Quantum criticality beyond the Landau-Ginzburg-Wilson paradigm

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Krishnendu Sengupta (Yale)
T. Senthil (MIT)
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Phys. Rev. Lett. **90**, 216403 (2003).
Science **303**, 1490 (2004).
cond-mat/0408xxx

Talk online: Google Sachdev

$T=0$

SDW

$$\langle \mathbf{S}_j \rangle = \mathbf{N}_1 \cos\left(\vec{K} \cdot \vec{r}_j\right) + \mathbf{N}_2 \sin\left(\vec{K} \cdot \vec{r}_j\right)$$

Collinear spins: $\mathbf{N}_1 \times \mathbf{N}_2 = 0$

Non-collinear spins: $\mathbf{N}_1 \times \mathbf{N}_2 \neq 0$

$$\langle \mathbf{S}_j \rangle = 0$$

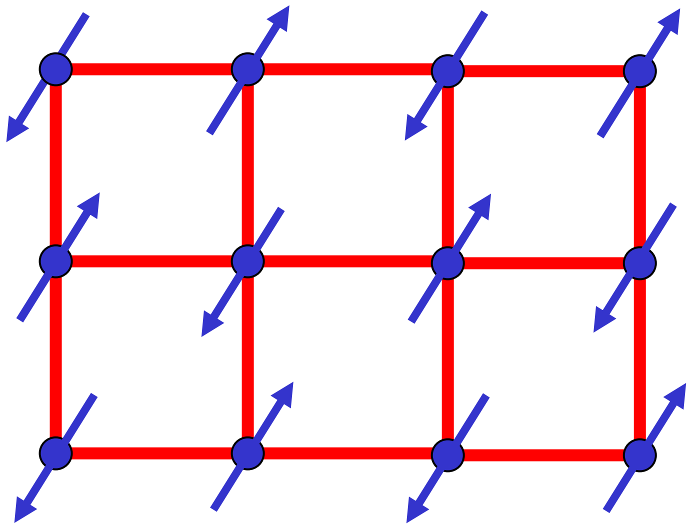
Pressure,
carrier concentration,….

Quantum critical point

States on both sides of critical point
could be either (A) Insulators
(B) Metals
(C) Superconductors

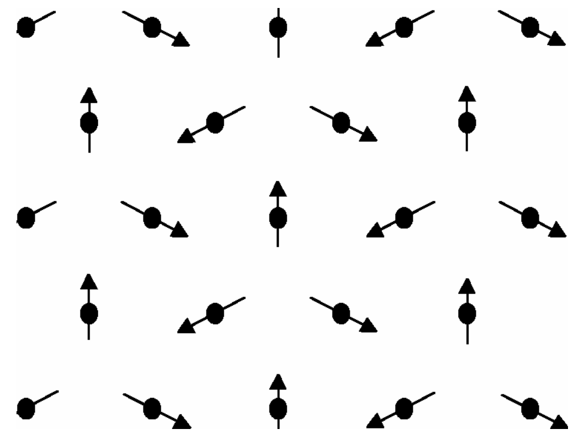
SDWs in Mott insulators

La_2CuO_4



$$\vec{K} = (\pi, \pi)$$

Collinear spins



$$\vec{K} = \left(4\pi/3, 4\pi/\sqrt{3}\right)$$

Non-collinear spins

"Disorder" the spins by enhancing quantum fluctuations in a variety of ways…..

Outline

A. "Dimerized" Mott insulators

Landau-Ginzburg-Wilson (LGW) theory.

B. Kondo lattice models

"Large" Fermi surfaces and the LGW SDW paramagnon theory.

C. Fractionalized Fermi liquids

Spin liquids and Fermi volume changing transitions with a topological order parameter.

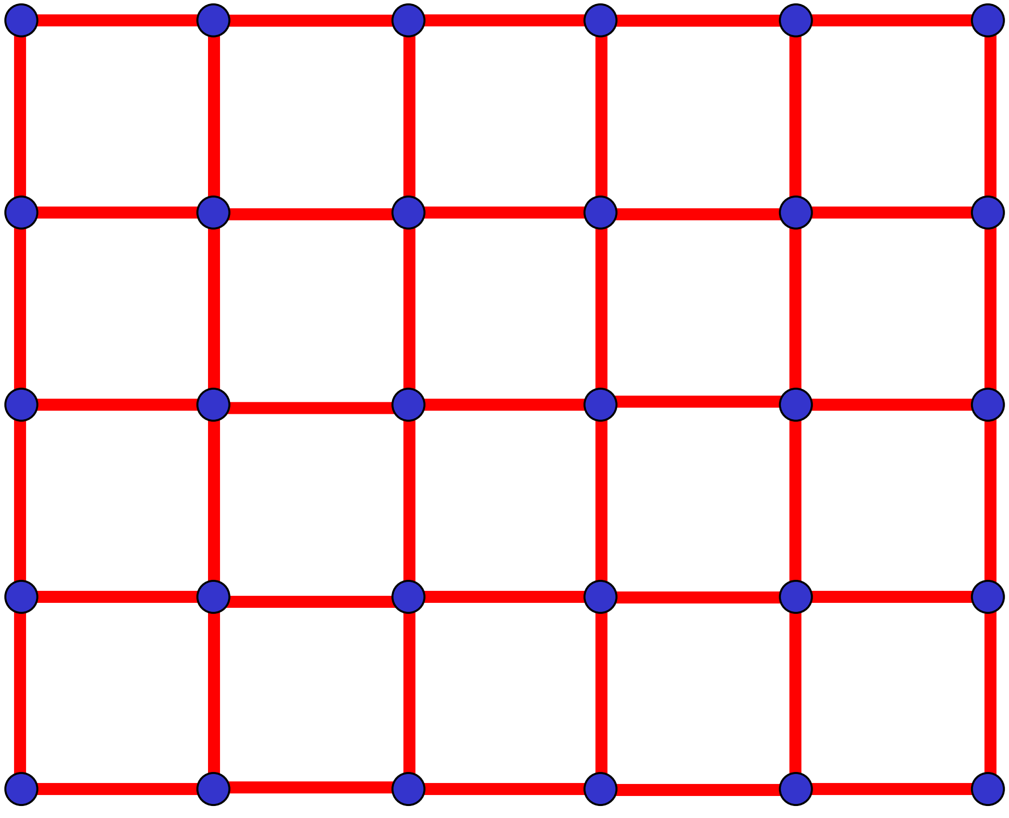
D. Multiple order parameters

LGW forbidden transitions

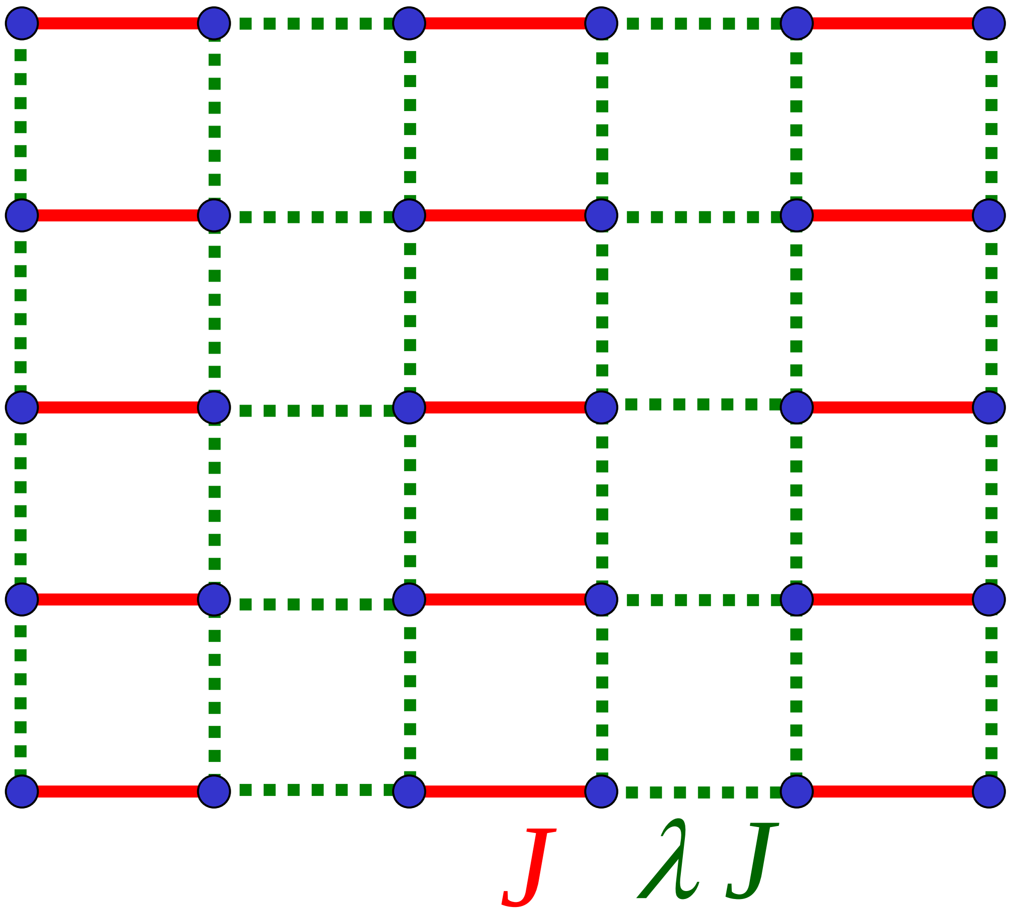
(A) Magnetic quantum phase transitions in "dimerized" Mott insulators

Landau-Ginzburg-Wilson (LGW) theory:

Second-order phase transitions described by fluctuations of an ***order parameter*** *associated with a* ***broken symmetry***



Coupled Dimer Antiferromagnet



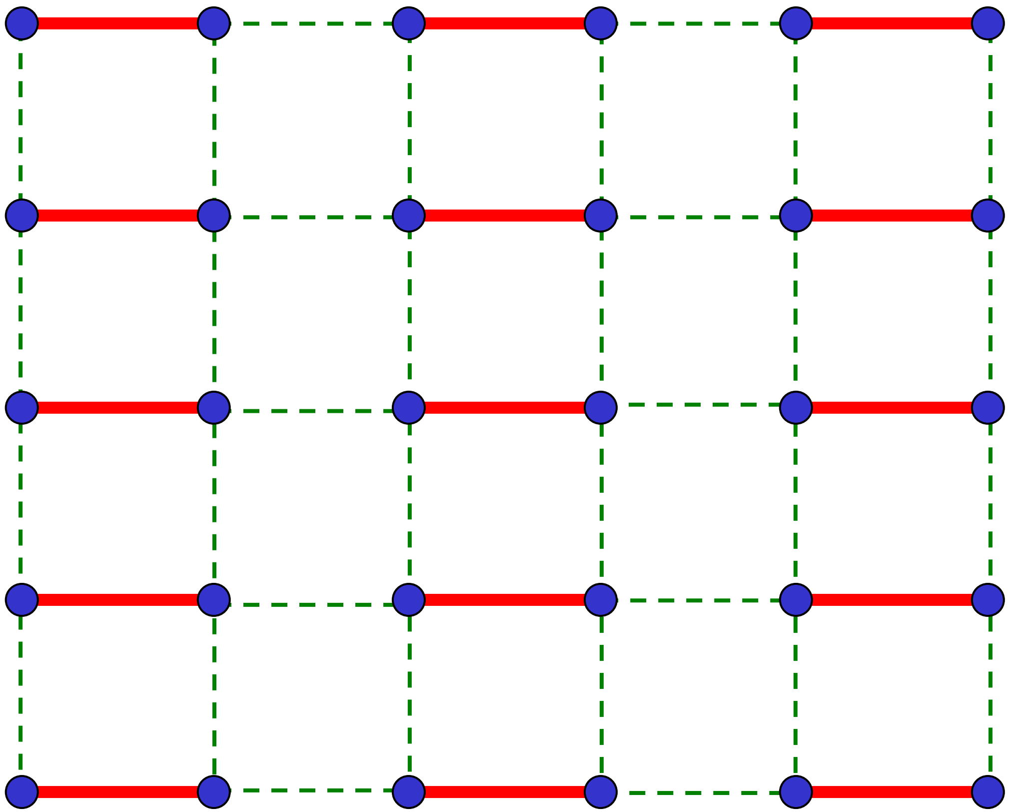
$$H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$0 \le \lambda \le 1$$

M. P. Gelfand, R. R. P. Singh, and D. A. Huse, *Phys. Rev. B* **40**, 10801-10809 (1989).
N. Katoh and M. Imada, *J. Phys. Soc. Jpn.* **63**, 4529 (1994).
J. Tworzydlo, O. Y. Osman, C. N. A. van Duin, J. Zaanen, *Phys. Rev.* B **59**, 115 (1999).
M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev.* B **65**, 014407 (2002).

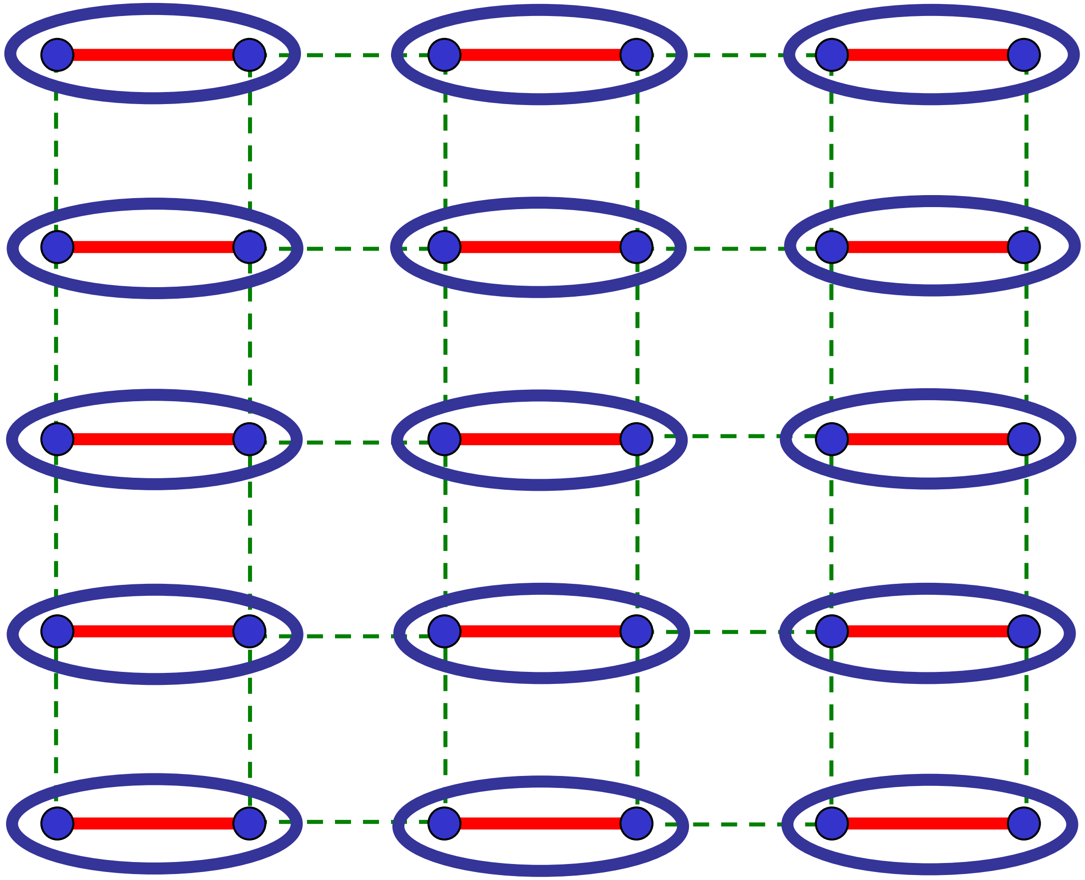
λ close to 0

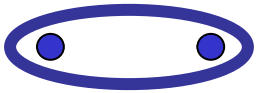
Weakly coupled dimers



λ close to 0

Weakly coupled dimers





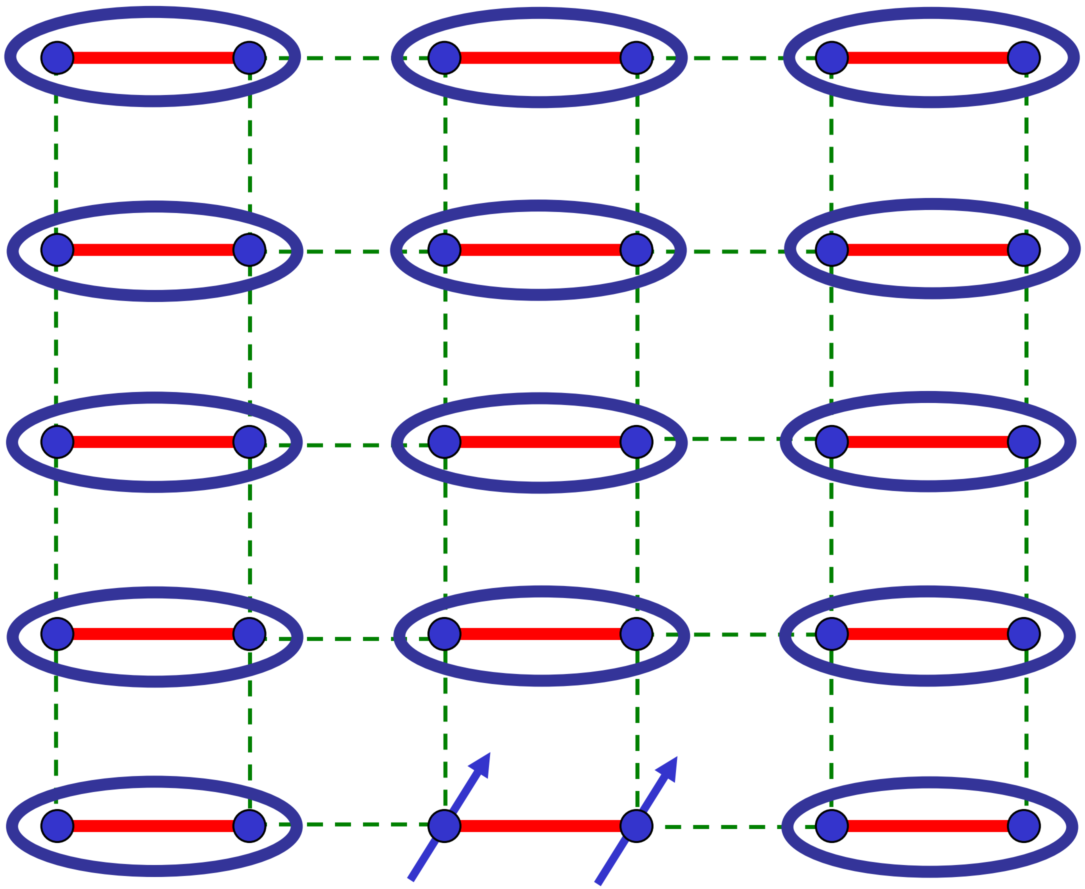
$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

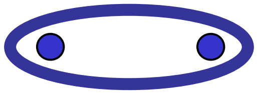
Paramagnetic ground state

$$\left\langle \vec{S}_i \right\rangle = 0, \ \left\langle \vec{\varphi} \right\rangle = 0$$

