector field \( \vec{\varphi} = \eta_i \vec{S}_i \)

\( \eta_i = \pm 1 \) on two sublattices

\( \langle \vec{\varphi} \rangle \neq 0 \) in SDW state.
The cuprate superconductors

Na-CCOC
- Cu
- Ca/Na
- O
- Cl

Temperature

- AFI
- PG
- ECG
- dSC

Hole concentration
- ~2-3%
- ~5-10%
- ~15%
Evolution of the (ARPES) Fermi surface on the cuprate phase diagram

K.M. Shen et al., Science 2005

M. Platé et al., PRL 2005

Smaller hole Fermi-pockets
Large hole Fermi surface
Overdoped SC State: Momentum-dependent Pair Energy Gap $\Delta(\vec{k})$

The SC energy gap $\Delta(\vec{k})$ has four nodes.

Shen et al  PRL 70, 3999 (1993)
Ding et al  PRB 54 9678 (1996)
Mesot et al  PRL 83 840 (1999)
Evolution of the (ARPES) Fermi surface on the cuprate phase diagram

Smaller hole Fermi-pockets

Large hole Fermi surface
Quantum oscillations and the Fermi surface in an underdoped high $T_c$ superconductor (ortho-II ordered YBa$_2$Cu$_3$O$_{6.5}$). The period corresponds to a carrier density $\approx 0.076$.

Electron pockets in the Fermi surface of hole-doped high-$T_c$ superconductors

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\textit{Nature} 450, 533 (2007)
Competition between superconductivity (SC) and spin-density wave (SDW) order


(b) Phenomenological quantum theory of competing orders
(b) Phenomenological quantum theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order

- Upper-critical field, \( H_{c2} \), decreases as SDW is enhanced with decreasing doping (\( r \))

(b) Phenomenological quantum theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order

Competing order predictions:

• Upper-critical field, $H_{c2}$, decreases as SDW is enhanced with decreasing doping ($r$)

• Onset of SDW order occurs at a larger doping in the normal state (line CM) than in the superconductor (point A).

(b) Phenomenological quantum theory of competing orders

Competition between superconductivity (SC) and spin-density wave (SDW) order

Competing order predictions:

- Upper-critical field, $H_{c2}$, decreases as SDW is enhanced with decreasing doping ($r$).

- Onset of SDW order occurs at a larger doping in the normal state (line CM) than in the superconductor (point A).

- For $r > r_c$, there is a field-induced quantum phase transition (line AM) at $H = H_{sdw}$ involving onset of SDW order.

Phase diagram

as a function of hole density $\delta \sim t$ and magnetic field $H$.

Exotic metal with “ghost” Fermi pockets

Metallic SDW

SDW + dSC

(Confinement + VBS order)

$H_{c2}$

$H_{sdw}$

Phase diagram
as a function of hole density $\delta \sim t$ and magnetic field $H$.

Metallic SDW

Exotic metal with “ghost” Fermi pockets

SDW + dSC

(Confinement + VBS order)

Neutron scattering on La$_{1.9}$Sr$_{0.1}$CuO$_4$
Phase diagram

as a function of hole density $\delta \sim t$ and magnetic field $H$.

Neutron scattering on La$_{1.855}$Sr$_{0.145}$CuO$_4$

J. Chang et al., arXiv:0902.1191

Phase diagram

as a function of hole density $\delta \sim t$ and magnetic field $H$.

Metallic SDW

Exotic metal with “ghost” Fermi pockets

SDW + dSC

(Confinement + VBS order)

$H_{c2}$

$H_{sdw}$

Neutron scattering on YBa$_2$Cu$_3$O$_{6.45}$

D. Haug et al., arXiv:0902.3335
Phase diagram

as a function of hole density $\delta \sim t$ and magnetic field $H$.

Phase diagram
as a function of hole density $\delta \sim t$ and magnetic field $H$.

Exhibit quantum oscillations without Zeeman splitting

Exotic metal with “ghost” Fermi pockets

SDW

Metallic SDW

SDW + dSC

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$H_{c2}$

$H_{sdw}$

$\delta_c$

$\delta$

Spin density wave theory in hole-doped cuprates

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Spin density wave theory in hole-doped cuprates

Spin density wave theory in hole-doped cuprates

Increasing SDW order

Hole pockets

Spin density wave theory in hole-doped cuprates

SDW order parameter is a vector, $\vec{\phi}$, whose amplitude vanishes at the transition to the Fermi liquid.

