Understanding correlated electron systems by a classification of Mott insulators

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Annals of Physics **303**, 226 (2003)

Talk online at http://pantheon.yale.edu/~subir

Strategy for analyzing correlated electron systems

(cuprate superconductors, heavy fermion compounds ……..)

Start from the point where the break down of the Bloch theory of metals is complete---the Mott insulator.

Classify ground states of Mott insulators using conventional and topological order parameters.

Correlated electron systems are described by phases and quantum phase transitions associated with order parameters of Mott insulator and the "orders" of Landau/BCS theory. Expansion away from quantum critical points allows description of states in which the order of Mott insulator is "fluctuating".

Outline

I. Order in Mott insulators

Class A: Compact U(1) gauge theory: collinear spins, bond order and confined spinons in d=2

Class B: Z_2 gauge theory: non-collinear spins, visons, topological order, and deconfined spinons

II. Class A in d=2

The cuprates

III. Class A in d=3

Deconfined spinons and quantum criticality in heavy fermion compounds

IV. Conclusions

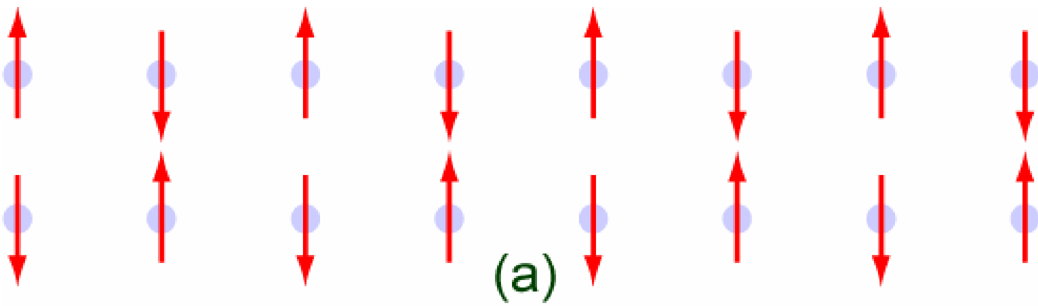
<u>Class A</u>:

Compact U(1) gauge theory: collinear spins, bond order and confined spinons in d=2

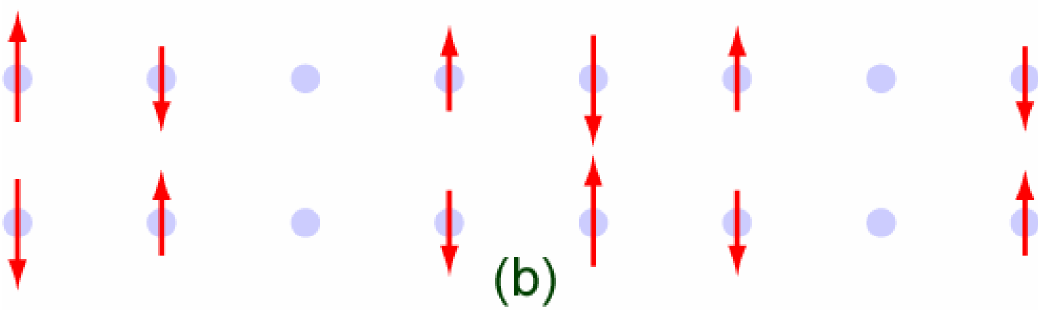
I. Order in Mott insulators

Magnetic order $\langle \boldsymbol{S}_j \rangle = \boldsymbol{N}_1 \cos\left(\vec{K} \cdot \vec{r}_j\right) + \boldsymbol{N}_2 \sin\left(\vec{K} \cdot \vec{r}_j\right)$

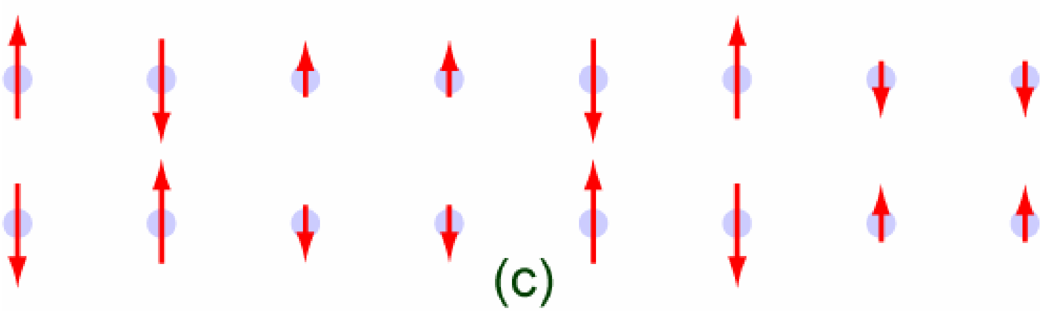
Class A. Collinear spins



$$\vec{K} = (\pi, \pi) \;\; ; \;\; \boldsymbol{N}_2 = 0$$



$$\vec{K} = (3\pi/4, \pi) \;\; ; \;\; \boldsymbol{N}_2 = 0$$



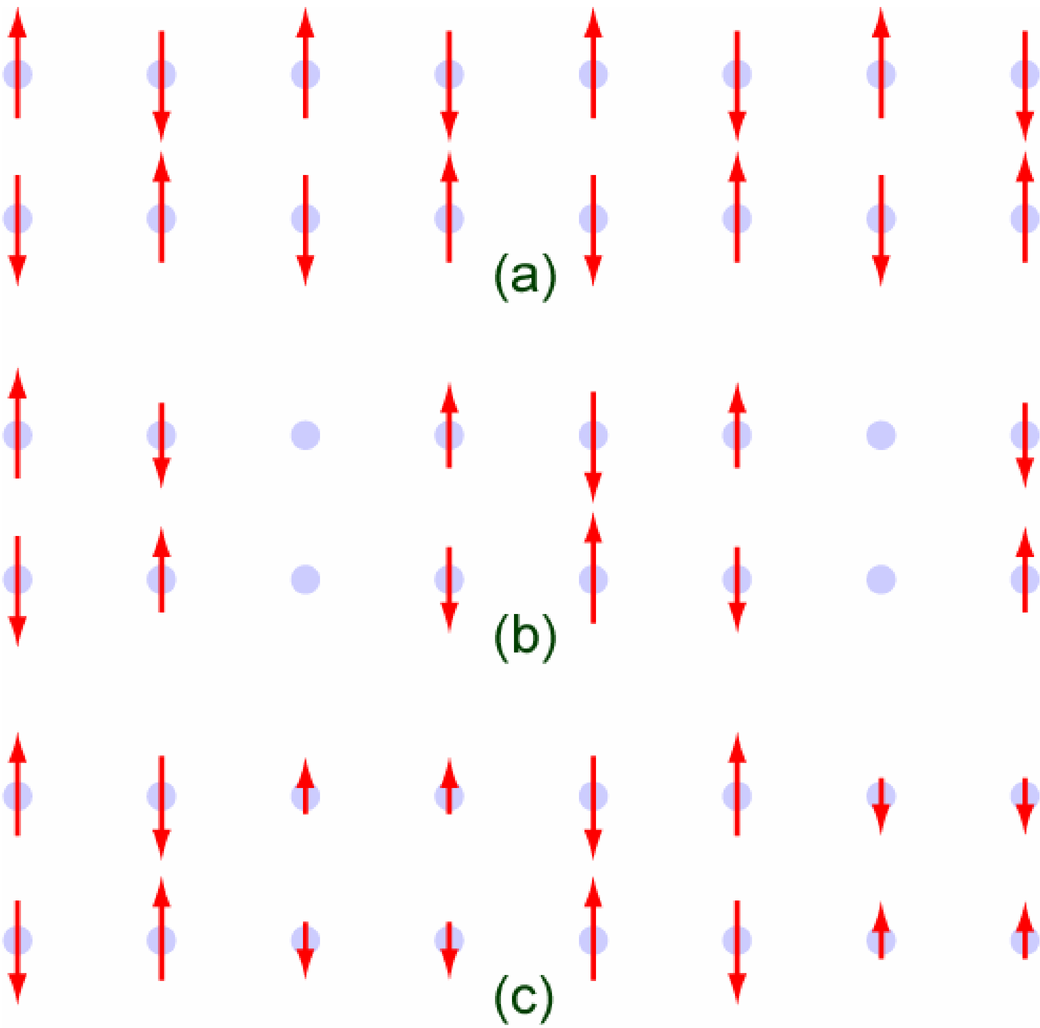
$$\vec{K} = (3\pi/4, \pi) \;\; ;$$

$$\boldsymbol{N}_2 = \left(\sqrt{2} - 1\right) \boldsymbol{N}_1$$

I. Order in Mott insulators

Magnetic order $\left\langle \boldsymbol{S}_j \right\rangle = \boldsymbol{N}_1 \cos\left(\vec{K} \, . \, \vec{r}_j\right) + \boldsymbol{N}_2 \sin\left(\vec{K} \, . \, \vec{r}_j\right)$

