Ordinary metals: the Fermi liquid

- Fermi surface separates empty and occupied states in momentum space.
- Area enclosed by Fermi surface = \( Q \). Momenta of low energy excitations fixed by density of all electrons.
- Long-lived electron-like quasi-particle excitations near the Fermi surface: lifetime of quasi-particles \( \sim 1/T^2 \).
**Ordinary metals: the Fermi liquid**

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- Area enclosed by Fermi surface = $Q$. Momenta of low energy excitations fixed by density of all electrons.

- Long-lived electron-like quasi-particle excitations near the Fermi surface: lifetime of quasi-particles $\sim 1/T^2$.

- \[
\frac{\text{(Thermal conductivity)}}{T \, \text{(Electrical conductivity)}} = \frac{\pi^2 k_B^2}{3e^2}
\]
Wiedemann-Franz law in a Fermi liquid:

\[
\frac{\kappa}{\sigma T} \approx \frac{\pi^2 k_B^2}{3e^2} \approx 2.45 \times 10^{-8} \frac{W \cdot \Omega}{K^2}.
\]

Graphene
Graphene

Electron Fermi surface

$\mu > 0$
Graphene

Hole Fermi surface

$\mu < 0$

Electron Fermi surface

$\mu > 0$
Graphene

$T(K)$

Quantum critical
Dirac liquid

Hole
Fermi liquid

Electron
Fermi liquid

$\mu < 0$

$\mu > 0$

M. Müller, L. Fritz, and S. Sachdev, PRB 78, 115406 (2008)
M. Müller and S. Sachdev, PRB 78, 115419 (2008)
Graphene

$T(K)$

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Key properties of a strange metal

- No quasiparticle excitations

- Shortest possible "collision time", or more precisely, fastest possible local equilibration time

- Continuously variable density, \( Q \) (conformal field theories are usually at fixed density, \( Q = 0 \))
Key properties of a strange metal

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  \[ \sim \frac{\hbar}{k_B T} \]
Key properties of a strange metal

- No quasiparticle excitations
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- Continuously variable density, \( Q \) (conformal field theories are usually at fixed density, \( Q = 0 \))
Prediction for transport in the graphene strange metal

Recall that in a Fermi liquid, the Lorenz ratio $L = \kappa / (T\sigma)$, where $\kappa$ is the thermal conductivity, and $\sigma$ is the conductivity, is given by $L = \pi^2 k_B^2 / (3e^2)$. 

For a strange metal with a "relativistic" Hamiltonian, hydrodynamic, holographic, and memory function methods yield

$$L = \pi^2 k_B^2 / (3e^2),$$

where $H$ is the enthalpy density, $\tau_{\text{imp}}$ is the momentum relaxation time (from impurities), while $\omega = \omega_Q$, an intrinsic, finite, "quantum critical" conductivity. Note that the limits $\omega_Q \to 0$ and $\tau_{\text{imp}} \to 1$ do not commute.
Prediction for transport in the graphene strange metal

Recall that in a Fermi liquid, the Lorenz ratio $L = \kappa/(T\sigma)$, where $\kappa$ is the thermal conductivity, and $\sigma$ is the conductivity, is given by $L = \pi^2 k_B^2/(3e^2)$.

For a strange metal with a “relativistic” Hamiltonian, hydrodynamic, holographic, and memory function methods yield

$$\sigma = \sigma_Q \left(1 + \frac{e^2 v_F^2 Q^2 \tau_{\text{imp}}}{\mathcal{H}\sigma_Q}\right), \quad \kappa = \frac{v_F^2 \mathcal{H}\tau}{T} \left(1 + \frac{e^2 v_F^2 Q^2 \tau_{\text{imp}}}{\mathcal{H}\sigma_Q}\right)^{-1}$$

$$L = \frac{v_F^2 \mathcal{H}\tau_{\text{imp}}}{T^2\sigma_Q} \left(1 + \frac{e^2 v_F^2 Q^2 \tau_{\text{imp}}}{\mathcal{H}\sigma_Q}\right)^{-2},$$

where $\mathcal{H}$ is the enthalpy density, $\tau_{\text{imp}}$ is the momentum relaxation time (from impurities), while $\sigma = \sigma_Q$, an intrinsic, finite, “quantum critical” conductivity. Note that the limits $Q \to 0$ and $\tau_{\text{imp}} \to \infty$ do not commute.

M. Müller and S. Sachdev, PRB 78, 115419 (2008)
Observation of the Dirac fluid
and the breakdown of the Wiedemann-Franz law in graphene

Jesse Crossno,1, 2 Jing K. Shi,1 Ke Wang,1 Xiaomeng Liu,1 Achim Harzheim,1 Andrew Lucas,1 Subir Sachdev,1, 3 Philip Kim,1, 2, * Takashi Taniguchi,4 Kenji Watanabe,4 Thomas A. Ohki,5 and Kin Chung Fong5, †

1 Department of Physics, Harvard University, Cambridge, MA 02138, USA
2 John A. Paulson School of Engineering and Applied Sciences, Harvard University, Cambridge, MA 02138, USA
3 Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2L 2Y5, Canada
4 National Institute for Materials Science, Namiki 1-1, Tsukuba, Ibaraki 305-0044, Japan
5 Raytheon BBN Technologies, Quantum Information Processing Group, Cambridge, Massachusetts 02138, USA

(Dated: September 28, 2015)

