Quantum matter without quasiparticles: strange metals and black holes

Stanford University
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Subir Sachdev

Talk online: sachdev.physics.harvard.edu
A quasiparticle is an “excited lump” in the many-electron state which responds just like an ordinary particle.
Quantum matter with quasiparticles:

The quasiparticle idea is the key reason for the many successes of quantum condensed matter physics:

- Fermi liquid theory of metals, insulators, semiconductors
- Theory of superconductivity (pairing of quasiparticles)
- Theory of disordered metals and insulators (diffusion and localization of quasiparticles)
- Theory of metals in one dimension (collective modes as quasiparticles)
- Theory of the fractional quantum Hall effect (quasiparticles which are `fractions’ of an electron)
Quantum matter without quasiparticles

Strange metal

Entangled electrons lead to “strange” temperature dependence of resistivity and other properties

Figure: K. Fujita and J. C. Seamus Davis
Quantum matter without quasiparticles

Resistivity \( \sim \rho_0 + AT^\alpha \)

Strange (or “bad”) Metal

\( \rho \sim T \)

\( \rho \gg h/e^2 \)

Superconductivity in Bad Metals

V. J. Emery and S. A. Kivelson
Phys. Rev. Lett. 74, 3253 – Published 17 April 1995

Thermal diffusivity measurements by the group of A. Kapitulnik in \((\text{Sm}_{1.839}\text{Ce}_{0.161})_2\text{CuO}_4\)
Quantum matter with quasiparticles:

- **Quasiparticles are additive excitations:** The low-lying excitations of the many-body system can be identified as a set \( \{n_\alpha\} \) of quasiparticles with energy \( \varepsilon_\alpha \)

\[
E = \sum_\alpha n_\alpha \varepsilon_\alpha + \sum_\alpha,\beta F_{\alpha\beta} n_\alpha n_\beta + \ldots
\]

In a lattice system of \( N \) sites, this parameterizes the energy of \( \sim e^{\alpha N} \) states in terms of poly\((N)\) numbers.
Quantum matter with quasiparticles:

- Quasiparticles eventually collide with each other. Such collisions eventually leads to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time diverges as

\[ \tau_{eq} \sim \frac{\hbar E_F}{(k_B T)^2}, \quad \text{as } T \to 0, \]

where \( E_F \) is the Fermi energy.
Quantum Ising models

Qubits with states $|\uparrow\rangle_i$, $|\downarrow\rangle_i$, on the sites, $i$, of a regular lattice.

$$
\sigma^z |\uparrow\rangle = |\uparrow\rangle, \quad \sigma^z |\downarrow\rangle = -|\downarrow\rangle
$$

$$
\sigma^x |\uparrow\rangle = |\downarrow\rangle, \quad \sigma^x |\downarrow\rangle = |\uparrow\rangle
$$

$$
H = -J \left( \sum_{\langle ij \rangle} \sigma^z_i \sigma^z_j + g \sum_i \sigma^x_i \right)
$$

For $g = 0$, ground state is a ferromagnet:

$$
|G\rangle = \cdots \uparrow\uparrow\uparrow\uparrow\cdots \quad \text{or} \quad \cdots \downarrow\downarrow\downarrow\downarrow\cdots
$$

For $g \gg 1$, unique ‘paramagnetic’ ground state:

$$
|G\rangle = \cdots \rightarrow\rightarrow\rightarrow\rightarrow\cdots
$$

where

$$
|\rightarrow\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle + |\downarrow\rangle \right), \quad |\leftarrow\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle - |\downarrow\rangle \right)
$$
Quantum Ising models

One dimension

- In one dimension, quasiparticles exist even at the quantum critical point: there is a non-local transformations from the qubits to a system of free fermions.
In two dimensions, the “quantum critical” region provides us the first example of a system without a quasiparticle description. This is described by a strongly-coupled conformal field theory (CFT) in 2+1 dimensions, and dynamic properties cannot be computed accurately.
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Quantum matter without quasiparticles:

- If there are no quasiparticles, then

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Quantum matter without quasiparticles:

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• If there are no quasiparticles, then

\[ \tau_{eq} = \# \frac{\hbar}{k_B T} \]

• Systems without quasiparticles are the fastest possible in reaching local equilibrium, and all many-body quantum systems obey, as \( T \to 0 \)

\[ \tau_{eq} > C \frac{\hbar}{k_B T} \]

– In Fermi liquids \( \tau_{eq} \sim 1/T^2 \), and so the bound is obeyed as \( T \to 0 \).
– This bound rules out quantum systems with e.g. \( \tau_{eq} \sim \hbar/(Jk_B T)^{1/2} \).
– There is no bound in classical mechanics (\( \hbar \to 0 \)). By cranking up frequencies, we can attain equilibrium as quickly as we desire.

