The quantum mechanics of superconductors

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Superfluids and superconductors: quantum mechanics on a macroscopic scale

Superfluidity or superconductivity – which is the preferred term if the fluid is made up of charged particles like electrons – is a fascinating phenomenon that allows us to observe a variety of quantum mechanical effects on the macroscopic scale. Besides being of tremendous interest in themselves and vehicles for developing key concepts and methods in theoretical physics, superfluids have found important applications in modern society. For instance, superconducting magnets are able to create strong enough magnetic fields for the magnetic resonance imaging technique (MRI) to be used for diagnostic purposes in medicine, for illuminating the structure of complicated molecules by nuclear magnetic resonance (NMR), and for confining plasmas in the context of fusion-reactor research. Superconducting magnets are also used for bending the paths of charged particles moving at speeds close to the speed of light into closed orbits in particle accelerators like the Large Hadron Collider (LHC) under construction at CERN.

The Ginzburg-Landau (GL) theory is based on Landau's theory of second order phase transitions from 1937. This was a natural starting point, since in the absence of a magnetic field the transition into the superconducting state at a critical temperature T_c is a second-order phase transition. Landau's theory describes the transition from a disordered to an ordered state in terms of an "order parameter", which is zero in the disordered phase and nonzero in the ordered phase. In the theory of ferromagnetism, for example, the order parameter is the spontaneous magnetisation. In order to describe the transition to a superconducting state, GL took the order parameter to be a certain complex function $\Psi(r)$, which they interpreted as the "effective" wave function of the "superconducting electrons", whose density n_s is given by $|\Psi|^2$; today we would say that $\Psi(r)$ is the macroscopic wave function of the superconducting condensate.

The quantum mechanics of

- Insulators
- Metals
- Semiconductors
- Superconductors

Technology in the 19th century

Technology of the 20th century

Technology of the 21st century ?

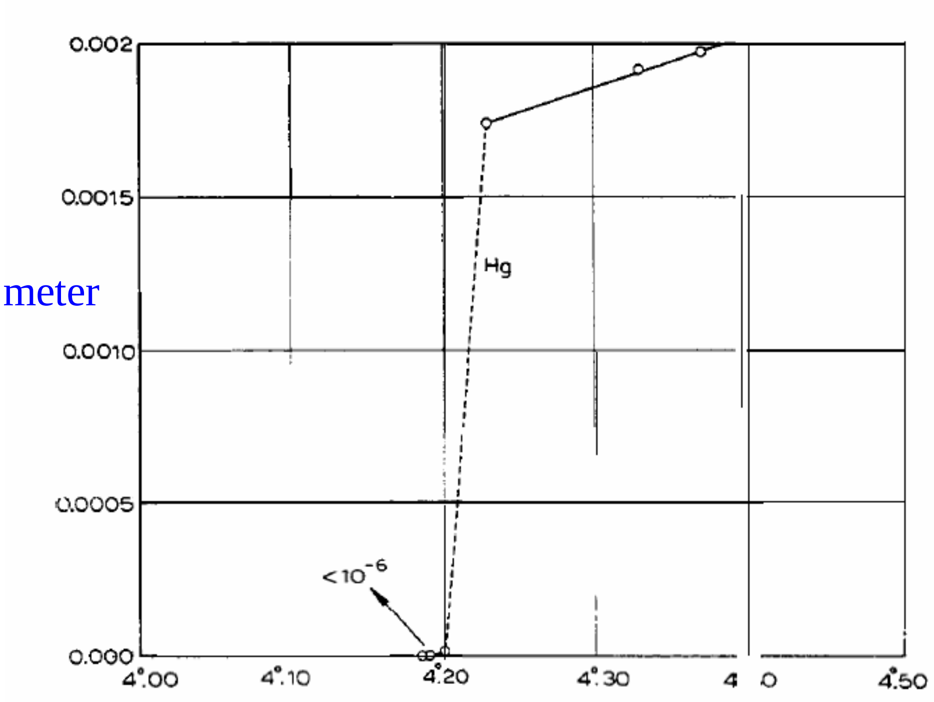
Electrical resistivity of Cu at 273 K: 1.56×10^{-10} Ohm meter

Electrical resistivity of C (diamond) at 273 K: 9×10^{21} Ohm meter

Ratio is comparable to the ratio $\dfrac{\text{size of atom } \sim10^{-10} \text{ meters}}{\text{size of galaxy } \sim10^{21} \text{ meters}}$

Resistance of Hg measured by Kammerlingh Onnes in 1911

$\times6\times10^{-8}$ Ohm meter



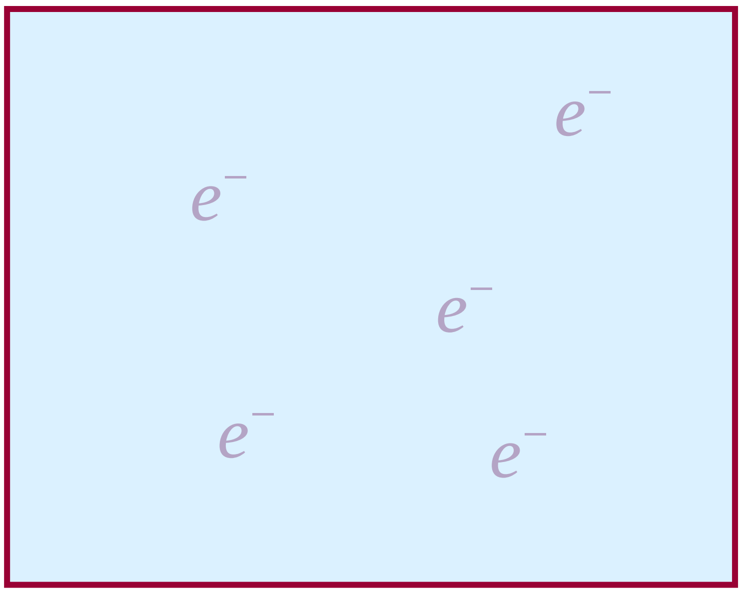
K

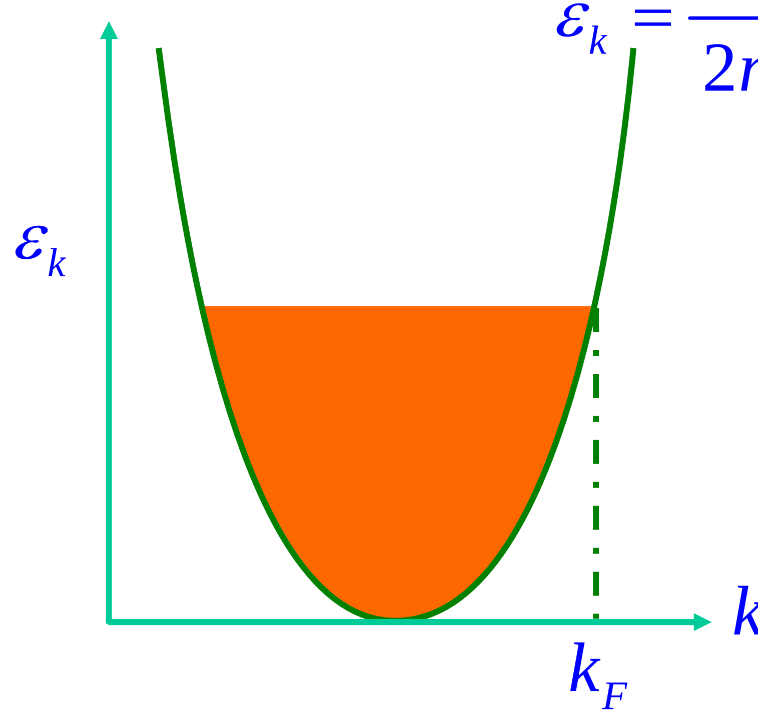
Quantum theory of metals

Valence electrons occupy plane wave states which extend across the entire material

$$H=\sum_i -\frac{\hbar^2\nabla_i^2}{2m^*}$$

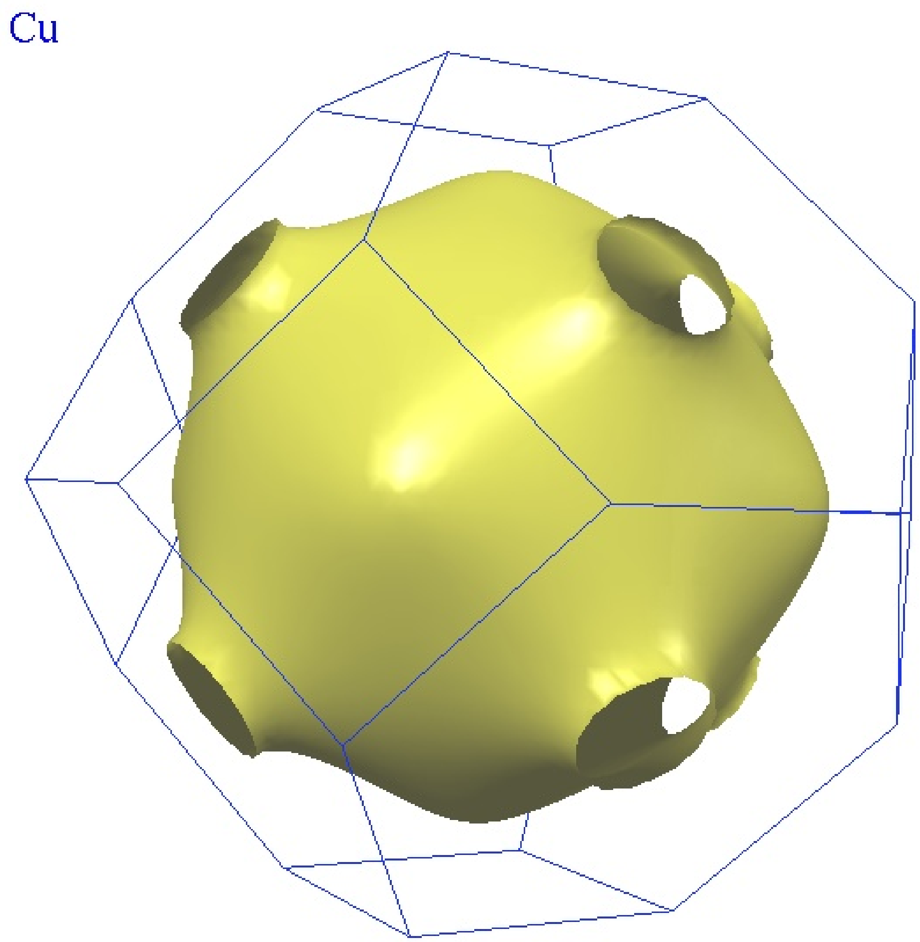
$$\varepsilon_k=\frac{\hbar^2k^2}{2m^*}$$