Interactions between particles in quantum many-body systems can lead to collective behavior described by hydrodynamics. One such system is the electron-hole plasma in graphene near the charge neutrality point which can form a strongly coupled Dirac fluid. This charge neutral plasma of quasi-relativistic fermions is expected to exhibit a substantial enhancement of the thermal conductivity, due to decoupling of charge and heat currents within hydrodynamics. Employing high sensitivity Johnson noise thermometry, we report the breakdown of the Wiedemann-Franz law in graphene, with a thermal conductivity an order of magnitude larger than the value predicted by Fermi liquid theory. This result is a signature of the Dirac fluid, and constitutes direct evidence of collective motion in a quantum electronic fluid.

arXiv:1509.04713
Graphene

\[
\sim \sqrt{n} (1 + \lambda \ln \Lambda \sqrt{n})
\]

\[
T(K)
\]

Quantum critical

Dirac liquid

Hole

Fermi liquid

Electron

Fermi liquid

M. Müller, L. Fritz, and S. Sachdev, PRB 78, 115406 (2008)
M. Müller and S. Sachdev, PRB 78, 115419 (2008)
Strange metal in graphene

J. Crossno *et al.* arXiv:1509.04713
Strange metal in graphene

Wiedemann-Franz obeyed

Strange metal in graphene

Wiedemann-Franz law violated!

Lorentz ratio \( L = \frac{\kappa_e}{(T\sigma)} \)

\[
= \frac{v_F^2 \mathcal{H} \tau_{\text{imp}}}{T^2 \sigma Q} \left( 1 + e^2 v_F^2 Q^2 \tau_{\text{imp}} / (\mathcal{H} \sigma Q) \right)^2
\]

Comparison to theory with a single momentum relaxation time $\tau_{\text{imp}}$. Best fit of density dependence to thermal conductivity does not capture the density dependence of electrical conductivity.

Figure 3: A cartoon of a nearly quantum critical fluid where our hydrodynamic description of transport is sensible. The local chemical potential $\mu(x)$ always obeys $|\mu| \ll k_B T$, and so the entropy density $s/k_B$ is much larger than the charge density $|n|$; both electrons and holes are everywhere excited, and the energy density $\epsilon$ does not fluctuate as much relative to the mean. Near charge neutrality the local charge density flips sign repeatedly. The correlation length of disorder $\xi$ is much larger than $l_{ee}$, the electron-electron interaction length.

Numerically solve the hydrodynamic equations of Hartnoll, Kovtun, Müller, Sachdev (PRB 76, 144502 (2007)) but in the presence of a $x$-dependent chemical potential. The thermoelectric transport properties will then depend upon the value of the shear viscosity, $\eta$. 

Note $n \equiv Q$
Figure 1: A comparison of our hydrodynamic theory of transport with the experimental results of [33] in clean samples of graphene at $T = 75$ K. We study the electrical and thermal conductances at various charge densities $n$ near the charge neutrality point. Experimental data is shown as circular red data markers, and numerical results of our theory, averaged over 30 disorder realizations, are shown as the solid blue line. Our theory assumes the equations of state described in (27) with the parameters $C_0 = 11$, $C_2 = 9$, $C_4 = 200$, $\mu_0 = 110$, $\eta = 1.7$, and (28) with $\mu_0 = 0.13$. The yellow shaded region shows where Fermi liquid behavior is observed and the Wiedemann-Franz law is restored, and our hydrodynamic theory is not valid in or near this regime. We also show the predictions of (2) as dashed purple lines, and have chosen the 3 parameter fit to be optimized for $L(n)$. where $e$ is the electron charge, $s$ is the entropy density, $n$ is the charge density (in units of length $^2$), $H$ is the enthalpy density, $\mu$ is a momentum relaxation time, and $q$ is a quantum critical effect, whose existence is a new effect in the hydrodynamic gradient expansion of a relativistic fluid. Note that up to $q$, $(n/\eta)$ is simply described by Drude physics. The Lorenz ratio then takes the general form $L(n) = L_{DF}(1 + (n/\eta)^2)^2$, (3)

$L_{DF} = v_F^2 H/\mu T^2 q$, (4a)

$n_2^2 = H q e^2 v_F^2/\mu$. (4b)

$L(n)$ can be parametrically larger than $L_{WF}$ (as $\mu \to 1$ and $n \mu \eta \to 0$), and much smaller ($n = n_0$).

Both of these predictions were observed in the recent experiment, and fits of the measured $L$ to (3) were quantitatively consistent, until large enough $n$ where Fermi liquid behavior was restored. However, the experiment also found that the conductivity did not grow rapidly away from $n = 0$ as predicted in (2), despite a large peak in $\mu(n)$ near $n = 0$, as we show in Figure 1. Furthermore, the theory of [25] does not make clear predictions for the temperature dependence of $\mu$, which determines $\mu(T)$. In this paper, we argue that there are two related reasons for the breakdown of (2). One is that the dominant source of disorder in graphene – fluctuations in the local charge density, commonly referred to as charge puddles [43, 44, 45, 46] – are not perturbatively weak, and therefore a non-perturbative treatment of their effect is necessary.

The second is that the parameter $\mu$, even when it is sharply defined, is its $T$ dependencies of other parameters also agree well with expectation.

Figure 2: A comparison of our hydrodynamic theory of transport with the experimental results of [33] in clean samples of graphene at the charge neutrality point ($n = 0$). We use no new fit parameters compared to Figure 1. The yellow shaded region denotes where Fermi liquid behavior is observed; the purple shaded region denotes the likely onset of electron-phonon coupling.