Spin density wave theory in hole-doped cuprates

Incommensurate order in YBa$_2$Cu$_3$O$_{6+x}$

Competition between the pseudogap and superconductivity in the high-Tc copper oxides

(c) Nodal-anti-nodal dichotomy in the underdoped cuprates
FIG. 1. Sketch of the Fermi line and region of the momentum space where pseudogap pairs is formed. The Fermi line shown here was obtained in the tight binding model with diagonal hopping $t' = -0.3t$; it is similar to the Fermi line observed in the underdoped Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ (Ref. 5). The shaded disks denote the part of the momentum space where a pseudogap was observed in the experiment. We shall assume that the fermions in these regions are paired into the bosons.

(c) Nodal-anti-nodal dichotomy in the underdoped cuprates

![Graphical representation of Fermi line and momentum space region]

\[ H = \sum_{q} \varepsilon b_{q}^{\dagger} b_{q} + \sum_{p,q}^{' \prime} V_{p,q} (b_{q}^{\dagger} c_{p}^{\dagger} c_{q-p} - H.c.) \]

\[ + \sum_{p} \xi_{p} c_{p,\sigma}^{\dagger} c_{p,\sigma}; \]

\[ V_{p,q} = Va^{2}(p_{x}^{2} - p_{y}^{2}) \]

-2e bosons at antinodes, +e fermion “arcs” at nodes, and proximity “Josephson” coupling

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(c) Nodal-anti-nodal dichotomy in the underdoped cuprates

-2e bosons at antinodes, +e fermion “arcs” at nodes, and proximity “Josephson” coupling

\[ H = \sum_q \varepsilon b_q^\dagger b_q + \sum_{p,q} V_{p,q} (b_q^\dagger c_p^\uparrow c_{q-p}^\downarrow + H.c.) \]

\[ + \sum_p \xi_p c_{p,\sigma}^\dagger c_{p,\sigma} ; \]

\[ V_{p,q} = V a^2(p_x^2 - p_y^2) \]

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(c) Nodal-anti-nodal dichotomy in the underdoped cuprates

\[ H = \sum_{\mathbf{q}} \varepsilon b_{\mathbf{q}}^\dagger b_{\mathbf{q}} + \sum_{\mathbf{p},\mathbf{q}} \ ' V_{\mathbf{p},\mathbf{q}} (b_{\mathbf{q}}^\dagger c_{\mathbf{p}} c_{\mathbf{q} - \mathbf{p}}) + \text{H.c.} \]
\[ + \sum_{\mathbf{p}} \xi_{\mathbf{p}} c_{\mathbf{p},\sigma}^\dagger c_{\mathbf{p},\sigma}; \]
\[ V_{\mathbf{p},\mathbf{q}} = V a^2 (p_x^2 - p_y^2) \]

-2e bosons at antinodes, +1 fermion “arcs” at nodes, and proximity “Josephson” coupling

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Attractive phenomenological model, but theoretical and microscopic basis is unclear.

\[
H = \sum_q \varepsilon b_q^\dagger b_q + \sum_{p,q} V_{p,q}(b_q^\dagger c_{p\uparrow} c_{q\downarrow} + \text{H.c.)} + \sum_{p} \xi_{p} c_{p,\sigma}^\dagger c_{p,\sigma} ;
\]
\[
V_{p,q} = Va^2(p_x^2 - p_y^2)
\]


FIG. 1. Sketch of the Fermi line and region of the momentum space where pseudogap pairs is formed. The Fermi line shown here was obtained in the tight binding model with diagonal hopping \( t' = -0.3t \); it is similar to the Fermi line observed in the underdoped Bi\(_2\)Sr\(_2\)CaCu\(_2\)O\(_{8+\delta}\) (Ref. 5). The shaded disks denote the part of the momentum space where a pseudogap was observed in the experiment. We shall assume that the fermions in these regions are paired into the bosons.
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3. Superconductivity of electron and hole pockets in a background of fluctuating antiferromagnetism
   Pairing by gauge forces the unusual d-wave superconductivity of the underdoped cuprates.
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Spin density wave theory in hole-doped cuprates