$$n=2\times\frac{1}{(2\pi)^3}\times\frac{4}{3}\pi k_F^3$$

Fermi velocity $v_F=\dfrac{\hbar k_F}{m^*}\sim 10^8\,\text{cm/sec}$

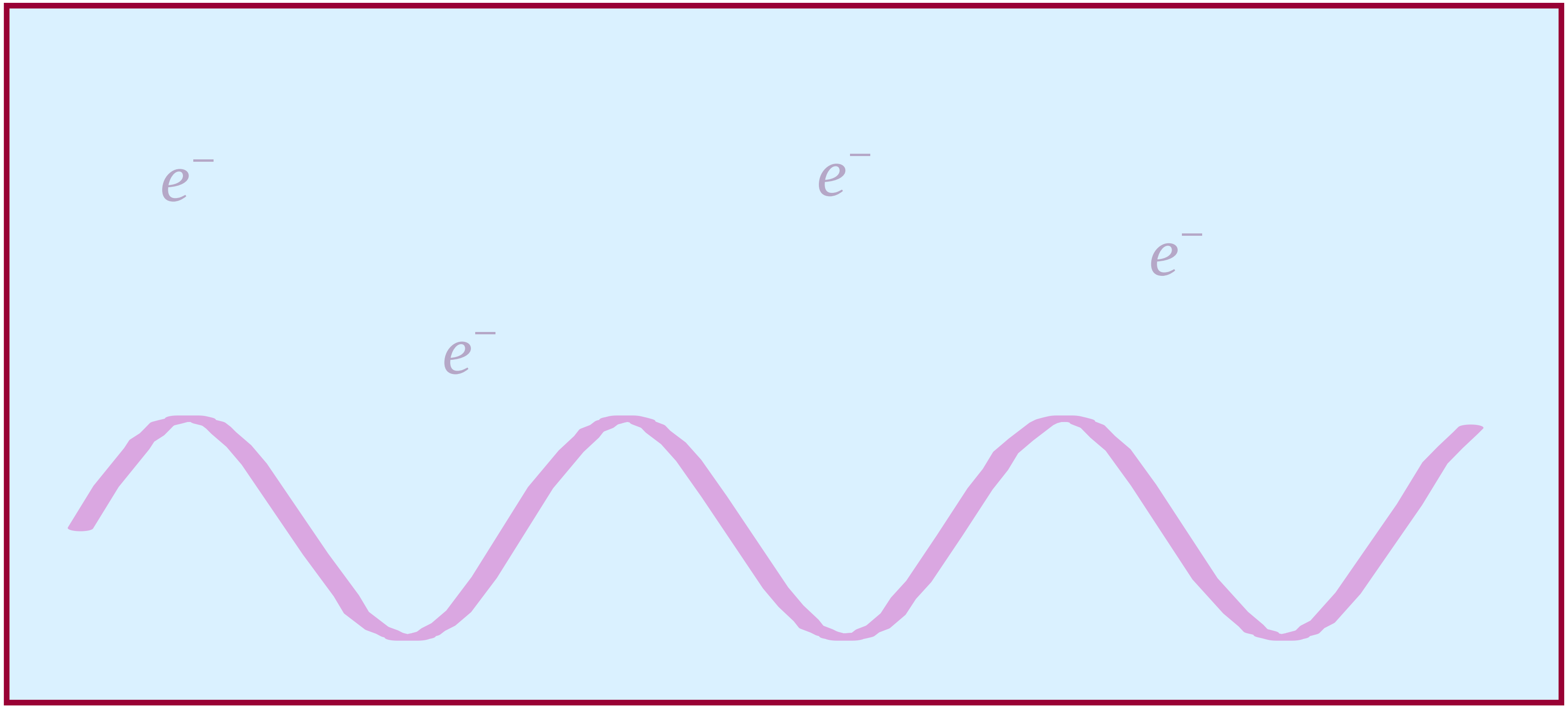


Fermi surface of copper

Electrical conduction occurs by acceleration of electrons near the Fermi surface

Quantum theory of metals

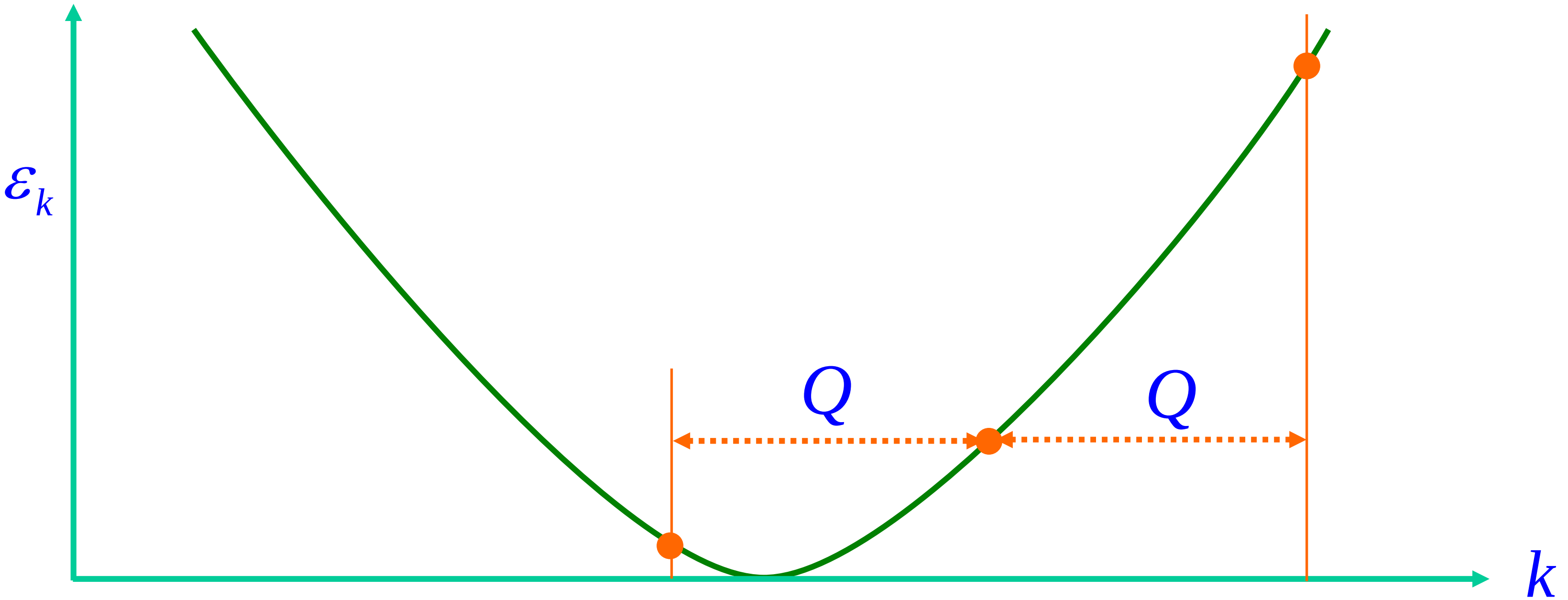
$$H = \sum_i -\frac{\hbar^2 \nabla_i^2}{2m^*} + \sum_i V_Q \cos(Q x_i)$$

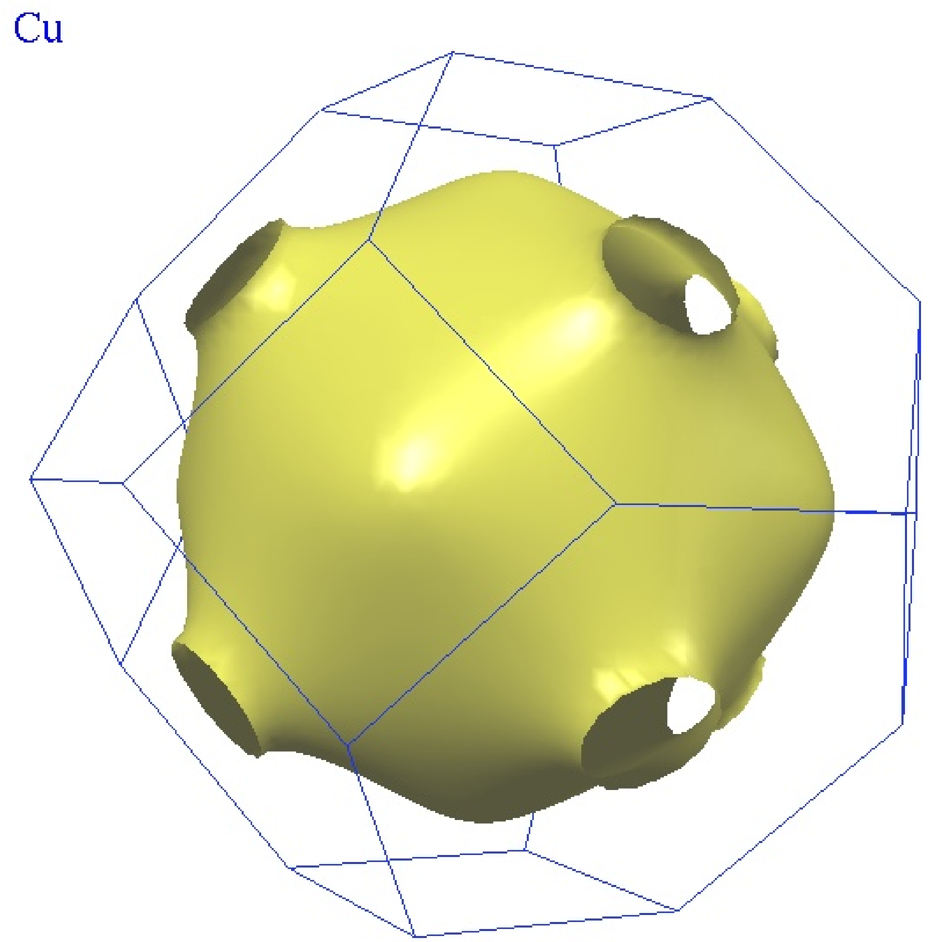


Quantum theory of metals

$$H = \sum_i -\frac{\hbar^2 \nabla_i^2}{2m^*} + \sum_i V_Q \cos\left(Q x_i\right)$$

Electrons can scatter off V_Q and change their momentum by $\pm\hbar Q$





Fermi surface of copper

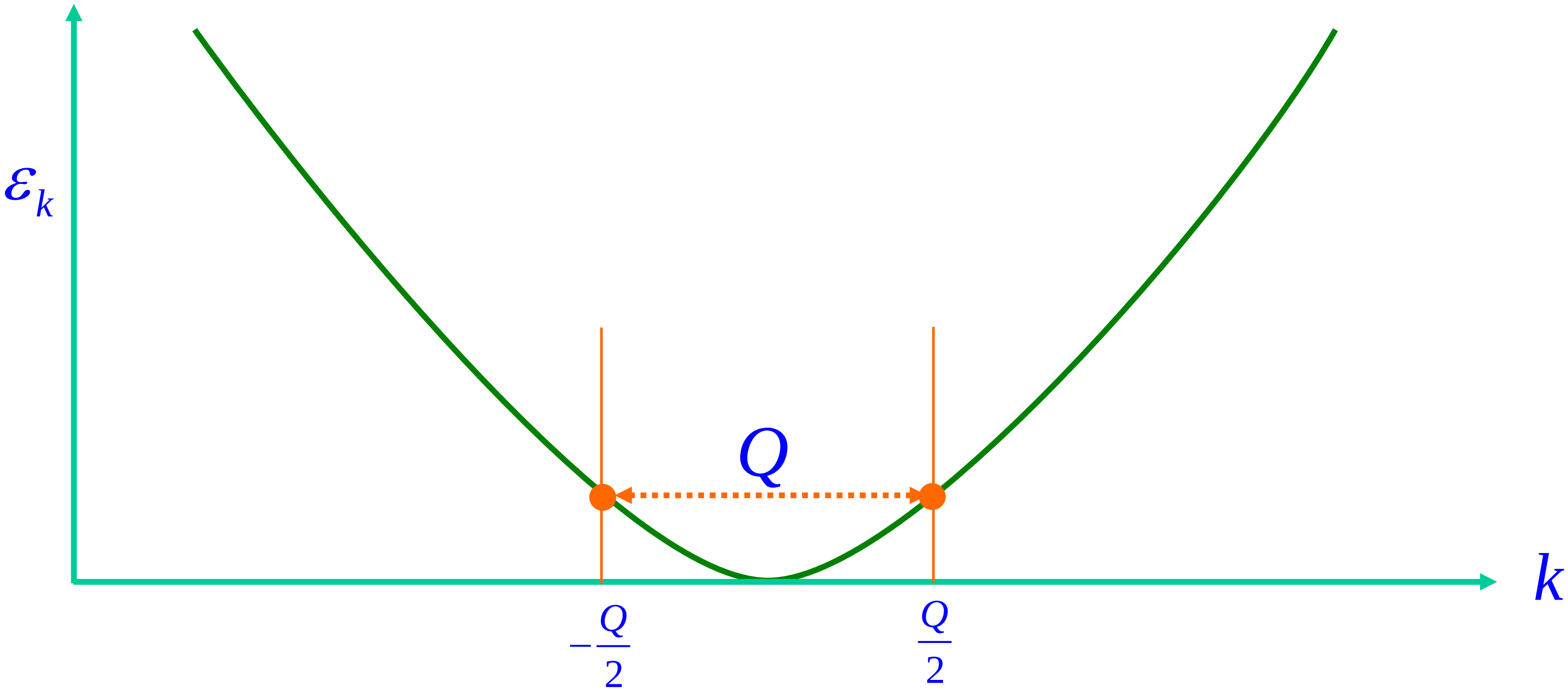
Deviation in shape from sphere is caused by non-resonant scattering off periodic potential

Quantum theory of insulators

$$\boxed{H = \sum_i -\frac{\hbar^2 \nabla_i^2}{2m^*} + \sum_i V_Q \cos\left(Q x_i\right)}$$