Solution of the hydrodynamic equations in the presence of a space-dependent chemical potential. Best fit of density dependence to thermal conductivity now gives a better fit to the density dependence of the electrical conductivity (for $\eta/s \approx 10$). The $T$ dependencies of other parameters also agree well with expectation.
Strange metal in graphene

Negative local resistance due to viscous electron backflow in graphene

D. A. Bandurin\textsuperscript{1}, I. Torre\textsuperscript{2,3}, R. Krishna Kumar\textsuperscript{1,4}, M. Ben Shalom\textsuperscript{1,5}, A. Tomadin\textsuperscript{6}, A. Principi\textsuperscript{7}, G. H. Auton\textsuperscript{5}, E. Khestanova\textsuperscript{1,5}, K. S. Novoselov\textsuperscript{5}, I. V. Grigorieva\textsuperscript{1}, L. A. Ponomarenko\textsuperscript{1,4}, A. K. Geim\textsuperscript{1}, M. Polini\textsuperscript{3,6}

\textbf{Figure 1.} Viscous backflow in doped graphene. (a,b) Steady-state distribution of current injected through a narrow slit for a classical conducting medium with zero \( v \) (a) and a viscous Fermi liquid (b). (c) Optical micrograph of one of our SLG devices. The schematic explains the measurement geometry for vicinity resistance. (d,e) Longitudinal conductivity \( \sigma_{xx} \) and \( R_V \) for this device as a function of \( n \) induced by applying gate voltage. \( I = 0.3 \ \mu\text{A}; \ L = 1 \ \mu\text{m} \). For more detail, see Supplementary Information.
Quantum matter without quasiparticles

1. Experiment and theory in graphene

2. A solvable model of a strange metal

3. Holography and charged black holes

4. Transport in strange metals
Quantum matter without quasiparticles

1. Experiment and theory in graphene

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4. Transport in strange metals
Infinite-range model of a Fermi liquid

\[
H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \ldots
\]

\[
c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}
\]

\[
\frac{1}{N} \sum_i c_i^\dagger c_i = Q
\]

\(t_{ij}\) are independent random variables with \(\overline{t_{ij}} = 0\) and \(|t_{ij}|^2 = t^2\)

Fermions occupying the eigenstates of a \(N \times N\) random matrix
Infinite-range model of a Fermi liquid

Feynman graph expansion in $t_{ij}$, and graph-by-graph average, yields exact equations in the large $N$ limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = t^2 G(\tau)$$

$$G(\tau = 0^-) = Q.$$

$G(\omega)$ can be determined by solving a quadratic equation.

Fermions occupying eigenstates with a “semi-circular” density of states
Infinite-range model of a strange metal

\[ H = \frac{1}{(NM)^{1/2}} \sum_{i,j=1}^{N} \sum_{\alpha,\beta=1}^{M} J_{ij} c_{i\alpha}^\dagger c_{i\beta} c_{j\beta}^\dagger c_{j\alpha} \]

\[ c_{i\alpha} c_{j\beta} + c_{j\beta} c_{i\alpha} = 0 \quad , \quad c_{i\alpha} c_{j\beta}^\dagger + c_{j\beta}^\dagger c_{i\alpha} = \delta_{ij} \delta_{\alpha\beta} \]

\[ \frac{1}{M} \sum_{\alpha} c_{i\alpha}^\dagger c_{i\alpha} = Q \]

\( J_{ij} \) are independent random variables with \( \overline{J_{ij}} = 0 \) and \( \overline{J_{ij}^2} = J^2 \)

\( N \to \infty \) at \( M = 2 \) yields spin-glass ground state.

\( N \to \infty \) and then \( M \to \infty \) yields critical strange metal

Infinite-range model of a strange metal

\[ H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;kl} c_i^\dagger c_j c_k c_\ell - \mu \sum_i c_i^\dagger c_i \]

\[ c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij} \]

\[ Q = \frac{1}{N} \sum_i c_i^\dagger c_i \]

\( J_{ij;kl} \) are independent random variables with \( \overline{J_{ij;kl}} = 0 \) and \( |J_{ij;kl}|^2 = J^2 \)

\( N \rightarrow \infty \) yields same critical strange metal; simpler to study numerically

Infinite-range strange metals

Feynman graph expansion in $J_{ij}$, and graph-by-graph average, yields exact equations in the large $N$ limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

$$G(\tau = 0^-) = Q.$$

Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{1}{A} \sqrt{z} + \ldots , \quad G(z) = \frac{A}{\sqrt{z}}$$

for some complex $A$.

Infinite-range strange metals

Local fermion density of states

\[ \rho(\omega) = -\text{Im} \ G(\omega) \sim \begin{cases} \omega^{-1/2}, & \omega > 0 \\ e^{-2\pi\mathcal{E}} \ |\omega|^{-1/2}, & \omega < 0. \end{cases} \]

\( \mathcal{E} \) encodes the particle-hole asymmetry

While \( \mathcal{E} \) determines the low energy spectrum, it is determined by the total fermion density \( Q \):

\[ Q = \frac{1}{4} (3 - \tanh(2\pi\mathcal{E})) - \frac{1}{\pi} \tan^{-1} (e^{2\pi\mathcal{E}}). \]

Analog of the relationship between \( Q \) and \( k_F \) in a Fermi liquid.

Infinite-range strange metals

At non-zero temperature, $T$, the Green’s function also fully determined by $\mathcal{E}$.

$$G^R(\omega) = \frac{-iC e^{-i\theta}}{(2\pi T)^{1-2\Delta}} \frac{\Gamma\left(\Delta - \frac{i\hbar \omega}{2\pi T} + i\mathcal{E}\right)}{\Gamma\left(1 - \Delta - \frac{i\hbar \omega}{2\pi T} + i\mathcal{E}\right)}$$

where $\Delta = 1/4$ and $e^{2\pi \mathcal{E}} = \frac{\sin(\pi \Delta + \theta)}{\sin(\pi \Delta - \theta)}$.