λ close to 0

Weakly coupled dimers

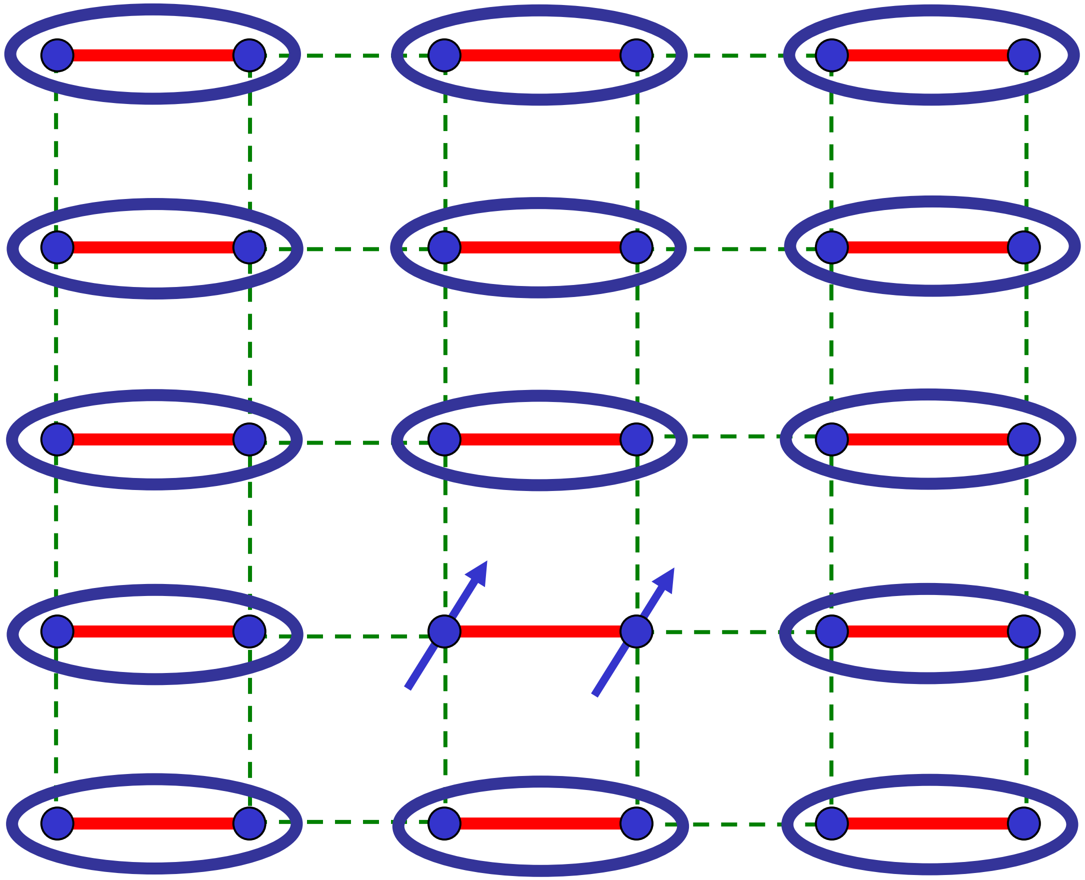


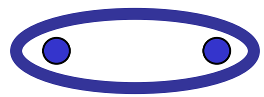
 $= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$

Excitation: S=1 ***triplon***

λ close to 0

Weakly coupled dimers





$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$

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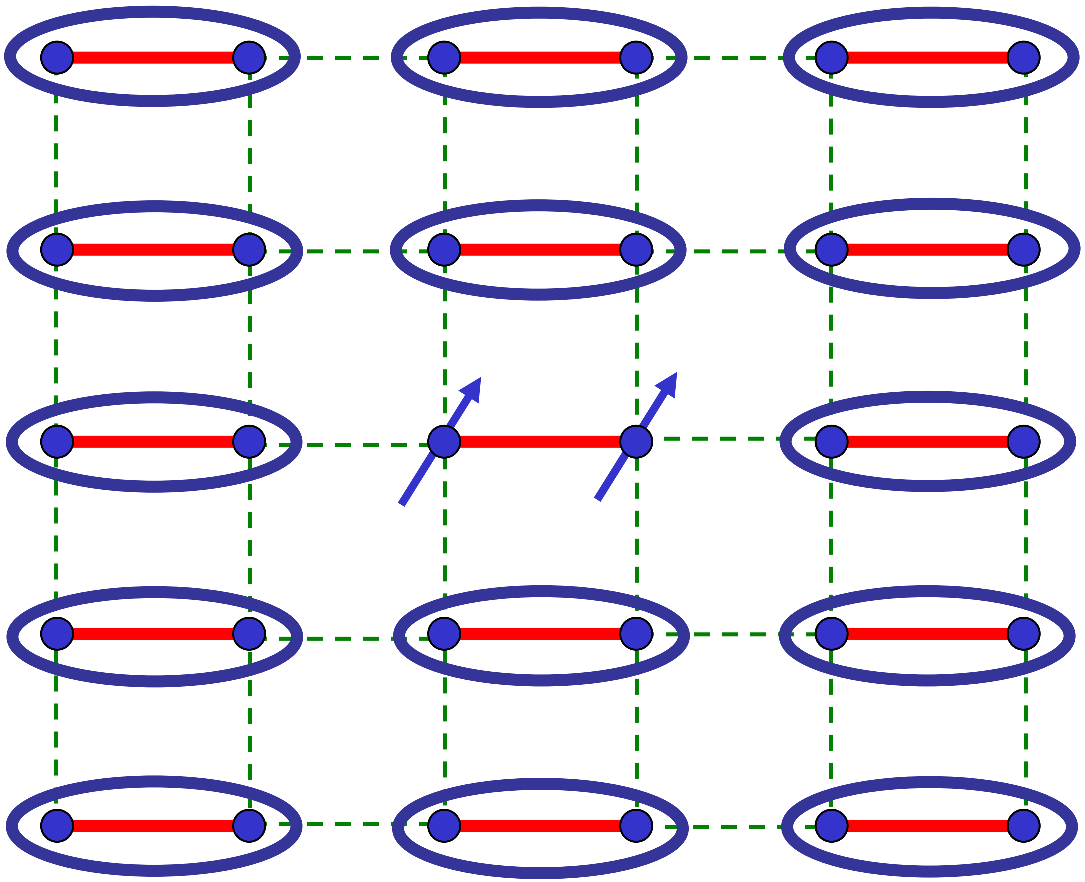
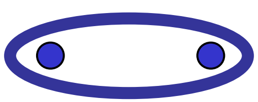
Weakly coupled dimers



 $= \frac{1}{\sqrt{2}}\Big(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\Big)$

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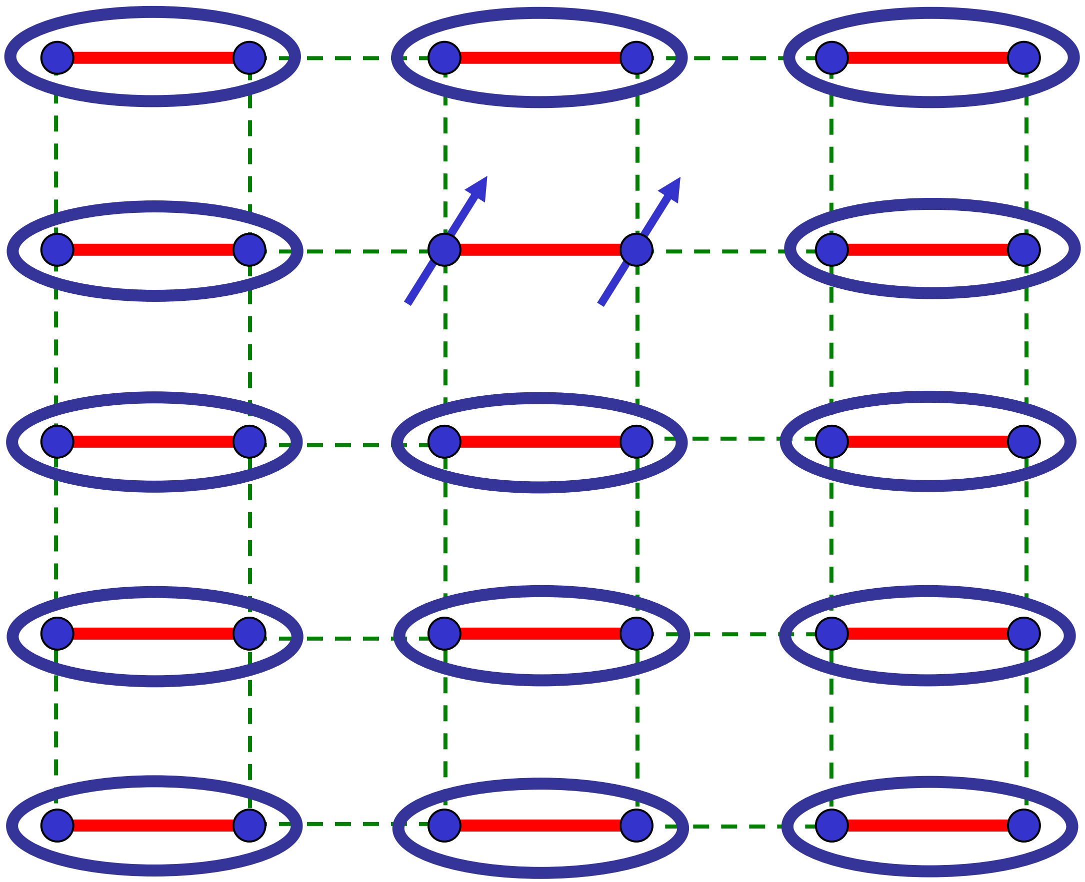
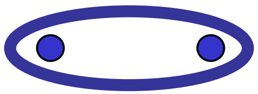
Weakly coupled dimers



 $= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle \right)$

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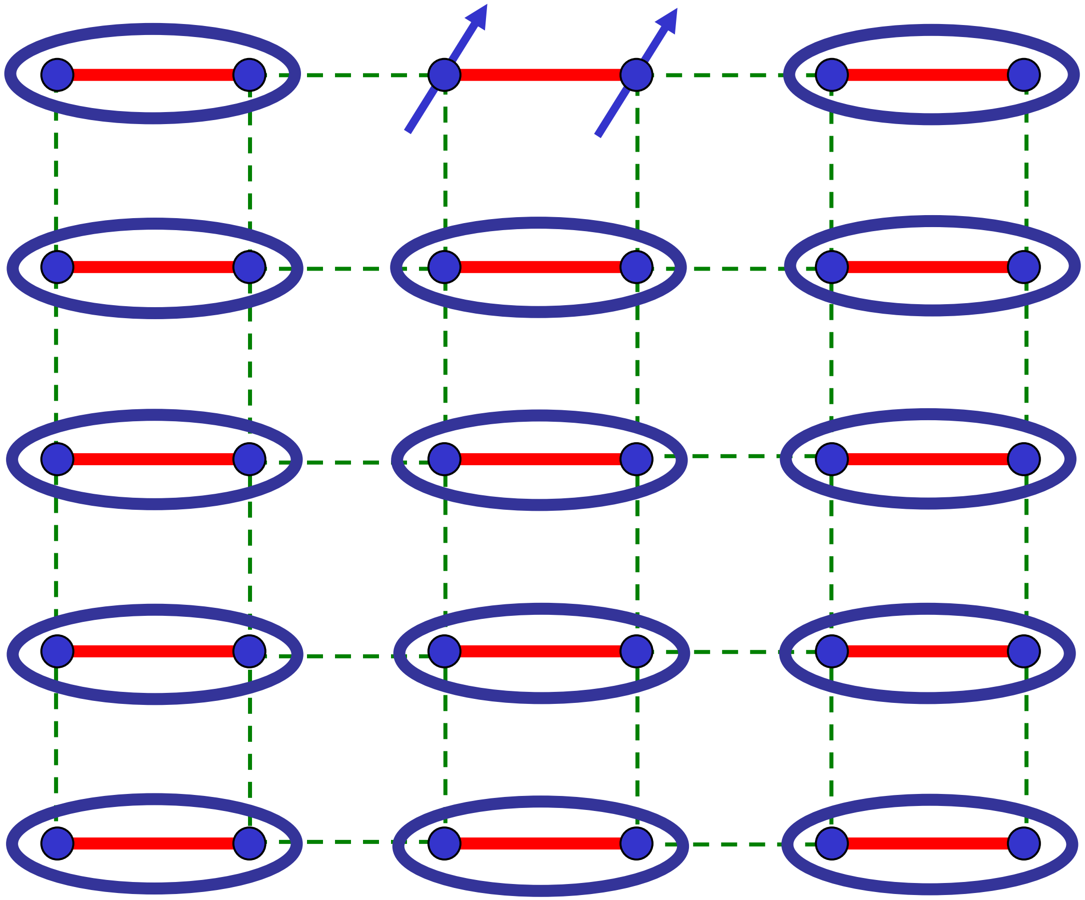
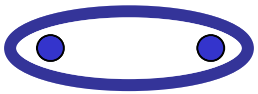
Weakly coupled dimers





$$= \frac{1}{\sqrt{2}}\Big(\big|\uparrow\downarrow\big\rangle - \big|\downarrow\uparrow\big\rangle \Big)$$

Excitation: S=1 ***triplon***

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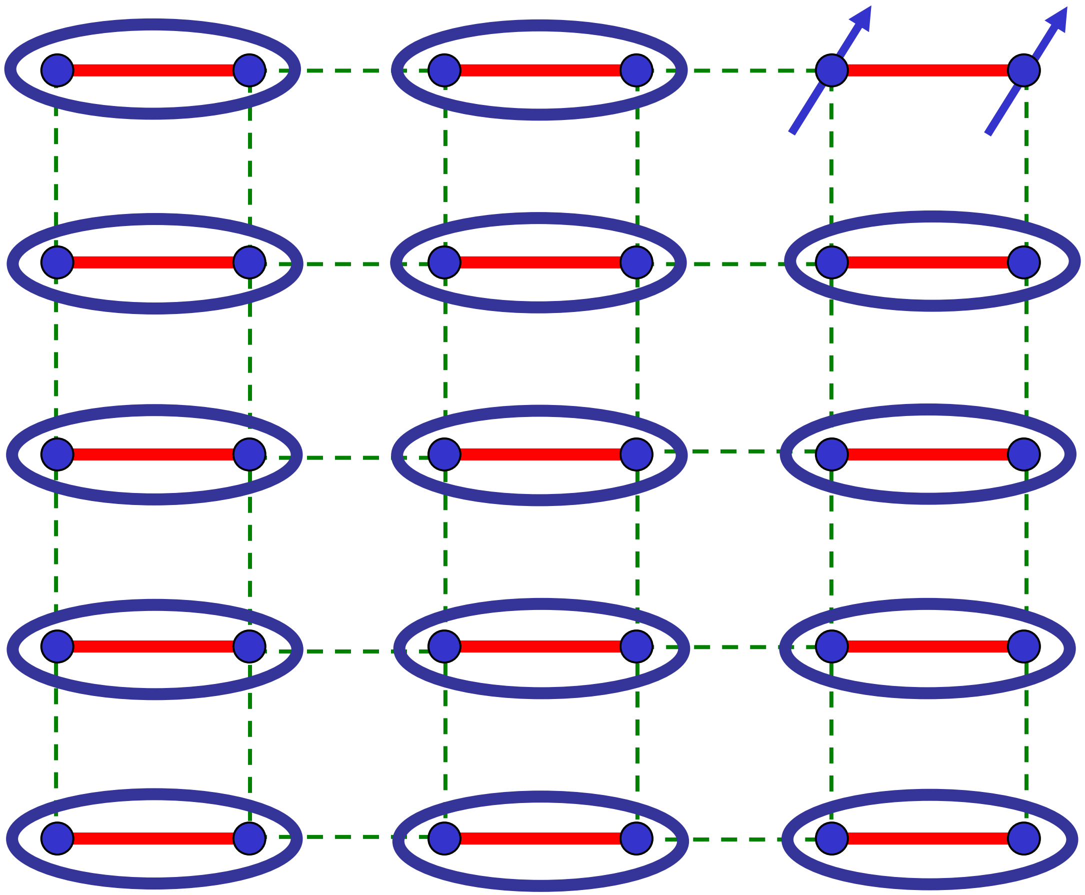
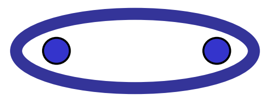
Weakly coupled dimers



 $= \frac{1}{\sqrt{2}}\left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$

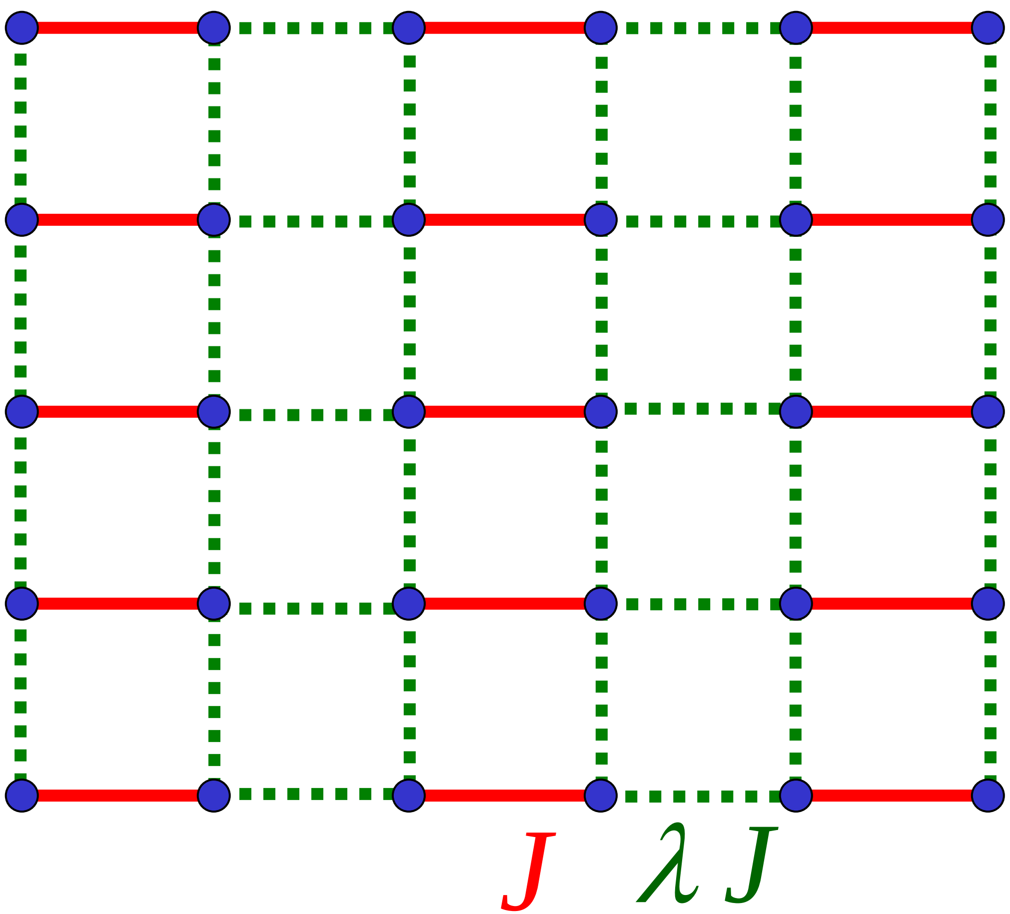
Excitation: S=1 ***triplon***
(***exciton***, spin collective mode)