Electrons can scatter off V_Q and change their momentum by $\pm\hbar Q$

Resonant scattering of electrons between momenta $k = \pm Q/2$



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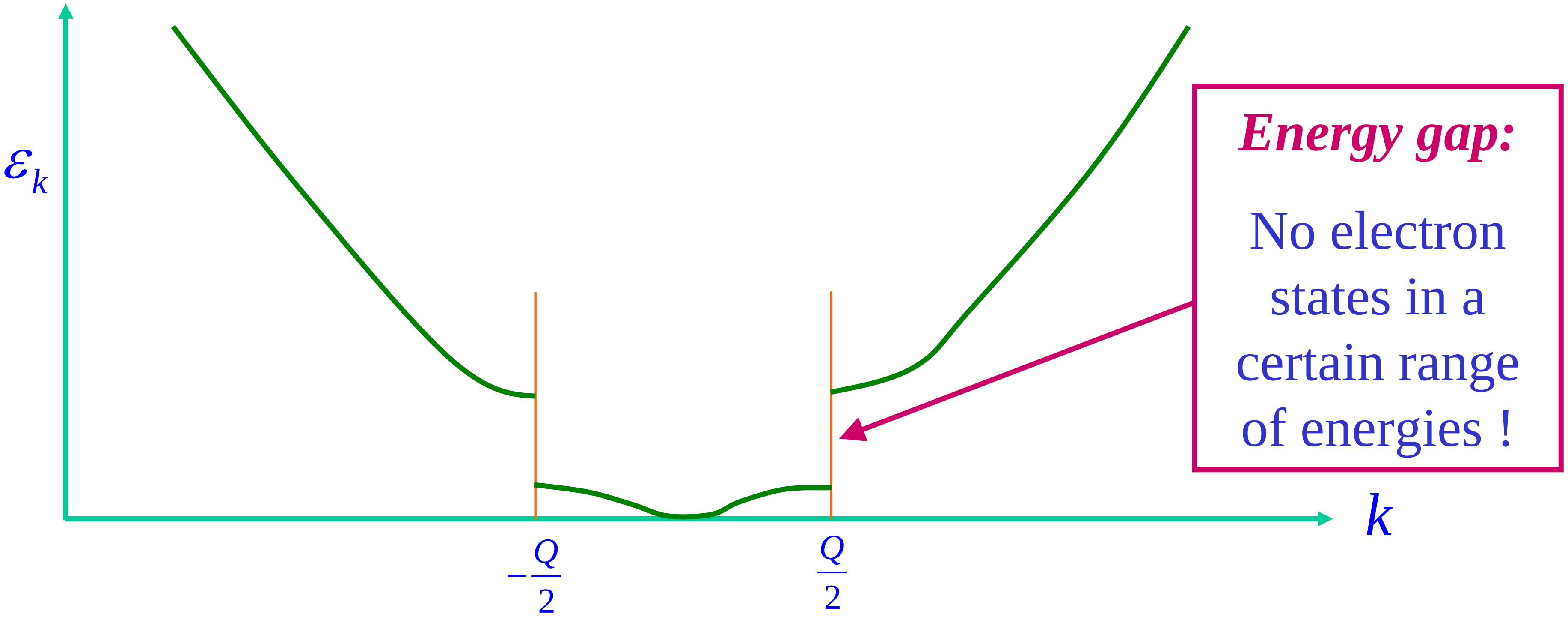
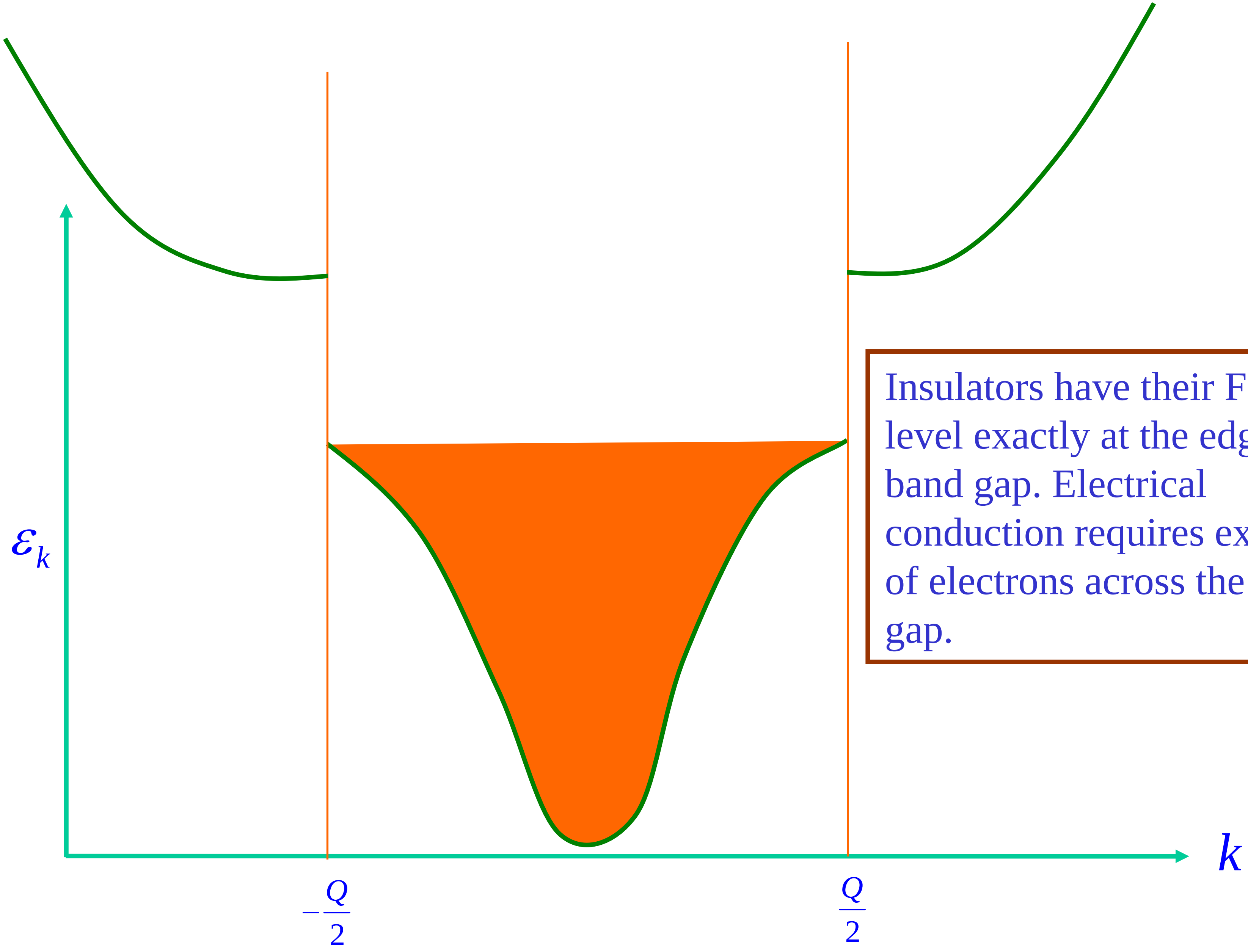


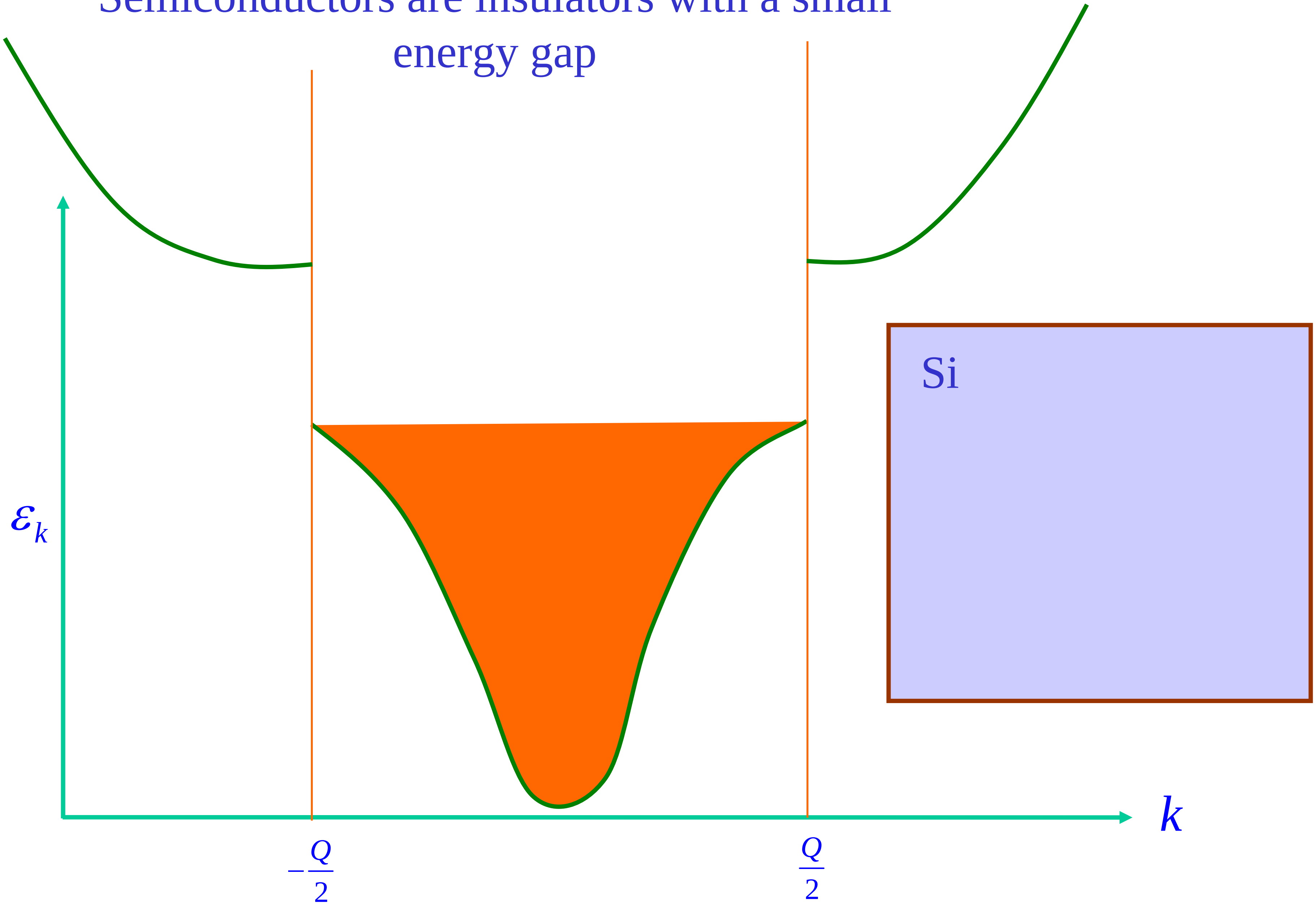
Quantum theory of insulators



Insulators have their Fermi level exactly at the edge of the band gap. Electrical conduction requires excitation of electrons across the band gap.