Note $G(\omega) \equiv f(\hbar \omega / k_B T)$

A. Georges and O. Parcollet PRB 59, 5341 (1999)
“Critically-screened” spin has “irrational” entropy

Infinite-range strange metals

The entropy per site, $S$, has a non-zero limit as $T \to 0$, and can be viewed as each site acquiring the universal boundary entropy of the multichannel Kondo problem.

This entropy obeys

$$\left( \frac{\partial S}{\partial Q} \right)_T = - \left( \frac{\partial \mu}{\partial T} \right)_Q = 2\pi \mathcal{E}$$

Note that $S$ and $\mathcal{E}$ involve low-lying states, while $Q$ depends upon all states, and details of the UV structure.


Quantum matter without quasiparticles

1. Experiment and theory in graphene
2. A solvable model of a strange metal
3. Holography and charged black holes
4. Transport in strange metals
AdS/CFT correspondence at zero temperature

Einstein gravity

\[ S_E = \int d^{d+2}x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( \mathcal{R} + \frac{d(d+1)}{L^2} \right) \right] \]

\[ ds^2 = \left( \frac{L}{r} \right)^2 \left[ dr^2 - dt^2 + d\mathbf{x}^2 \right] \]
AdS/CFT correspondence at non-zero temperature

Einstein gravity \( S_E = \int d^{d+2}x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( \mathcal{R} + \frac{d(d+1)}{L^2} \right) \right] \)

AdS-Schwarzschild

AdS-Schwarzschild

CFT\(_{d+1}\) at Hawking temperature \( T \)

Entropy density of CFT\(_{d+1}\), \( S \sim T^d \)

Bekenstein-Hawking entropy density, \( S_{BH} \sim T^d \)
AdS/CFT correspondence at non-zero temperature

Einstein gravity \( S_E = \int d^{d+2}x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( \mathcal{R} + \frac{d(d+1)}{L^2} \right) \right] \)

AdS-Schwarzschild

CFT\(_{d+1}\) at Hawking temperature \( T \) of horizon

For SU(\( N \)) SYM in \( d = 3 \), \( S_{\text{BH}} = (\pi^2/2)N^2T^3 \). But there is (still) no confirmation of this from a field-theory computation on SYM.
Charged black branes

Einstein-Maxwell theory

\[ S_{EM} = \int d^{d+2}x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( \mathcal{R} + \frac{d(d+1)}{L^2} - \frac{R^2}{g_F^2} F^2 \right) \right] \]

AdS-Reissner-Nordstrom

Quantum matter on the boundary with a variable charge density \( Q \) of a global U(1) symmetry.

Realizes a strange metal: a state with an unbroken global U(1) symmetry with a continuously variable charge density, \( Q \), at \( T = 0 \) which does not have any quasiparticle excitations.

A. Chamblin, R. Emparan, C. V. Johnson, and R. C. Myers, 99
• Near-horizon metric is AdS$_2$, with near-horizon electric field $\mathcal{E}$.
Quantum fields on charged black branes

\[ ds^2 = (d\zeta^2 - dt^2)/\zeta^2 + d\vec{x}^2 \]

Gauge field: \( A = (\mathcal{E}/\zeta) dt \)

\[ \zeta = \infty \]

Boundary Green’s function of \( \psi \) at \( T = 0 \)

\[ \text{Im} G(\omega) \sim \begin{cases} \omega^{-(1 - 2\Delta)} & , \ \omega > 0 \\ e^{-2\pi\mathcal{E} |\omega|^{-(1 - 2\Delta)}} & , \ \omega < 0. \end{cases} \]

where the fermion scaling dimension \( \Delta \) is a function of \( m \)

\( \mathcal{E} \) encodes the particle-hole asymmetry

T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh, PRD 83, 125002 (2011)
Quantum fields on charged black branes

Conformal mapping to $T > 0$

$\zeta = \zeta_0$

$$ds^2 = \left[ d\zeta^2 / (1 - \zeta^2 / \zeta_0^2) - (1 - \zeta^2 / \zeta_0^2) dt^2 \right] / \zeta^2 + d\vec{x}^2$$

Gauge field: $A = E (1 / \zeta - 1 / \zeta_0) dt$ with $\zeta_0 = 1 / (2\pi T)$

$\zeta = \infty$

Boundary Green’s function of $\psi$ at $T > 0$

is fully determined by $\mathcal{E}$

$$G^R(\omega) = \frac{-iC e^{-i\theta}}{(2\pi T)^{1-2\Delta}} \frac{\Gamma \left( \Delta - \frac{i\hbar\omega}{2\pi T} + i\mathcal{E} \right)}{\Gamma \left( 1 - \Delta - \frac{i\hbar\omega}{2\pi T} + i\mathcal{E} \right)}$$

where $e^{2\pi\mathcal{E}} = \frac{\sin(\pi\Delta + \theta)}{\sin(\pi\Delta - \theta)}$.