S. Sachdev, Quantum Phase Transitions, Cambridge (1999)
A simple model of a metal with quasiparticles

Pick a set of random positions
A simple model of a metal with quasiparticles

Place electrons randomly on some sites
A simple model of a metal with quasiparticles

Electrons move one-by-one randomly
A simple model of a metal with quasiparticles

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\[ H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \ldots \]

\[ c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij} \]

\[ \frac{1}{N} \sum_i c_i^\dagger c_i = Q \]

\( t_{ij} \) are independent random variables with \( \overline{t_{ij}} = 0 \) and \( |t_{ij}|^2 = t^2 \)

Fermions occupying the eigenstates of a \( N \times N \) random matrix
Let $\varepsilon_\alpha$ be the eigenvalues of the matrix $t_{ij}/\sqrt{N}$. The fermions will occupy the lowest $NQ$ eigenvalues, upto the Fermi energy $E_F$. The density of states is $\rho(\omega) = (1/N) \sum_\alpha \delta(\omega - \varepsilon_\alpha)$. 

![Diagram of density of states](chart.png)
A simple model of a metal with quasiparticles

There are $2^N$ many body levels with energy

$$E = \sum_{\alpha=1}^{N} n_\alpha \varepsilon_\alpha,$$

where $n_\alpha = 0, 1$. Shown are all values of $E$ for a single cluster of size $N = 12$. The $\varepsilon_\alpha$ have a level spacing $\sim 1/N$. 

Many-body level spacing $\sim 2^{-N}$

Quasiparticle excitations with spacing $\sim 1/N$
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Quasiparticle excitations with spacing $\sim 1/N$
The Sachdev-Ye-Kitaev (SYK) model

Pick a set of random positions
Place electrons randomly on some sites
Entangle electrons pairwise randomly
The SYK model

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Entangle electrons pairwise randomly
This describes both a strange metal and a black hole!
The SYK model

(See also: the “2-Body Random Ensemble” in nuclear physics; did not obtain the large $N$ limit;

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell} c_i^\dagger c_j^\dagger c_k c_{\ell} - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j c_i^\dagger = \delta_{ij}$$

$$Q = \frac{1}{N} \sum_i c_i^\dagger c_i$$

$U_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $|U_{ij;k\ell}|^2 = U^2$

$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL 70, 3339 (1993)
Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

Non-quasiparticle excitations with spacing $\sim e^{-Ns_0}$

There are $2^N$ many body levels with energy $E$, which do not admit a quasiparticle decomposition. Shown are all values of $E$ for a single cluster of size $N = 12$. The $T \to 0$ state has an entropy $S_{GPS} = Ns_0$ with

$$s_0 = \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848 \ldots$$

$$< \ln 2$$

where $G$ is Catalan’s constant, for the half-filled case $Q = 1/2$.

GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB 63, 134406 (2001)

W. Fu and S. Sachdev, PRB 94, 035135 (2016)
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No quasiparticles!

$$E \neq \sum_{\alpha} n_\alpha \varepsilon_\alpha + \sum_{\alpha, \beta} F_{\alpha\beta} n_\alpha n_\beta + \ldots$$
The SYK model

- Low energy, many-body density of states
  \[ \rho(E) \sim e^{Ns_0} \sinh(\sqrt{2(E - E_0)}N\gamma) \]

  A. Georges, O. Parcollet, and S. Sachdev, PRB 63, 134406 (2001)
  D. Stanford and E. Witten, 1703.04612
  A. M. Garica-Garcia, J.J.M. Verbaarschot, 1701.06593
  D. Bagrets, A. Altland, and A. Kamenev, 1607.00694
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- Low temperature entropy
  \[ S = Ns_0 + N\gamma T + \ldots \]

A. Kitaev, unpublished
J. Maldacena and D. Stanford, 1604.07818
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- The last property indicates \( \tau_{eq} \sim \hbar/(k_B T) \), and this has been found in a recent numerical study.

A. Eberlein, V. Kasper, S. Sachdev, and J. Steinberg, arXiv:1706.07803
The basic features can be determined by a simple power-counting. Considering for simplicity scale-incondition heavy fermion systems. We assume like the previous higher dimensional SYK models where low-fermion “pair hopping” interactions. They obtained electrical provide a powerful framework to study such physics. The exactly soluble SYK models

\( t = 2 \)  

\( \mu = 4 \)

\( \langle x x' \rangle \)

\( \sigma \)

\( \bar{\sigma} \)

\( \Theta \)

\( \alpha \)

\( \beta \)

\( \gamma \)

\( \delta \)

\( \epsilon \)

\( \zeta \)

\( \eta \)

\( \theta \)

\( \vartheta \)

\( \chi \)

\( \psi \)

\( \omega \)

\( \Omega \)

\( \Phi \)

\( \Psi \)

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Title: A strongly correlated metal built from Sachdev-Ye-Kitaev models

Authors: Xue-Yang Song, Chao-Ming Jian, Leon Balents

Low ‘coherence’ scale

\[ E_c \sim \frac{t_0^2}{U} \]
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For \( E_c < T < U \), the resistivity, \( \rho \), and entropy density, \( s \), are

\[ \rho \sim \frac{\hbar}{e^2} \left( \frac{T}{E_c} \right), \quad s = s_0 \]
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Low ‘coherence’ scale

\[ E_c \sim \frac{t_0^2}{U} \]

For \( T < E_c \), the resistivity, \( \rho \), and entropy density, \( s \), are

\[ \rho = \frac{h}{e^2} \left[ c_1 + c_2 \left( \frac{T}{E_c} \right)^2 \right] \]

\[ s \sim s_0 \left( \frac{T}{E_c} \right) \]
- Black holes have an entropy and a temperature, $T_H$.
- The entropy is proportional to their surface area.
The Hawking temperature, $T_H$, influences the radiation from the black hole at the very last stages of the ring-down (not observed so far). The ring-down (approach to thermal equilibrium) happens very rapidly in a time $\sim \frac{\hbar}{k_B T_H} = \frac{8\pi GM}{c^3} \sim 8$ milliseconds.
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Black holes have an entropy and a temperature, $T_H$.