Energy dispersion away from antiferromagnetic wavevector

$$\varepsilon_p = \Delta + \frac{c_x^2 p_x^2 + c_y^2 p_y^2}{2\Delta}$$

$\Delta \rightarrow$ spin gap

Coupled Dimer Antiferromagnet



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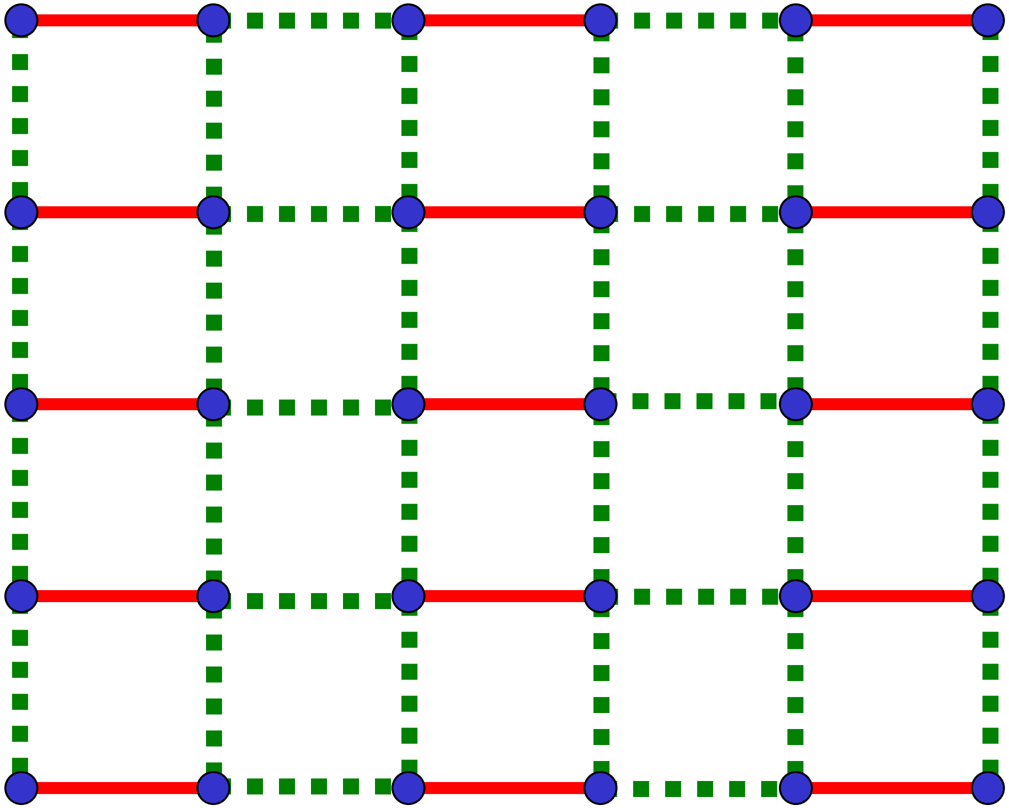
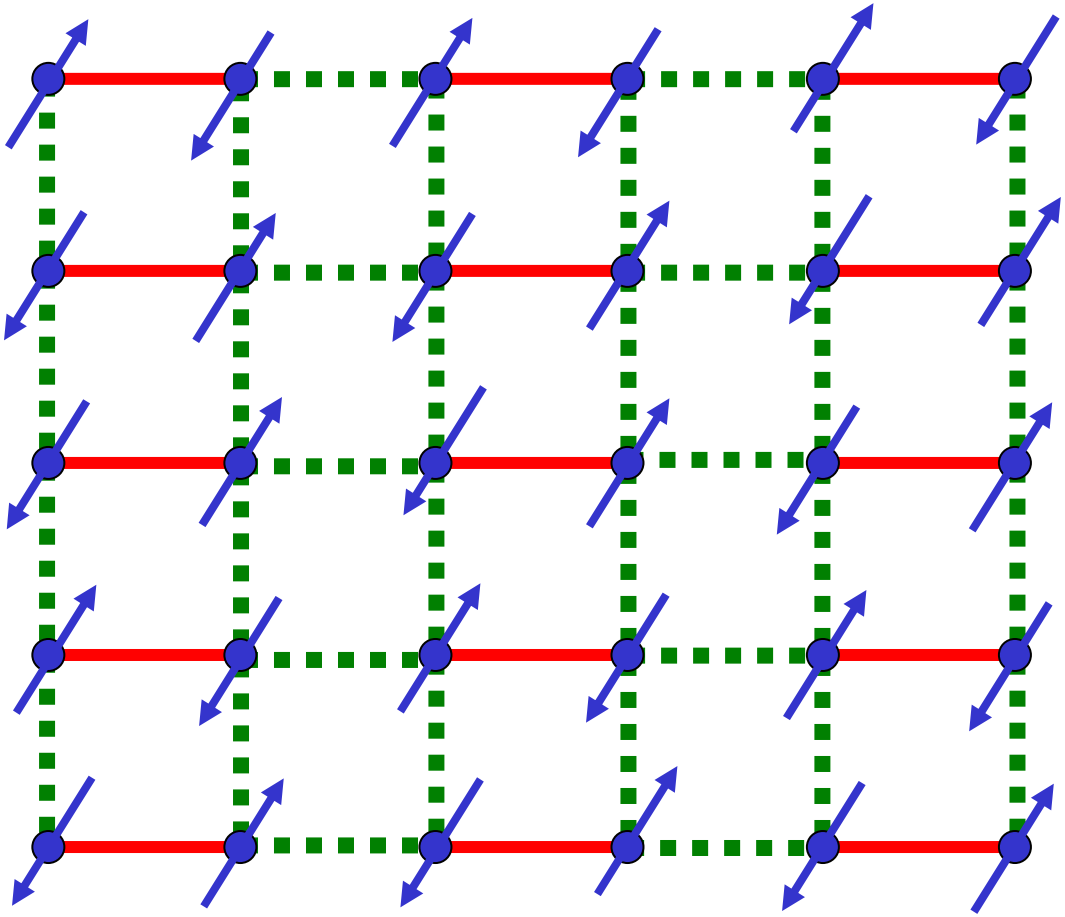
λ close to 1

Weakly dimerized square lattice



λ close to 1

Weakly dimerized square lattice



Excitations:
2 spin waves (***magnons***)

$$\varepsilon_p = \sqrt{c_x{}^2 p_x{}^2 + c_y{}^2 p_y{}^2}$$

Ground state has long-range spin density wave (Néel) order at wavevector ***K***= (π,π)

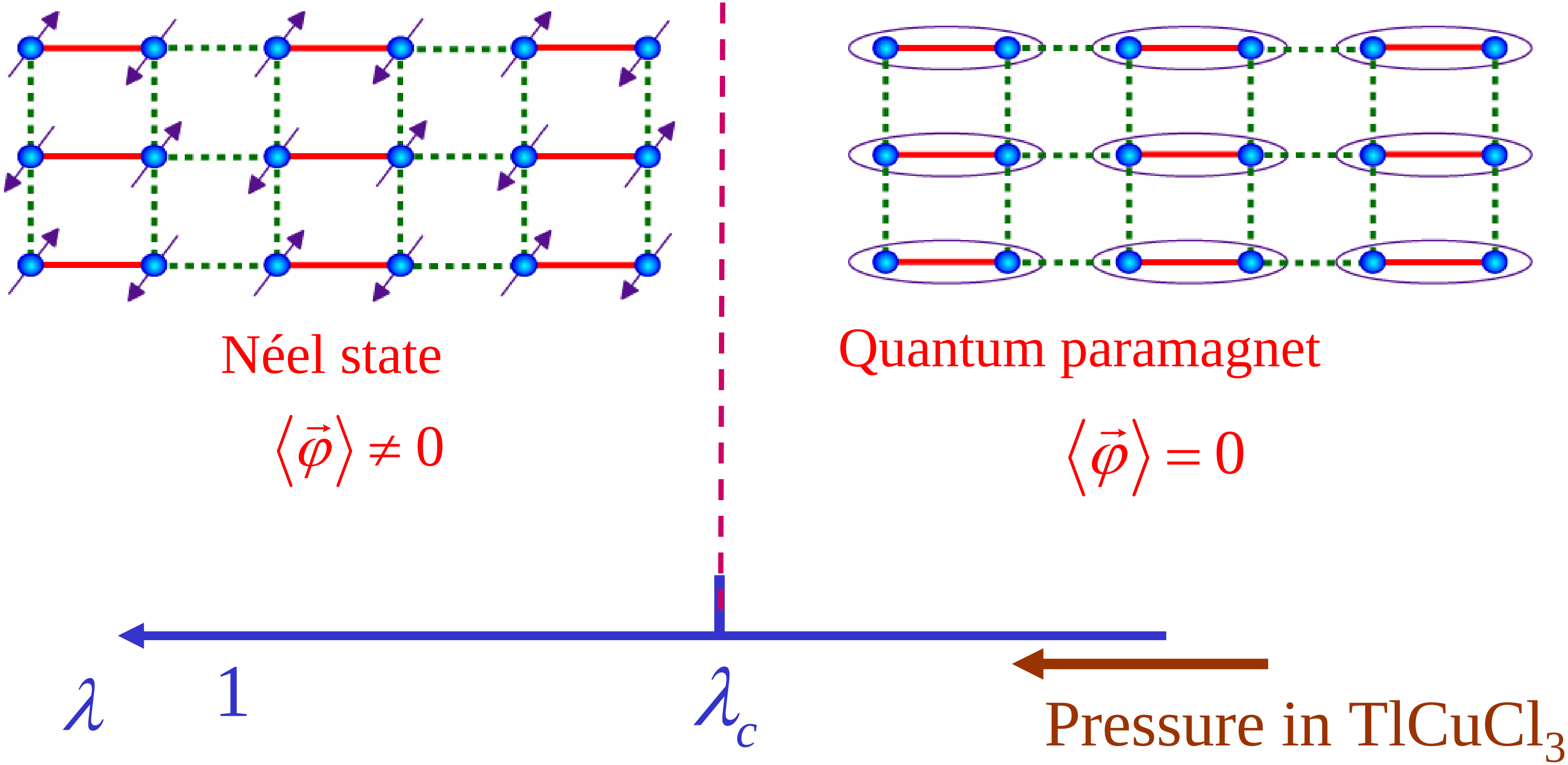
$$\langle \vec{\varphi} \rangle \neq 0$$

spin density wave order parameter: $\vec{\varphi} = \eta_i \frac{\vec{S}_i}{S}$; $\eta_i = \pm 1$ on two sublattices

T=0

$\lambda_c = 0.52337(3)$

M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev.* B **65**, 014407 (2002)



The method of bond operators (S. Sachdev and R.N. Bhatt, *Phys. Rev.* B **41**, 9323 (1990)) provides a quantitative description of spin excitations in $TlCuCl_3$ across the quantum phase transition (M. Matsumoto, B. Normand, T.M. Rice, and M. Sigrist, *Phys. Rev. Lett.* **89**, 077203 (2002))

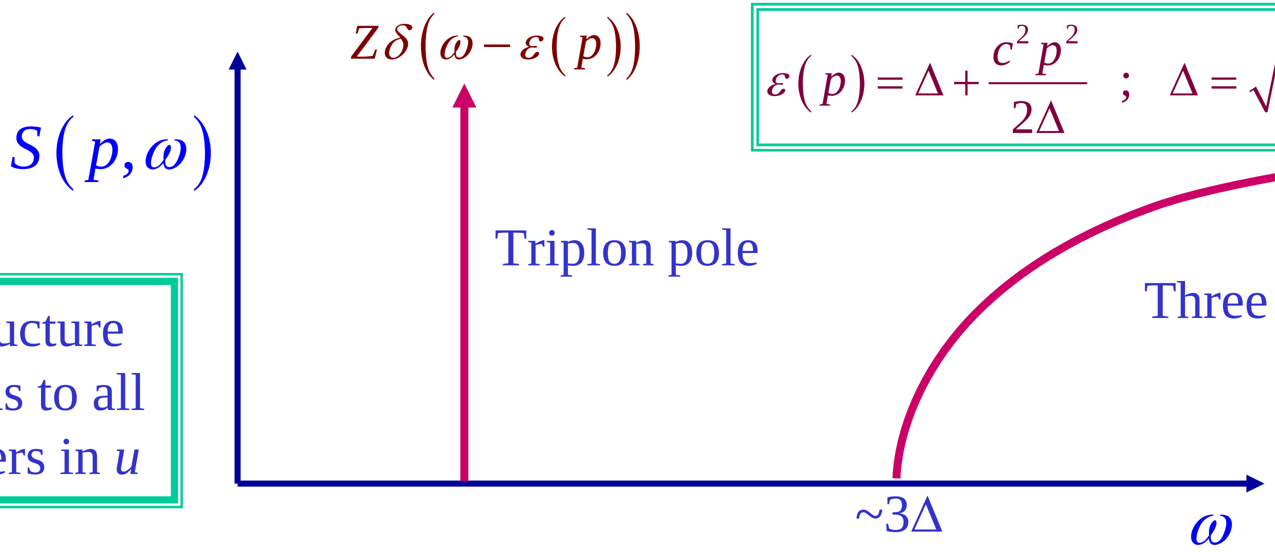
LGW theory for quantum criticality

Landau-Ginzburg-Wilson theory: write down an effective action for the antiferromagnetic order parameter $\vec{\varphi}$ by expanding in powers of $\vec{\varphi}$ and its spatial and temporal derivatives, while preserving all symmetries of the microscopic Hamiltonian

$$S_\varphi = \int d^2x d\tau \left[\frac{1}{2}\left(\left(\nabla_x \vec{\varphi}\right)^2 + c^2 \left(\partial_\tau \vec{\varphi}\right)^2 + \left(\lambda_c - \lambda\right) \vec{\varphi}^2 \right) + \frac{u}{4!}\left(\vec{\varphi}^2\right)^2 \right]$$

S. Chakravarty, B.I. Halperin, and D.R. Nelson, *Phys. Rev.* B **39**, 2344 (1989)

For $\lambda < \lambda_c$ oscillations of $\vec{\varphi}$ about $\vec{\varphi} = 0$ lead to the following structure in the dynamic structure factor $S(p,\omega)$



$$\varepsilon(p) = \Delta + \frac{c^2 p^2}{2\Delta} \quad ; \quad \Delta = \sqrt{\lambda_c - \lambda}/c$$

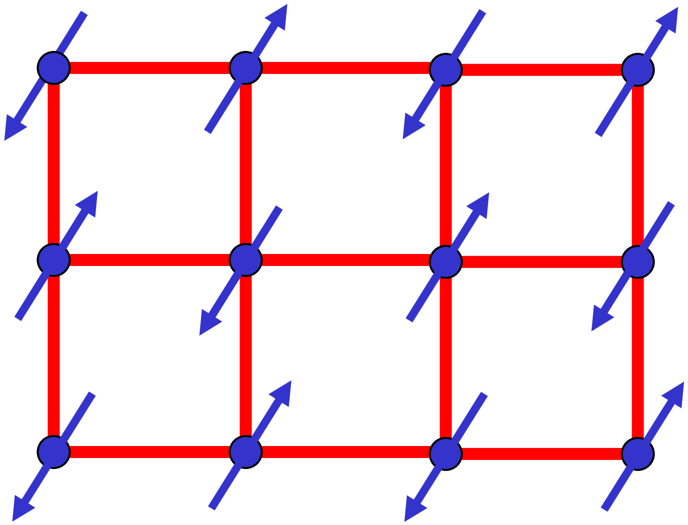
Structure holds to all orders in u

A.V. Chubukov, S. Sachdev, and J.Ye, *Phys. Rev.* B **49**, 11919 (1994)

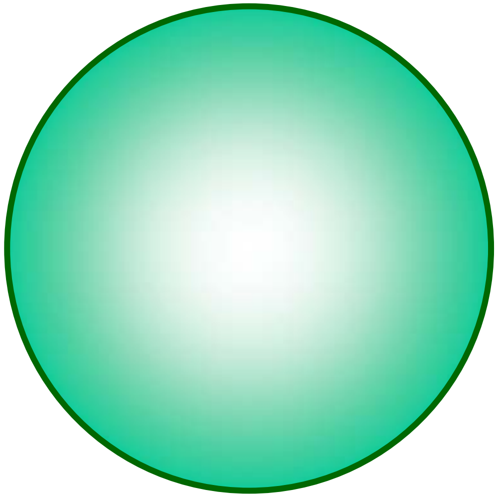
(B) Kondo lattice models

"Large" Fermi surfaces and the Landau-Ginzburg-Wilson spin-density-wave paramagnon theory

Kondo lattice



+



Local moments f_σ

Conduction electrons c_σ

$$H_K = \sum_{i<j} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + J_K \sum_i c^\dagger_{i\sigma} \vec{\tau}_{\sigma\sigma'} c_{i\sigma} \cdot \vec{S}_{fi} + J \sum_{\langle ij \rangle} \vec{S}_{fi} \cdot \vec{S}_{fj}$$

At large J_K, magnetic order is destroyed, and we obtain a non-magnetic Fermi liquid (FL) ground state

S. Doniach, *Physica* B **91**, 231 (1977).

Luttinger's Fermi volume on a d-dimensional lattice for the FL phase

Let v_0 be the volume of the unit cell of the ground state,
n_T be the total number density of electrons per volume v_0.
(need not be an integer)

$$n_T = n_f + n_c = 1 + n_c$$

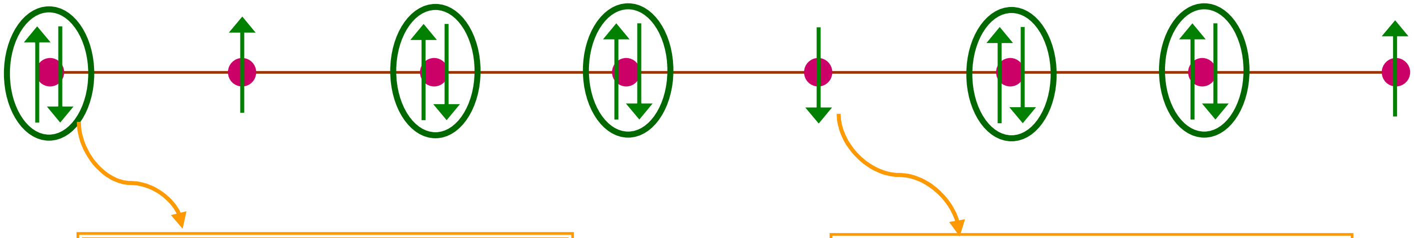
$$2 \times \frac{v_0}{(2\pi)^d} \left(\text{Volume enclosed by Fermi surface} \right)$$

$$= n_T \left(\text{mod } 2 \right)$$

A "large" Fermi surface

Argument for the Fermi surface volume of the FL phase

Single ion Kondo effect implies $J_K \to \infty$ at low energies



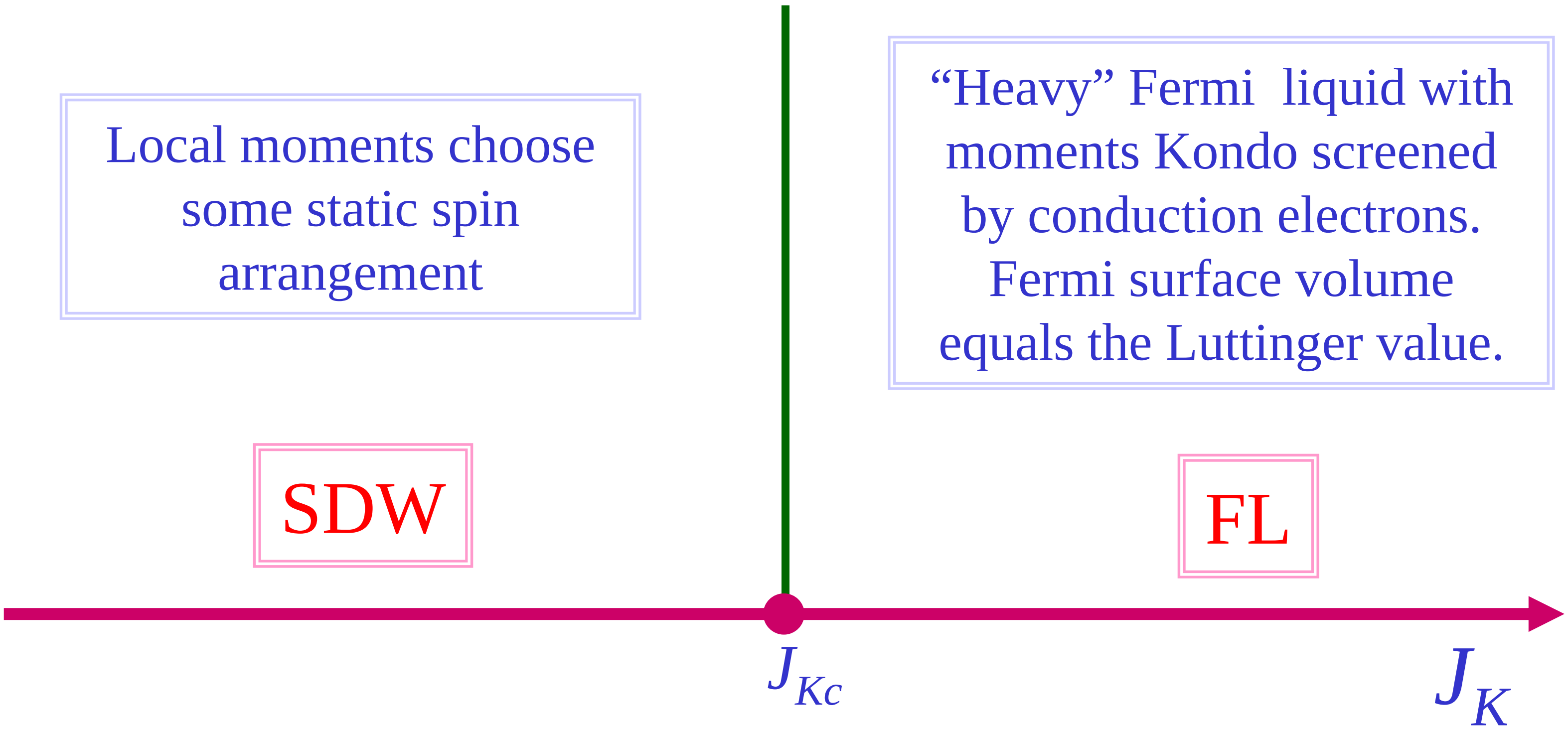
$$\left(c_{i\uparrow}^{\dagger} f_{i\downarrow}^{\dagger} - c_{i\downarrow}^{\dagger} f_{i\uparrow}^{\dagger}\right)\left|0\right\rangle$$

$$f_{i\downarrow}^{\dagger}\left|0\right\rangle, \; S=1/2 \text{ hole}$$

Fermi liquid of S=1/2 holes with hard-core repulsion

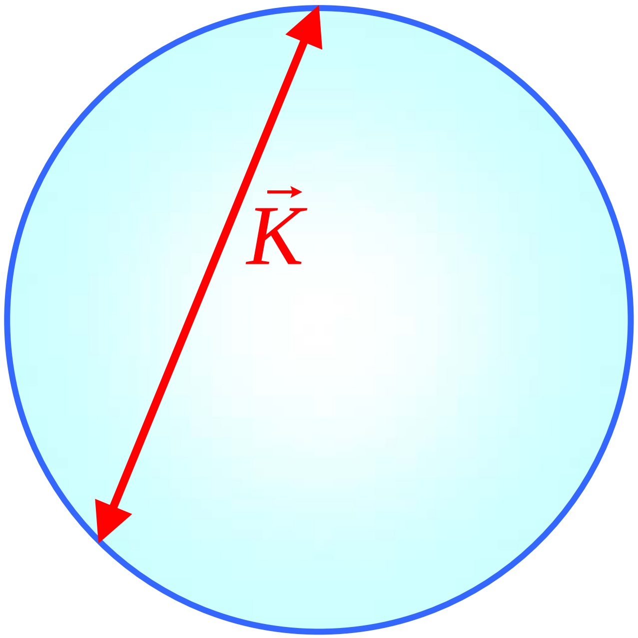
$$\begin{aligned}\text{Fermi surface volume } &= -\left(\text{density of holes}\right) \bmod 2 \\ &= -\left(1-n_c\right) = \left(1+n_c\right) \bmod 2\end{aligned}$$