T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh, PRD 83, 125002 (2011)
**General Relativity of charged black branes**

Conformal mapping to $T > 0$

\[ \zeta = \zeta_0 \]

\[ \frac{ds^2}{\zeta^2} = \left[ \frac{d\zeta^2}{1 - \zeta^2 / \zeta_0^2} - (1 - \zeta^2 / \zeta_0^2)dt^2 \right] / \zeta^2 + d\vec{x}^2 \]

Gauge field: $A = \mathcal{E}(1/\zeta - 1/\zeta_0)dt$ with $\zeta_0 = 1/(2\pi T)$

\[ \zeta = \infty \]

- As $T \to 0$, there is a non-zero Bekenstein-Hawking entropy, $S_{BH}$.

- Using Gauss’s Law, it can be shown that $\mu(T) = -2\pi \mathcal{E}T +$ constant as $T \to 0$.

- Using a thermodynamic Maxwell relation (also obeyed by gravity),

\[
\left( \frac{\partial S_{BH}}{\partial Q} \right)_T = - \left( \frac{\partial \mu}{\partial T} \right)_Q = 2\pi \mathcal{E}
\]

A. Sen
hep-th/0506177

S. Sachdev
1506.05111
General Relativity of charged black branes

Conformal mapping to $T > 0$

$\zeta = \zeta_0$

$ds^2 = \left[ d\zeta^2 / (1 - \zeta^2 / \zeta_0^2) - (1 - \zeta^2 / \zeta_0^2)dt^2 \right] / \zeta^2 + d\vec{x}^2$

Gauge field: $A = \mathcal{E}(1/\zeta - 1/\zeta_0)dt$ with $\zeta_0 = 1/(2\pi T)$

$\zeta = \infty$

- As $T \to 0$, there is a non-zero Bekenstein-Hawking entropy, $S_{BH}$.
- Using Gauss’s Law, it can be shown that $\mu(T) = -2\pi \mathcal{E}T + \text{constant}$ as $T \to 0$.
- Using a thermodynamic Maxwell relation (also obeyed by gravity)

$$\left( \frac{\partial S_{BH}}{\partial Q} \right)_T = - \left( \frac{\partial \mu}{\partial T} \right)_Q = 2\pi \mathcal{E}$$

Also obeyed by Wald entropy in higher-derivative gravity.
\[ H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;kl} c_i^\dagger c_j^\dagger c_k c_\ell \]

\[ Q = \frac{1}{N} \sum_i \langle c_i^\dagger c_i \rangle. \]

Local fermion density of states
\[ \rho(\omega) \sim \begin{cases} 
\omega^{-1/2}, & \omega > 0 \\
e^{-2\pi\varepsilon |\omega|^{-1/2}}, & \omega < 0.
\end{cases} \]

Known ‘equation of state’ determines \( \mathcal{E} \) as a function of \( Q \)

Microscopic zero temperature entropy density, \( S \), obeys
\[ \frac{\partial S}{\partial Q} = 2\pi \mathcal{E} \]
\[ H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;kl} c_i^\dagger c_j^\dagger c_k c_\ell \]

Microscopic zero temperature entropy density, \( S \), obeys
\[ \frac{\partial S}{\partial Q} = 2\pi \mathcal{E} \]

Einstein-Maxwell theory + cosmological constant

Horizon area \( \mathcal{A}_h \);
\( \text{AdS}_2 \times \mathbb{R}^d \)
\[ ds^2 = \frac{(d\zeta^2 - dt^2)}{\zeta^2} + d\bar{x}^2 \]
Gauge field: \( A = (\mathcal{E}/\zeta) dt \)

\[ Q = \frac{1}{N} \sum_i \langle c_i^\dagger c_i \rangle. \]

Local fermion density of states
\[ \rho(\omega) \sim \begin{cases} \omega^{-1/2}, & \omega > 0 \\ e^{-2\pi \mathcal{E}} |\omega|^{-1/2}, & \omega < 0. \end{cases} \]

Known ‘equation of state’ determines \( \mathcal{E} \) as a function of \( Q \)

\( \zeta = \infty \)
\[ \zeta \]
\[ \zeta \]

\[ \mathcal{L} = \bar{\psi} \Gamma^\alpha D_\alpha \psi + m \bar{\psi} \psi \]

Local fermion density of states
\[ \rho(\omega) \sim \begin{cases} \omega^{-1/2}, & \omega > 0 \\ e^{-2\pi \mathcal{E}} |\omega|^{-1/2}, & \omega < 0. \end{cases} \]

‘Equation of state’ relating \( \mathcal{E} \) and \( Q \) depends upon the geometry of spacetime far from the \( \text{AdS}_2 \)

Black hole thermodynamics (classical general relativity) yields
\[ \frac{\partial S_{\text{BH}}}{\partial Q} = 2\pi \mathcal{E} \]

\[ H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{i,j;k,\ell} c_i^\dagger c_j^\dagger c_k c_\ell \]

\[ Q = \frac{1}{N} \sum_i \langle c_i^\dagger c_i \rangle. \]

Local fermion density of states
\[ \rho(\omega) \sim \begin{cases} \omega^{-1/2}, & \omega > 0 \\ e^{-2\pi \epsilon} |\omega|^{-1/2}, & \omega < 0. \end{cases} \]

Known ‘equation of state’ determines \( \mathcal{E} \) as a function of \( Q \)

Microscopic zero temperature entropy density, \( S \), obeys
\[ \frac{\partial S}{\partial Q} = 2\pi \mathcal{E} \]

Evidence for AdS\(_2\) gravity dual of \( H \)

Einstein-Maxwell theory + cosmological constant

Horizon area \( \mathcal{A}_h \):
\( \text{AdS}_2 \times \mathbb{R}^d \)
\[ ds^2 = (d\xi^2 - dt^2)/\xi^2 + dx^2 \]
Gauge field: \( A = (\mathcal{E}/\xi)dt \)

