Exotic phases of the Kondo lattice, and holography

Stanford, July 15, 2010

Talk online: sachdev.physics.harvard.edu
Outline

1. The Anderson/Kondo lattice models
   Luttinger’s theorem

2. Fractionalized Fermi liquids
   Metallic spin-liquid states

3. A mean field theory of a fractionalized Fermi liquid
   Marginal Fermi liquid physics

4. An AdS/CFT perspective
   Holographic metals as fractionalized Fermi liquids
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   Holographic metals as fractionalized Fermi liquids
Anderson/Kondo lattice models

Anderson model Hamiltonian for intermetallic compound with conduction electrons, $c_{i\sigma}$, and localized orbitals, $f_{i\sigma}$

\[
H = -\sum_{i<j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_i \left( V c_{i\sigma}^\dagger f_{i\sigma} + V f_{i\sigma}^\dagger c_{i\sigma} + \varepsilon_f (n_{fi\uparrow} + n_{fi\downarrow}) + U n_{fi\uparrow} n_{fi\downarrow} \right)
\]

\[
n_{fi\sigma} = f_{i\sigma}^\dagger f_{i\sigma} \quad ; \quad n_{ci\sigma} = c_{i\sigma}^\dagger c_{i\sigma} \quad ; \quad n_T = n_f + n_c
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$$n_{fi\sigma} = f_{i\sigma}^\dagger f_{i\sigma} ; \quad n_{ci\sigma} = c_{i\sigma}^\dagger c_{i\sigma} ; \quad n_T = n_f + n_c$$

In the limit of large $U$, this maps onto the Kondo lattice model of conduction electrons, $c_{i\sigma}$, and spins $\vec{S}_{fi} = f_{i\sigma}^\dagger \vec{\tau}_{\sigma\sigma'} f_{i\sigma'}$

$$H_K = - \sum_{i<j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_i J_K c_{i\sigma}^\dagger \vec{\tau}_{\sigma\sigma'} c_{i\sigma'} \cdot \vec{S}_{fi} + \sum_{i<j} J_H(i,j) \vec{S}_{fi} \cdot \vec{S}_{fj}$$
Luttinger’s theorem on a $d$-dimensional lattice

For simplicity, we consider systems with SU(2) spin rotation invariance, which is preserved in the ground state.

Let $v_0$ be the volume of the unit cell of the ground state, $n_T$ be the total number density of electrons per volume $v_0$. (need not be an integer)

Then, in a metallic Fermi liquid state with a sharp electron-like Fermi surface:

$$2 \times \frac{v_0}{(2\pi)^d} \left( \text{Volume enclosed by Fermi surface} \right) = n_T \, (\text{mod} \, 2)$$
Luttinger’s theorem on a \( d \)-dimensional lattice

For simplicity, we consider systems with SU(2) spin rotation invariance, which is preserved in the ground state.

Let \( \nu_0 \) be the volume of the unit cell of the ground state, \( n_T \) be the total number density of electrons per volume \( \nu_0 \). (need not be an integer)

Then, in a metallic Fermi liquid state with a sharp electron-like Fermi surface:

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2 \times \frac{\nu_0}{(2\pi)^d} \text{(Volume enclosed by Fermi surface)} = n_T \mod 2
\]

A “large” Fermi surface
Adiabatically insert flux $\Phi=2\pi$ (units $\hbar=c=\varepsilon=1$) acting on $\uparrow$ electrons. State changes from $|\Psi\rangle$ to $|\Psi'\rangle$, and $UH(0)U^{-1} = H(\Phi)$, where

$$U = \exp\left[\frac{2\pi i}{L_x} \sum_r x \hat{n}_{r\uparrow}\right].$$

Adiabatic process commutes with the translation operator $T_x$, so momentum $P_x$ is conserved.