SDW order parameter is a vector, $\vec{\phi}$, whose amplitude vanishes at the transition to the Fermi liquid.
Spin-fluctuation exchange theory of d-wave superconductivity in the cuprates

Fermions at the large Fermi surface exchange fluctuations of the SDW order parameter $\vec{\phi}$.
Spin-fluctuation exchange theory of d-wave superconductivity in the cuprates
Approaching the onset of antiferromagnetism in the spin-fluctuation theory
Approaching the onset of antiferromagnetism in the spin-fluctuation theory

- $T_c$ increases upon approaching the SDW transition. SDW and SC orders do not compete, but attract each other.

- No simple mechanism for nodal-anti-nodal dichotomy.

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Spin density wave theory in hole-doped cuprates

SDW order parameter is a vector, $\vec{\phi}$, whose amplitude vanishes at the transition to the Fermi liquid.

Begin with SDW ordered state, and focus on fluctuations in the orientation of $\bar{\varphi}$, by using a unit-length bosonic spinor $z_\alpha$

$$\bar{\varphi} = z_\alpha^* \bar{\sigma}_{\alpha \beta} z_\beta$$
Charge carriers in the lightly-doped cuprates with Neel order

Electron pockets

Hole pockets
For a uniform SDW order with \( \vec{\phi} \propto (0, 0, 1) \), write

\[
\begin{pmatrix}
    c_{1\uparrow} \\
    c_{1\downarrow}
\end{pmatrix} = \begin{pmatrix}
    g_+ \\
    g_-
\end{pmatrix}
\]
For a spacetime dependent SDW order, $\vec{\varphi} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$,

\[
\begin{pmatrix}
c_{1\uparrow} \\
c_{1\downarrow}
\end{pmatrix} = R_z \begin{pmatrix} g_+ \\ g_- \end{pmatrix}; \quad R_z \equiv \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix}.
\]

So $g_\pm$ are the “up/down” electron operators in a rotating reference frame defined by the local SDW order.
For a spacetime dependent SDW order, \( \vec{\phi} = z^\ast \alpha \vec{\sigma}_{\alpha\beta} z^\beta \),

\[
\left( c_{2\uparrow}^\alpha \right) = \left( \begin{array}{c} g_+ \\ -g_- \end{array} \right) .
\]

SDW theory also specifies electrons at second pocket for \( \vec{\varphi} \propto (0, 0, 1) \).

Electron operator

\( c_{2\alpha} \)
For a spacetime dependent SDW order, \( \vec{\varphi} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta \),

\[
\begin{pmatrix}
  c_{2\uparrow} \\
  c_{2\downarrow}
\end{pmatrix} = \mathcal{R}_z \begin{pmatrix}
  g_+ \\
  -g_-
\end{pmatrix}; \quad \mathcal{R}_z \equiv \begin{pmatrix}
  z^\uparrow & -z^*_\downarrow \\
  z^\downarrow & z^*_\uparrow
\end{pmatrix}.
\]

Same SU(2) matrix also rotates electrons in second pocket.
Low energy theory for spinless, charge $-e$ fermions $g_\pm$, and spinful, charge 0 bosons $z_\alpha$:

\[
\mathcal{L} = \mathcal{L}_z + \mathcal{L}_g \\
\mathcal{L}_z = \frac{1}{t} \left[ |(\partial_\tau - iA_\tau)z_\alpha|^2 + v^2|\nabla - iA)z_\alpha|^2 \right] \\
+ \text{ Berry phases of monopoles in } A_\mu.
\]

$\text{CP}^1$ field theory for $z_\alpha$ and an emergent U(1) gauge field $A_\mu$. Coupling $t$ tunes the strength of SDW orientation fluctuations.
Low energy theory for spinless, charge $-e$ fermions $g_{\pm}$, and spinful, charge 0 bosons $z_\alpha$:

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$$

$$
\mathcal{L}_z = \frac{1}{t} \left[ \left| (\partial_\tau - iA_\tau)z_\alpha \right|^2 + v^2 \left| \nabla - i\mathbf{A} \right| z_\alpha \right]^2
+ \text{ Berry phases of monopoles in } A_\mu.
$$

CP$^1$ field theory for $z_\alpha$ and an emergent $\text{U}(1)$ gauge field $A_\mu$. Coupling $t$ tunes the strength of SDW orientation fluctuations.

$$
\mathcal{L}_g = g_+^\dagger \left[ (\partial_\tau - iA_\tau) - \frac{1}{2m^*} (\nabla - i\mathbf{A})^2 - \mu \right] g_+
+ g_-^\dagger \left[ (\partial_\tau + iA_\tau) - \frac{1}{2m^*} (\nabla + i\mathbf{A})^2 - \mu \right] g_-
$$

Two Fermi surfaces coupled to the emergent $\text{U}(1)$ gauge field $A_\mu$ with opposite charges.
- Gauge forces lead to a $s$-wave paired state with a $T_c$ of order the Fermi energy of the pockets. Inelastic scattering from low energy gauge modes lead to very singular $g_{\pm}$ self energy, but is not pair-breaking.

$$\langle g_+ g_- \rangle = \Delta$$
Strong pairing of the $g_{\pm}$ electron pockets

Electron operator $c_{2\alpha}$

Electron operator $c_{1\alpha}$
Increasing SDW order

Electron operator $c_{2\alpha}$

Electron operator $c_{1\alpha}$

Strong pairing of the $g_\pm$ electron pockets

- Transforming back to the physical fermions:

\[
\begin{pmatrix}
c_{1\uparrow} \\
c_{1\downarrow}
\end{pmatrix} =
\begin{pmatrix}
z_{\uparrow} & -z_{\downarrow}^* \\
z_{\downarrow} & z_{\uparrow}^*
\end{pmatrix}
\begin{pmatrix}
g_+ \\
g_-
\end{pmatrix};
\begin{pmatrix}
c_{2\uparrow} \\
c_{2\downarrow}
\end{pmatrix} =
\begin{pmatrix}
z_{\uparrow} & -z_{\downarrow}^* \\
z_{\downarrow} & z_{\uparrow}^*
\end{pmatrix}
\begin{pmatrix}
g_+ \\
-g_-
\end{pmatrix},
\]

we find: $\langle c_{1\uparrow}c_{1\downarrow} \rangle = -\langle c_{2\uparrow}c_{2\downarrow} \rangle \sim |z_{\uparrow}|^2 + |z_{\downarrow}|^2 \langle g_+g_- \rangle$;
Strong pairing of the $g_{\pm}$ electron pockets

- Transforming back to the physical fermions:

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\begin{pmatrix}
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    g_+ \\
    -g_-
\end{pmatrix},
\]

we find:  \[\langle c_{1\uparrow} c_{1\downarrow} \rangle = -\langle c_{2\uparrow} c_{2\downarrow} \rangle \sim \langle |z_{\uparrow}|^2 + |z_{\downarrow}|^2 \rangle \langle g_+ g_- \rangle ;\]

i.e. the pairing signature for the electrons is $d$-wave.
Increasing SDW order

\( g^\pm \)
Increasing SDW order

\[ f_{\pm v} \]

\[ g_{\pm} \]
Low energy theory for spinless, charge $+e$ fermions $f_{\pm v}$:

$$
\mathcal{L}_f = \sum_{v=1,2} \left\{ \begin{array}{c}
 f_{+v}^\dagger \left[ (\partial_\tau - iA_\tau) - \frac{1}{2m^*} (\nabla - iA)^2 - \mu \right] f_{+v} \\
 + f_{-v}^\dagger \left[ (\partial_\tau + iA_\tau) - \frac{1}{2m^*} (\nabla + iA)^2 - \mu \right] f_{-v} \end{array} \right\}
$$
Weak pairing of the $f_{\pm}$ hole pockets

\[
\mathcal{L}_{\text{Josephson}} = iJ \left[ g_+ g_- \right] \left[ f_{+1} \hat{\partial}_x f_{-1} - f_{+1} \hat{\partial}_y f_{-1} + f_{+2} \hat{\partial}_x f_{-2} + f_{+2} \hat{\partial}_y f_{-2} \right] + \text{H.c.}
\]