Class A. Collinear spins



Key property

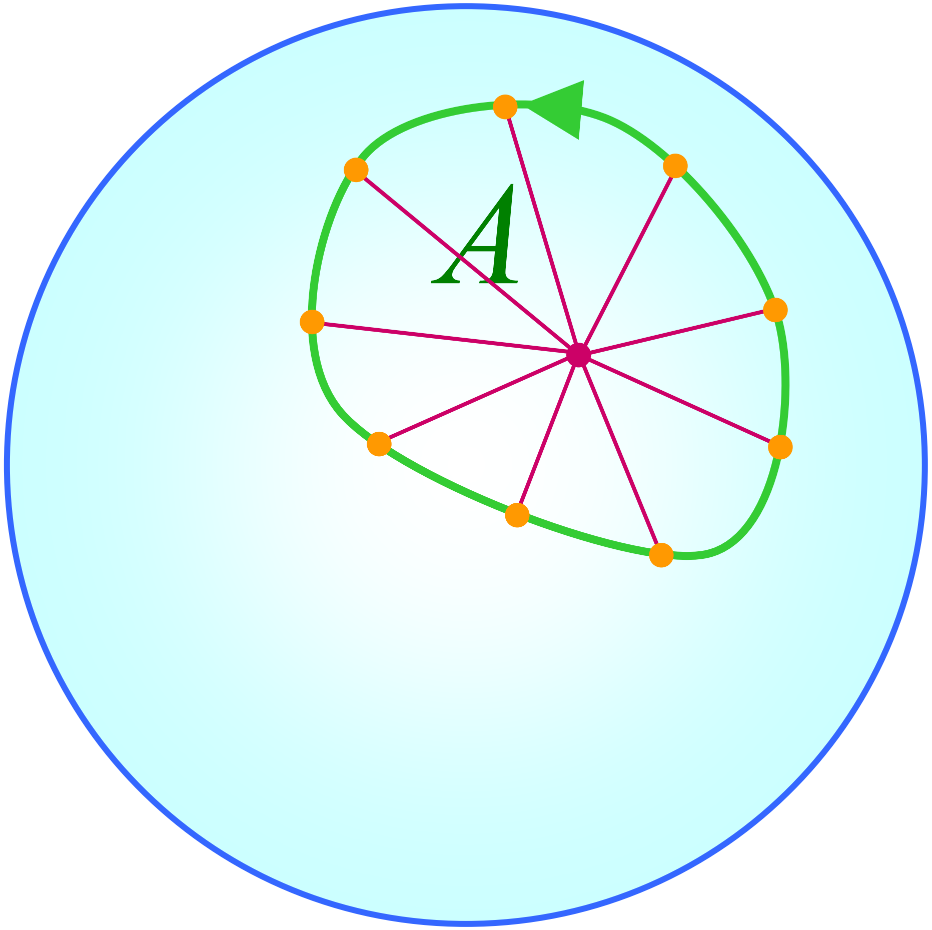
Order specified by a single vector $\boldsymbol{N}$.

Quantum fluctuations leading to loss of magnetic order should produce a paramagnetic state with a vector (S=1) quasiparticle excitation.

Class A: Collinear spins and compact U(1) gauge theory

Write down path integral for quantum spin fluctuations

Key ingredient: Spin Berry Phases



$$e^{iSA}$$

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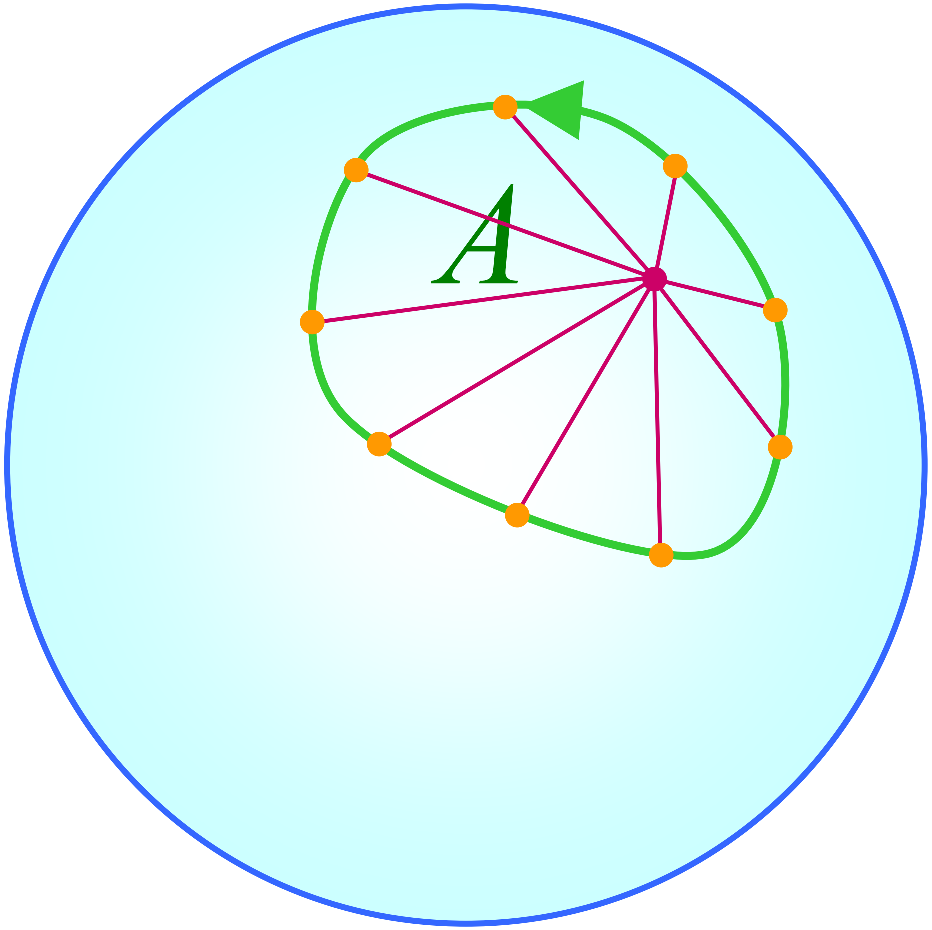
Write down path integral for quantum spin fluctuations

Key ingredient: Spin Berry Phases



$$e^{iSA}$$

Class A: Collinear spins and compact U(1) gauge theory

S=1/2 square lattice antiferromagnet with non-nearest neighbor exchange

$$H = \sum_{i<j} J_{ij} \; \vec{S}_i \cdot \vec{S}_j$$

Include Berry phases after discretizing coherent state path integral on a cubic lattice in spacetime