However $U^{-1}T_x U = T_x \exp \left[ \frac{2\pi i}{L_x} \sum_r \hat{n}_{Tr\uparrow} \right]$;

so shift in momentum $\Delta P_x$ between states $U|\Psi'\rangle$ and $|\Psi\rangle$ is

$$\Delta P_x = \frac{\pi L_y}{v_0} n_T \left( \text{mod} \frac{2\pi}{a_x} \right) \quad (1).$$

Alternatively, we can compute $\Delta P_x$ by assuming it is absorbed by quasiparticles of a Fermi liquid. Each quasiparticle has its momentum shifted by $2\pi/L_x$, and so

$$\Delta P_x = \frac{2\pi}{L_x} \left( \text{Volume enclosed by Fermi surface} \right) \left( \text{mod} \frac{2\pi}{a_x} \right) \quad (2).$$

From (1) and (2), same argument in $y$ direction, using coprime $L_x/a_x, L_y/a_y$:

$$2 \times \frac{v_0}{(2\pi)^2} \left( \text{Volume enclosed by Fermi surface} \right) = n_T \left( \text{mod} \ 2 \right)$$

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n_{fi\sigma} = f_{i\sigma}^\dagger f_{i\sigma} ; \quad n_{ci\sigma} = c_{i\sigma}^\dagger c_{i\sigma} ; \quad n_T = n_f + n_c
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\[
n_{f_i\sigma} = f_{i\sigma}^\dagger f_{i\sigma} \quad ; \quad n_{c_i\sigma} = c_{i\sigma}^\dagger c_{i\sigma} \quad ; \quad n_T = n_f + n_c
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For small \( U \), we obtain a Fermi liquid ground state, with a “large” Fermi surface volume determined by \( n_T \pmod{2} \)
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n_{f i \sigma} = f_{i\sigma}^\dagger f_{i\sigma} \quad ; \quad n_{c i \sigma} = c_{i\sigma}^\dagger c_{i\sigma} \quad ; \quad n_T = n_f + n_c
\]

For small $U$, we obtain a Fermi liquid ground state, with a “large” Fermi surface volume determined by $n_T \pmod{2}$

This is adiabatically connected to a Fermi liquid ground state at large $U$, where $n_f = 1$, and whose Fermi surface volume must also be determined by

\[
n_T \pmod{2} = (1 + n_c) \pmod{2}
\]
Anderson/Kondo lattice models

\[ H_K = - \sum_{i<j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_i J_K c_{i\sigma}^\dagger \vec{S}_{\sigma^\prime} \cdot \vec{S}_{fi} \]

\[ + \sum_{i<j} J_H(i, j) \vec{S}_{fi} \cdot \vec{S}_{fj} \]

We can also use the Kondo lattice model to argue for a Fermi liquid ground state whose “large” Fermi surface volume is \((1 + n_c)(\text{mod } 2)\)
Arguments for the Fermi surface volume of the FL phase

Single ion Kondo effect implies $J_K \to \infty$ at low energies

$\left( c_{i \uparrow}^\dagger f_{i \downarrow}^\dagger - c_{i \downarrow}^\dagger f_{i \uparrow}^\dagger \right) |0\rangle$

$f_{i \downarrow}^\dagger |0\rangle$, $S=1/2$ hole

Fermi liquid of $S=1/2$ holes with hard-core repulsion

\begin{align*}
\text{Fermi surface volume} &= - (\text{density of holes}) \mod 2 \\
&= -(1 - n_c) = (1 + n_c) \mod 2
\end{align*}
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   *Holographic metals as fractionalized Fermi liquids*
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There exist “topologically ordered” ground states in dimensions $d > 1$ with a Fermi surface of electron-like quasiparticles for which

$$2 \times \frac{v_0}{(2\pi)^d} \left( \text{Volume enclosed by Fermi surface} \right) = (n_T - 1)(\text{mod } 2)$$
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A Fractionalized Fermi Liquid
Start with the Kondo lattice, with $J_H \gg J_K$. Assume $J_H$ are so that the $\vec{S}_f$ spins form a spin liquid

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\frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)
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$$\frac{1}{\sqrt{2}} \left( |\uparrow \downarrow \rangle - |\downarrow \uparrow \rangle \right)$$

Effect of flux-piercing on a spin liquid

N. E. Bonesteel, 

G. Misguich, C. Lhuillier, 
M. Mambrini, and P. Sindzingre, 

\[
|\Psi\rangle = \sum_{D} a_{D} |D\rangle
\]
Effect of flux-piercing on a spin liquid

\[ |D\rangle = \sum_D a_D |D\rangle \]

After flux insertion, \( |D\rangle \Rightarrow (-1)^{\text{Number of bonds cutting dashed line}} |D\rangle \); equivalent to inserting a \textit{vison} inside hole of the torus.