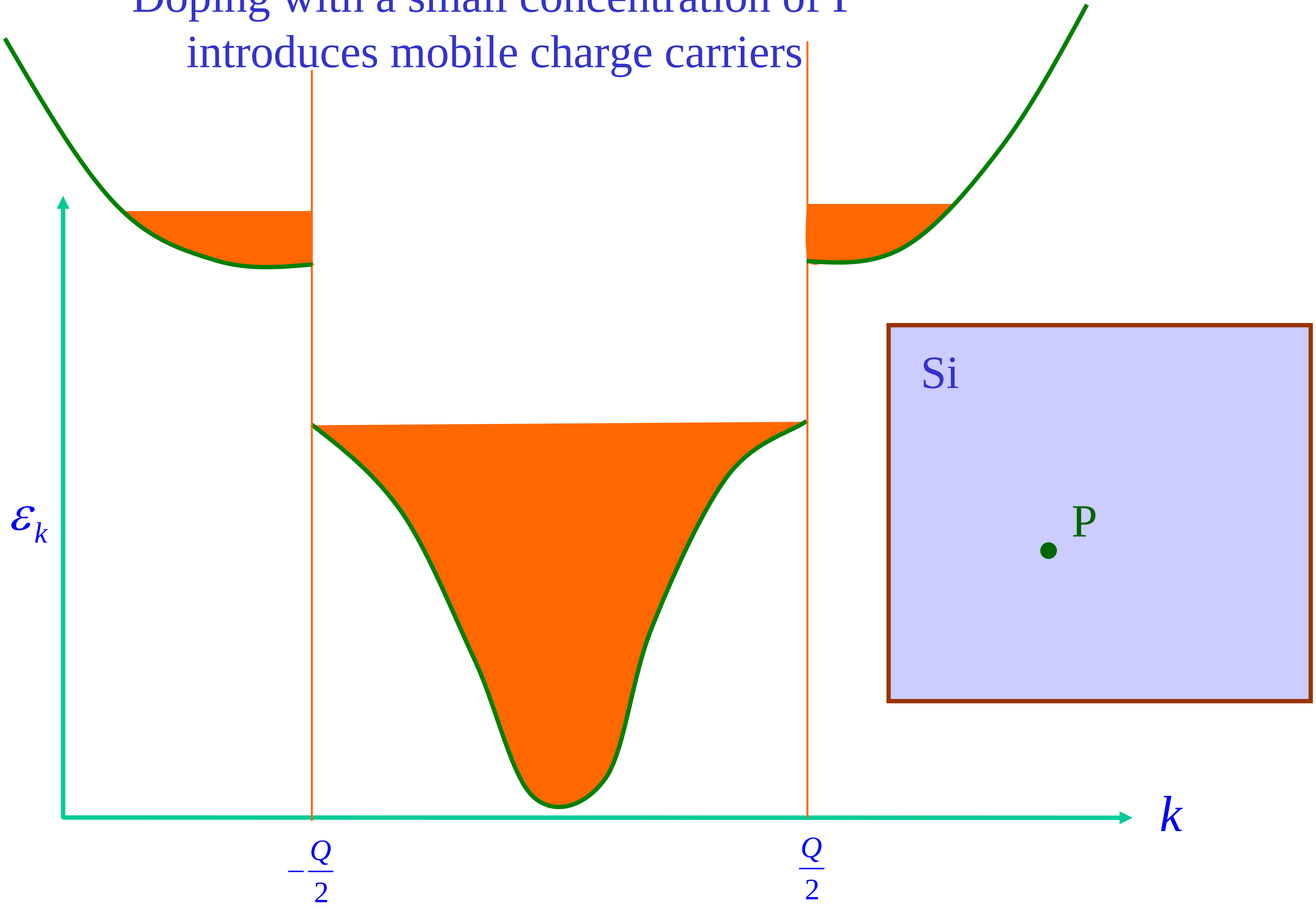
Quantum theory of semiconductors

Semiconductors are insulators with a small energy gap



Quantum theory of semiconductors

Doping with a small concentration of P introduces mobile charge carriers



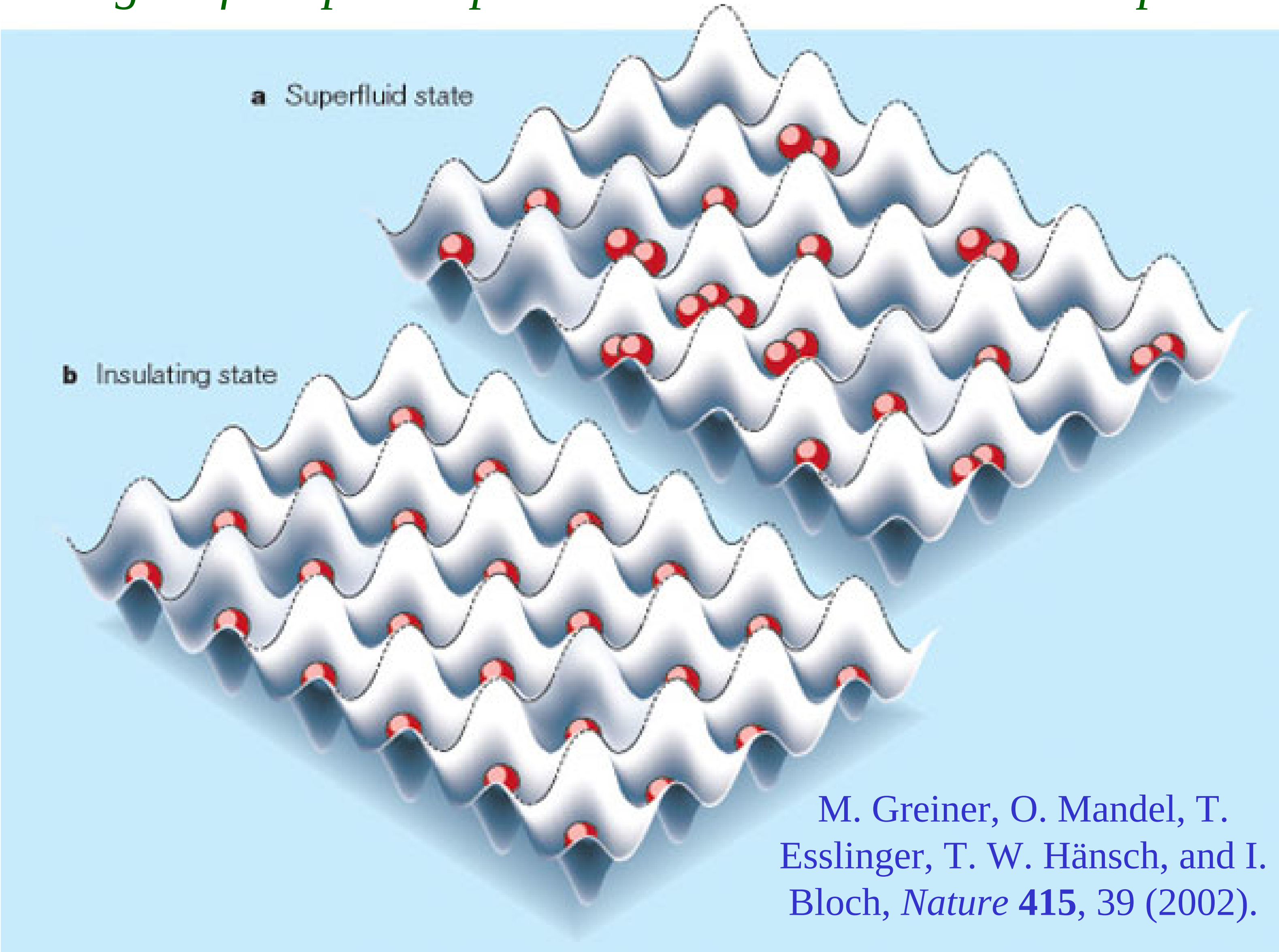
We have so far considered the quantum theory of a large number of *fermions* moving in a periodic potential and found ground states which are

metals, insulators and semiconductors

Now consider the quantum theory of a large number of *bosons* moving in a periodic potential

^{87}Rb bosonic atoms in a magnetic trap and an optical lattice potential

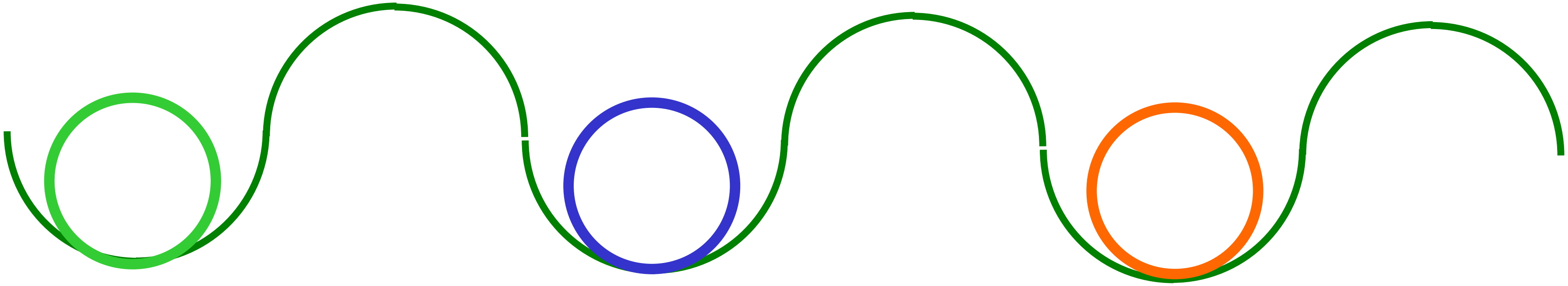
The strength of the period potential can be varied in the experiment



M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Strong periodic potential

"Eggs in an egg carton"



Tunneling between neighboring minima is negligible and atoms remain localized in a well. However, the total wavefunction must be symmetric between exchange

Strong periodic potential

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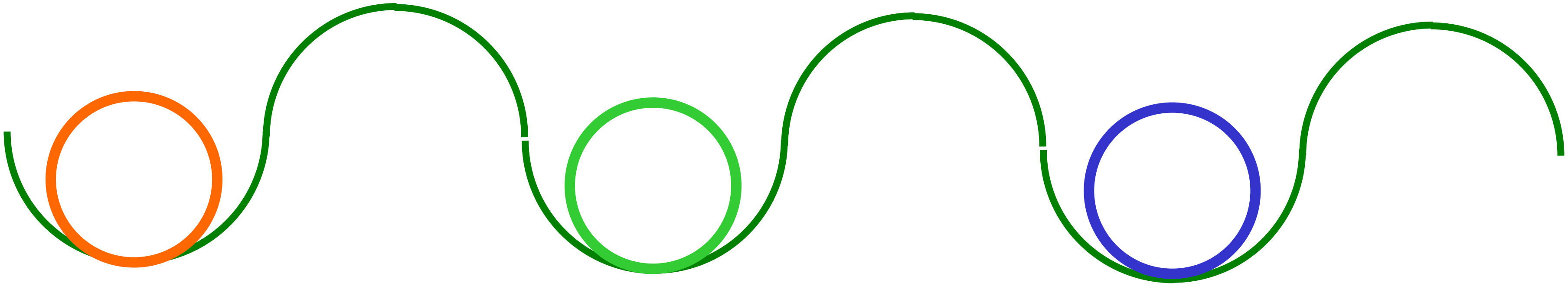

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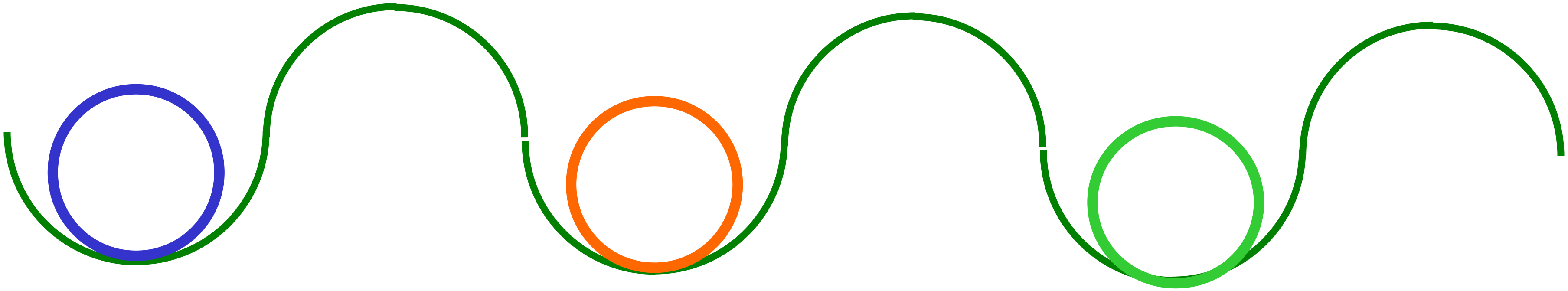
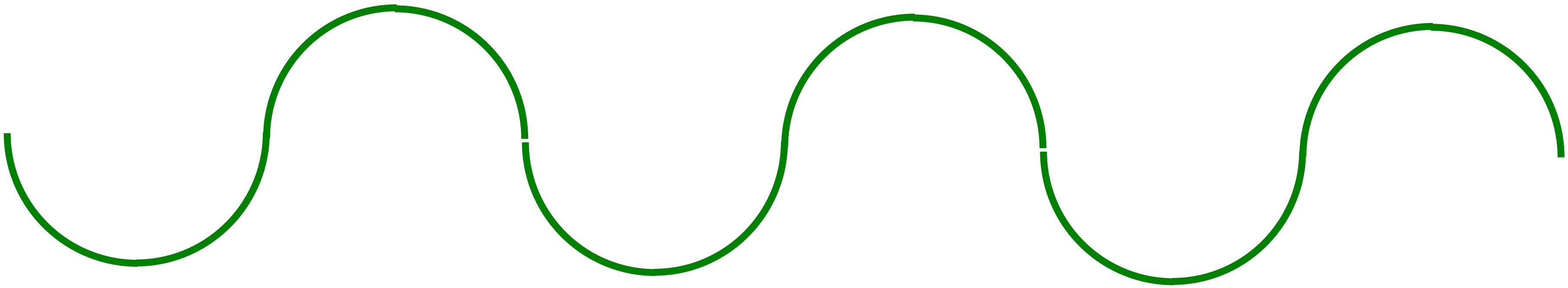


Tunneling between neighboring minima is negligible and atoms remain localized in a well. However, the total wavefunction must be symmetric between exchange

$$\left|\text{Insulator}\right\rangle = \left|\,|{\color{blue}\bigcirc}|{\color{red}\bigcirc}|{\color{green}\bigcirc}|\right\rangle + \left|\,|{\color{red}\bigcirc}|{\color{blue}\bigcirc}|{\color{green}\bigcirc}|\right\rangle + \left|\,|{\color{blue}\bigcirc}|{\color{green}\bigcirc}|{\color{red}\bigcirc}|\right\rangle$$