- The entropy is proportional to their surface area.
- They relax to thermal equilibrium in a time $\sim \frac{\hbar}{k_B T_H}$. 
AdS/CFT correspondence at zero temperature

Quantum gravity in 3+1 dimensions

Maximally supersymmetric Yang-Mills theory in 2+1 dimensions (a CFT3)

Maldacena, Gubser, Klebanov, Polyakov, Witten

\(ds^2 = \left( \frac{L}{r} \right)^2 \left[ dr^2 - dt^2 + dx^2 + dy^2 \right]\)
AdS/CFT correspondence at zero temperature

Quantum gravity in 3+1 dimensions

Maximally supersymmetric Yang-Mills theory in 2+1 dimensions (a CFT3)

This spacetime is a solution of Einstein gravity with a negative cosmological constant

\[ S_E = \int d^4 x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) \right] \]
AdS/CFT correspondence at non-zero temperatures

**AdS\(_4\)-Schwarzschild black-brane**

There is a family of solutions of Einstein gravity which describe non-zero temperatures.

\[
ds^2 = \left( \frac{L}{r} \right)^2 \left[ \frac{dr^2}{f(r)} - f(r) dt^2 + dx^2 + dy^2 \right]
\]

with \( f(r) = 1 - \left( \frac{r}{R} \right)^3 \)

Maximally supersymmetric Yang-Mills at a temperature

\[
k_B T = \frac{3\hbar}{4\pi R}.
\]

Maldacena, Gubser, Klebanov, Polyakov, Witten
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Black hole horizon at a Hawking temperature \( T_H = T \)

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Black hole entropy = Entropy of Yang-Mills theory

Maldacena, Gubser, Klebanov, Polyakov, Witten
SYK and black holes

Is there a holographic quantum gravity dual of the SYK model?
The SYK model

- Low energy, many-body density of states
  \[ \rho(E) \sim e^{Ns_0} \sinh(\sqrt{2(E - E_0)}N\gamma) \]

- Low temperature entropy \( S = Ns_0 + N\gamma T + \ldots \)

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- Is there any black hole which holographically matches these properties?
Yes, a charged black hole in Einstein-Maxwell theory

S = \int d^4 x \sqrt{-\hat{g}} \left( \hat{R} + 6/L^2 - \frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \right)

S. Sachdev, PRL 105, 151602 (2010)
The leading low temperature properties of an AdS-Reissner-Nordstrom black hole (as computed by T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694) match those of the SYK model. The mapping applies when temperature $\ll 1/(\text{size of} \ T^2)$. 

S. Sachdev, PRL 105, 151602 (2010)
Quantum gravity on the 1+1 dimensional spacetime $\text{AdS}_2$ (when embedded in $\text{AdS}_4$) is holographically matched to the 0+1 dimensional SYK model.

$S. \text{Sachdev, PRL} \_105, \ 151602 \ (2010); \ A. \text{Kitaev (unpublished); J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857}$
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All these properties of the SYK model match those of the AdS\(_2\) horizon in Einstein-Maxwell theory

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Schwarzian theory of quantum gravity fluctuations also matches these corrections

Many-body quantum chaos

- Using holographic analogies, Shenker and Stanford introduced the “Lyapunov time”, $\tau_L$, the time over which a generic many-body quantum system loses memory of its initial state.

  S. Shenker and D. Stanford, arXiv:1306.0622
Many-body quantum chaos

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- A shortest-possible time to reach quantum chaos was established

\[
\tau_L \geq \frac{\hbar}{2\pi k_B T}
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  \( \text{S. Shenker and D. Stanford, arXiv:1306.0622} \)

- A shortest-possible time to reach quantum chaos was established

- The SYK model, and black holes in Einstein gravity, saturate the bound on the Lyapunov time

  \[ \tau_L = \frac{\hbar}{2\pi k_B T} \]


  \( \text{A. Kitaev, unpublished} \)

  \( \text{J. Maldacena and D. Stanford, arXiv:1604.07818} \)
Quantum matter without quasiparticles:

- No quasiparticle decomposition of low-lying states:
  \[ E \neq \sum_{\alpha} n_{\alpha} \varepsilon_{\alpha} + \sum_{\alpha, \beta} F_{\alpha \beta} n_{\alpha} n_{\beta} + \ldots \]

- Thermalization and many-body chaos in the shortest possible time of order \( \hbar/(k_B T) \).
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- These are also characteristics of black holes in quantum gravity.