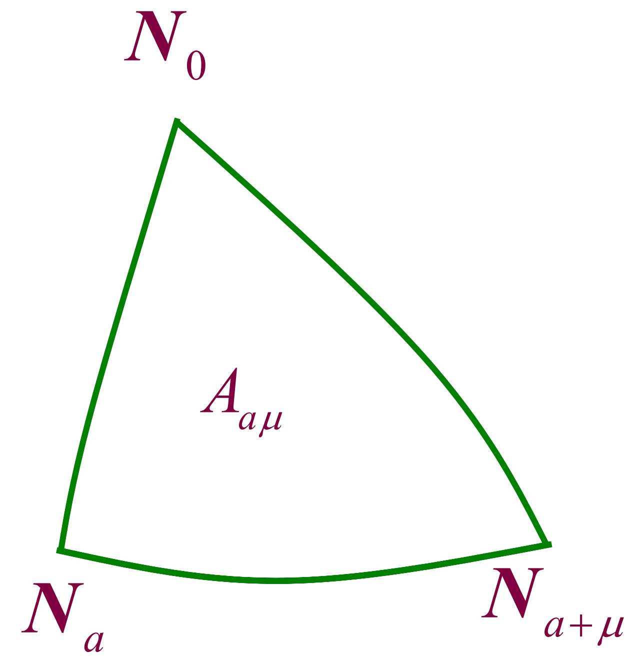
$$Z = \prod_a \int d\boldsymbol{n}_a \delta\left(\boldsymbol{n}_a^2 - 1\right) \exp\left(\frac{1}{g} \sum_{a,\mu} \boldsymbol{N}_a \cdot \boldsymbol{N}_{a+\mu} - \frac{i}{2} \sum_a \eta_a A_{a\tau} \right)$$

$\eta_{\mathrm{a}} \to \pm 1$ on two square sublattices ;

$\boldsymbol{N}_a \sim \eta_a \vec{S}_a \to$ Neel order parameter;

$A_{a\mu} \to$ oriented area of spherical triangle

formed by $\boldsymbol{N}_a$, $\boldsymbol{N}_{a+\mu}$, and an arbitrary reference point $\boldsymbol{N}_0$

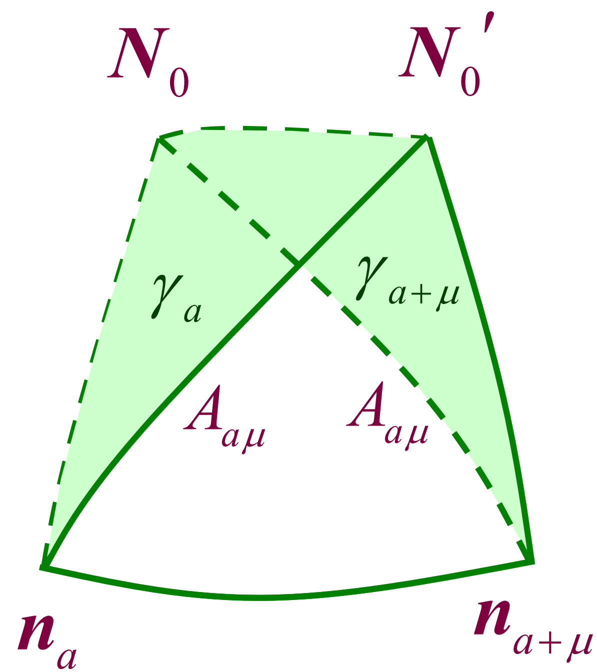


$\boldsymbol{N}_0$
$A_{a\mu}$
$\boldsymbol{N}_a$
$\boldsymbol{N}_{a+\mu}$

Change in choice of $\boldsymbol{n}_0$ is like a "gauge transformation"

$$A_{a\mu} \to A_{a\mu} - \gamma_{a+\mu} + \gamma_a$$

(γ_a is the oriented area of the spherical triangle formed by $\boldsymbol{N}_a$ and the two choices for $\boldsymbol{N}_0$).



The area of the triangle is uncertain modulo 4π, and the action is invariant under

$$A_{a\mu} \to A_{a\mu} + 4\pi$$

These principles strongly constrain the effective action for $A_{a\mu}$ which provides description of the large g phase

Simplest large g effective action for the $A_{a\mu}$

$$Z = \prod_{a,\mu} \int dA_{a\mu} \exp\left(\frac{1}{2e^2} \sum_{\square} \cos\left(\frac{1}{2} \left(\Delta_\mu A_{a\nu} - \Delta_\nu A_{a\mu} \right) \right) - \frac{i}{2} \sum_a \eta_a A_{a\tau} \right)$$

with $e^2 \sim g^2$

This is compact QED in $d+1$ dimensions with static charges ± 1 on two sublattices.

This theory can be reliably analyzed by a duality mapping.

d=2: The gauge theory is *always* in a *confining* phase and there is bond order in the ground state.

d=3: A deconfined phase with a gapless "photon" is possible.

N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).

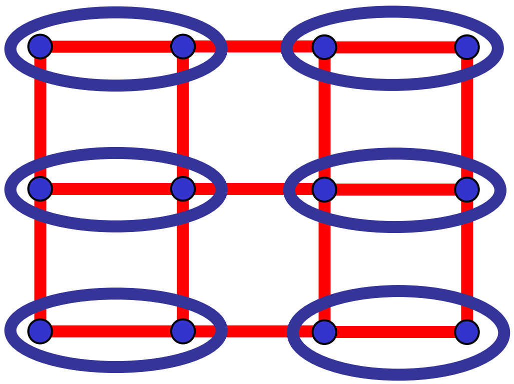
S. Sachdev and R. Jalabert, *Mod. Phys. Lett.* B **4**, 1043 (1990).

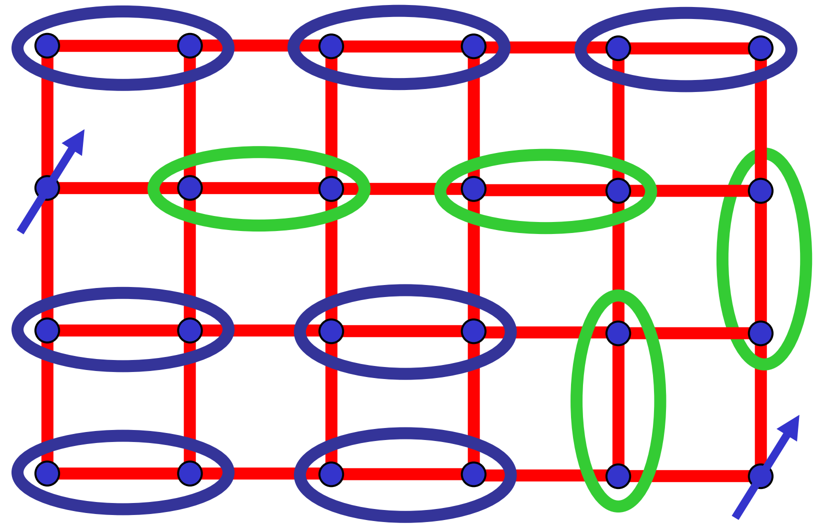
K. Park and S. Sachdev, *Phys. Rev.* B **65**, 220405 (2002).

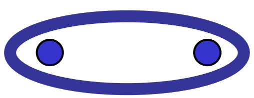
I. Order in Mott insulators

Paramagnetic states $\left\langle \boldsymbol{S}_j \right\rangle = 0$

Class A. Bond order and spin excitons in d=2







$$= \frac{1}{\sqrt{2}}\left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$$

S=1/2 spinons are ***confined*** by a linear potential into a S=1 spin exciton

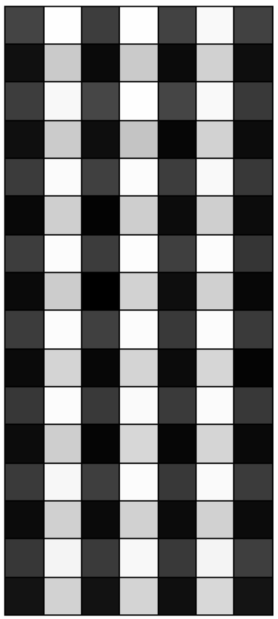
Spontaneous bond-order leads to vector S=1 spin excitations

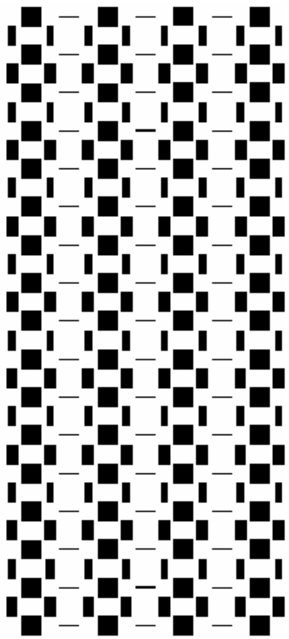
N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).