Doniach's *T=0* phase diagram for the Kondo lattice



LGW theory for quantum critical point

Write down effective action for SDW order parameter $\vec{\varphi}$



$\vec{\varphi}$ fluctuations are damped by mixing with fermionic quasiparticles near the Fermi surface

$$S_\varphi = \int \frac{d^d q d\omega}{(2\pi)^{d+1}} \left|\vec{\varphi}(q,\omega)\right|^2 \left(q^2 + |\omega| + \left(J_K - J_{Kc}\right)\right) + \frac{u}{4}\int d^d r d\tau \left(\vec{\varphi}^2\right)^2$$

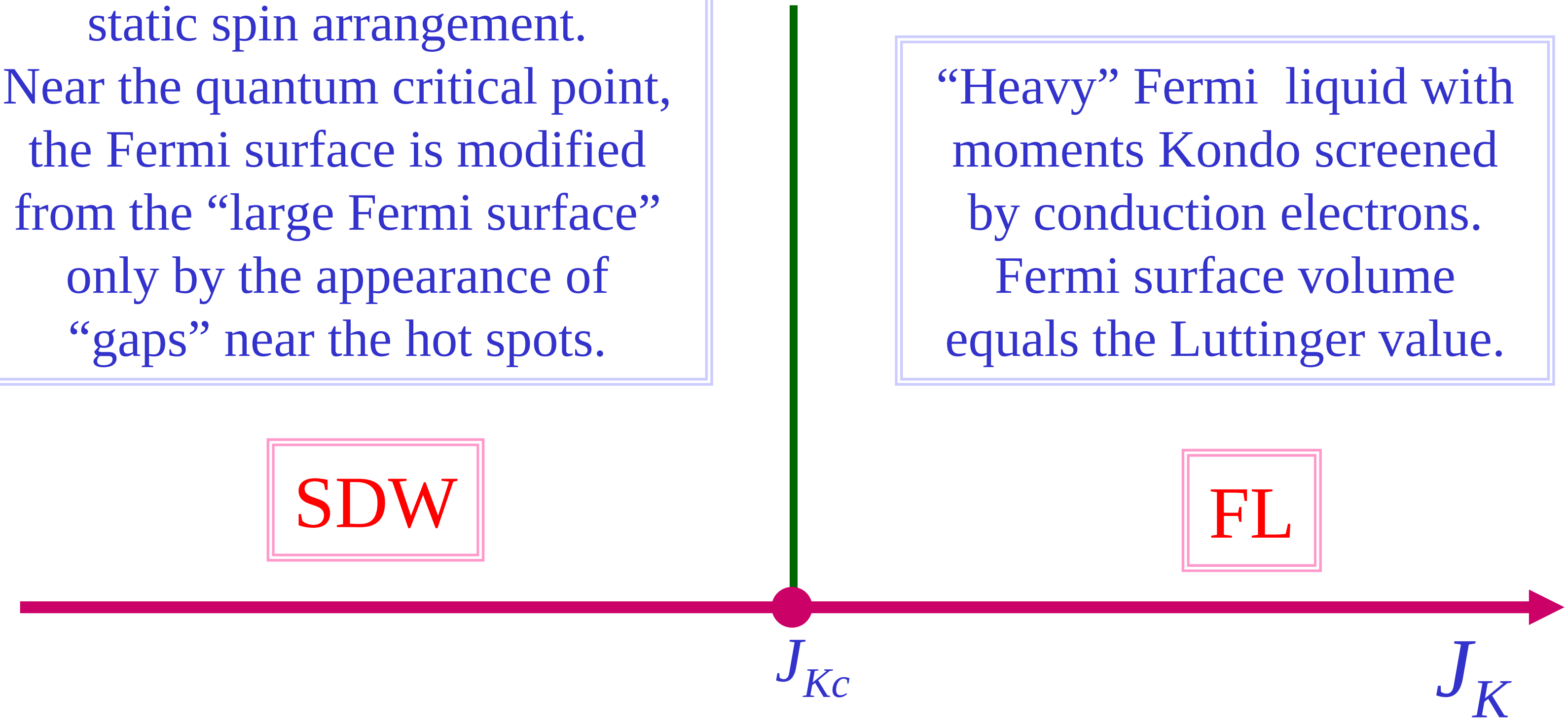
Fluctuations of $\vec{\varphi}$ about $\vec{\varphi} = 0 \Rightarrow$ the triplon is now a paramagnon

J. Mathon, *Proc. R. Soc. London* A, **306**, 355 (1968); T.V. Ramakrishnan, *Phys. Rev.* B **10**, 4014 (1974);
M. T. Beal-Monod and K. Maki, *Phys. Rev. Lett.* **34**, 1461 (1975); J.A. Hertz, *Phys. Rev.* B **14**, 1165 (1976).
T. Moriya, *Spin Fluctuations in Itinerant Electron Magnetism*, Springer-Verlag, Berlin (1985);
G. G. Lonzarich and L. Taillefer, *J. Phys.* C **18**, 4339 (1985); A.J. Millis, *Phys. Rev.* B **48**, 7183 (1993).

Doniach's *T=0* phase diagram for the Kondo lattice

Local moments choose some static spin arrangement. Near the quantum critical point, the Fermi surface is modified from the "large Fermi surface" only by the appearance of "gaps" near the hot spots.

"Heavy" Fermi liquid with moments Kondo screened by conduction electrons. Fermi surface volume equals the Luttinger value.

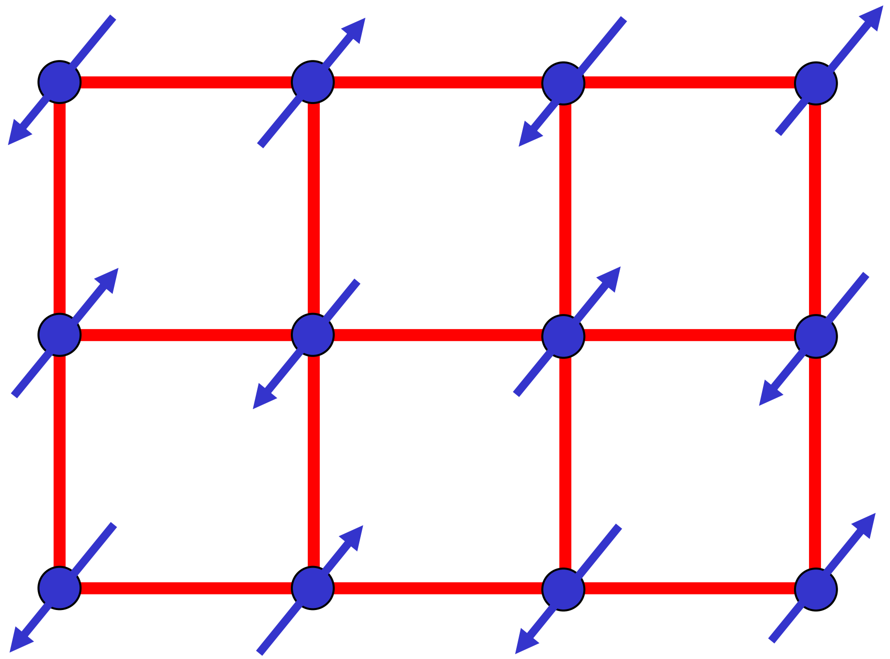


(C) Fractionalized Fermi liquids (FL*)

Spin liquids and Fermi volume changing transitions with a <u>topological</u> order parameter

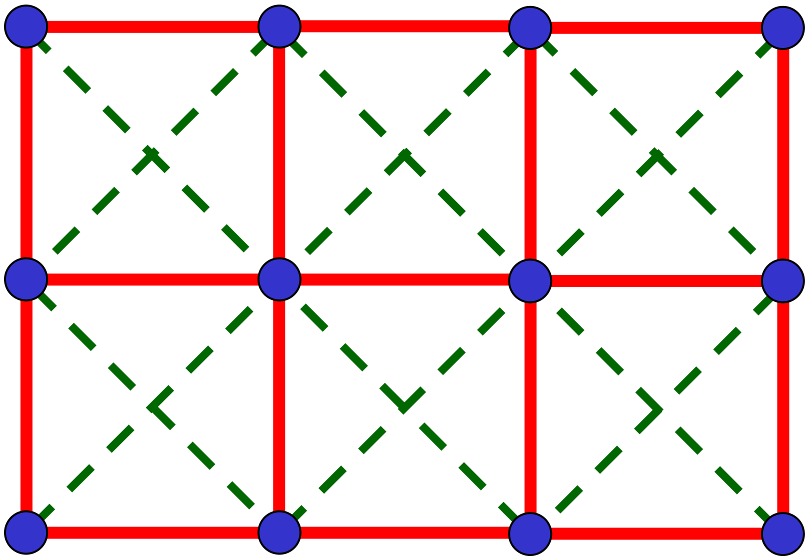
Beyond LGW: quantum phases and phase transitions with emergent gauge excitations and fractionalization

Work in the regime with small J_K, and consider destruction of magnetic order by frustrating (RKKY) exchange interactions between *f* moments



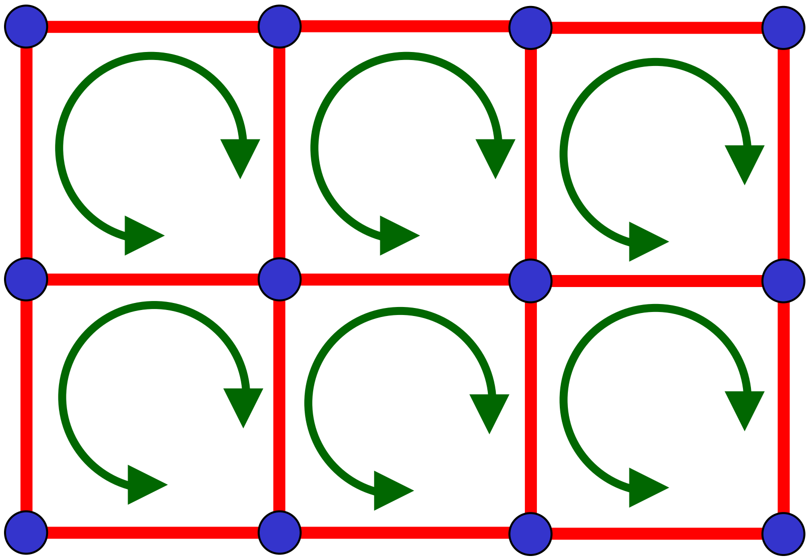
Ground state has Neel order with $\vec{\varphi} \neq 0$

Work in the regime with small J_K, and consider destruction of magnetic order by frustrating (RKKY) exchange interactions between *f* moments



Destroy SDW order by perturbations which preserve full square lattice symmetry *e.g.* second-neighbor or ring exchange.

Work in the regime with small J_K, and consider destruction of magnetic order by frustrating (RKKY) exchange interactions between *f* moments



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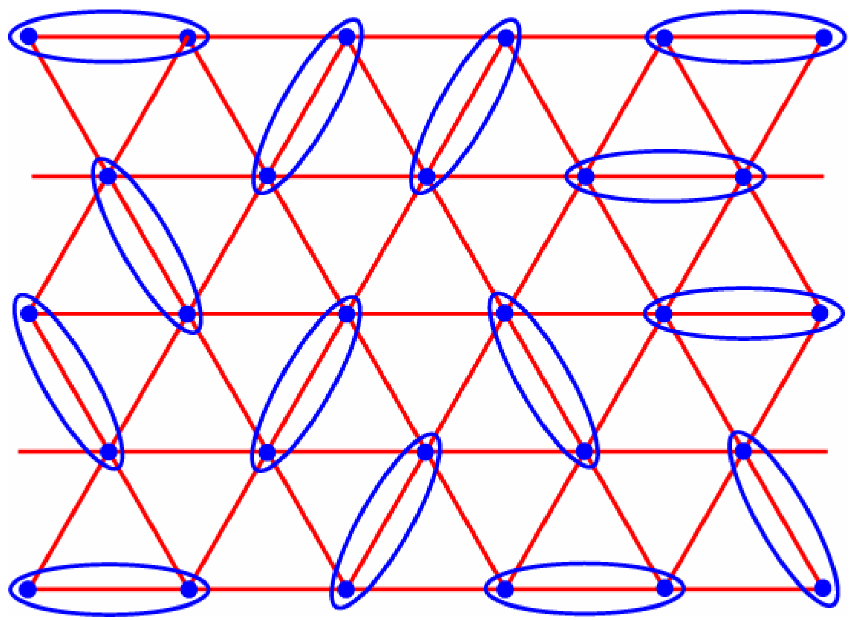
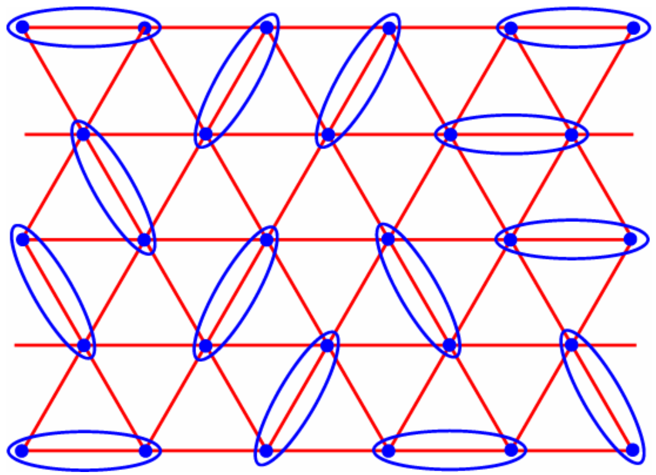


A *spin liquid* ground state with $\langle \vec{\varphi} \rangle = 0$ and no broken lattice symmetries.

Such a state has emergent excitations described by a Z_2 or U(1) gauge theory

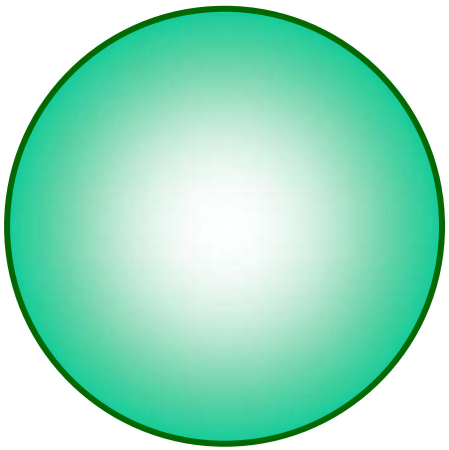
P. Fazekas and P.W. Anderson, *Phil Mag* **30**, 23 (1974).
N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991);
X. G. Wen, *Phys. Rev.* B **44**, 2664 (1991).

Influence of conduction electrons



Local moments f_σ

+



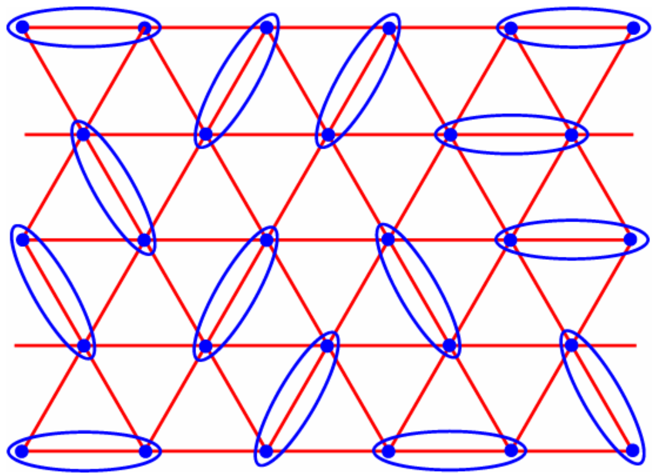
Conduction electrons c_σ

$$H = \sum_{i<j} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + \sum_i \left(J_K c^\dagger_{i\sigma} \vec{\tau}_{\sigma\sigma'} c_{i\sigma} \cdot \vec{S}_{fi} \right) + \sum_{i<j} J_H (i,j) \vec{S}_{fi} \cdot \vec{S}_{fj}$$

Determine the ground state of the quantum antiferromagnet defined by J_H, and then couple to conduction electrons by J_K

Choose J_H so that ground state of antiferromagnet is a Z_2 or U(1) spin liquid

Influence of conduction electrons



Local moments f_σ

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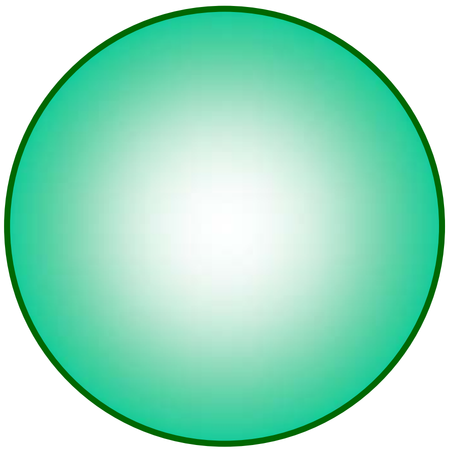


Conduction electrons c_σ

At J_K= 0 the conduction electrons form a Fermi surface on their own with volume determined by n_c.

Perturbation theory in J_K is regular, and so this state will be stable for finite J_K.