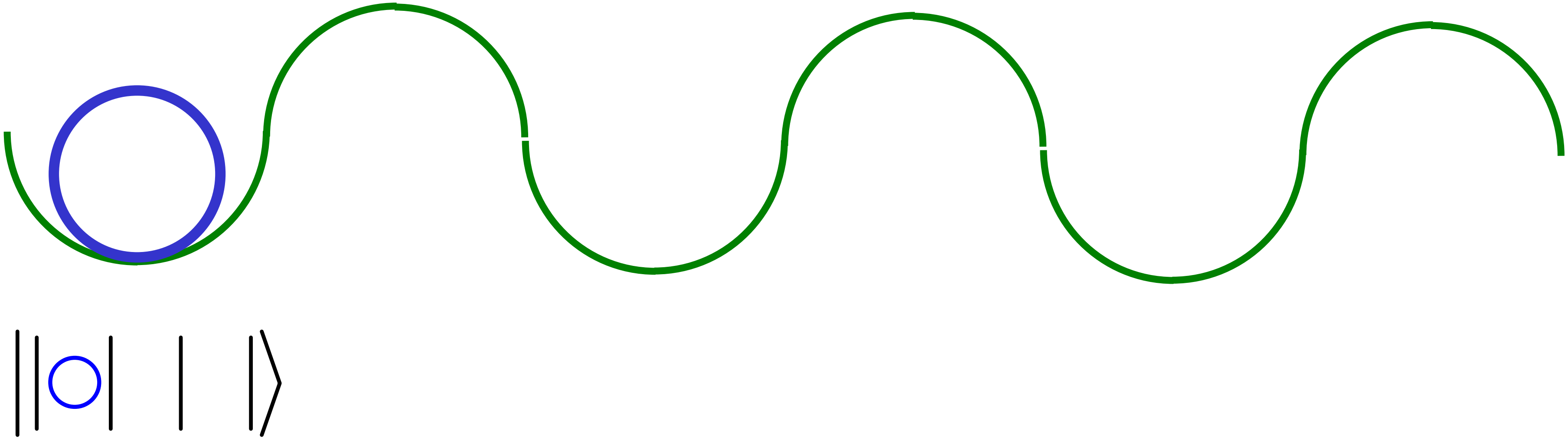
$$+ \left|\,|{\color{green}\bigcirc}|{\color{blue}\bigcirc}|{\color{red}\bigcirc}|\right\rangle + \left|\,|{\color{red}\bigcirc}|{\color{green}\bigcirc}|{\color{blue}\bigcirc}|\right\rangle + \left|\,|{\color{green}\bigcirc}|{\color{red}\bigcirc}|{\color{blue}\bigcirc}|\right\rangle$$

Weak periodic potential



A single atom can tunnel easily between neighboring minima

Weak periodic potential



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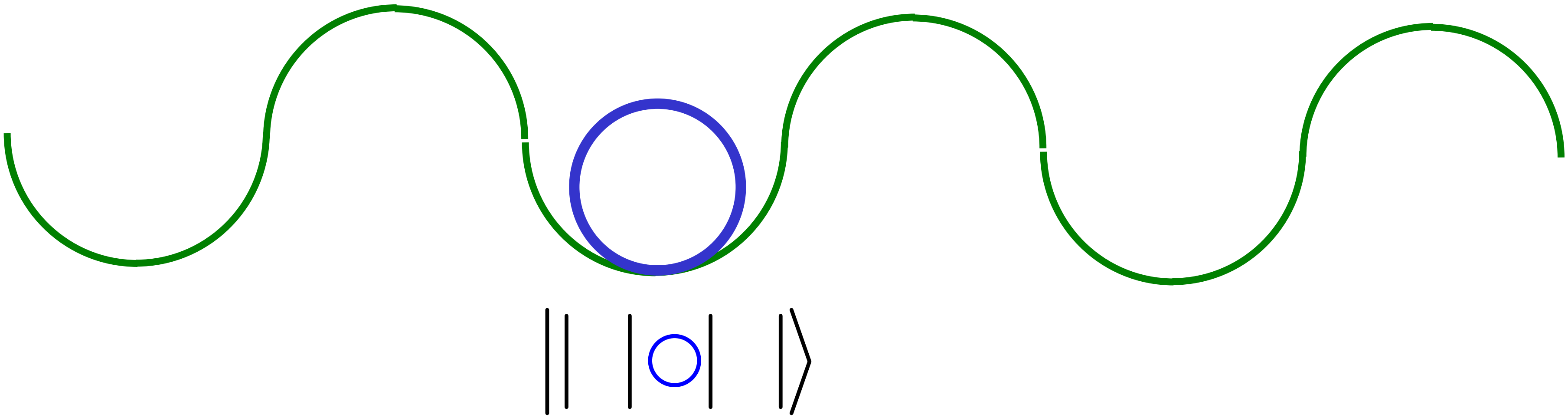

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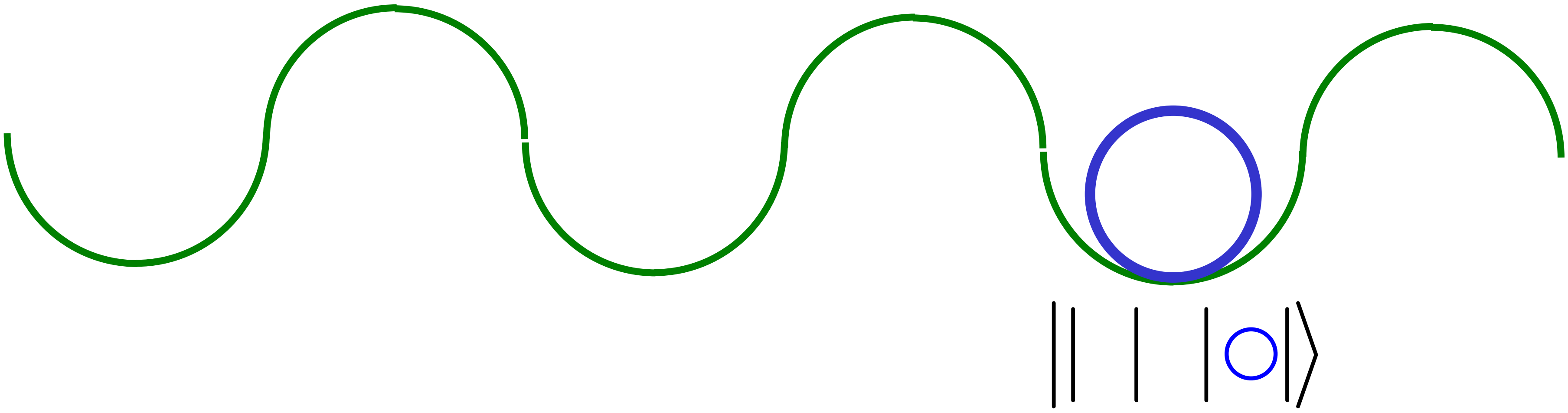

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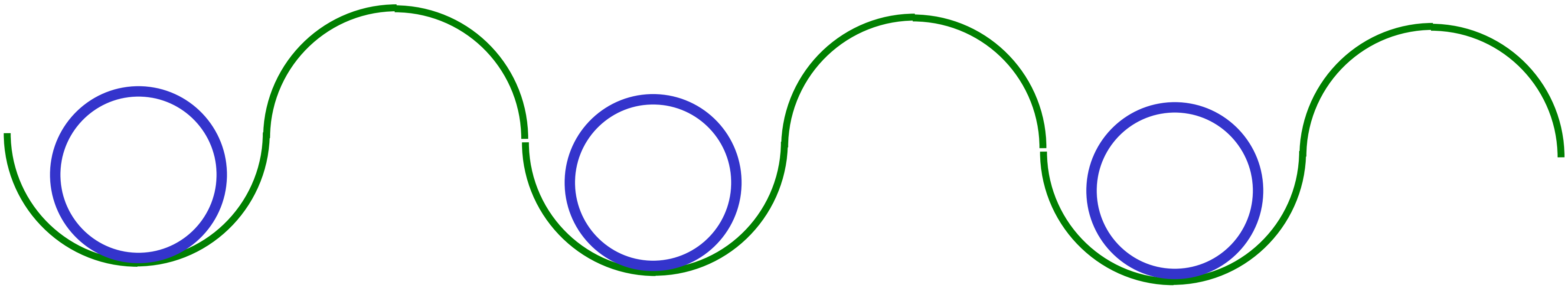


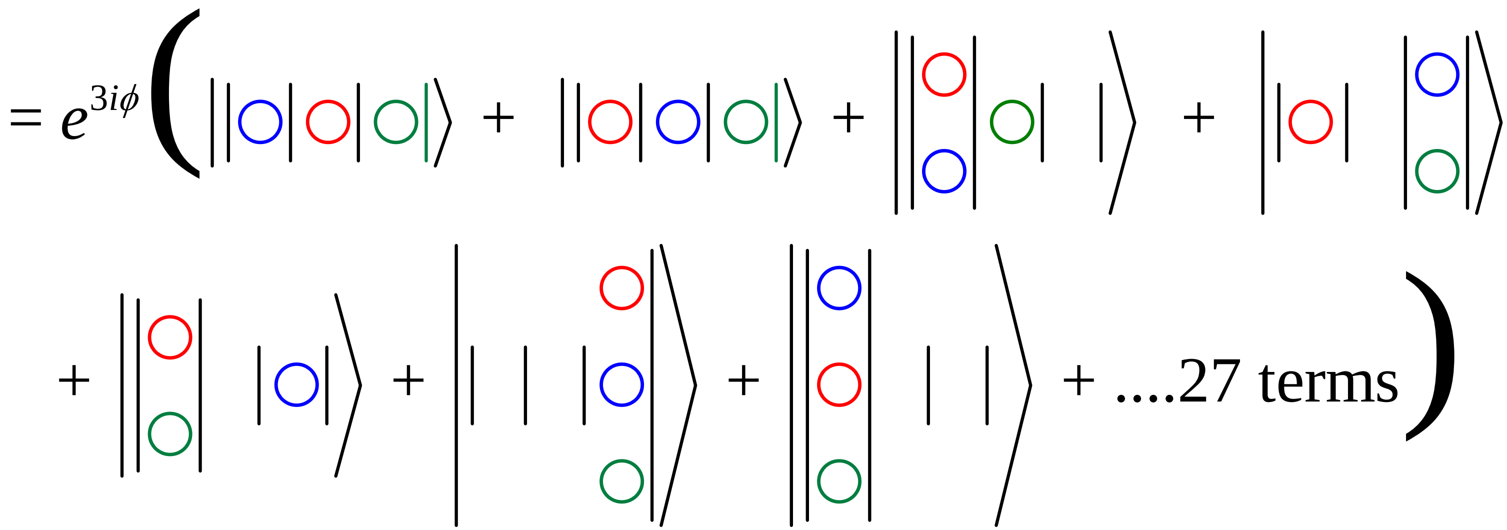
$$|\mathrm{G}\rangle = e^{i\phi}\left(\big|\,|\bigcirc|\;\;|\;\;|\,\big\rangle + \big|\,|\;\;|\bigcirc|\;\;|\,\big\rangle + \big|\,|\;\;|\;\;|\bigcirc|\,\big\rangle \right)$$

The ground state of a single particle is a zero momentum state, which is a quantum superposition of states with different particle locations.