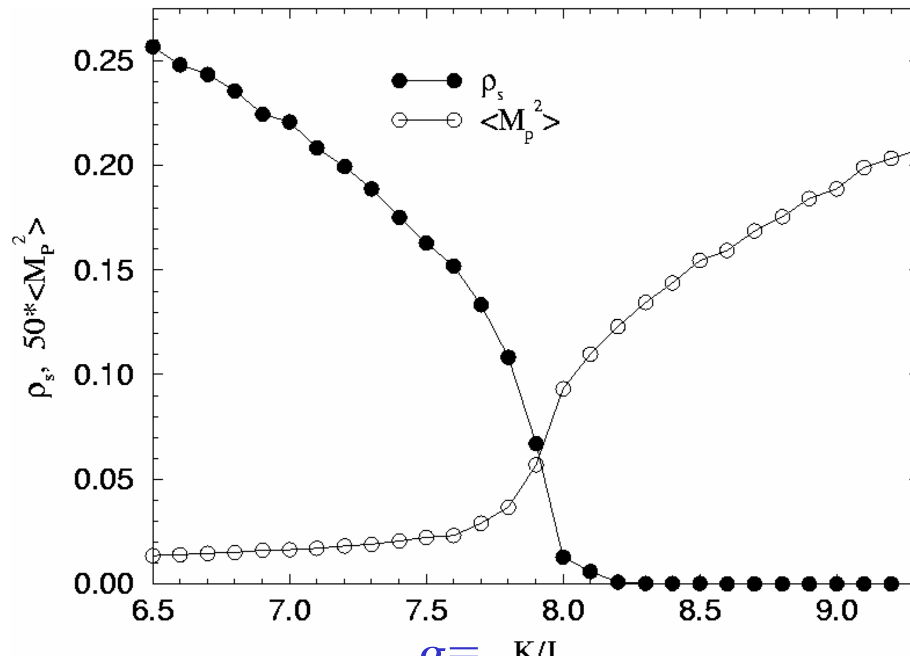
Bond order in a frustrated $S=1/2$ XY magnet

A. W. Sandvik, S. Daul, R. R. P. Singh, and D. J. Scalapino, *Phys. Rev. Lett*. **89**, 247201 (2002)

First *large scale* numerical study of the destruction of Neel order in a $S=1/2$ antiferromagnet with full square lattice symmetry







$g=$

$$H = 2J \sum_{\langle ij \rangle} \left(S_i^x S_j^x + S_i^y S_j^y \right) - K \sum_{\langle ijkl \rangle \subset \square} \left(S_i^+ S_j^- S_k^+ S_l^- + S_i^- S_j^+ S_k^- S_l^+ \right)$$

Class B:

Z_2 gauge theory: non-collinear spins, visons, topological order, and deconfined spinons

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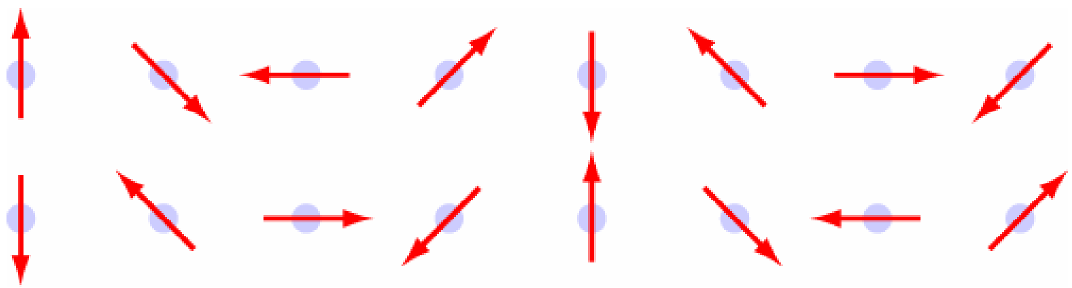
Magnetic order $\left\langle \boldsymbol{S}_j \right\rangle = \boldsymbol{N}_1 \cos\left(\vec{K} \cdot \vec{r}_j\right) + \boldsymbol{N}_2 \sin\left(\vec{K} \cdot \vec{r}_j\right)$

Class B. Noncollinear spins

(B.I. Shraiman and E.D. Siggia, *Phys. Rev. Lett.* **61**, 467 (1988))



$$\vec{K} = \left(3\pi/4, \pi\right) \;;$$

$$\boldsymbol{N}_2^2 = \boldsymbol{N}_1^2 \,,\; \boldsymbol{N}_1 \cdot \boldsymbol{N}_2 = 0$$

Solve constraints by expressing $\boldsymbol{N}_{1,2}$ in terms of two complex numbers $z_\uparrow$, $z_\downarrow$

$$\boldsymbol{N}_1 + i\boldsymbol{N}_2 = \begin{pmatrix} z_\downarrow^2 - z_\uparrow^2 \\ i\left(z_\downarrow^2 + z_\uparrow^2\right) \\ 2 z_\uparrow z_\downarrow \end{pmatrix}$$

Order in ground state specified by a spinor $\left(z_\uparrow, z_\downarrow\right)$ (modulo an overall sign).

This spinor can become a S=1/2 spinon in paramagnetic state.

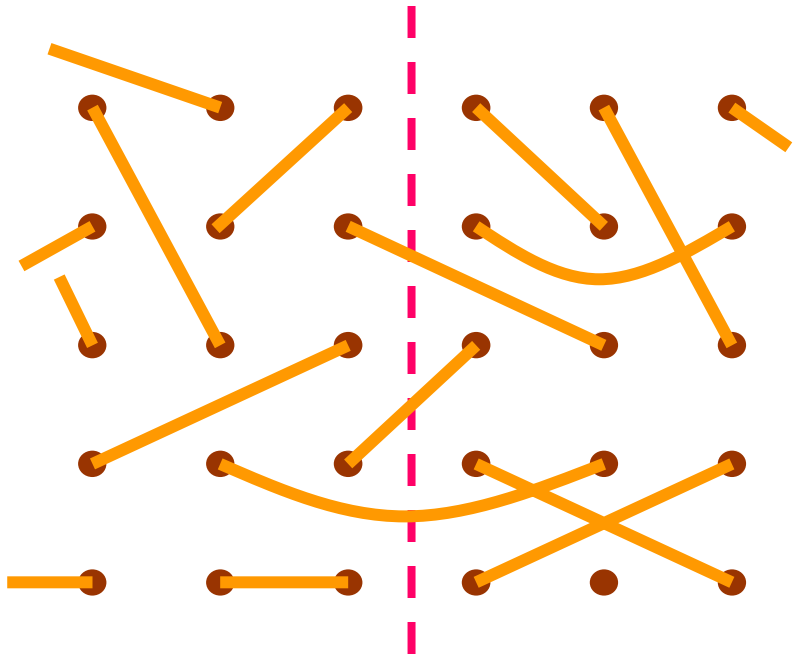
Theory of spinons must obey the Z_2 gauge symmetry $z_a \rightarrow -z_a$

A. V. Chubukov, S. Sachdev, and T. Senthil *Phys. Rev. Lett.* **72,** 2089 (1994)

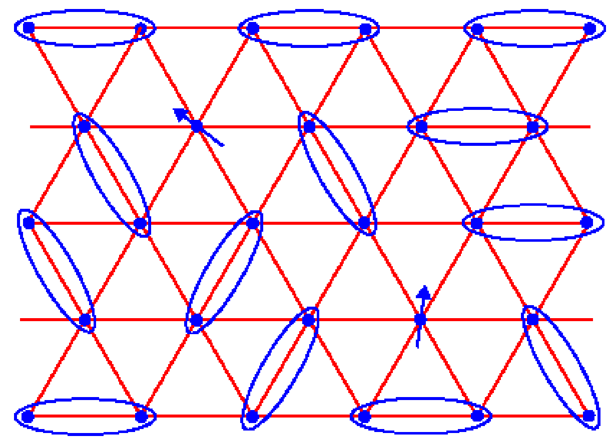
I. Order in Mott insulators

Paramagnetic states $\langle \boldsymbol{S}_j \rangle = 0$

Class B. Topological order and deconfined spinons



Number of valence bonds cutting line is conserved modulo 2 – this is described by the same Z_2 gauge theory as non-collinear spins