So volume of Fermi surface is determined by
$(n_T - 1) = n_c (\text{mod } 2)$, and does not equal the Luttinger value.

The (U(1) or Z_2) FL* state

A new phase: FL*

This phase preserves spin rotation invariance, and has a Fermi surface of *sharp* electron-like quasiparticles.

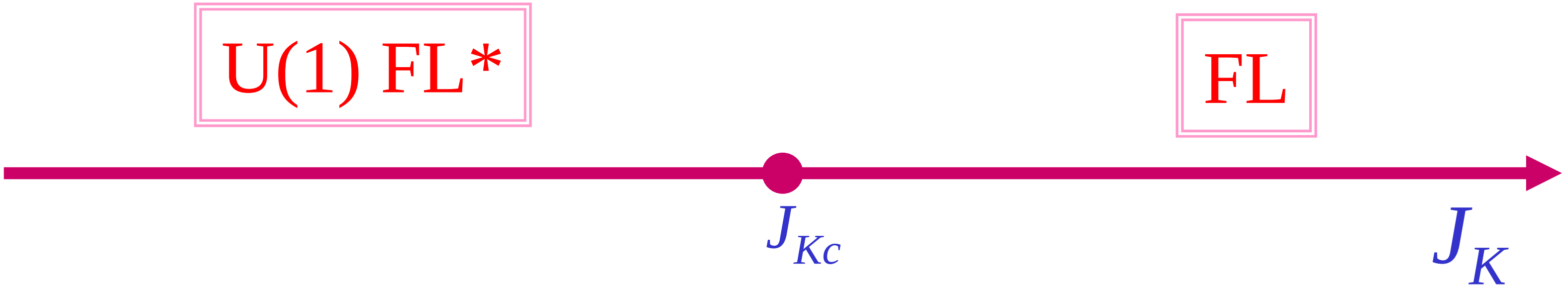
The state has "*topological order*" and associated neutral excitations. The topological order can be detected by the violation of Luttinger's Fermi surface volume. It can only appear in dimensions $d > 1$

$$2 \times \frac{v_0}{(2\pi)^d} \left(\text{Volume enclosed by Fermi surface} \right)$$

$$= (n_T - 1) (\text{mod } 2)$$

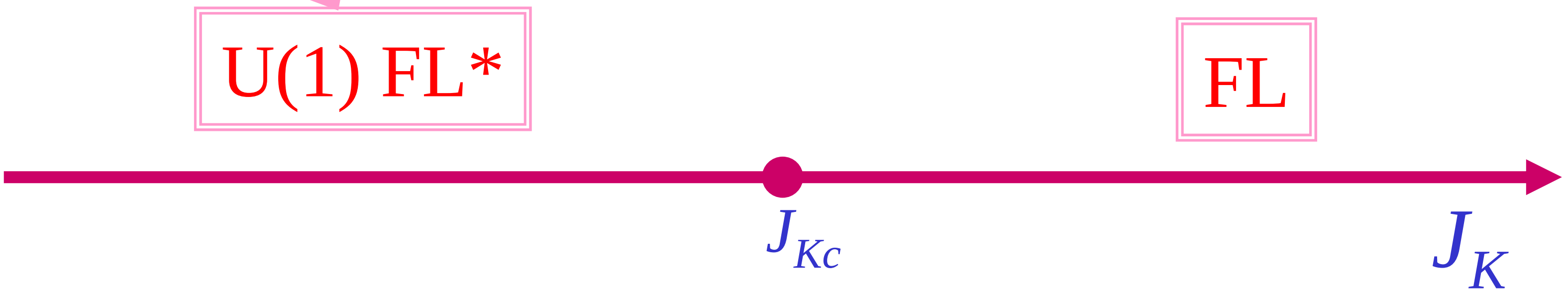
Precursors: N. Andrei and P. Coleman, *Phys. Rev. Lett*. **62**, 595 (1989).
Yu. Kagan, K. A. Kikoin, and N. V. Prokof'ev, *Physica* B **182**, 201 (1992).
Q. Si, S. Rabello, K. Ingersent, and L. Smith, *Nature* **413**, 804 (2001).
S. Burdin, D. R. Grempel, and A. Georges, *Phys. Rev*. B 66, 045111 (2002).
L. Balents and M. P. A. Fisher and C. Nayak, *Phys. Rev*. B **60**, 1654, (1999);
T. Senthil and M.P.A. Fisher, *Phys. Rev*. B **62**, 7850 (2000).
F. H. L. Essler and A. M. Tsvelik, *Phys. Rev*. B **65**, 115117 (2002).

Phase diagram

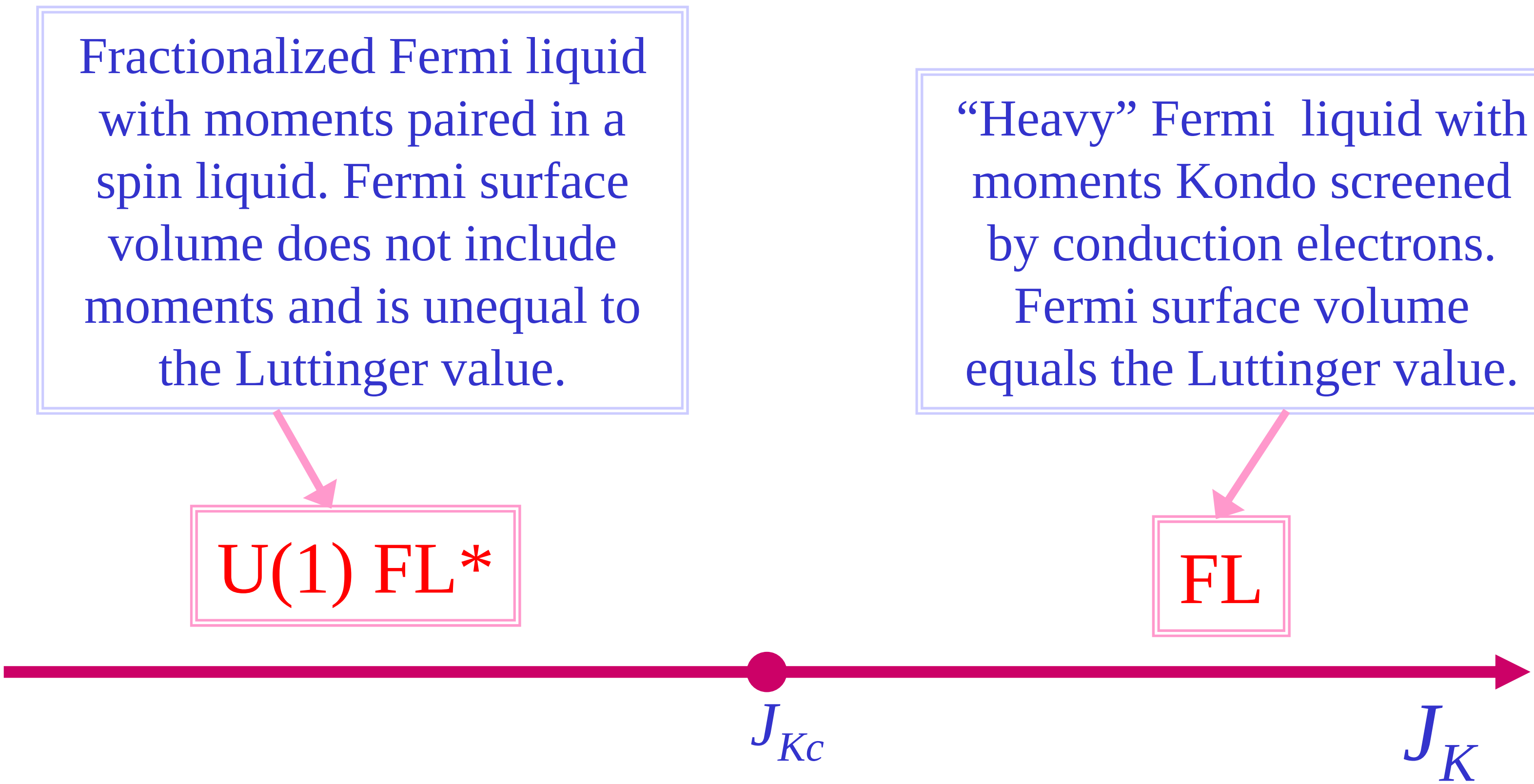


Phase diagram

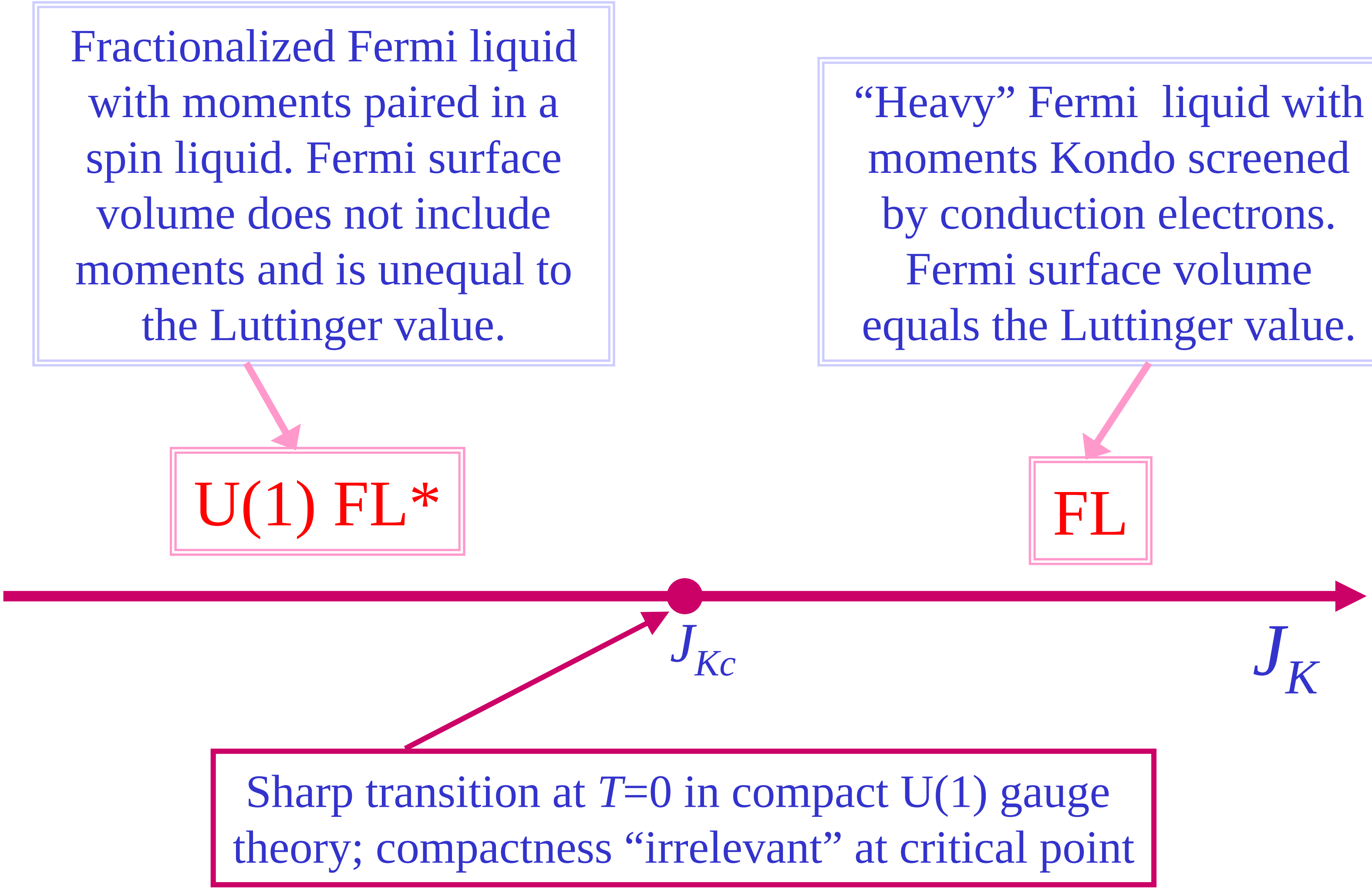
Fractionalized Fermi liquid with moments paired in a spin liquid. Fermi surface volume does not include moments and is unequal to the Luttinger value.