\textit{Vison} carries momentum \( \pi L_y / v_0 \)

Flux piercing argument in Kondo lattice

Shift in momentum is carried by $n_T$ electrons, where

$$n_T = n_f + n_c$$

Treat the Kondo lattice perturbatively in $J_K$. In the spin liquid, momentum associated with $n_f=1$ electron is absorbed by creation of vison. The remaining momentum is absorbed by Fermi surface quasiparticles, which enclose a volume associated with $n_c$ electrons.
There exist “topologically ordered” ground states in dimensions $d > 1$ with a Fermi surface of electron-like quasiparticles for which

$$2 \times \frac{v_0}{(2\pi)^d} (\text{Volume enclosed by Fermi surface})$$

$$= (n_T - 1) (\text{mod } 2)$$

A Fractionalized Fermi Liquid

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   Holographic metals as fractionalized Fermi liquids
Focus on a single $\vec{S}_f$ spin, and represent its imaginary time fluctuations by a unit vector $\vec{S}_f = \vec{n}(\tau)/2$ which is controlled by the partition function

$$Z = \int \mathcal{D}\vec{n}(\tau) \delta(\vec{n}^2(\tau) - 1) \exp(-S)$$

$$S = \frac{i}{2} \int_0^1 du \int_0^{1/T} d\tau \vec{n} \cdot \left( \frac{\partial \vec{n}}{\partial u} \times \frac{\partial \vec{n}}{\partial \tau} \right) - \int_0^{1/T} d\tau \vec{h}(\tau) \cdot \vec{n}(\tau)$$

The first term is a Wess-Zumino term, with the “extra dimension” $u$ defined so that $\vec{n}(\tau, u = 1) \equiv \vec{n}(\tau)$ and $\vec{n}(\tau, u = 0) = (0, 0, 1)$.

The field $\vec{h}(\tau)$ represents the “environment” which has to be determined self-consistently.
Simplest self-consistency condition:
\( \vec{h}(\tau) \) is a Gaussian random variable with two-point correlation

\[
\langle \vec{h}(\tau) \cdot \vec{h}(0) \rangle \propto \langle \vec{n}(\tau) \cdot \vec{n}(0) \rangle
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\]

Solution:

\[
\langle \vec{n}(\tau) \cdot \vec{n}(0) \rangle \sim \frac{\pi T}{\sin(\pi T \tau)} \quad \text{at large } \tau.
\]

This has the structure of correlations on a conformally-invariant 0+1 dimensional boundary of a CFT2.

There is also a non-zero ground state entropy per spin.

Effective low energy theory for conduction electrons

The operators acting on the low energy subspace are $c_i$ and $\tilde{S}_{fi}$. For the $c_i$ we have the effective theory

$$S_c = \int \frac{d^d k}{(2\pi)^d} \int d\tau \left[ c_{k\sigma}^\dagger \left( \frac{\partial}{\partial \tau} - \varepsilon_k \right) c_{k\sigma} - VF_{k\sigma}^\dagger c_{k\sigma} - V F_{k\sigma}^\dagger F_{k\sigma} \right]$$

Here the $F_{i\sigma}$ are strongly renormalized operators on the $f$ orbitals, which project onto the low energy theory as

$$F_{i\sigma} \sim \frac{1}{U} \left( \vec{r}_{\sigma\sigma'} \cdot \vec{S}_{fi} \right) c_{i\sigma'}$$
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\[
F_{i\sigma} \sim \frac{1}{U} \left( \vec{t}_{\sigma\sigma'} \cdot \vec{S}_{fi} \right) c_{i\sigma'}
\]