Boundary area \( \mathcal{A}_b \): charge density \( Q \)

\[ \zeta = \infty \]
\[ \mathcal{L} = \bar{\psi} \Gamma^\alpha D_\alpha \psi + m \bar{\psi} \psi \]

Local fermion density of states
\[ \rho(\omega) \sim \begin{cases} \omega^{-1/2}, & \omega > 0 \\ e^{-2\pi \epsilon} |\omega|^{-1/2}, & \omega < 0. \end{cases} \]

‘Equation of state’ relating \( \mathcal{E} \)
and \( Q \) depends upon the geometry of spacetime far from the AdS\(_2\)

Black hole thermodynamics (classical general relativity) yields
\[ \frac{\partial S_{\text{BH}}}{\partial Q} = 2\pi \mathcal{E} \]

S. Sachdev, arXiv:1506.05111
Quantum matter without quasiparticles

1. Experiment and theory in graphene
2. A solvable model of a strange metal
3. Holography and charged black holes
4. Transport in strange metals
Beyond Perturbation Theory

**Summary**

- Hydrodynamics
- Memory matrix
- Holography
- Universal constraints on transport
- Few conserved quantities
- Perturbative limit
- Long time dynamics;
  “renormalized IR fluid” emerges
- Matrix large N theory;
  Non-perturbative computations

**Figure** from [Lucas, Sachdev, Physical Review B91 195122 (2015)]

**References**

more generally, measure thermoelectric transport:

\[
\begin{pmatrix}
  \delta J_i \\
  \delta Q_i
\end{pmatrix} = \begin{pmatrix}
  \sigma_{ij} & \alpha_{ij} \\
  T\bar{\alpha}_{ij} & \bar{\kappa}_{ij}
\end{pmatrix} \begin{pmatrix}
  \delta E_j \\
  -\partial_j \delta T \equiv T \delta \zeta_j
\end{pmatrix}.
\]

\( \sigma = \) easy experiment; related to QFT correlators:

\[
\sigma_{ij}(\omega) = \frac{i}{\omega} \langle J_i(-\omega)J_j(\omega) \rangle, \quad \text{etc.}
\]
Thermoelectric transport coefficients

Transport has two components: a “momentum drag” term, and a “quantum critical” term.

\[ \sigma = \frac{Q^2}{\mathcal{M}} \pi \delta(\omega) + \sigma_Q(\omega) \]

\[ \alpha = \frac{SQ}{\mathcal{M}} \pi \delta(\omega) + \alpha_Q(\omega) \]

\[ \bar{\kappa} = \frac{TS^2}{\mathcal{M}} \pi \delta(\omega) + \bar{\kappa}_Q(\omega) \]

with entropy density $S$, $Q \equiv \chi_{J_x, P_x}$, and $\mathcal{M} \equiv \chi_{P_x, P_x}$.

Obtained in hydrodynamics, holography, and by memory functions

Relativistic hydrodynamics

- hydrodynamics when \( l \gg l_{ee}, t \gg t_{ee} \)
- long time dynamics governed by conservation laws:

\[
\partial_{\nu} T^{\mu\nu} = J_{\nu} \left( F_{\text{ext}}^{\mu\nu} \right), \quad \partial_{\mu} J^{\mu} = 0.
\]

- dynamics of relaxation to equilibrium

- expand \( T^{\mu\nu}, J^{\mu} \) in perturbative parameter \( l_{ee} \partial_{\mu} \):

\[
T^{\mu\nu} = P \eta^{\mu\nu} + (\epsilon + P) u^{\mu} u^{\nu} - 2 P^{\mu\rho} P^{\nu\sigma} \eta \partial_{(\rho} u_{\sigma)} - P^{\mu\nu} \left( \zeta - \frac{2\eta}{d} \right) \partial_{\rho} u^{\rho} + \cdots,
\]

\[
J^{\mu} = Q u^{\mu} - \sigma_{Q} P^{\mu\rho} \left( \partial_{\rho} \mu - \frac{\mu}{T} \partial_{\rho} T - u^{\nu} F_{\rho\nu}^{\text{ext}} \right) + \cdots,
\]

\[
P^{\mu\nu} \equiv \eta^{\mu\nu} + u^{\mu} u^{\nu},
\]

\[
Q^{i} = J^{i} - \mu T^{ti}
\]

- quantum physics \( \rightarrow \) values of \( P, \sigma_{Q}, \text{etc...} \)

Transport has two components: a “momentum drag” term, and a “quantum critical” term.

\[
\sigma = \frac{Q^2}{\mathcal{M}} \pi \delta(\omega) + \sigma_Q(\omega)
\]

\[
\alpha = \frac{SQ}{\mathcal{M}} \pi \delta(\omega) + \alpha_Q(\omega)
\]

\[
\bar{\kappa} = \frac{TS^2}{\mathcal{M}} \pi \delta(\omega) + \bar{\kappa}_Q(\omega)
\]

with entropy density \( S \), \( Q \equiv \chi_{J_x,P_x} \), and \( \mathcal{M} \equiv \chi_{P_x,P_x} \).

In theories which are relativistic at high energies (including graphene), \( T\alpha_Q(\omega) = -\mu\sigma_Q(\omega) \), \( T\bar{\kappa}_Q(\omega) = \mu^2\sigma_Q(\omega) \), \( \mathcal{M} = TS + \mu Q = \mathcal{H} \) the enthalpy density, and \( Q = n \) the electron density.