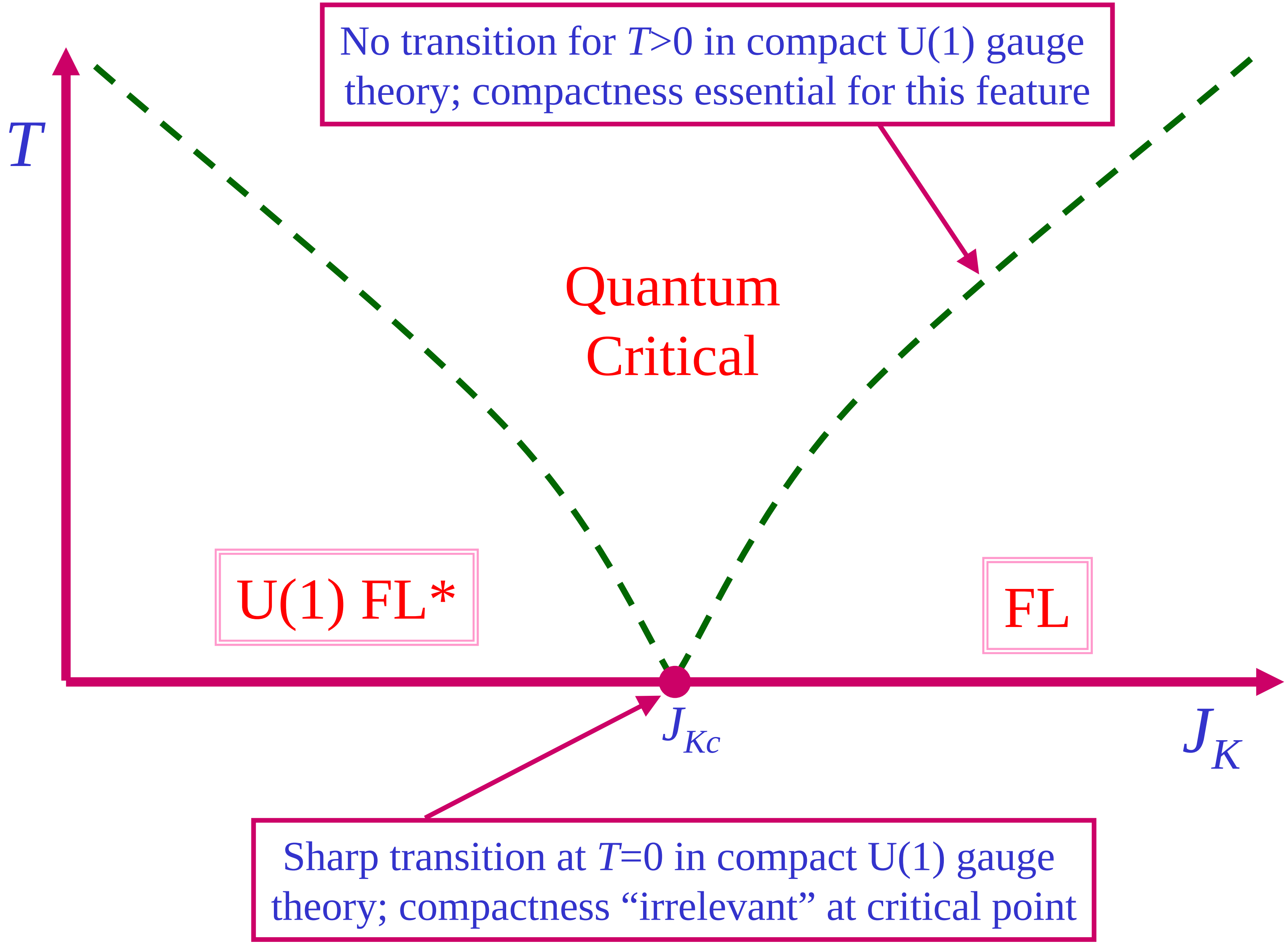
Phase diagram



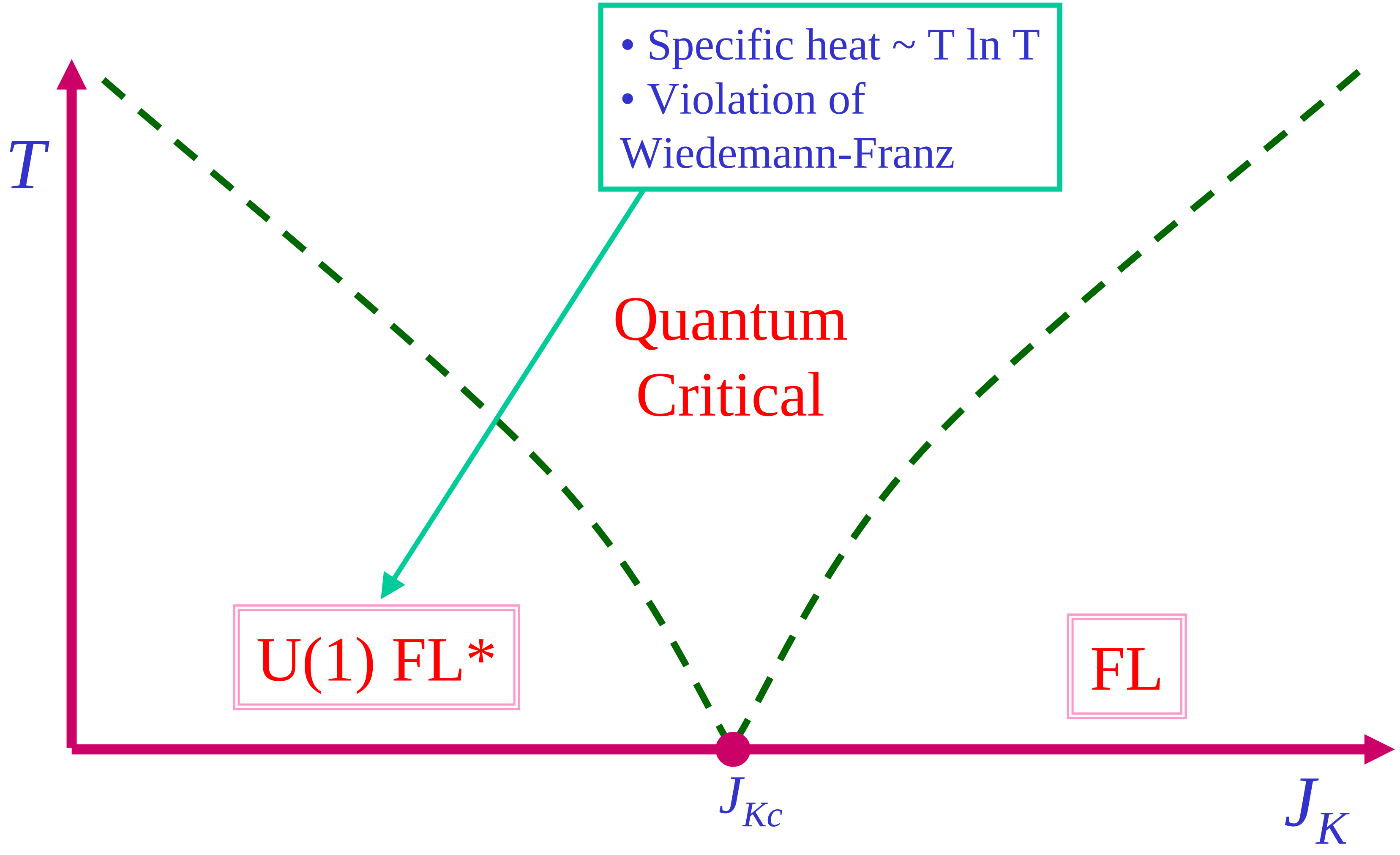
Phase diagram



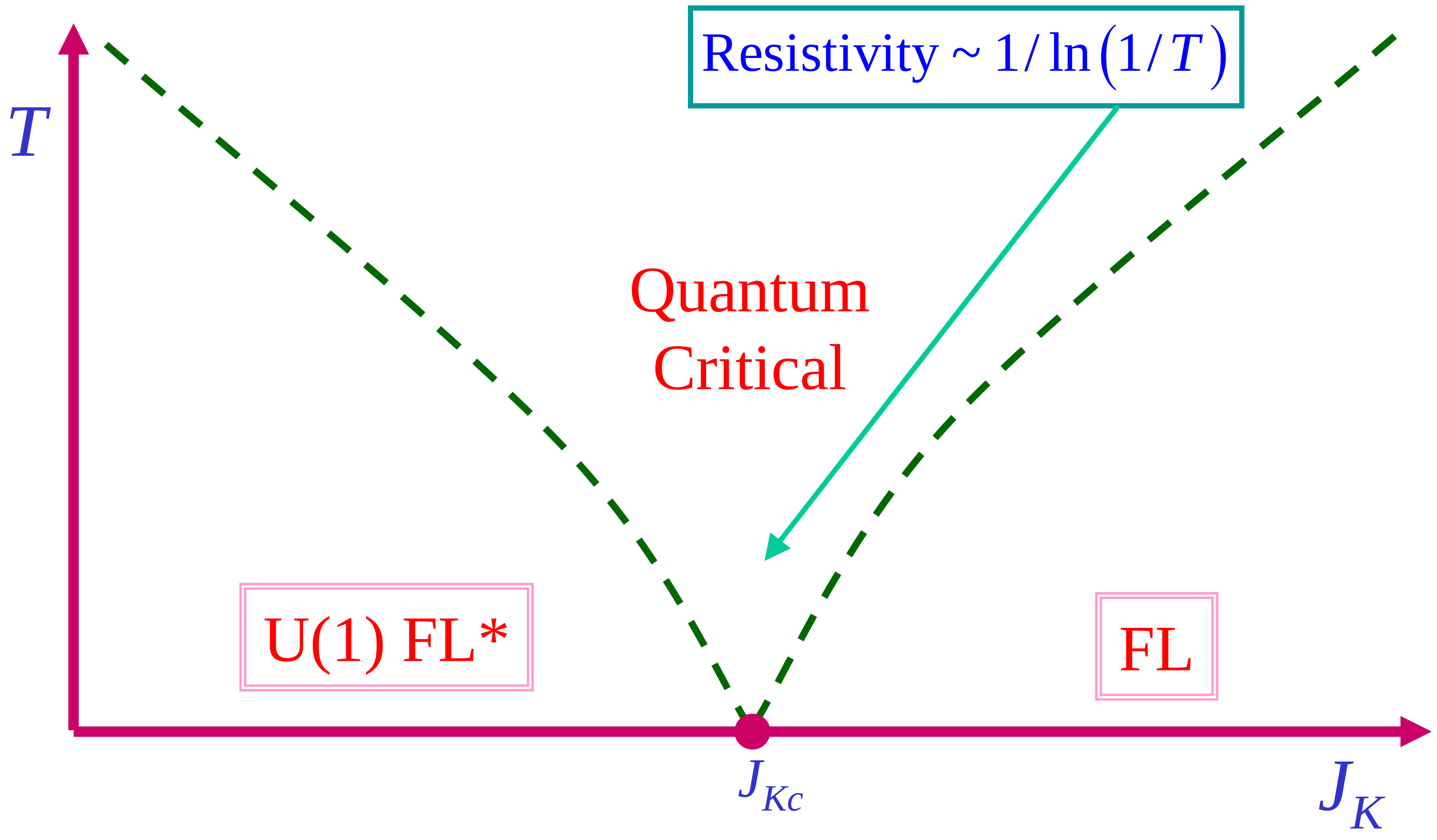
Phase diagram



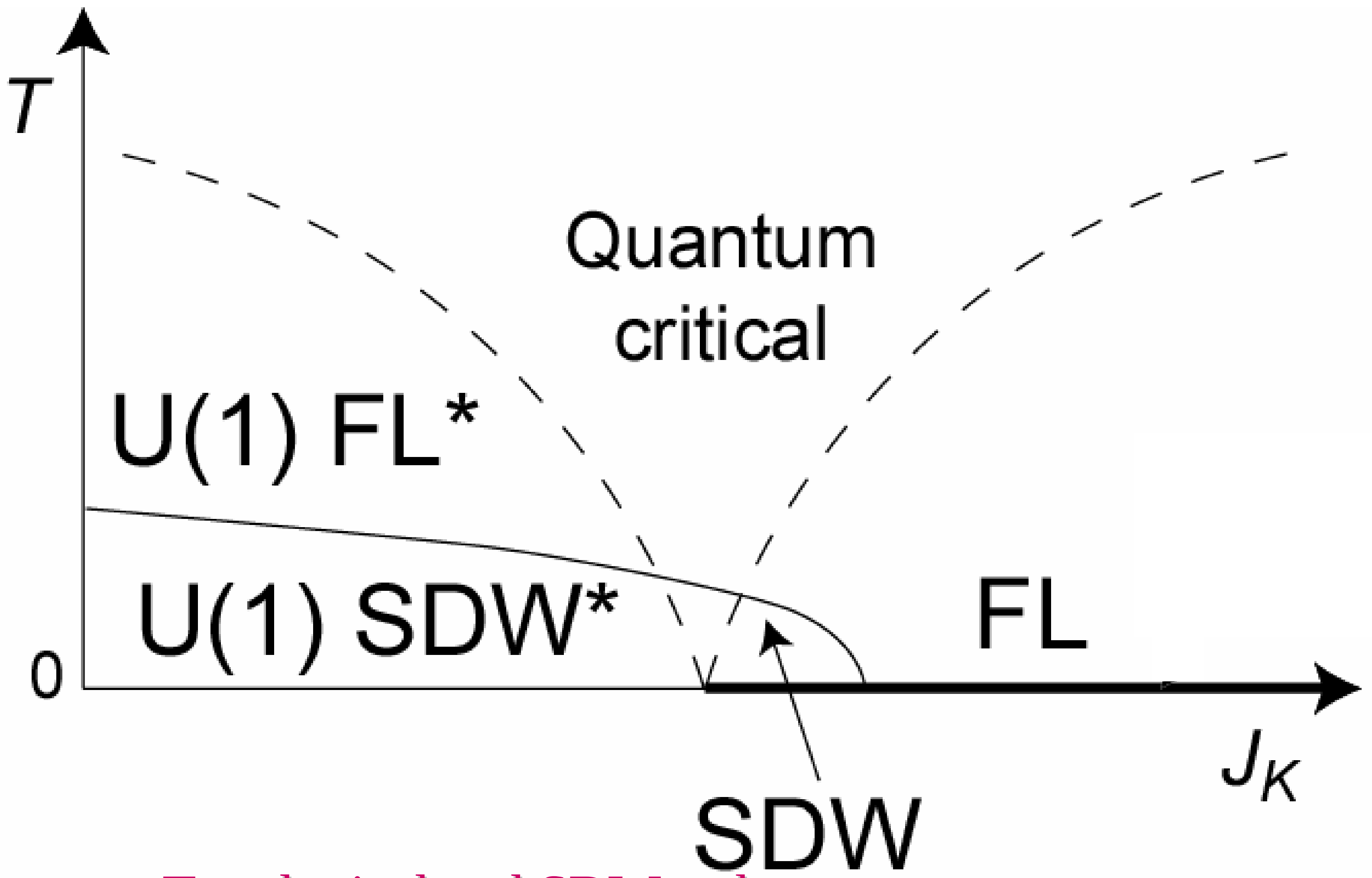
Phase diagram



Phase diagram



Phase diagram
(after allowing for conventional magnetic order)

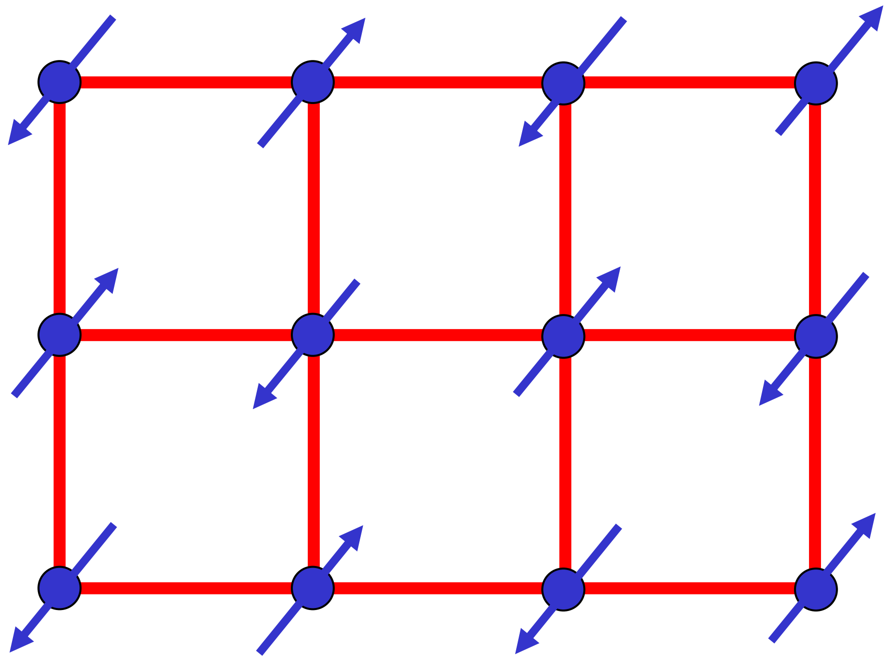


Topological and SDW order parameters suggest two separate quantum critical points

(D) Multiple order parameters

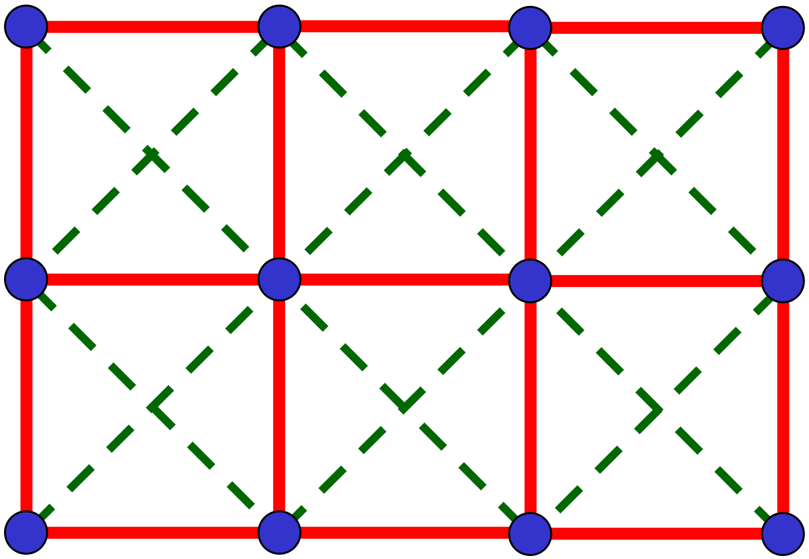
Berry phases and the breakdown of the LGW paradigm

Work in the regime with small J_K, and consider destruction of magnetic order by frustrating (RKKY) exchange interactions between f moments



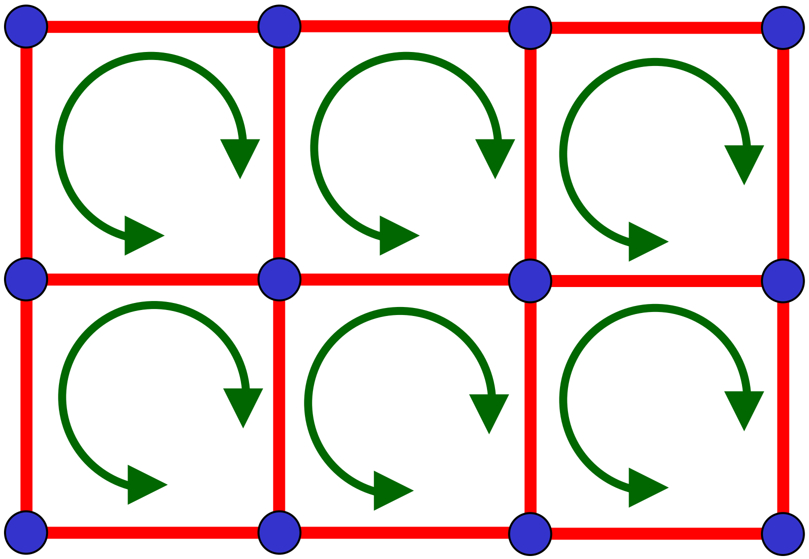
Ground state has Neel order with $\vec{\varphi} \neq 0$

Work in the regime with small J_K, and consider destruction of magnetic order by frustrating (RKKY) exchange interactions between f moments



Destroy SDW order by perturbations which preserve full square lattice symmetry *e.g.* second-neighbor or ring exchange.

Work in the regime with small J_K, and consider destruction of magnetic order by frustrating (RKKY) exchange interactions between *f* moments



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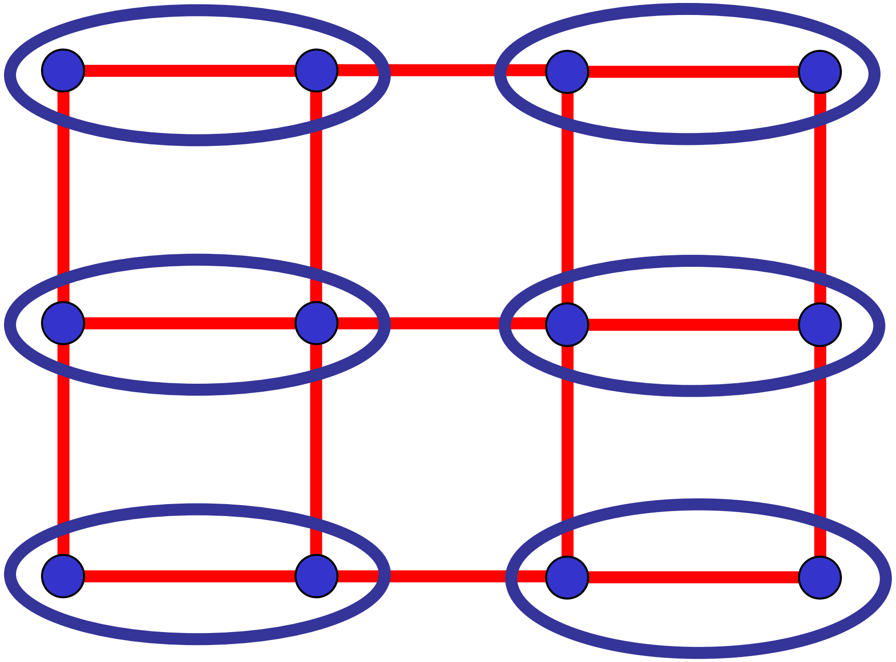
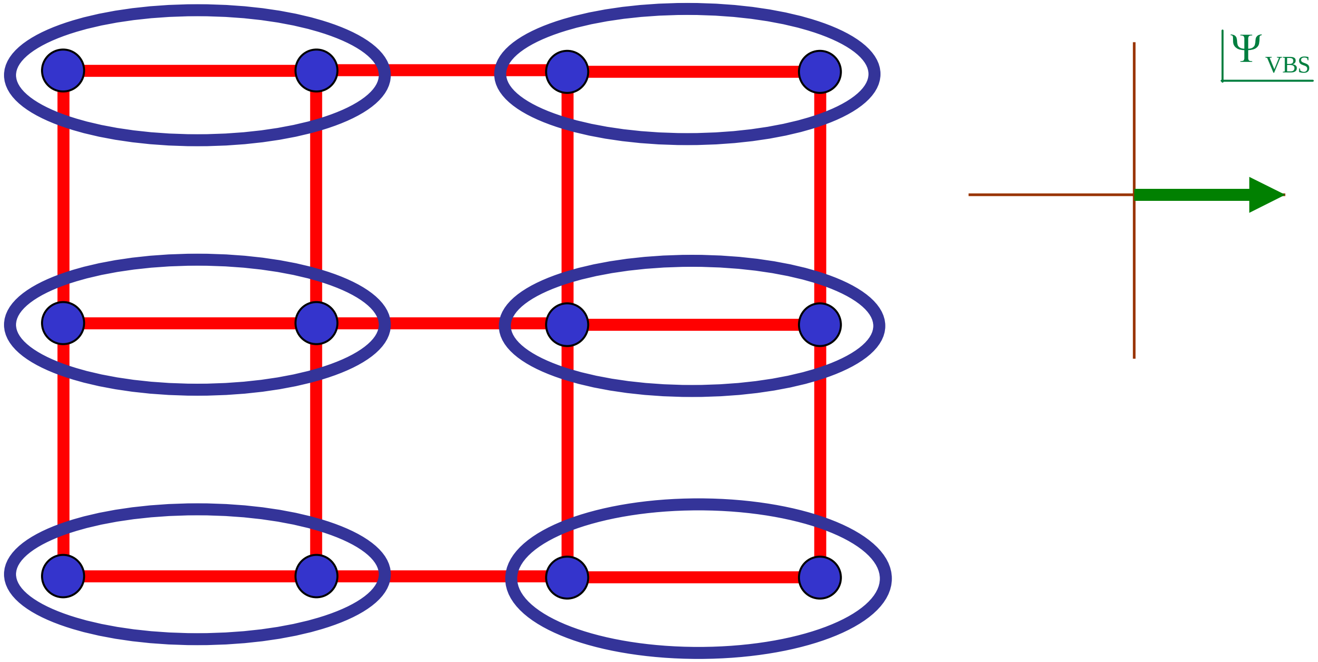


Possible paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Work in the regime with small J_K, and consider destruction of magnetic order by frustrating (RKKY) exchange interactions between f moments

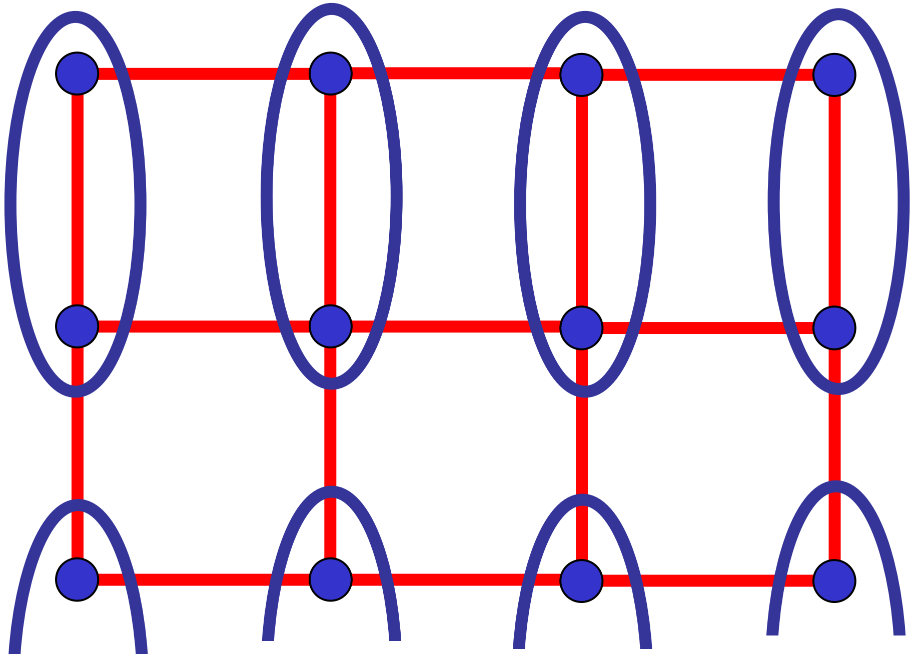


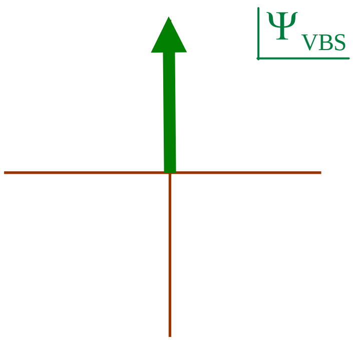
Possible paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Such a state breaks lattice symmetry and has $\langle \Psi_{\text{VBS}} \rangle \neq 0$,

where Ψ_{VBS} is the ***valence bond solid (VBS) order parameter***

Work in the regime with small J_K, and consider destruction of magnetic order by frustrating (RKKY) exchange interactions between f moments