The Bose-Einstein condensate in a weak periodic potential

Lowest energy state for many atoms

$$|\mathrm{BEC}\rangle = |G\rangle|G\rangle|G\rangle$$



Large fluctuations in number of atoms in each potential well – *superfluidity* (atoms can “flow” without dissipation)

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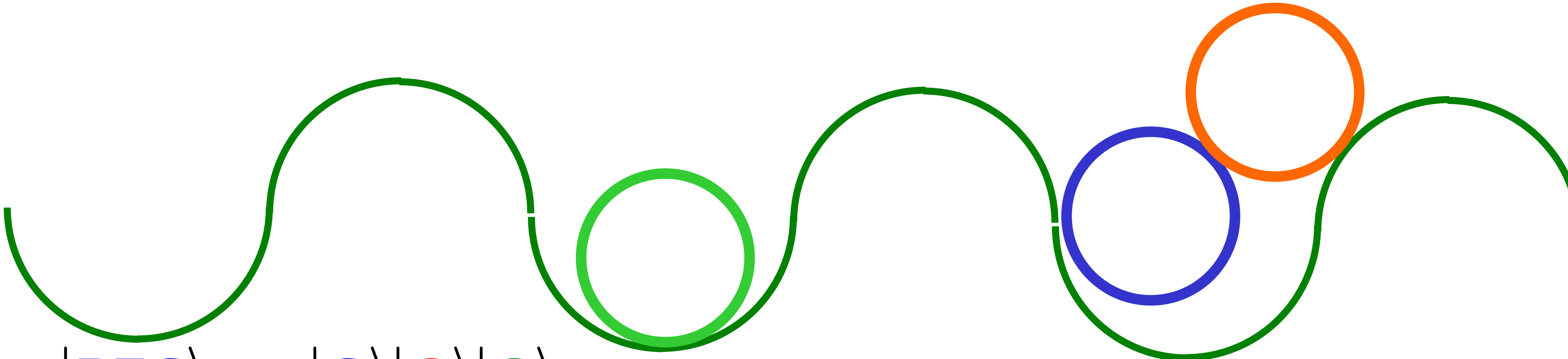

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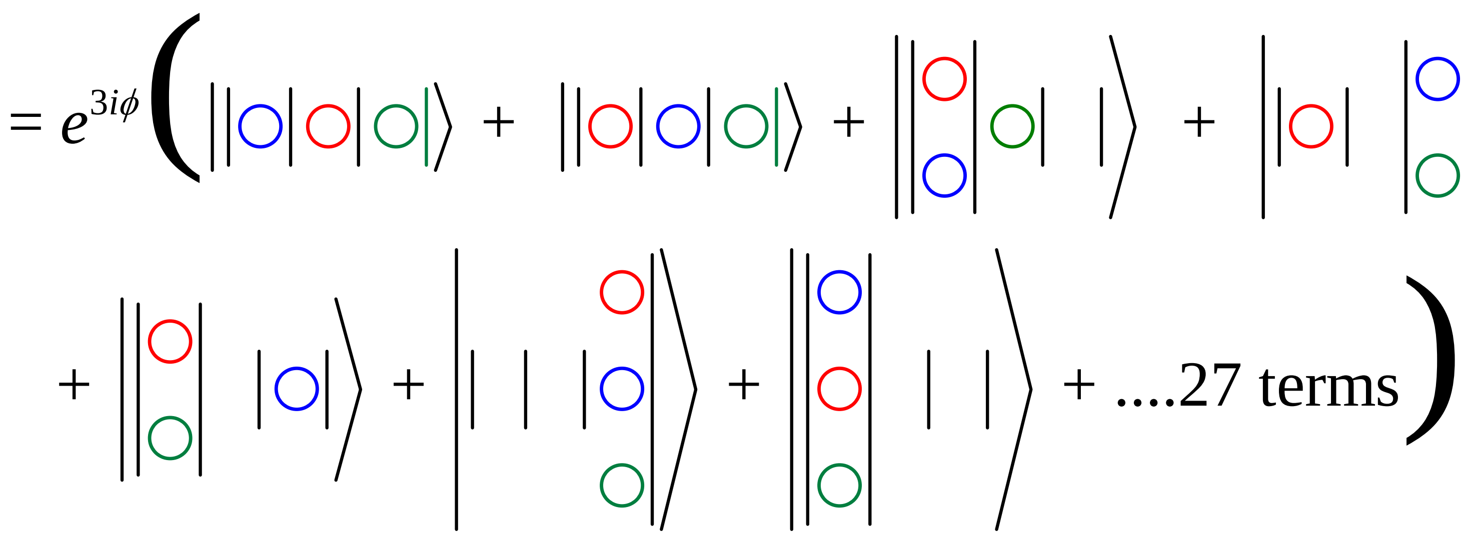

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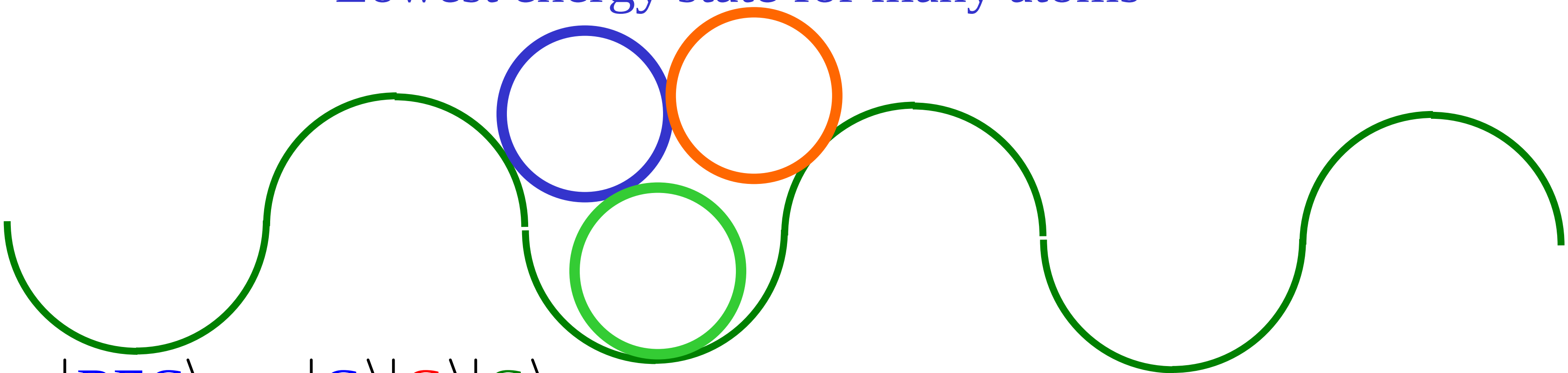

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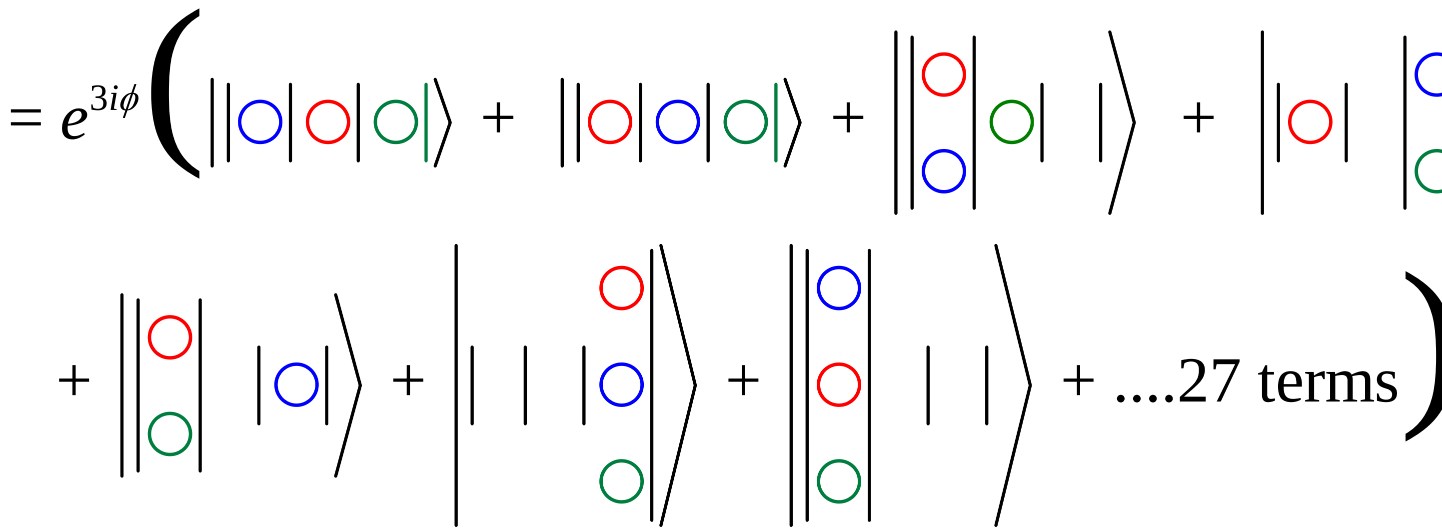

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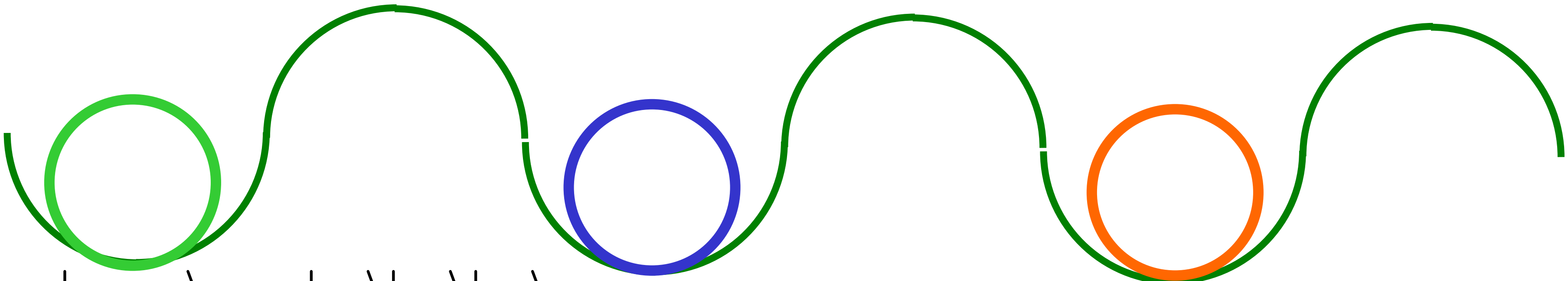
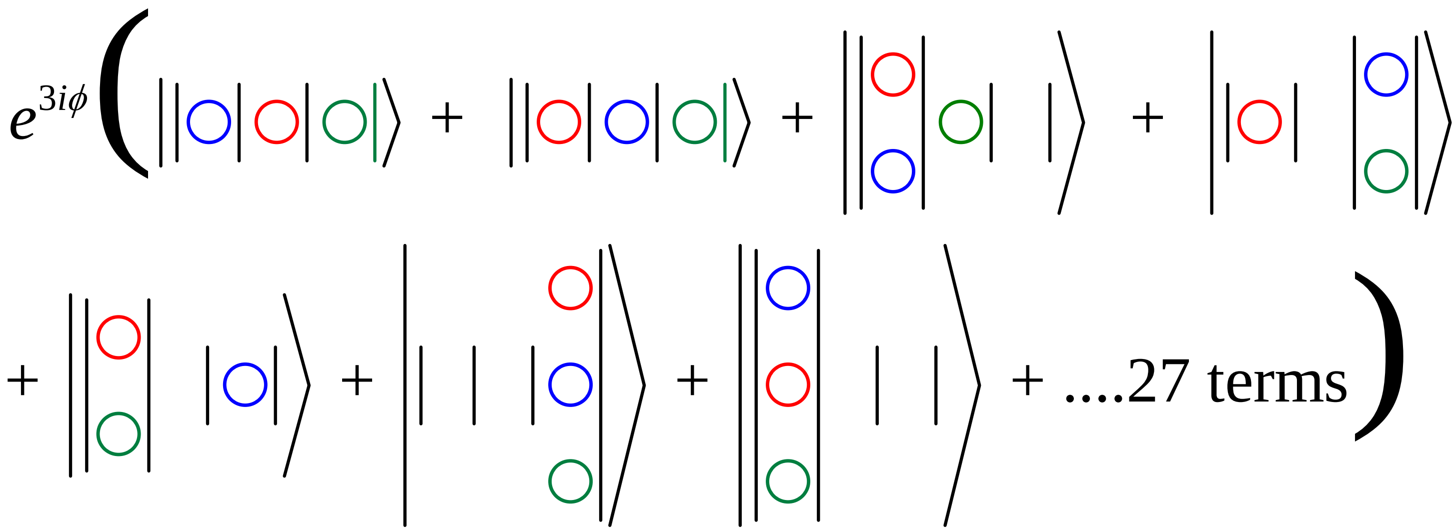


$$|\mathrm{BEC}\rangle = |G\rangle|G\rangle|G\rangle$$

$$= e^{3i\phi}\Big(\dots$$



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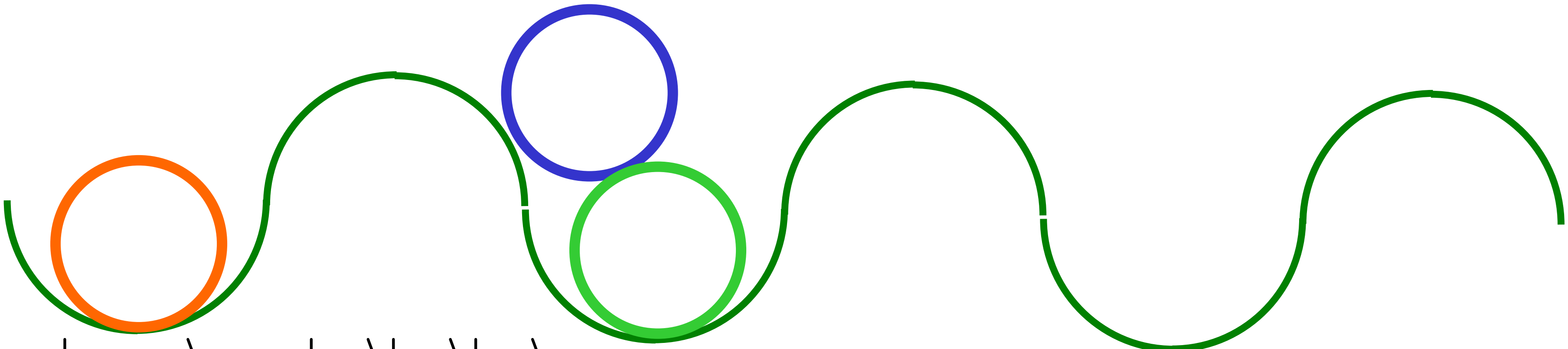
The Bose-Einstein condensate in a weak periodic potential