RVB state with free spinons

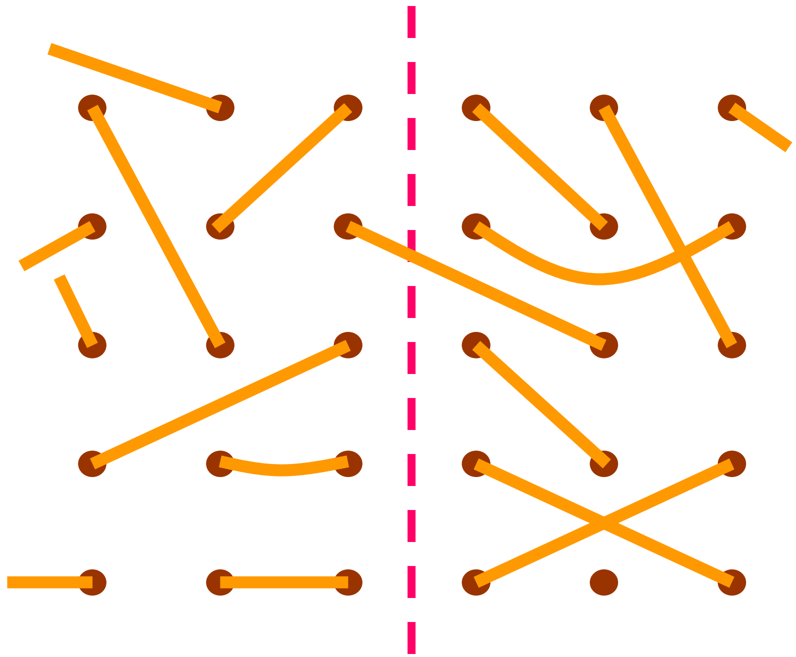
P. Fazekas and P.W. Anderson, *Phil Mag* **30**, 23 (1974).

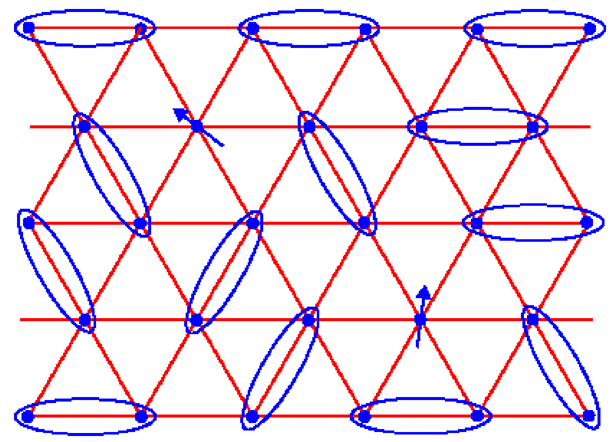
D.S. Rokhsar and S. Kivelson, *Phys. Rev. Lett*. **61**, 2376 (1988)
N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991);
R. Jalabert and S. Sachdev, *Phys. Rev*. B **44**, 686 (1991);
X. G. Wen, *Phys. Rev*. B **44**, 2664 (1991).
T. Senthil and M.P.A. Fisher, *Phys. Rev*. B **62**, 7850 (2000).

I. Order in Mott insulators

Paramagnetic states $\langle \boldsymbol{S}_j \rangle = 0$

Class B. Topological order and deconfined spinons





RVB state with free spinons

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D.S. Rokhsar and S. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988)
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X. G. Wen, *Phys. Rev.* B **44**, 2664 (1991).
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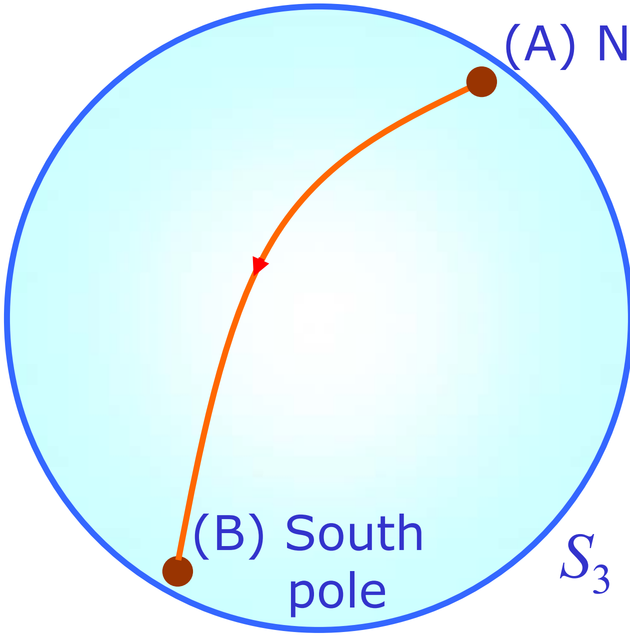
I. Order in Mott insulators

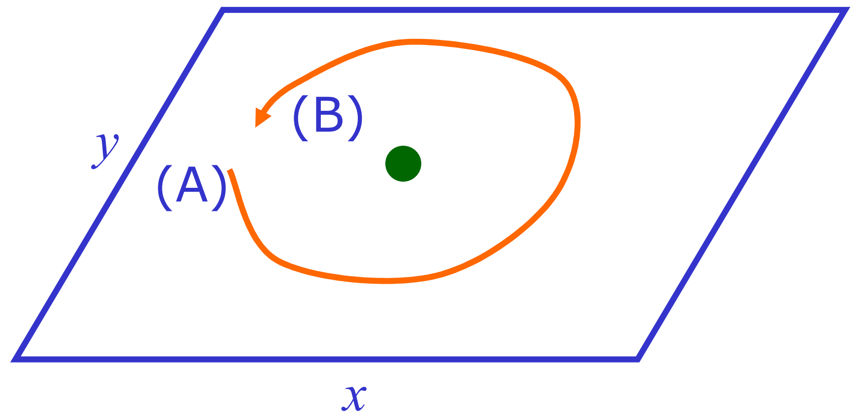
Paramagnetic states $\langle \boldsymbol{S}_j \rangle = 0$

Class B. Topological order and deconfined spinons

Order parameter space: S_3/Z_2

Vortices associated with $\pi_1(S_3/Z_2)=Z_2$ (visons) have gap in the paramagnet.
This gap survives doping and leads to stable *hc/e* vortices at low doping.





N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991)
T. Senthil and M.P.A. Fisher, *Phys. Rev.* B **62**, 7850 (2000).
S. Sachdev, *Physical Review* B **45**, 389 (1992)
N. Nagaosa and P.A. Lee, *Physical Review* B **45**, 966 (1992)