Proximity Josephson coupling $J$ to $g_{\pm}$ fermions leads to $p$-wave pairing of the $f_{\pm \nu}$ fermions. The $A_\mu$ gauge forces are pair-breaking, and so the pairing is weak.

\[
\langle f_{+1}(\mathbf{k}) f_{-1}(-\mathbf{k}) \rangle \sim (k_x - k_y) J \langle g_+ g_- \rangle;
\]

\[
\langle f_{+2}(\mathbf{k}) f_{-2}(-\mathbf{k}) \rangle \sim (k_x + k_y) J \langle g_+ g_- \rangle;
\]

\[
\langle f_{+1}(\mathbf{k}) f_{-2}(-\mathbf{k}) \rangle = 0,
\]
Weak pairing of the $f_{\pm}$ hole pockets

$\langle f_{+1}(k)f_{-1}(-k) \rangle \sim (k_x - k_y)J\langle g_+ g_- \rangle$;
$\langle f_{+2}(k)f_{-2}(-k) \rangle \sim (k_x + k_y)J\langle g_+ g_- \rangle$;
$\langle f_{+1}(k)f_{-2}(-k) \rangle = 0$, 

Increasing SDW order
Increasing SDW order $d$-wave pairing of the electrons is associated with

- **Strong s-wave** pairing of $g_{\pm}$
- **Weak p-wave** pairing of $f_{\pm\nu}$.
Emergence of preformed Cooper pairs from the doped Mott insulating state in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8+\delta$

H.-B. Yang$^1$, J. D. Rameau$^1$, P. D. Johnson$^1$, T. Valla$^1$, A. Tsvelik$^1$ & G. D. Gu$^1$

Here we report a photoemission study of the underdoped copper oxide $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8+\delta$ that shows the opening of a symmetric gap only in the anti-nodal region, contrary to the expectation that pairing would take place in the nodal region. It is therefore evident that the pseudogap does reflect the formation of preformed pairs of electrons and that the pairing occurs only in well-defined directions of the underlying lattice.

Universal theory of superconductivity

\[ \mathcal{L} = \frac{1}{t} \left[ |(\partial_\tau - iA_\tau) z_\alpha|^2 + v^2 |\nabla - iA| z_\alpha|^2 \right] \\
+ g_+^\dagger \left[ (\partial_\tau - iA_\tau) - \frac{1}{2m^*} (\nabla - iA)^2 - \mu \right] g_+ \\
+ g_-^\dagger \left[ (\partial_\tau + iA_\tau) - \frac{1}{2m^*} (\nabla + iA)^2 - \mu \right] g_-

- Complex, ‘relativistic’ bosons \( z_\alpha \), with \(|z_\alpha|^2 = 1\)
- Fermions \( g_\pm \)
- Gauge field \((A_\tau, A)\)
Universal theory of superconductivity

\[ \mathcal{L} = \frac{1}{t} \left[ |( \partial_\tau - iA_\tau )z_\alpha |^2 + v^2 |\nabla - iA| z_\alpha |^2 \right] \\
+ \ g_+^\dagger \left[ ( \partial_\tau - iA_\tau ) - \frac{1}{2m^*} ( \nabla - iA )^2 - \mu \right] g_+ \\
+ \ g_-^\dagger \left[ ( \partial_\tau + iA_\tau ) - \frac{1}{2m^*} ( \nabla + iA )^2 - \mu \right] g_-

- Complex, ‘relativistic’ bosons \( z_\alpha \), with \( |z_\alpha|^2 = 1 \)
- Fermions \( g_\pm \)
- Gauge field \((A_\tau, A)\)

- Theory fully characterized by two dimensionless parameters:
  - \( (1/t_c - 1/t)/m^* \) measures distance from SDW ordering quantum transition
  - \( k_F/(m^*v) \)
Universal theory of superconductivity

\[ \mathcal{L} = \frac{1}{t} \left[ |(\partial_\tau - iA_\tau)z_\alpha|^2 + v^2|\nabla - iA|z_\alpha|^2 \right] + g^+_± \left[ (\partial_\tau - iA_\tau) - \frac{1}{2m^*}(\nabla - iA)^2 - \mu \right] g_± \]

\[ + \ g^\dagger_± \left[ (\partial_\tau + iA_\tau) - \frac{1}{2m^*}(\nabla + iA)^2 - \mu \right] g_- \]