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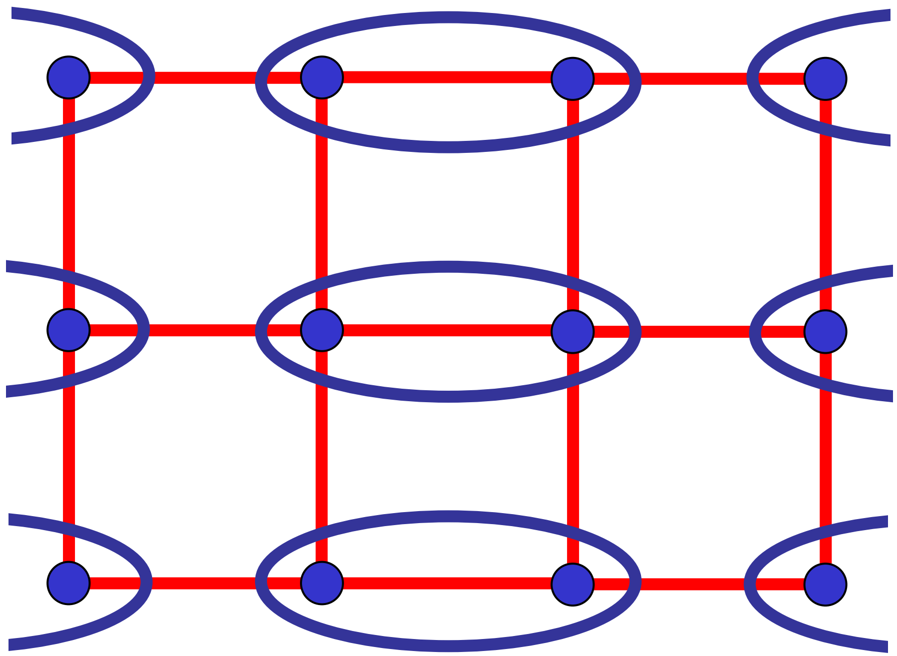

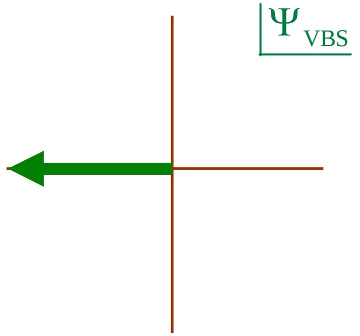

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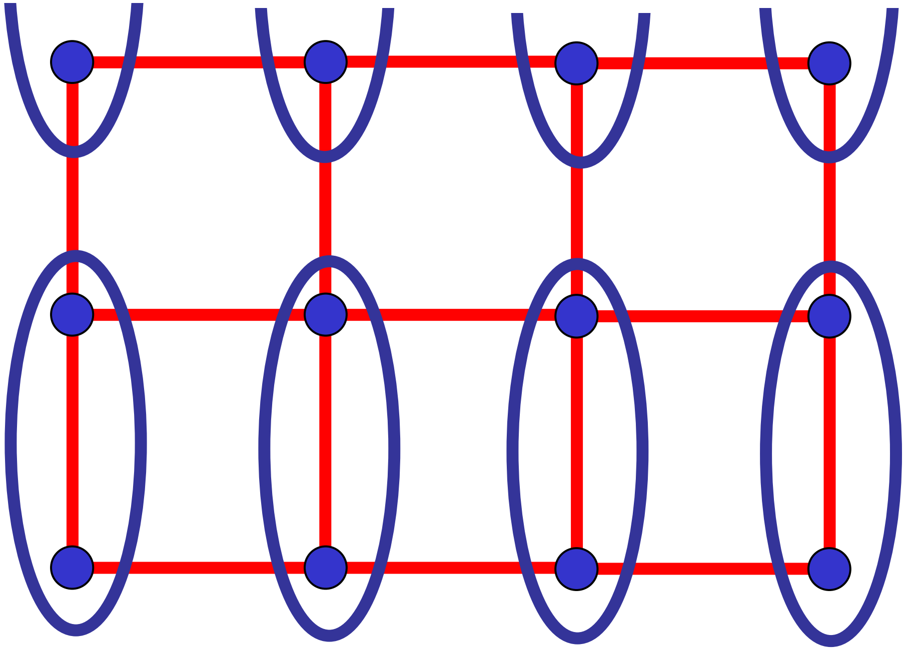

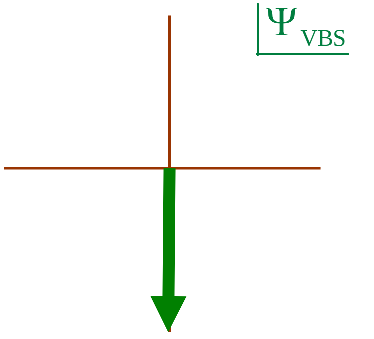

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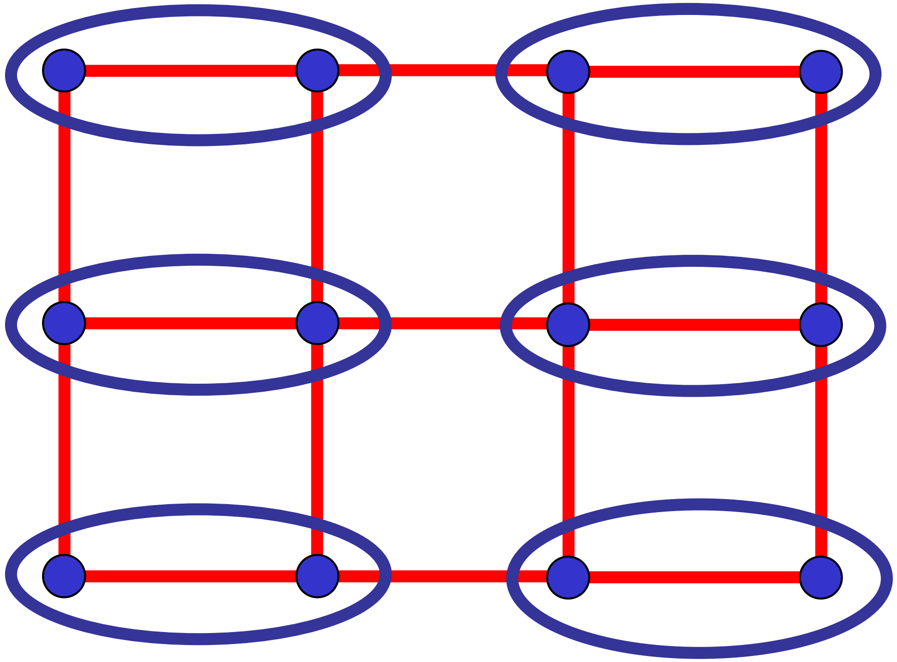

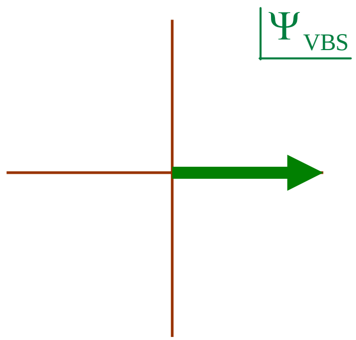

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).

Possible paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Such a state breaks lattice symmetry and has $\langle \Psi_{VBS} \rangle \neq 0$, where Ψ_{VBS} is the ***valence bond solid (VBS) order parameter***

VBS order (and confinement) appear for collinear spins in *d=2*

Naïve approach: add VBS order parameter to LGW theory "by hand"

Neel order

$\langle \vec{\varphi} \rangle \neq 0, \langle \Psi_{\text{VBS}} \rangle = 0$

First order transition

VBS order

$\langle \vec{\varphi} \rangle = 0, \langle \Psi_{\text{VBS}} \rangle \neq 0$

g

Neel order

$\langle \vec{\varphi} \rangle \neq 0, \langle \Psi_{\text{VBS}} \rangle = 0$

Coexistence

$\langle \vec{\varphi} \rangle \neq 0,$

$\langle \Psi_{\text{VBS}} \rangle \neq 0$

VBS order

$\langle \vec{\varphi} \rangle = 0, \langle \Psi_{\text{VBS}} \rangle \neq 0$

g

Neel order

$\langle \vec{\varphi} \rangle \neq 0, \langle \Psi_{\text{VBS}} \rangle = 0$

"*Disordered*"

($\neq$ topologically ordered)

$\langle \vec{\varphi} \rangle = 0, \; \langle \Psi_{\text{VBS}} \rangle = 0$

VBS order

$\langle \vec{\varphi} \rangle = 0, \langle \Psi_{\text{VBS}} \rangle \neq 0$

g

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First order transition

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g

Neel order

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VBS order

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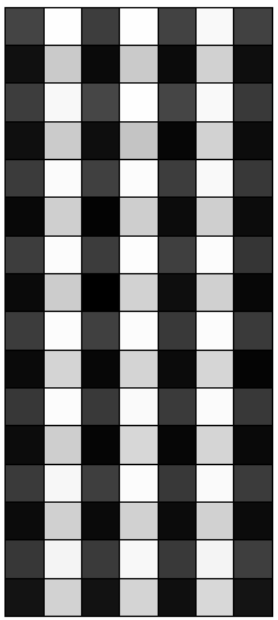
g

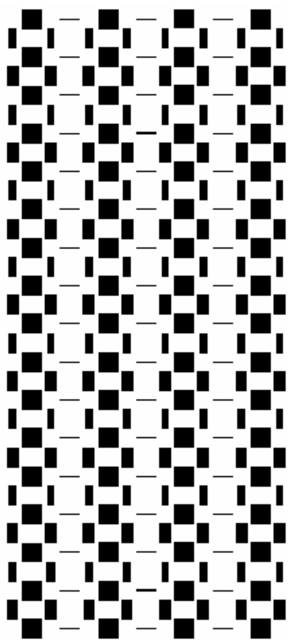
Superfluid-insulator transition of hard core bosons at *f*=1/2 (Neel-valence bond solid transition of S=1/2 AFM)

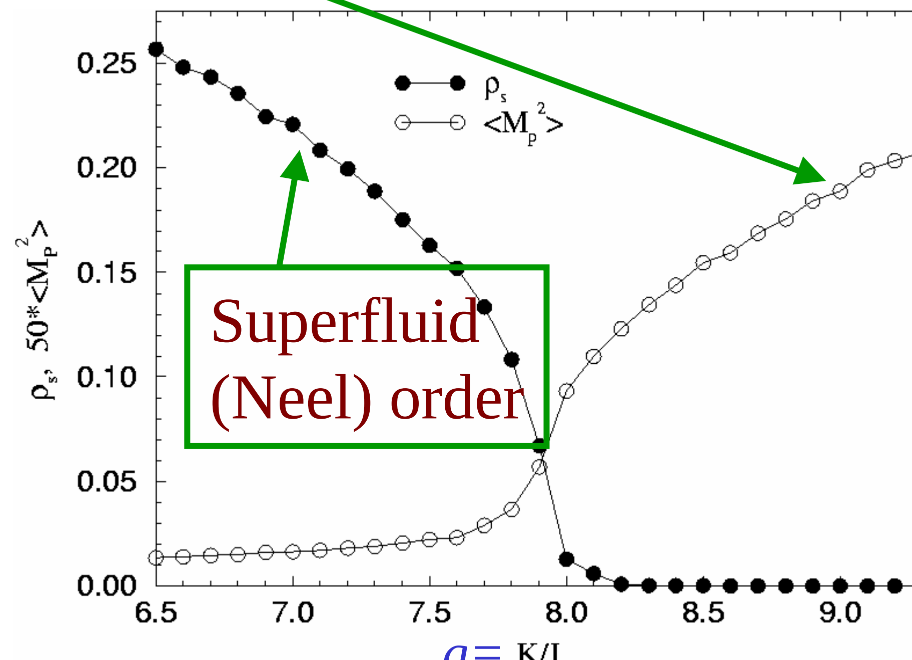
A. W. Sandvik, S. Daul, R. R. P. Singh, and D. J. Scalapino, *Phys. Rev. Lett.* **89**, 247201 (2002)

Large scale (> 8000 sites) numerical study of the destruction of superfluid (i.e. magnetic Neel) order at half filling with full square lattice symmetry

VBS insulator

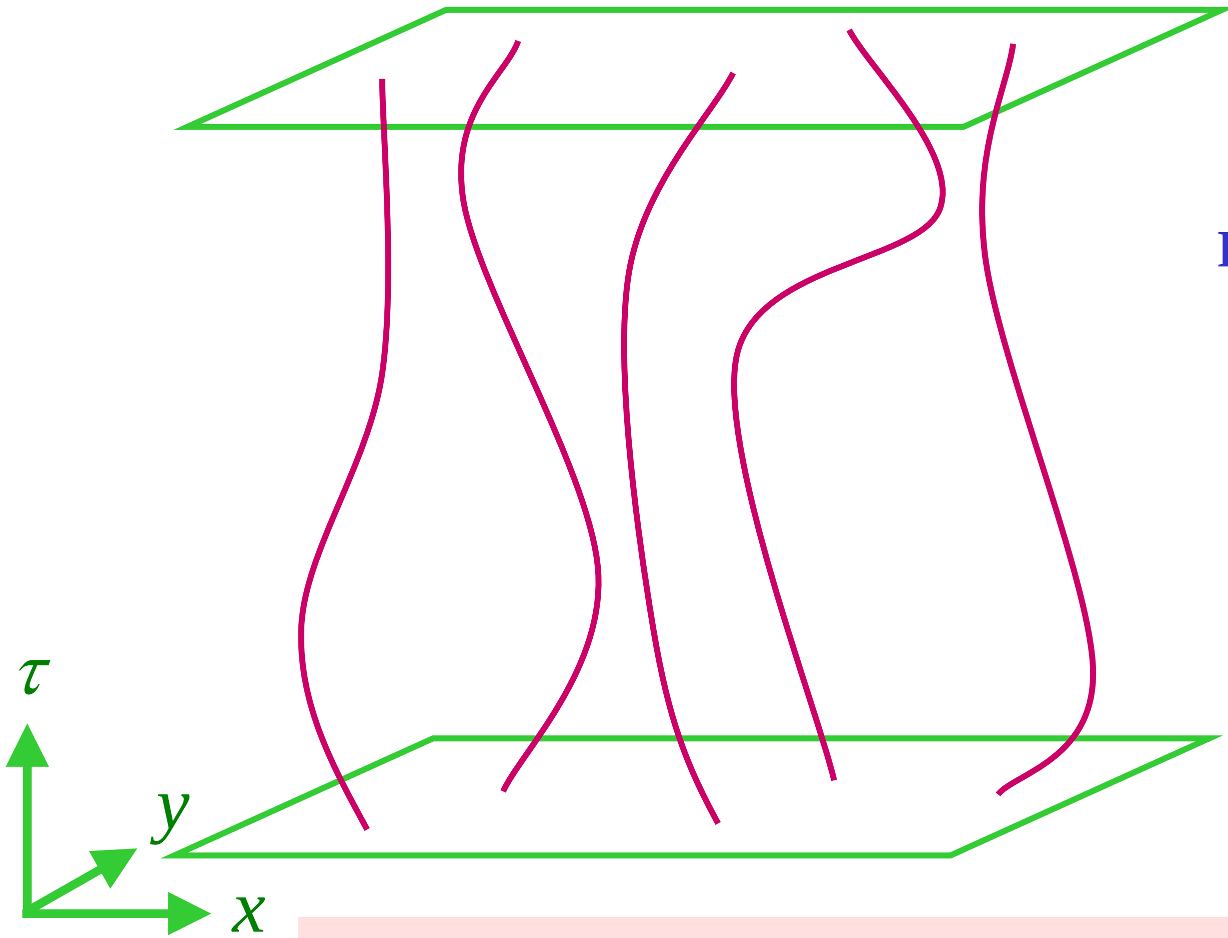






$$H = J\sum_{\langle ij\rangle}\left(S_i^+S_j^- + S_i^-S_j^+\right) - K\sum_{\langle ijkl\rangle\subset\square}\left(S_i^+S_j^-S_k^+S_l^- + S_i^-S_j^+S_k^-S_l^+\right)$$

Boson-vortex duality

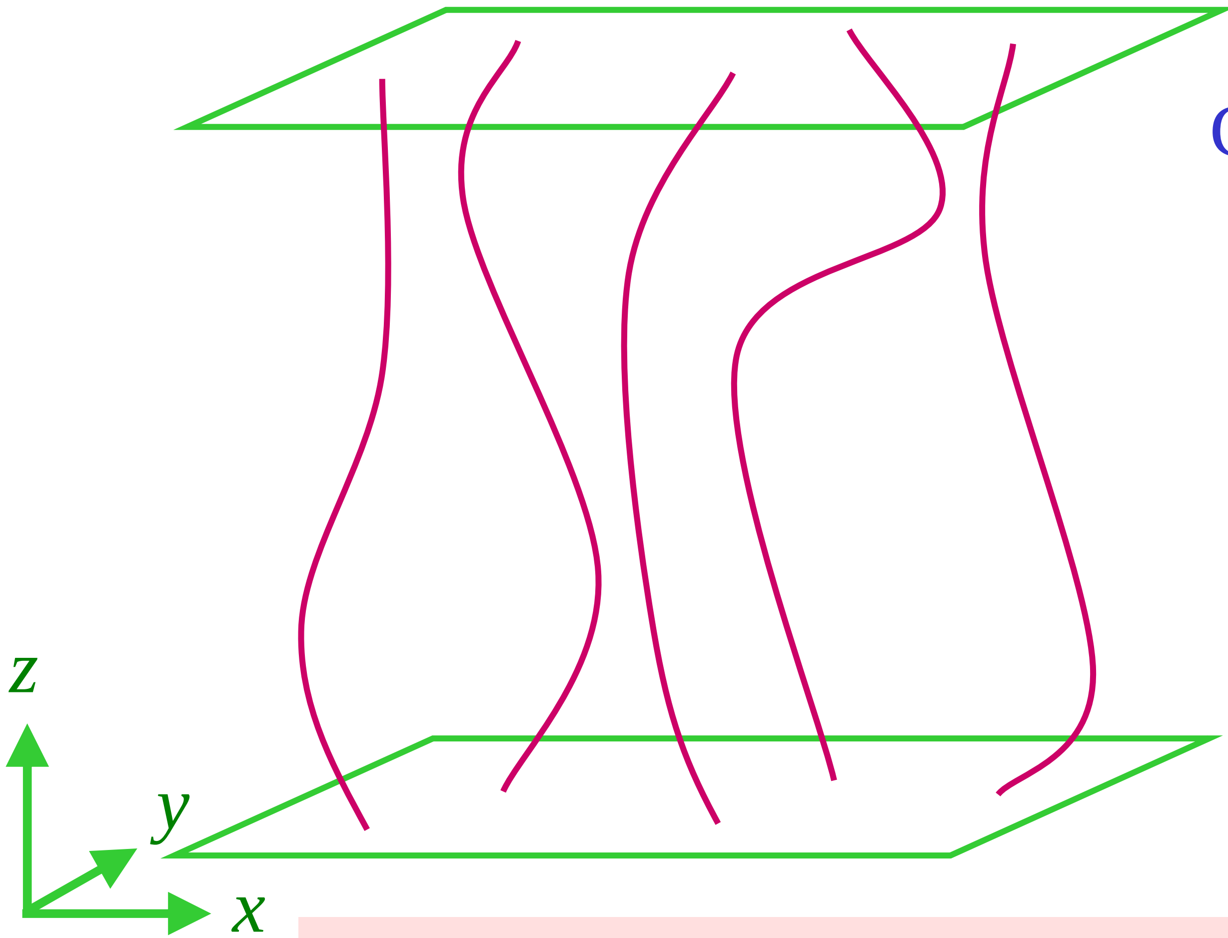


Quantum mechanics of two-dimensional bosons: world lines of bosons in spacetime

Express theory of S=1/2 AFM as a theory of S_z= -1 "spin down" bosons at filling f=1/2