Thermoelectric transport coefficients

Transport has two components: a “momentum drag” term, and a “quantum critical” term.

\[
\begin{align*}
\sigma &= \frac{Q^2}{\mathcal{M}} \left( \frac{1}{-i\omega + 1/\tau} \right) + \sigma_Q(\omega) \\
\alpha &= \frac{S Q}{\mathcal{M}} \left( \frac{1}{-i\omega + 1/\tau} \right) + \alpha_Q(\omega) \\
\bar{\kappa} &= \frac{T S^2}{\mathcal{M}} \left( \frac{1}{-i\omega + 1/\tau} \right) + \bar{\kappa}_Q(\omega)
\end{align*}
\]

Momentum relaxation by an external source \( h \) coupling to the operator \( \mathcal{O} \)

\[
H = H_0 - \int d^d x \ h(x) \mathcal{O}(x).
\]

\[
\frac{\mathcal{M}}{\tau} = \lim_{\omega \to 0} \int d^d q |h(q)|^2 q_x^2 \frac{\text{Im} \left( G_{\mathcal{O}\mathcal{O}}^R(q, \omega) \right) H_0}{\omega} + \text{higher orders in } h
\]

Magneto-electric transport

Transport has two components: a “momentum drag” term, and a “quantum critical” term.

\[
\sigma_{xx} = \frac{(\tau^{-1} - i\omega)M\sigma_Q + Q^2 + B^2\sigma_Q^2}{Q^2B^2 + ((\tau^{-1} - i\omega)M + B^2\sigma_Q)^2}M\left(\frac{1}{\tau} - i\omega\right),
\]

\[
\sigma_{xy} = \frac{2(\tau^{-1} - i\omega)M\sigma_Q + Q^2 + B^2\sigma_Q^2}{Q^2B^2 + ((\tau^{-1} - i\omega)M + B^2\sigma_Q)^2}BQ.
\]

Electrical and thermal magnetotransport

in a magnetic field \(B\) with no additional parameters

(assuming \(\sigma_Q\) is field-independent)

Figure 1: A comparison of our hydrodynamic theory of transport with the experimental results of \[33\] in clean samples of graphene at $T = 75$ K. We study the electrical and thermal conductances at various charge densities $n$ near the charge neutrality point. Experimental data is shown as circular red data markers, and numerical results of our theory, averaged over 30 disorder realizations, are shown as the solid blue line. Our theory assumes the equations of state described in (27) with the parameters $C_0 \equiv 11$, $C_2 \equiv 9$, $C_4 \equiv 200$, $\alpha_0 \equiv 110$, $\alpha_7 \equiv 1.7$, and (28) with $u_0 \equiv 0.13$. The yellow shaded region shows where Fermi liquid behavior is observed and the Wiedemann-Franz law is restored, and our hydrodynamic theory is not valid in or near this regime. We also show the predictions of (2) as dashed purple lines, and have chosen the 3 parameter fit to be optimized for $L(n)$. Where $e$ is the electron charge, $s$ is the entropy density, $n$ is the charge density (in units of length $^2$), $H$ is the enthalpy density, $\tau$ is a momentum relaxation time, and $q$ is a quantum critical effect, whose existence is a new effect in the hydrodynamic gradient expansion of a relativistic fluid. Note that up to $q$, $(n^2)$ is simply described by Drude physics. The Lorenz ratio then takes the general form $L(n) = L_{DF}(1 + (n/n_0)^2)^2$, (3) where $L_{DF} = v_F^2 H \tau T^2 q$, (4a) $n_0 = H q e^2 v_F \tau$. (4b) $L(n)$ can be parametrically larger than $L_{WF}$ (as $\tau \to 1$ and $n \to \infty$) and much smaller ($n \neq n_0$). Both of these predictions were observed in the recent experiment, and fits of the measured $L$ to (3) were quantitatively consistent, until large enough $n$ where Fermi liquid behavior was restored. However, the experiment also found that the conductivity did not grow rapidly away from $n = 0$ as predicted in (2), despite a large peak in $L(n)$ near $n = 0$, as we show in Figure 1. Furthermore, the theory of \[25\] does not make clear predictions for the temperature dependence of $\alpha$, which determines $L(T)$. In this paper, we argue that there are two related reasons for the breakdown of (2). One is that the dominant source of disorder in graphene – fluctuations in the local charge density, commonly referred to as charge puddles \[43, 44, 45, 46\] – are not perturbatively weak, and therefore a non-perturbative treatment of their effect is necessary. The second is that the parameter $\tau$, even when it is sharply defined, is $3$. See \[47, 48\] for a theory of electrical conductivity in charge puddle dominated graphene at low temperatures. A. Lucas, J. Crossno, K.C. Fong, P. Kim, and S. Sachdev, arXiv:1510.01738.