II. Evidence cuprates are in **class A**

Competing order parameters

1. Pairing order of BCS theory (SC)

Bose-Einstein condensation of *d*-wave Cooper pairs

Orders associated with proximate Mott insulator in class A

2. Collinear magnetic order (CM)

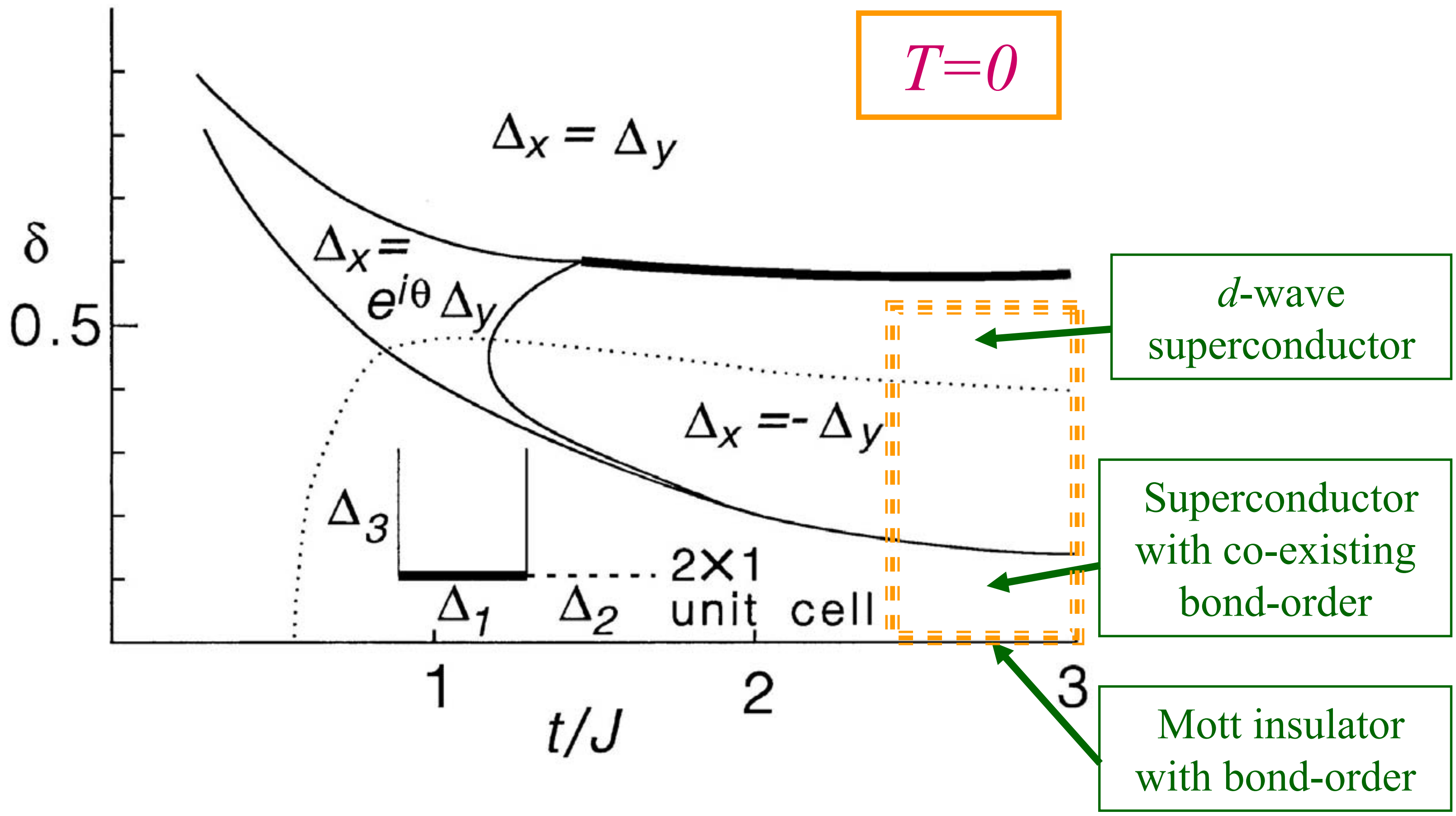
3. Bond order (B)

S. Sachdev and N. Read, *Int. J. Mod. Phys.* B **5**, 219 (1991).

II. Doping Class A

Doping a paramagnetic bond-ordered Mott insulator

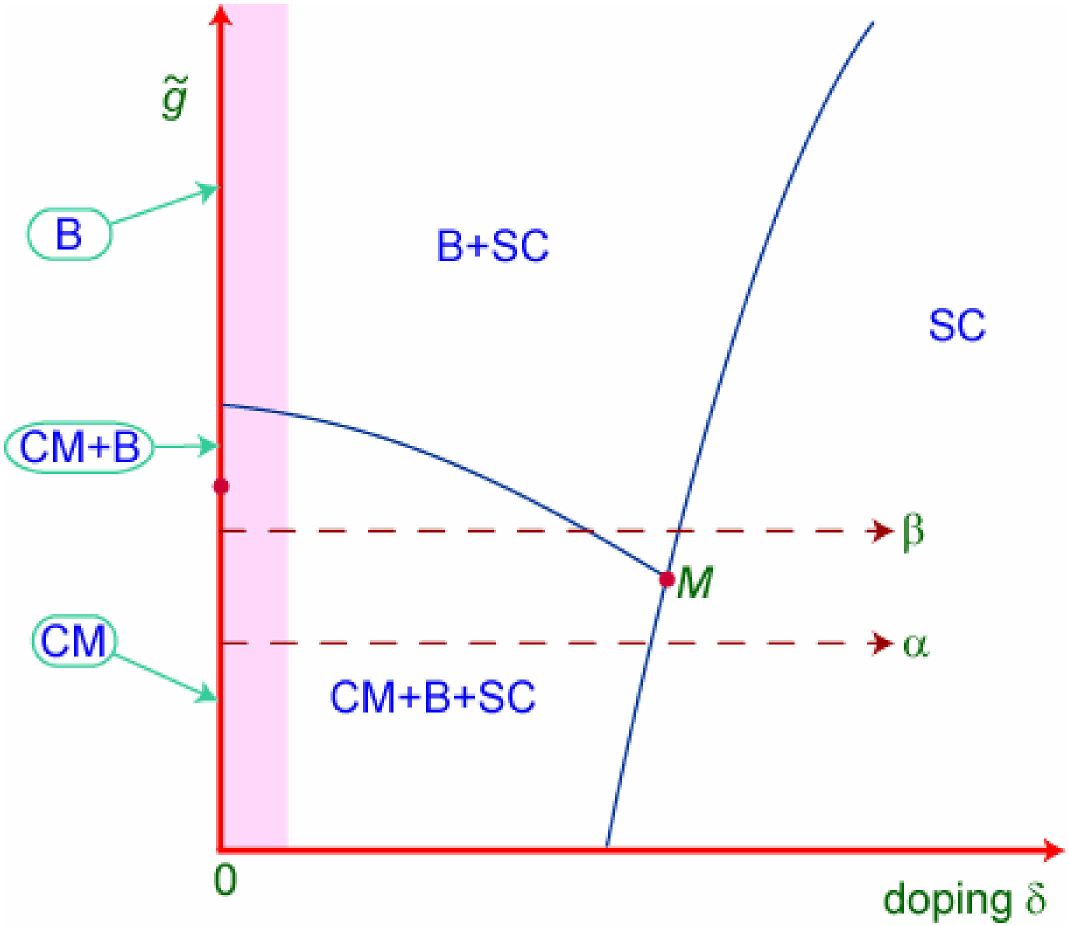
systematic Sp(N) theory of translational symmetry breaking, while preserving spin rotation invariance.

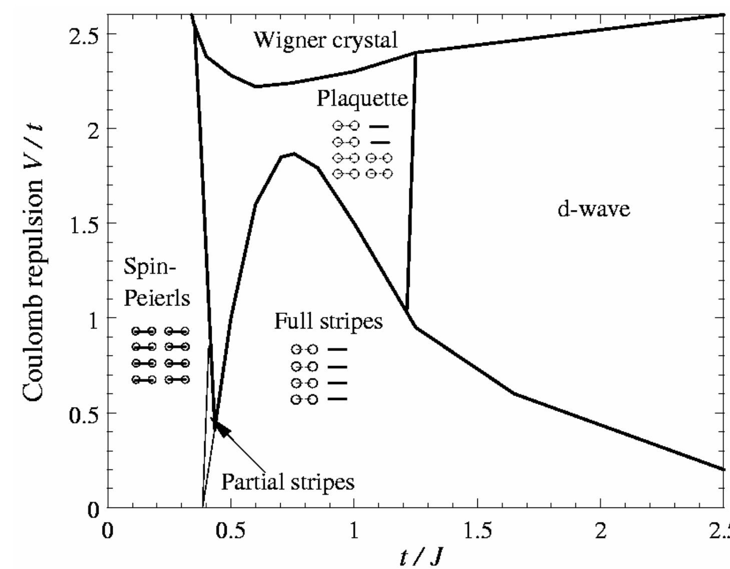


S. Sachdev and N. Read, *Int. J. Mod. Phys.* B **5**, 219 (1991).

A phase diagram

Vertical axis is any microscopic parameter which suppresses CM order





Microscopic theory for the interplay of bond (B) and *d*-wave superconducting (SC) order

- **Pairing order of BCS theory (SC)**
- **Collinear magnetic order (CM)**
- **Bond order (B)**

S. Sachdev and N. Read, *Int. J. Mod. Phys.* B **5**, 219 (1991).
M. Vojta and S. Sachdev, *Phys. Rev. Lett.* **83**, 3916 (1999);
M. Vojta, Y. Zhang, and S. Sachdev, *Phys. Rev.* B **62**, 6721 (2000); M. Vojta, Phys. Rev. B **66**, 104505 (2002).