C. Dasgupta and B.I. Halperin, *Phys. Rev. Lett.* **47**, 1556 (1981); D.R. Nelson, *Phys. Rev. Lett.* **60**, 1973 (1988); M.P.A. Fisher and D.-H. Lee, *Phys. Rev.* B **39**, 2756 (1989);

Boson-vortex duality



Classical statistical mechanics of a "dual" three-dimensional superconductor: vortices in a "magnetic" field

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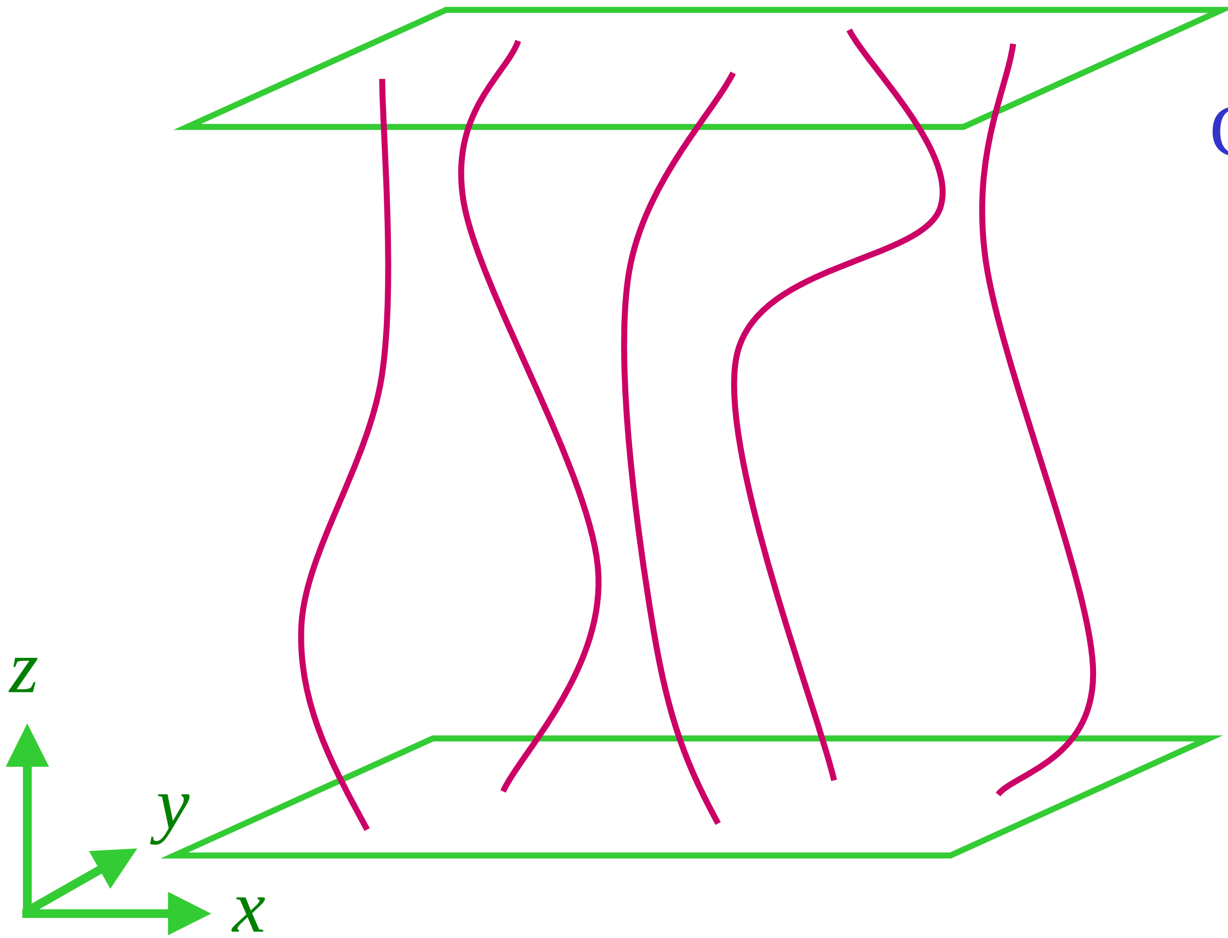
Boson-vortex duality



Classical statistical mechanics of a "dual" three-dimensional superconductor: vortices in a "magnetic" field

Strength of "magnetic" field = density of bosons
= f flux quanta per plaquette

C. Dasgupta and B.I. Halperin, *Phys. Rev. Lett.* **47**, 1556 (1981); D.R. Nelson, *Phys. Rev. Lett.* **60**, 1973 (1988); M.P.A. Fisher and D.-H. Lee, *Phys. Rev.* B **39**, 2756 (1989);

Boson-vortex duality

Statistical mechanics of dual superconductor is invariant under the square lattice space group:

T_x, T_y : Translations by a lattice spacing in the x, y directions

R : Rotation by 90 degrees.

Magnetic space group:

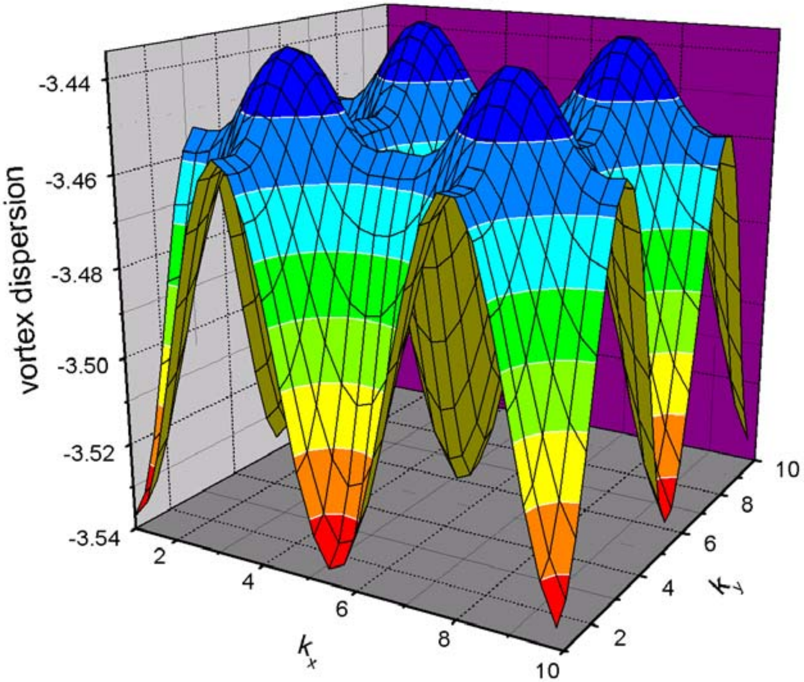
$$T_x T_y = e^{2\pi i f} T_y T_x \quad ;$$

$$R^{-1} T_y R = T_x \quad ; \quad R^{-1} T_x R = T_y^{-1} \quad ; \quad R^4 = 1$$

Strength of "magnetic" field = density of bosons
= f flux quanta per plaquette

Boson-vortex duality

Hofstäder spectrum of dual “superconducting” order



At density $f = p/q$ (p, q relatively prime integers) there are q species of vortices, φ_ℓ (with $\ell = 1 \ldots q$), associated with q gauge-equivalent regions of the Brillouin zone

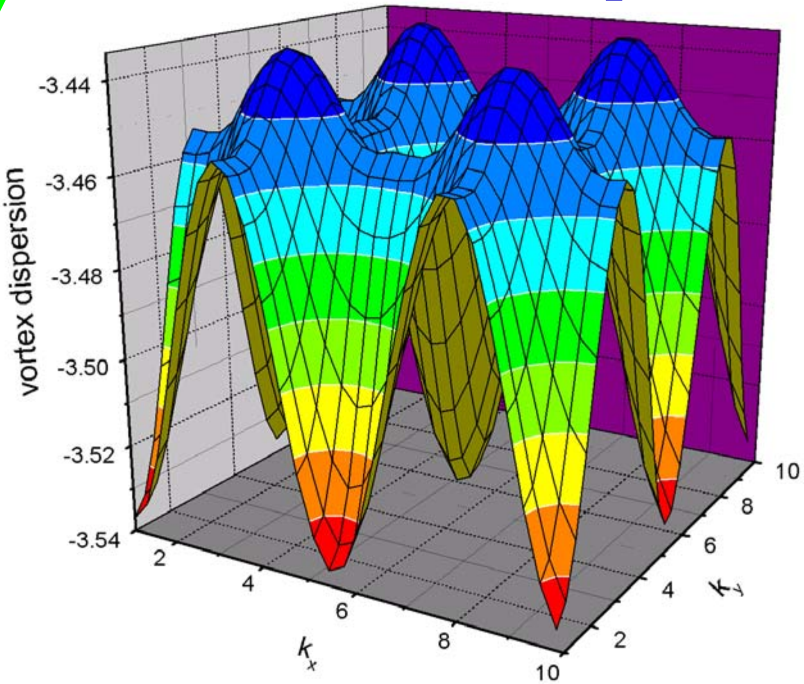
Magnetic space group:

$$T_x T_y = e^{2\pi i f} T_y T_x \quad ;$$

$$R^{-1} T_y R = T_x \quad ; \quad R^{-1} T_x R = T_y^{-1} \quad ; \quad R^4 = 1$$

Boson-vortex duality

Hofstäder spectrum of dual "superconducting" order



At density $f = p/q$ (p, q relatively prime integers) there are q species of vortices, φ_ℓ (with $\ell = 1 \ldots q$), associated with q gauge-equivalent regions of the Brillouin zone

The q vortices form a *projective* representation of the space group

$$T_x : \varphi_\ell \to \varphi_{\ell+1} \quad ; \quad T_y : \varphi_\ell \to e^{2\pi i \ell f} \varphi_\ell$$

$$R : \varphi_\ell \to \frac{1}{\sqrt{q}} \sum_{m=1}^{q} \varphi_m e^{2\pi i \ell m f}$$

See also X.-G. Wen, *Phys. Rev.* B **65**, 165113 (2002)

Boson-vortex duality

The φ_ℓ fields characterize *both* Neel (superfluid) and VBS order

Neel (superfluid) order: $\langle \varphi_\ell \rangle = 0$

VBS (charge) order:

Status of space group symmetry determined by density operators $\rho_{\mathbf{Q}}$ at wavevectors $\mathbf{Q}_{mn} = \frac{2\pi p}{q}(m,n)$

$$\rho_{mn} = e^{i\pi mnf} \sum_{\ell=1}^{q} \varphi_\ell^* \varphi_{\ell+n} e^{2\pi i \ell mf}$$

$$T_x : \rho_{\mathbf{Q}} \to \rho_{\mathbf{Q}} e^{i\mathbf{Q}\cdot\hat{x}} \quad ; \qquad T_y : \rho_{\mathbf{Q}} \to \rho_{\mathbf{Q}} e^{i\mathbf{Q}\cdot\hat{y}}$$

$$R : \rho(\mathbf{Q}) \to \rho(R\mathbf{Q})$$

Naïve approach: add VBS order parameter to LGW theory "by hand"

Neel order

$\langle \vec{\varphi} \rangle \neq 0, \langle \Psi_{\mathrm{VBS}} \rangle = 0$

First order transition

VBS order

$\langle \vec{\varphi} \rangle = 0, \langle \Psi_{\mathrm{VBS}} \rangle \neq 0$

g

Neel order

$\langle \vec{\varphi} \rangle \neq 0, \langle \Psi_{\mathrm{VBS}} \rangle = 0$

Coexistence

$\langle \vec{\varphi} \rangle \neq 0,$

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VBS order

$\langle \vec{\varphi} \rangle = 0, \langle \Psi_{\mathrm{VBS}} \rangle \neq 0$

g

Predictions of extended "LGW" theory with projective symmetry

Neel order

$$\langle \vec{\varphi} \rangle \neq 0, \langle \Psi_{\text{VBS}} \rangle = 0$$

First order transition

VBS order

$$\langle \vec{\varphi} \rangle = 0, \langle \Psi_{\text{VBS}} \rangle \neq 0$$

g

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$$\langle \vec{\varphi} \rangle \neq 0,$$

$$\langle \Psi_{\text{VBS}} \rangle \neq 0$$

VBS order

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Neel order

$$\langle \vec{\varphi} \rangle \neq 0, \langle \Psi_{\text{VBS}} \rangle = 0$$

Second order transition

VBS order

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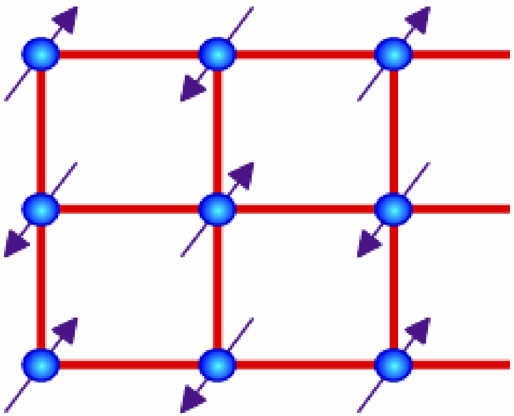
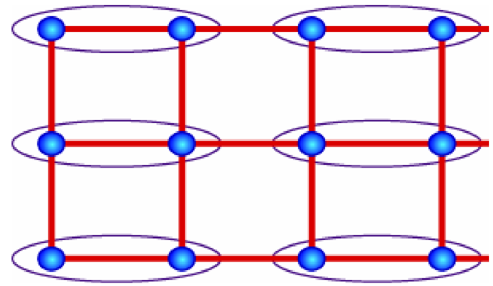
g

Phase diagram of S=1/2 square lattice antiferromagnet

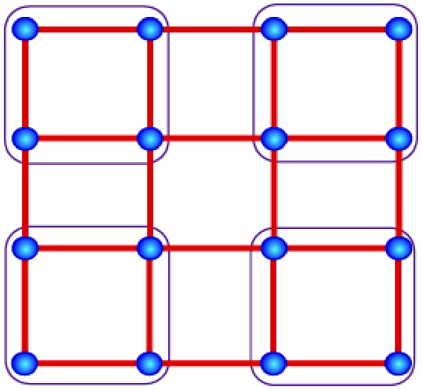


Neel order

$\vec{\varphi} \sim z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta \neq 0$



or



VBS order $\Psi_{\text{VBS}} \neq 0$,

$S = 1/2$ spinons z_α confined,

$S = 1$ triplon excitations

g

Second-order critical point described by a theory of deconfined spinons

$$\mathcal{S}_{\text{critical}} = \int d^2x d\tau \left[\left|(\partial_\mu - iA_\mu) z_\alpha\right|^2 + r \left|z_\alpha\right|^2 + \frac{u}{2}\left(\left|z_\alpha\right|^2\right)^2 + \frac{1}{4e^2}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 \right]$$

where the U(1) gauge field A_μ is *non-compact*. Deconfinement also happens in critical *phases* (Hermele *et al.* cond-mat/0404751) and monopole screening arguments (Herbut, Sachdev *et al.*) fail at critical points/phases.

Main lesson:

Novel second-order quantum critical point between phases with conventional order parameters.

A direct second-order transition between such phases is forbidden by symmetry in LGW theory.

Key question for metallic systems:

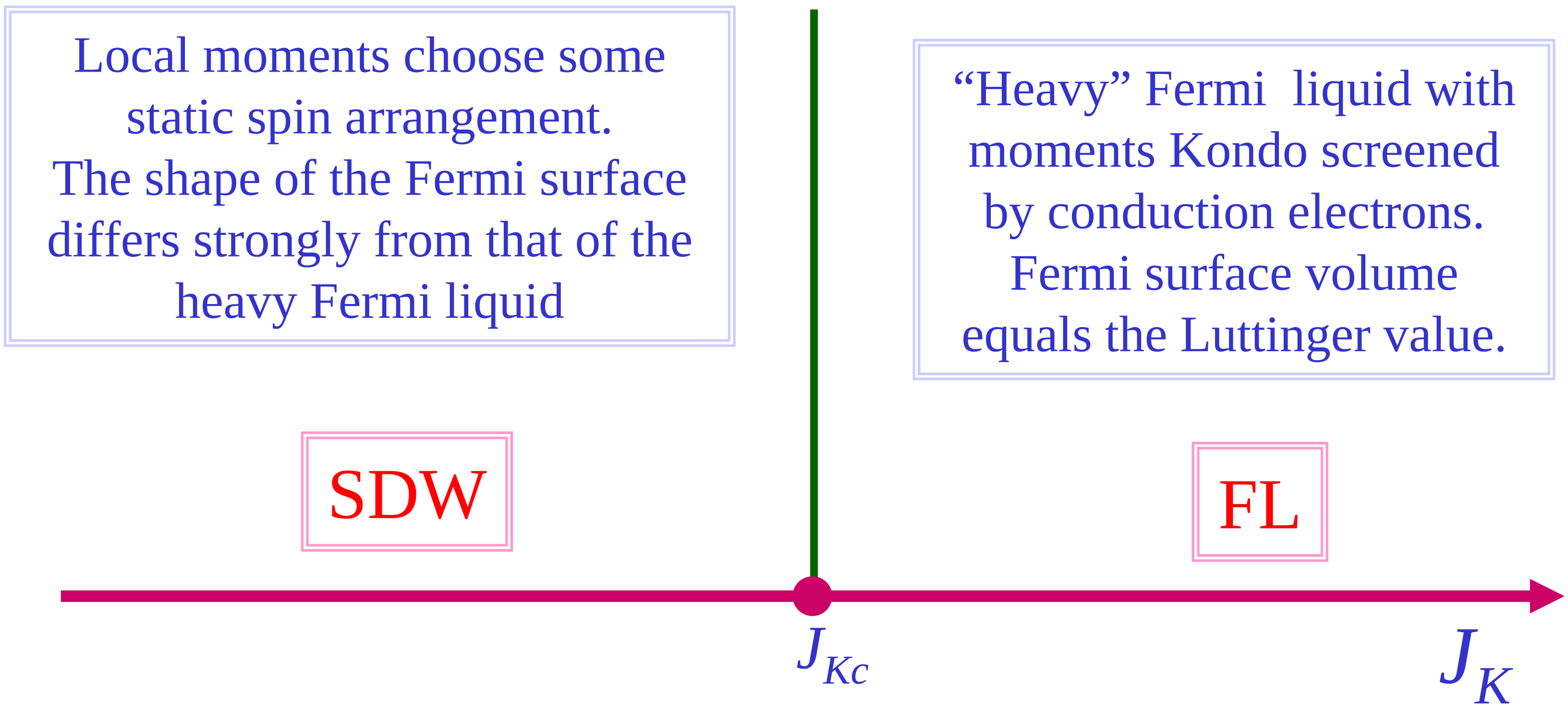
Is a direct second-order quantum critical point possible between metallic states distinguished by two conventional order parameters:

(A) SDW order

(B) Shape of Fermi surface ?

Such a transition is obtained if the FL phase is unstable to confinement to a SDW state at low energies.*

Phase diagram for the Kondo lattice ?



See also Q. Si, S. Rabello, K. Ingersent, and J. L. Smith, *Nature* 413, 804 (2001);
S. Paschen, T. Luehmann, C. Langhammer, O. Trovarelli, S. Wirth, C. Geibel, F. Steglich,
Acta Physica Polonica B **34**, 359 (2003).

Conclusions

Two possible routes to "exotic" quantum criticality

I. New FL* phase with a Fermi surface of electron-like quasiparticles (whose volume violates the Luttinger theorem), topological order, emergent gauge excitations, and neutral fractionalized quasiparticles.

Novel Fermi-volume-changing quantum criticality in the transition between the FL and FL* phases (and associated SDW and SDW* phases).

Conclusions

Two possible routes to "exotic" quantum criticality

II. Conventional FL and SDW phases (but with very different shapes of Fermi surfaces) undergo a direct quantum phase transition.

Analogous quantum critical point found in a direct transition between Neel and VBS states in *S*=1/2 Mott insulators in two dimensions. Mapping to this scenario to metals we obtain the above scenario if the FL* phase is unstable to confinement to a SDW phase.