Lowest energy state for many atoms



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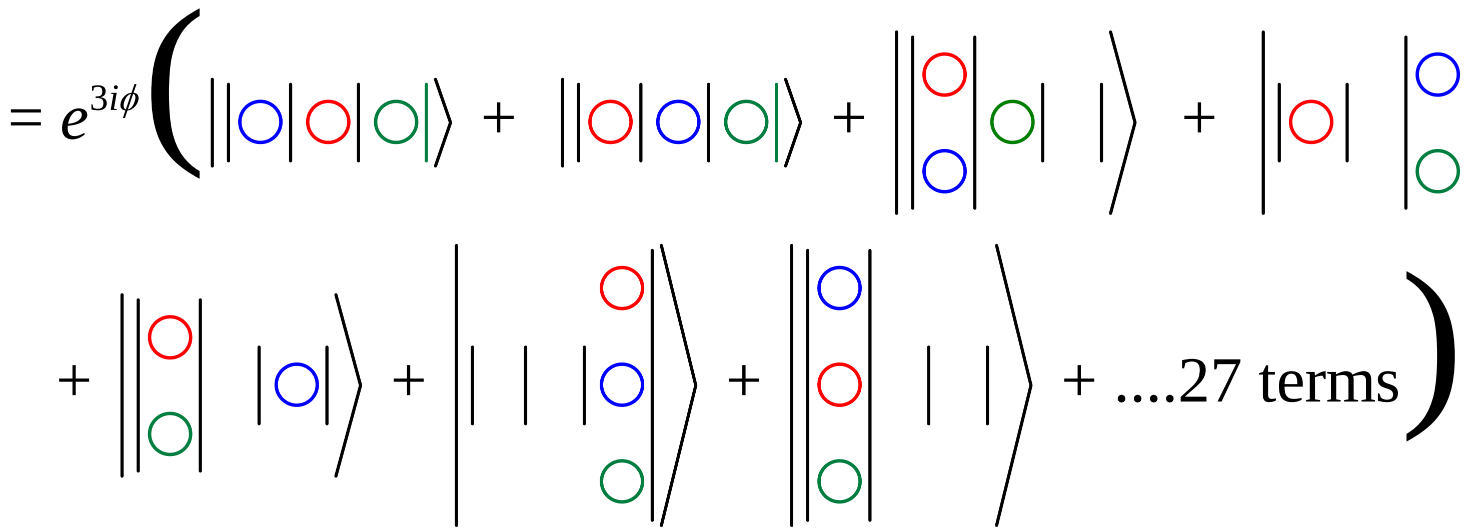

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The Bose-Einstein condensate in a weak periodic potential

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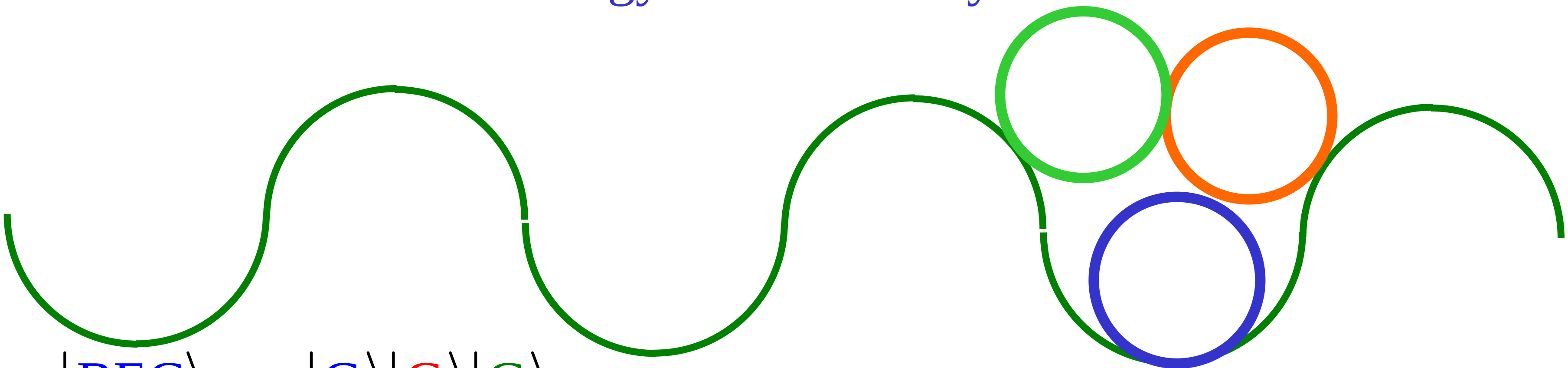

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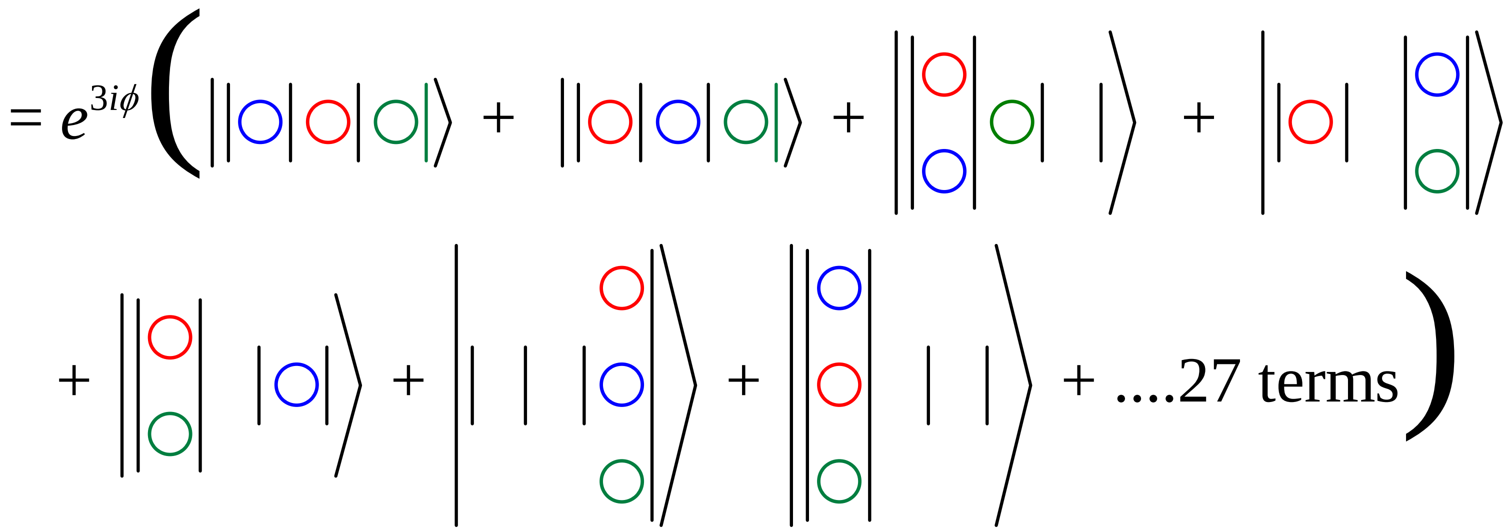

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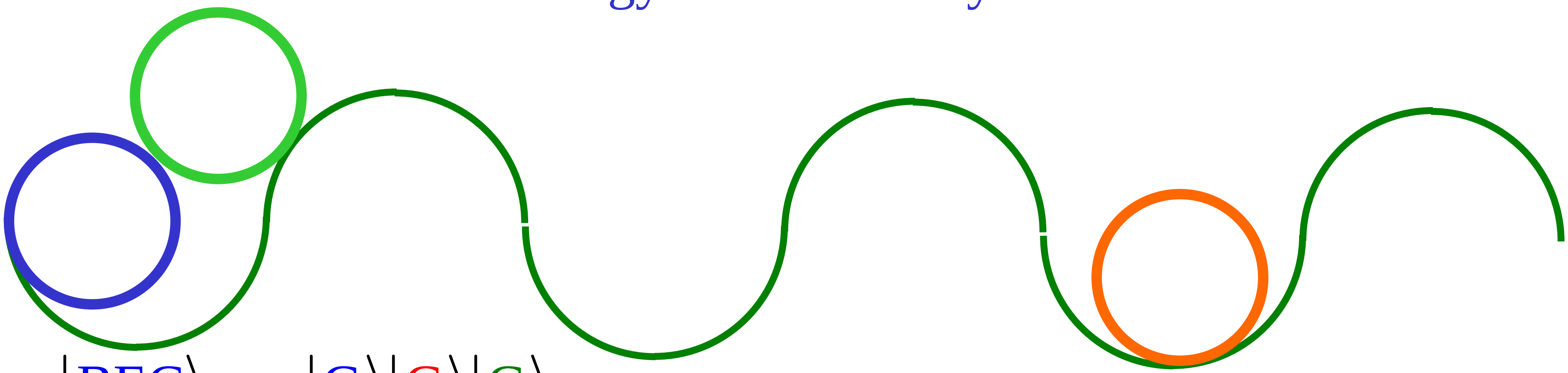