Evidence cuprates are in class A

Evidence cuprates are in class A

- Neutron scattering shows collinear magnetic order co-existing with superconductivity

J. M. Tranquada *et al*., *Phys*. *Rev*. B **54**, 7489 (1996).
Y.S. Lee, R. J. Birgeneau, M. A. Kastner *et al., Phys*. *Rev*. B **60**, 3643 (1999).
S. Wakimoto, R.J. Birgeneau, Y.S. Lee, and G. Shirane, *Phys*. *Rev*. B **63**, 172501 (2001).

Evidence cuprates are in class A

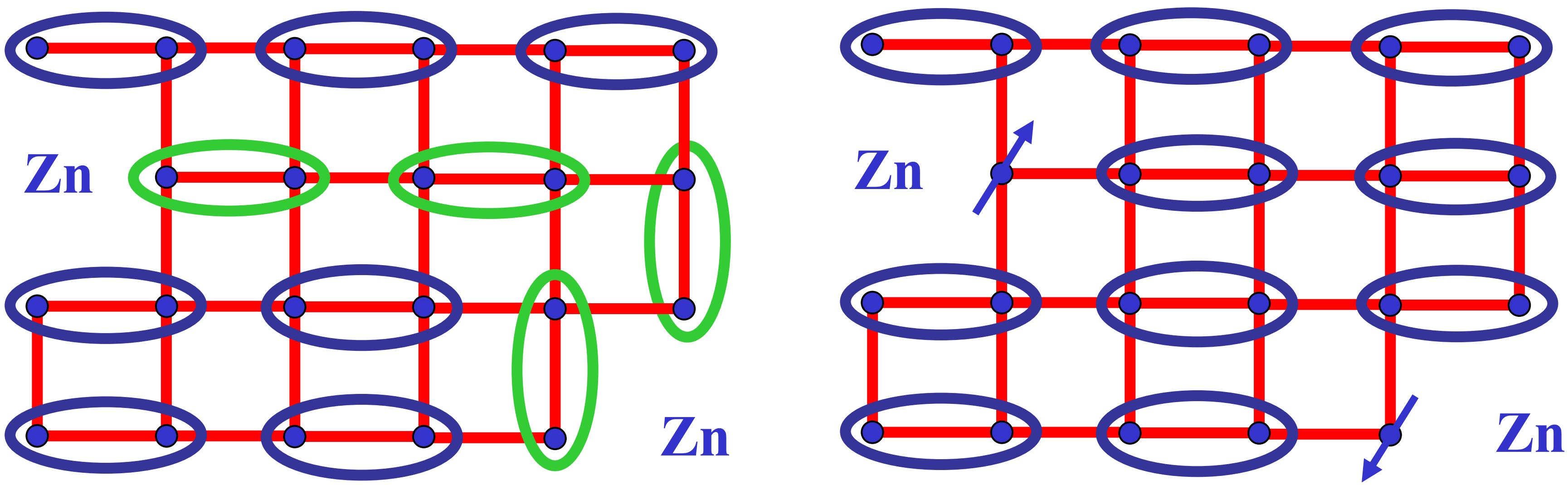
- Neutron scattering shows collinear magnetic order co-existing with superconductivity
- Proximity of Z_2 Mott insulators requires stable *hc/e* vortices, vison gap, and Senthil flux memory effect

S. Sachdev, *Physical Review* B **45**, 389 (1992)
N. Nagaosa and P.A. Lee, *Physical Review* B **45**, 966 (1992)
T. Senthil and M. P. A. Fisher, *Phys. Rev. Lett.* **86**, 292 (2001).
D. A. Bonn, J. C. Wynn, B. W. Gardner, Y.-J. Lin, R. Liang, W. N. Hardy, J. R. Kirtley, and K. A. Moler, *Nature* **414**, 887 (2001).
J. C. Wynn, D. A. Bonn, B. W. Gardner, Y.-J. Lin, R. Liang, W. N. Hardy, J. R. Kirtley, and K. A. Moler, *Phys. Rev. Lett.* **87**, 197002 (2001).

Evidence cuprates are in class A

- Neutron scattering shows collinear magnetic order co-existing with superconductivity
- Proximity of Z_2 Mott insulators requires stable *hc/e* vortices, vison gap, and Senthil flux memory effect
- Non-magnetic impurities in underdoped cuprates acquire a $S=1/2$ moment

Effect of static non-magnetic impurities (Zn or Li)

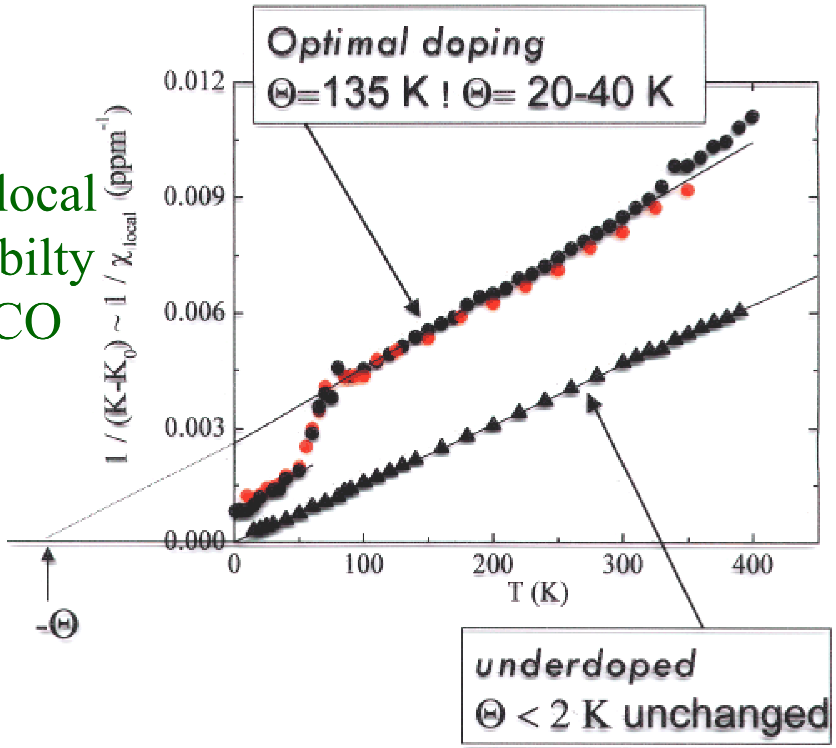


Spinon confinement implies that free S=1/2 moments form near each impurity

$$\chi_{\text{impurity}}(T \to 0) = \frac{S(S+1)}{3k_B T}$$

Spatially resolved NMR of Zn/Li impurities in the superconducting state

Inverse local susceptibilty in YBCO



^{7}Li NMR below T_c

J. Bobroff, H. Alloul, W.A. MacFarlane, P. Mendels, N. Blanchard, G. Collin, and J.-F. Marucco, *Phys. Rev. Lett*. **86**, 4116 (2001).

Measured $\chi_{impurity}(T \to 0) = \dfrac{S(S+1)}{3k_B T}$ with $S = 1/2$ in underdoped sample.

This behavior does not emerge out of BCS theory.

A.M Finkelstein, V.E. Kataev, E.F. Kukovitskii, G.B. Teitel'baum, *Physica* C **168**, 370 (1990).