- Complex, ‘relativistic’ bosons \( z_\alpha \), with \( |z_\alpha|^2 = 1 \)
- Fermions \( g_± \)
- Gauge field \( (A_\tau, A) \)

- Theory fully characterized by two dimensionless parameters:
  - \( (1/t_c - 1/t)/m^* \) measures distance from SDW ordering quantum transition
  - \( k_F/(m^*v) \)

- Characteristic length scale: \( 1/k_F \).
- Characteristic energy scale: \( m^*v^2 \).
Universal theory of superconductivity

\[ \mathcal{L} = \frac{1}{t} \left[ |(\partial_\tau - iA_\tau)z_\alpha|^2 + v^2 |\nabla - iA|z_\alpha|^2 \right] + g_+^\dagger \left[ (\partial_\tau - iA_\tau) - \frac{1}{2m^*} (\nabla - iA)^2 - \mu \right] g_+
\]
\[ + g_-^\dagger \left[ (\partial_\tau + iA_\tau) - \frac{1}{2m^*} (\nabla + iA)^2 - \mu \right] g_- \]

- Complex, ‘relativistic’ bosons \( z_\alpha \), with \( |z_\alpha|^2 = 1 \)
- Fermions \( g_\pm \)
- Gauge field \((A_\tau, A)\)

---

Exotic metal with “ghost” Fermi pockets

SDW

dSC

\[ \left( \frac{2eH}{\hbar c} \right) \left( \frac{\hbar}{m^* v} \right)^2 \]
Universal theory of superconductivity

SDW order is suppressed in the superconductor, i.e. $E_z > 0$, by enhancement of gauge field fluctuations, which are screened only in the metallic phases.
Phase diagram

as a function of hole density $\delta \sim t$ and magnetic field $H$.

Exotic metal with “ghost” Fermi pockets

SDW (Confinement + VBS order)

Metallic SDW

Phase diagram as a function of hole density $\delta \sim t$ and magnetic field $H$.

Phase diagram

as a function of hole density $\delta \sim t$ and magnetic field $H$.

Exhibit quantum oscillations without Zeeman splitting

Exotic metal with “ghost” Fermi pockets

SDW + dSC

(Confinement + VBS order)

$H_{c2}$

$H_{sdw}$

Conclusions

★ Gauge theory for pairing in the underdoped cuprates, describing “angular” fluctuations of spin-density-wave order

★ Natural route to $d$-wave pairing with strong pairing at the antinodes and weak pairing at the nodes
Conclusions

★ Gauge theory for pairing in the underdoped cuprates, describing "angular" fluctuations of spin-density-wave order

★ Natural route to $d$-wave pairing with strong pairing at the antinodes and weak pairing at the nodes

★ Explains characteristic "competing order" features of field-doping phase diagram: SDW order is more stable in the metal than in the superconductor.
Phase diagram

as a function of hole density $\delta \sim t$ and magnetic field $H$.

Conclusions

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- Paired electron pockets are expected to lead to valence-bond-solid modulations at low temperature
Tunneling Asymmetry (TA)-map at $E=150\text{meV}$

$\text{Ca}_{1.90}\text{Na}_{0.10}\text{CuO}_2\text{Cl}_2$ and $\text{Bi}_{2.2}\text{Sr}_{1.8}\text{Ca}_{0.8}\text{Dy}_{0.2}\text{Cu}_2\text{O}_y$

Indistinguishable bond-centered TA contrast
with disperse $4a_0$-wide nanodomains

TA Contrast is at oxygen site (Cu-O-Cu bond-centered)

$\text{Ca}_1.88\text{Na}_{0.12}\text{CuO}_2\text{Cl}_2$, 4 K

$4a_0$

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Evidence for a predicted valence bond supersolid

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★ Needed: theory for transition to “large” Fermi surface at higher doping