From this we obtain the marginal Fermi liquid behavior in the conduction electron self energy

\[
\Sigma_c(\tau) \sim \left[ \frac{\pi T}{\sin(\pi T \tau)} \right]^2
\]

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   \textit{Holographic metals as fractionalized Fermi liquids}
Begin with a CFT3 e.g. the ABJM theory with a SO(8) global symmetry

The CFT3 is dual to a gravity theory on AdS$_4 \times S^7$
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Add some SO(8) charge by turning on a chemical potential (this will break the SO(8) symmetry)

The CFT3 is dual to a gravity theory on $\text{AdS}_4 \times S^7$
In the Einstein-Maxwell theory, the chemical potential leads to an extremal Reissner-Nordstrom black hole in the $\text{AdS}_4$ spacetime.
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The CFT3 is dual to a gravity theory on AdS$_4 \times S^7$
In the Einstein-Maxwell theory, the chemical potential leads to an extremal Reissner-Nordstrom black hole in the AdS$_4$ spacetime.
The near-horizon geometry of the RN black hole is AdS$_2 \times R^2$. There has been no clear interpretation of the AdS$_2$ theory, the $R^2$ degeneracy, and the finite ground state entropy density
What is the meaning of $\text{AdS}_2 \times \mathbb{R}^2$?

The $\text{AdS}_2$ represents the dynamics of a quantum “spin” with partition function

$$Z = \int \mathcal{D}\vec{n}(\tau) \exp (-S)$$

$$S = S_{\text{Wess–Zumino}}[\vec{n}(\tau, u)] - \int_0^{1/T} d\tau \vec{h}(\tau) \cdot \vec{n}(\tau)$$

The environment field $\vec{h}(\tau)$ represents the dynamics of the CFT$_3$. The fixed point theory of $Z$ is analogous to a critical Kondo fixed point, and has a finite “boundary” entropy. The spin carries a global $\text{SO}(8)$ charge. The $\mathbb{R}^2$ represents a finite density of such spins. In the classical gravity theory, these spins do not interact with each other: this leads to the $\mathbb{R}^2$ degeneracy and the finite ground state entropy density.
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S. Sachdev, arXiv:1006.3794
**Effective low energy theory for “conduction electrons”**

The operators acting on the low energy subspace are the probe fermions $c_i$, and the $F_i$ with the effective theory

$$S_c = \int \frac{d^d k}{(2\pi)^d} \int d\tau \left[ c^\dagger_{k\sigma} \left( \frac{\partial}{\partial \tau} - \varepsilon_k \right) c_{k\sigma} - V F^\dagger_{k\sigma} c_{k\sigma} - V c^\dagger_{k\sigma} F_k\sigma \right]$$

Here the $F_{i\sigma}$ are gauge-invariant operators in the AdS$_2$ with the same quantum numbers as the electron, with the correlator

$$\left\langle F_{k\sigma}(\tau) F_{k\sigma}^\dagger(0) \right\rangle \sim \left( \frac{\pi T}{\sin(\pi T\tau)} \right)^{2\Delta_k}$$

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$$\langle F_{k\sigma}(\tau)F_{k\sigma}^\dagger(0) \rangle \sim \left( \frac{\pi T}{\sin(\pi T\tau)} \right)^{2\Delta_k}$$

From this we obtain the non-Fermi liquid behavior in the conduction electron self energy

$$\Sigma_c(\tau) \sim \left[ \frac{\pi T}{\sin(\pi T\tau)} \right]^{2\Delta_k}$$

Conclusions

There is a close correspondence between the theory of holographic metals, and the fractionalized Fermi liquid phase of the Anderson/Kondo lattice.
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The correspondence suggests that the ground state of AdS$_4$ (or AdS$_5$) CFTs at non-zero $R$-charge chemical potential is a Kondo lattice of spins carrying the $R$-charge.
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There is a close correspondence between the theory of holographic metals, and the fractionalized Fermi liquid phase of the Anderson/Kondo lattice.

The correspondence suggests that the ground state of $\text{AdS}_4$ (or $\text{AdS}_5$) CFTs at non-zero $R$-charge chemical potential is a Kondo lattice of spins carrying the $R$-charge.