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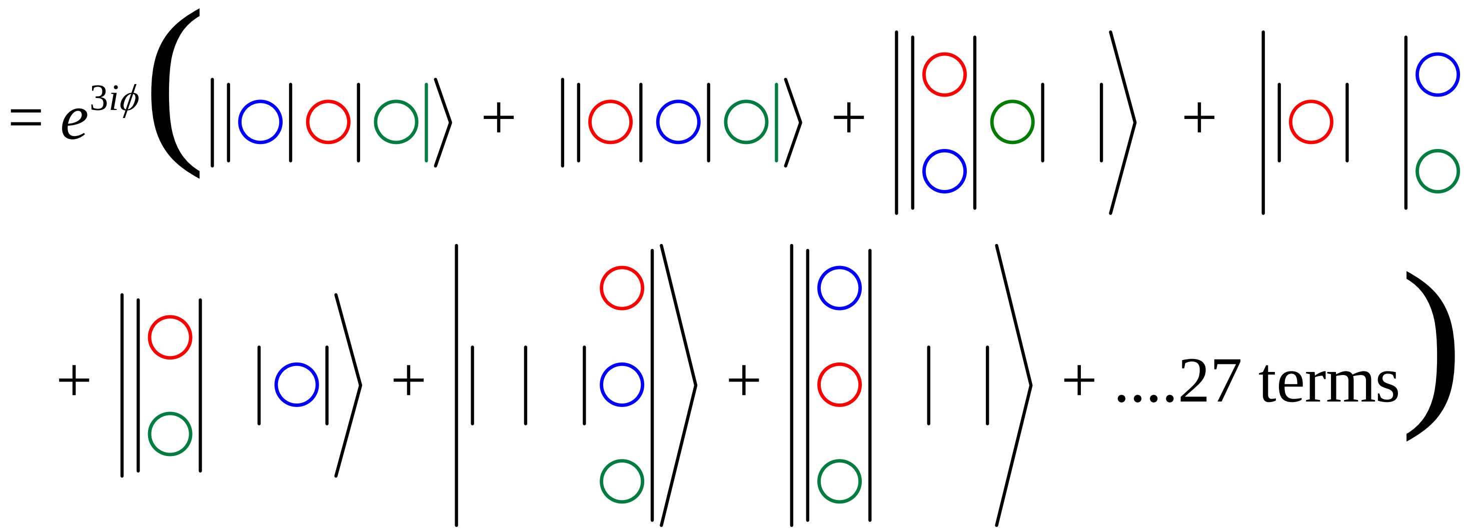
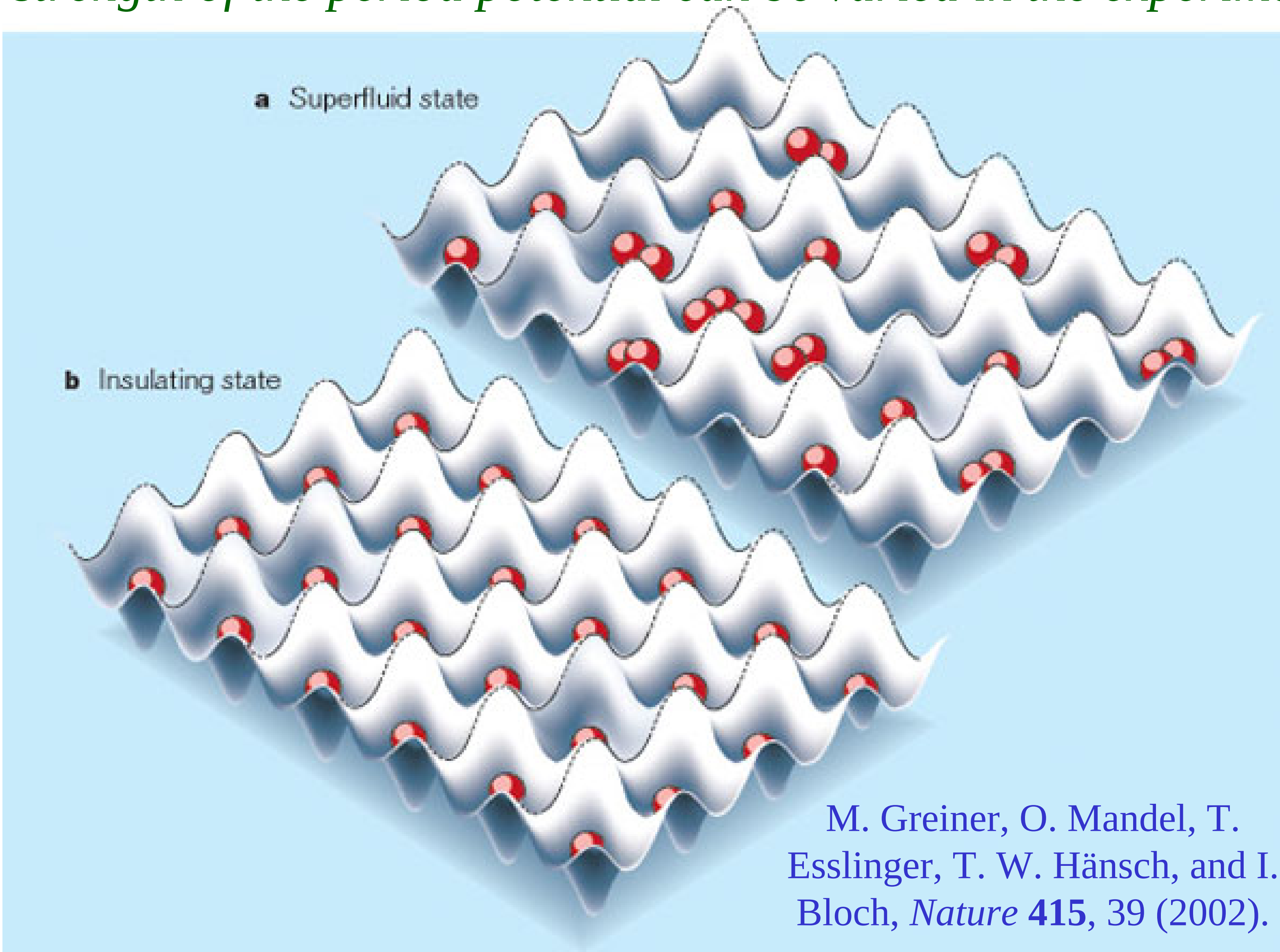


Large fluctuations in number of atoms in each potential well – *superfluidity* (atoms can “flow” without dissipation)

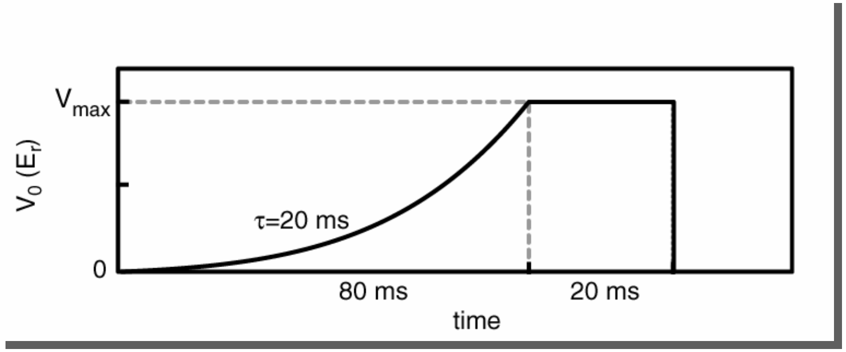
^{87}Rb bosonic atoms in a magnetic trap and an optical lattice potential

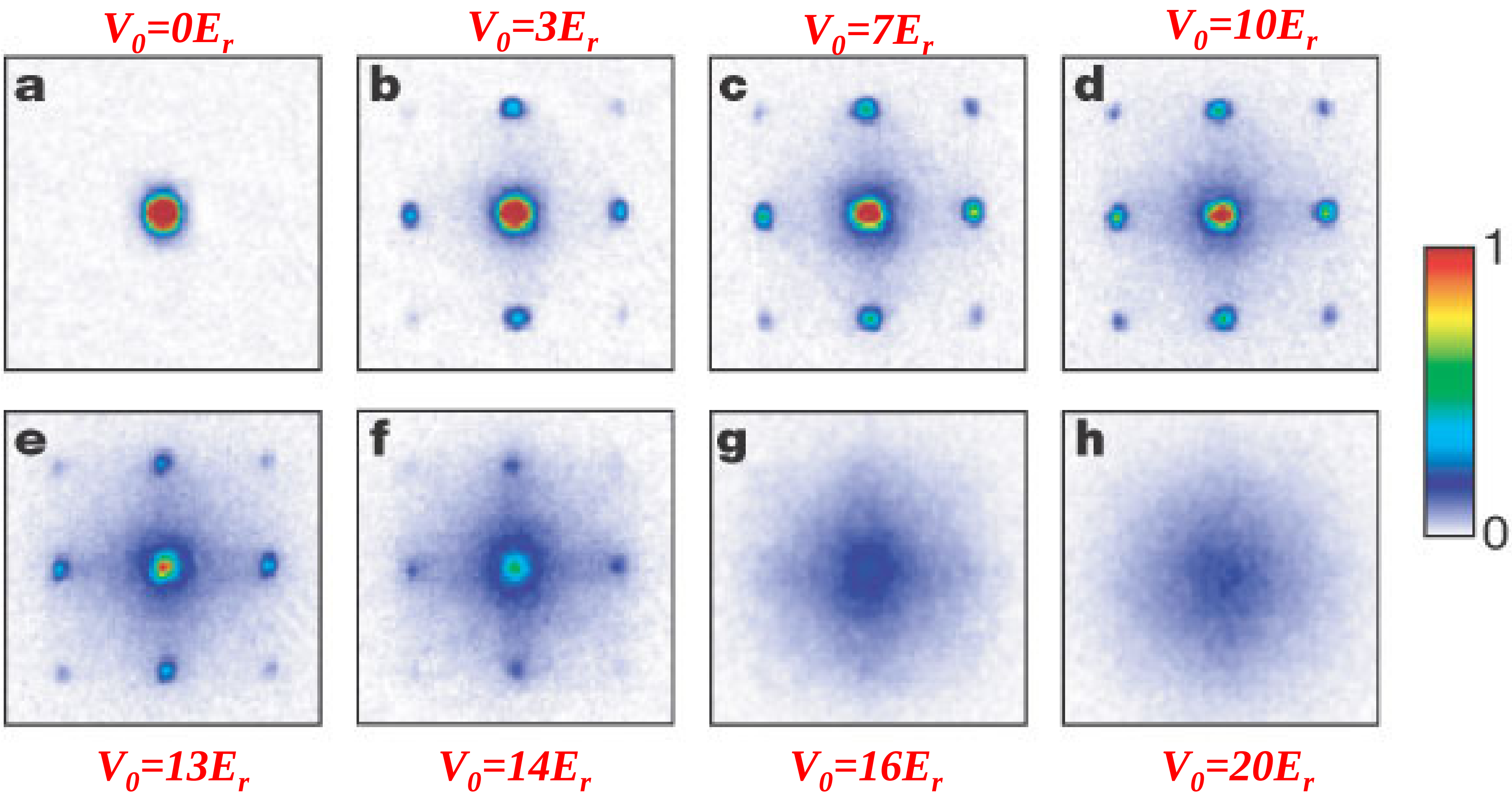
The strength of the period potential can be varied in the experiment



M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Superfluid-insulator transition





In any subvolume of the superfluid, there are large fluctuations in the number of atoms but a definite phase ϕ

Physical meaning of ϕ

Consider two superfluids connected by a weak link

Phase ϕ_1 ; $e^{iN_1\phi_1}|N_1\rangle$ —— w —— Phase ϕ_2 ; $e^{iN_2\phi_2}|N_2\rangle$

$$H = -w\left(|N_1+1, N_2-1\rangle\langle N_1, N_2| + |N_1-1, N_2+1\rangle\langle N_1, N_2|\right)$$

$$\text{(Super)current in weak link} = \frac{d\widehat{N}_1}{dt} = -\frac{i}{\hbar}\left[\widehat{N}_1, H\right]$$

$$= \frac{iw}{\hbar}\left(|N_1, N_2\rangle\langle N_1+1, N_2-1| - |N_1, N_2\rangle\langle N_1-1, N_2+1|\right)$$

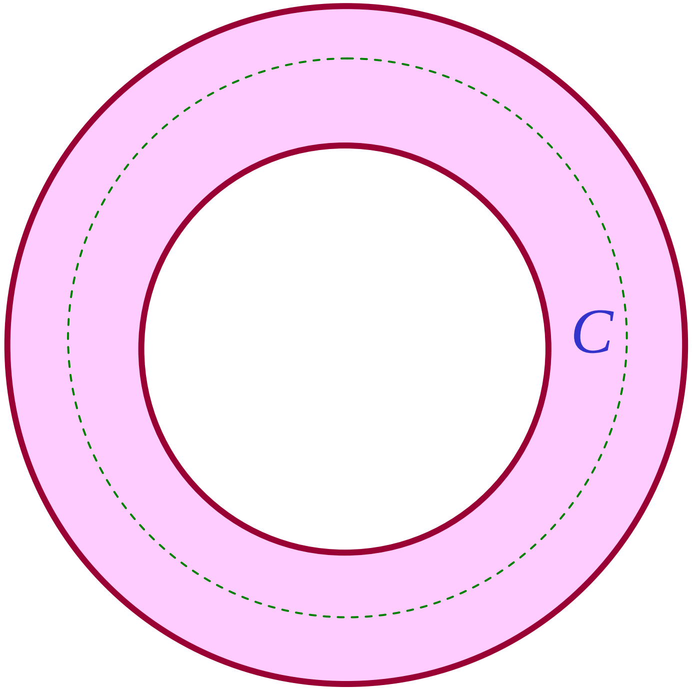
$$= \frac{2w}{\hbar}\sin\left(\phi_1 - \phi_2\right)$$

A superfluid state is characterized by the Ginzburg-Landau complex "order parameter" $\Psi(x)$

$$\Psi(x) \sim e^{i\phi(x)}$$

$$\text{Supercurrent} \sim \nabla\phi$$

Persistent currents



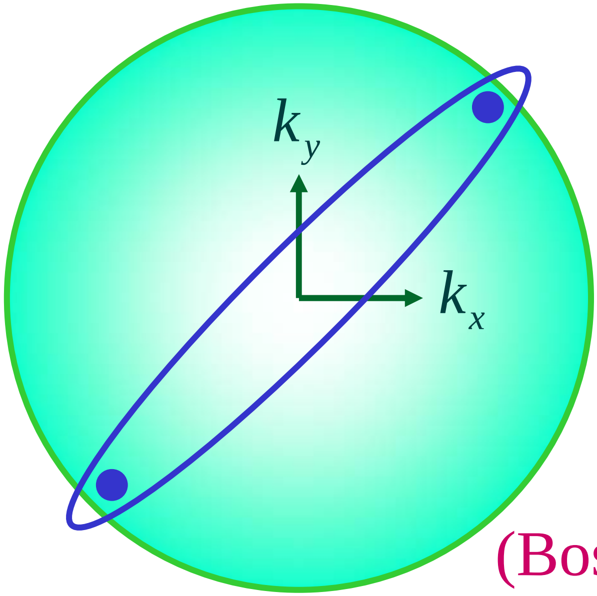
$$\oint_C \nabla\phi \, dx = 2\pi n$$

No local change of the wavefunction can change the value of n

Supercurrent flows "forever"

How do metals form superconductors ?

Electrons form (Cooper) pairs at low temperatures,
and these pairs act like bosons



Pair wavefunction

$$\Psi = \varphi(k)\Big(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\Big)$$

$$\left\langle \vec{S} \right\rangle = 0$$

(Bose-Einstein) condensation of Cooper pairs

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