Solution of the HKMS equations in the presence of a space-dependent chemical potential. Best fit of density dependence to thermal conductivity now gives a better fit to the density dependence of the electrical conductivity (for $\eta/s \approx 10$). The $T$ dependencies of other parameters also agree well with expectation.
Quantum matter without quasiparticles

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5. Transport in stranger metals
Quantum matter without quasiparticles

1. Experiment and theory in graphene
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High temperature superconductors

\[ \text{YBa}_2\text{Cu}_3\text{O}_{6+x} \]

CuO\(_2\) plane

\[ 11.6802 \text{ Å} \]
\[ 3.8872 \text{ Å} \]
\[ 3.8227 \text{ Å} \]
Anti-ferromagnet with $p$ holes per square
$\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$

Figure: K. Fujita and J. C. Seamus Davis
Fermi liquid (FL): a conventional metal
A new metal — a fractionalized Fermi liquid (FL*) — with electron-like quasiparticles on a Fermi surface of size $p$

Strange metal

No quasiparticle excitations
Thermoelectric transport coefficients

Transport has two components: a “momentum drag” term, and a “quantum critical” term.

\[
\sigma = \frac{Q^2}{\mathcal{M}} \pi \delta(\omega) + \sigma_Q(\omega)
\]
\[
\alpha = \frac{SQ}{\mathcal{M}} \pi \delta(\omega) + \alpha_Q(\omega)
\]
\[
\bar{\kappa} = \frac{TS^2}{\mathcal{M}} \pi \delta(\omega) + \bar{\kappa}_Q(\omega)
\]

with entropy density \( S \), \( Q \equiv \chi_{J_x, P_x} \), and \( \mathcal{M} \equiv \chi_{P_x, P_x} \).

Obtained in hydrodynamics, holography, and by memory functions

Thermoelectric transport coefficients

Transport has two components: a “momentum drag” term, and a “quantum critical” term.

\[ \sigma = \frac{Q^2}{\mathcal{M}} \frac{1}{(-i\omega + 1/\tau)} + \sigma_Q(\omega) \]
\[ \alpha = \frac{SQ}{\mathcal{M}} \frac{1}{(-i\omega + 1/\tau)} + \alpha_Q(\omega) \]
\[ \bar{k} = \frac{TS^2}{\mathcal{M}} \frac{1}{(-i\omega + 1/\tau)} + \bar{k}_Q(\omega) \]

Momentum relaxation by an external source \( h \) coupling to the operator \( \mathcal{O} \)

\[ H = H_0 - \int d^dx \ h(x) \mathcal{O}(x). \]
\[ \frac{\mathcal{M}}{\tau} = \lim_{\omega \to 0} \int d^d q |h(q)|^2 q_x^2 \frac{\text{Im} (G^R_{\mathcal{O}\mathcal{O}}(q, \omega))}{\omega} H_0 + \text{higher orders in } h \]

Transport has two components: a “momentum drag” term, and a “quantum critical” term.

\[
\sigma_{xx} = \frac{(\tau^{-1} - i\omega)\mathcal{M}\sigma_Q + Q^2 + B^2\sigma_Q^2}{Q^2B^2 + ((\tau^{-1} - i\omega)\mathcal{M} + B^2\sigma_Q)^2}\mathcal{M}\left(\frac{1}{\tau} - i\omega\right),
\]

\[
\sigma_{xy} = \frac{2(\tau^{-1} - i\omega)\mathcal{M}\sigma_Q + Q^2 + B^2\sigma_Q^2}{Q^2B^2 + ((\tau^{-1} - i\omega)\mathcal{M} + B^2\sigma_Q)^2}BQ.
\]

Electrical and thermal magnetotransport in a magnetic field \(B\) with no additional parameters

(assuming \(\sigma_Q\) is field-independent)

Magneto-electric transport

Transport has two components: a “momentum drag” term, and a “quantum critical” term.

\[ \sigma_{xx} = \frac{(\tau^{-1} - i\omega)M\sigma_Q + Q^2 + B^2\sigma_Q^2}{Q^2B^2 + ((\tau^{-1} - i\omega)M + B^2\sigma_Q)^2}M\left(\frac{1}{\tau} - i\omega\right), \]

\[ \sigma_{xy} = \frac{2(\tau^{-1} - i\omega)M\sigma_Q + Q^2 + B^2\sigma_Q^2}{Q^2B^2 + ((\tau^{-1} - i\omega)M + B^2\sigma_Q)^2}BQ. \]

Blake and Donos: With \( \sigma_Q \sim 1/T, \tau_L \sim 1/T^2 \) and assuming momentum drag dominates longitudinal transport, we obtain \( \sigma_{xx} \sim 1/T \) and \( \tan(\theta_H) = \sigma_{xy}/\sigma_{xx} \sim 1/T^2 \), in agreement with strange metal data on cuprates (such data cannot be explained in a quasiparticle model).
Computations on non-Fermi liquids

Transport has two components: a “momentum drag” term, and a “quantum critical” term.

Spin density wave critical point

Momentum drag conductivity: $\sigma \sim T^{-1}$ from disorder-induced shifts in the position of the critical point

S. A. Hartnoll, D. M. Hofman, M. A. Metlitski and S. Sachdev, PRB 84, 125115 (2011)

Quantum critical conductivity: $\sigma \sim T^0$

A. A. Patel and S. Sachdev, PRB 90, 165146 (2014)

Computations on non-Fermi liquids

Transport has two components: a “momentum drag” term, and a “quantum critical” term.

Ising-nematic critical point

Momentum drag conductivity: $\sigma \sim T^{-2/3}$ from disorder-induced shifts in the position of the critical point

A. A. Patel and S. Sachdev, PRB 90, 165146 (2014)

Momentum drag conductivity: $\sigma \sim T^{1/2}$ from random-field disorder


Quantum critical conductivity: $\sigma \sim ??$, in progress...
Quantum matter without quasiparticles

- No quasiparticle excitations

- Shortest possible “collision time”, or more precisely, fastest possible local equilibration time $\sim \frac{\hbar}{k_B T}$

- Continuously variable density, $Q$
  (conformal field theories are usually at fixed density, $Q = 0$)

- Theory built from hydrodynamics/holography /memory-functions/strong-coupled-field-theory

- Exciting experimental realization in graphene.