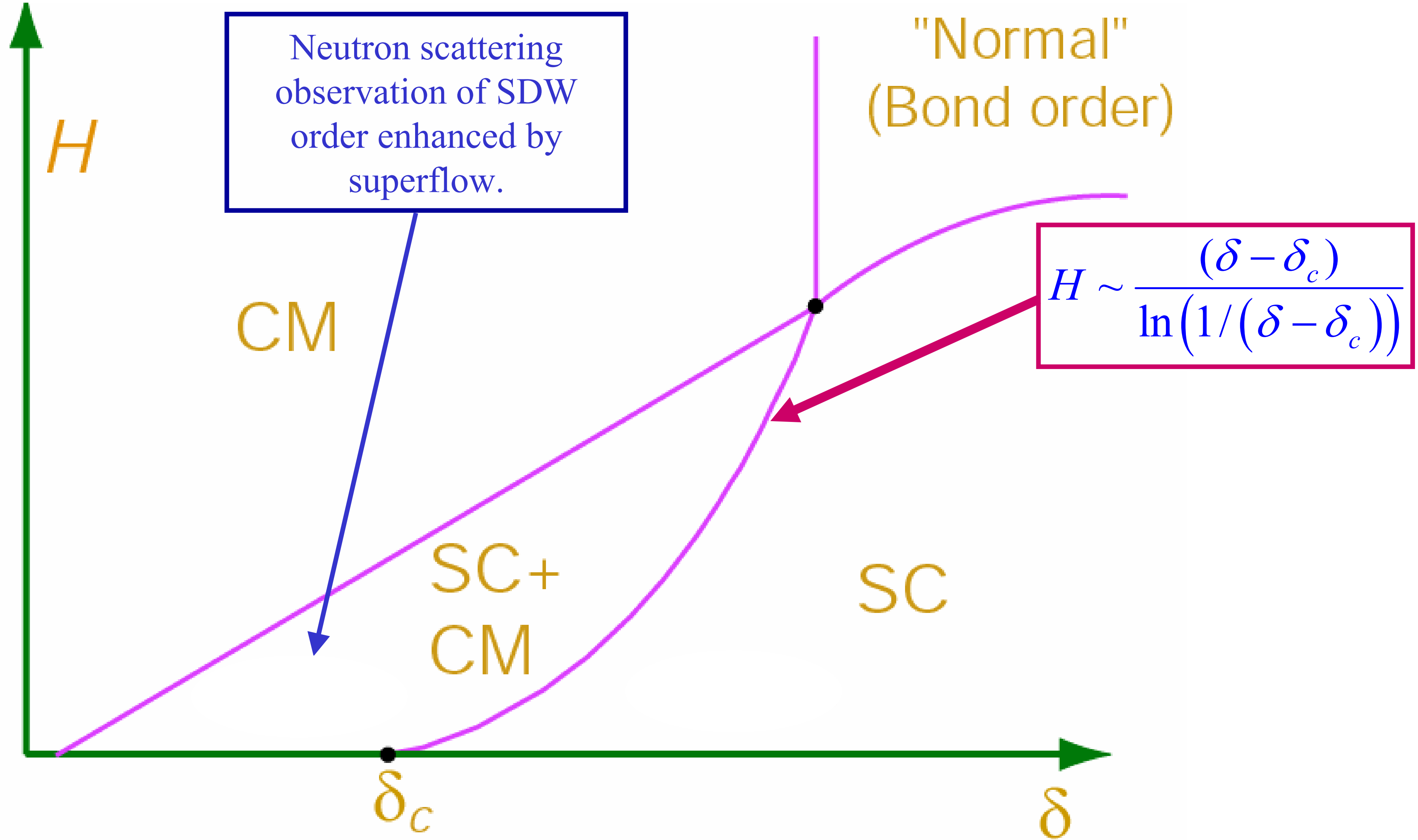
Evidence cuprates are in class A

- Neutron scattering shows collinear magnetic order co-existing with superconductivity
- Proximity of Z_2 Mott insulators requires stable *hc/e* vortices, vison gap, and Senthil flux memory effect
- Non-magnetic impurities in underdoped cuprates acquire a S=1/2 moment

Evidence cuprates are in class A

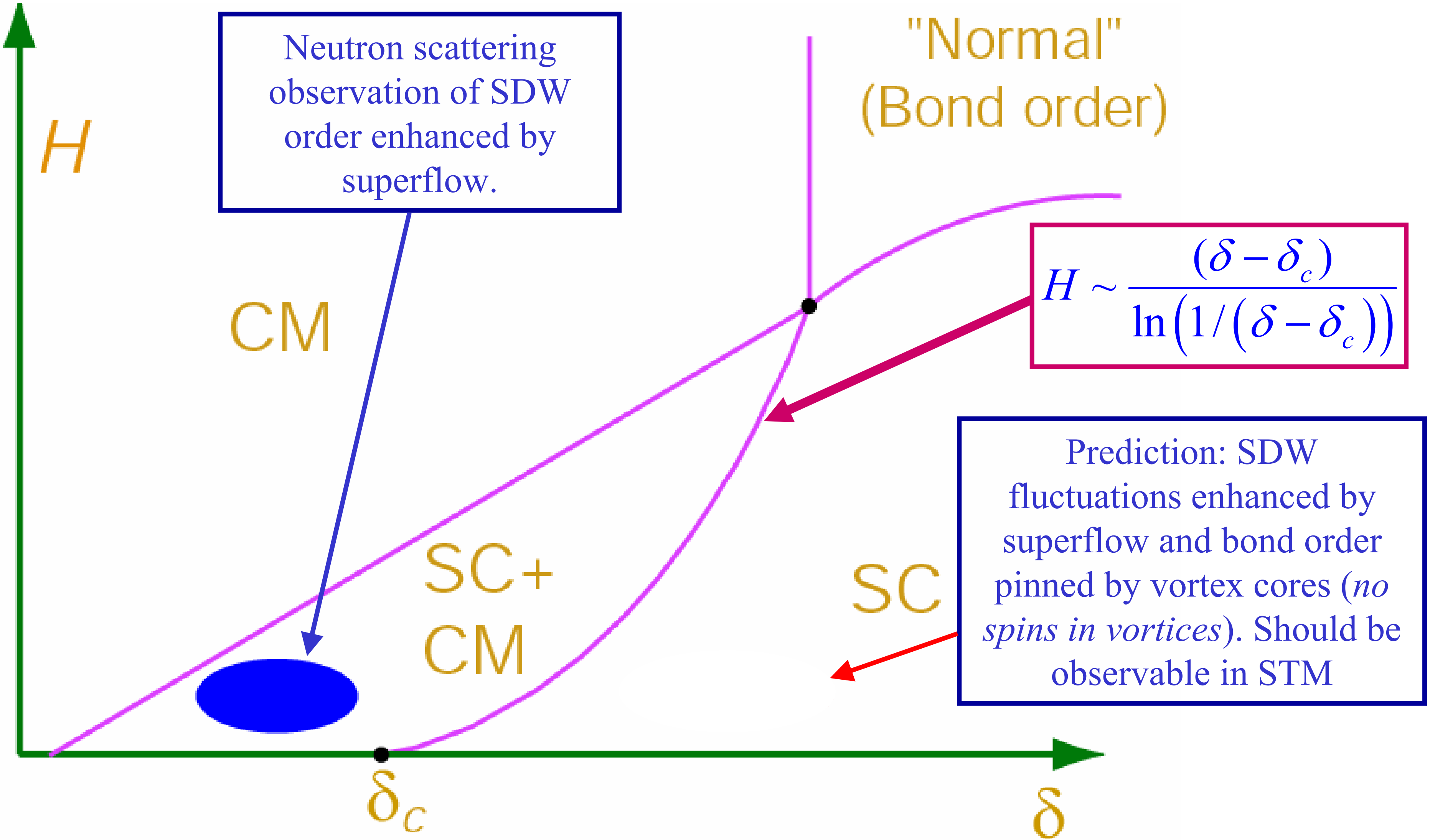
- Neutron scattering shows collinear magnetic order co-existing with superconductivity
- Proximity of Z_2 Mott insulators requires stable *hc/e* vortices, vison gap, and Senthil flux memory effect
- Non-magnetic impurities in underdoped cuprates acquire a S=1/2 moment
- Tests of phase diagram in a magnetic field (talk by E. Demler, Microsymposium MS IV, May 28, 11:40)

Superflow kinetic energy $\left\langle v_s^2 \right\rangle \propto \frac{H}{H_{c2}} \ln \frac{3H_{c2}}{H} \Rightarrow \delta_{\text{eff}}\left(H\right) = \delta - C\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$



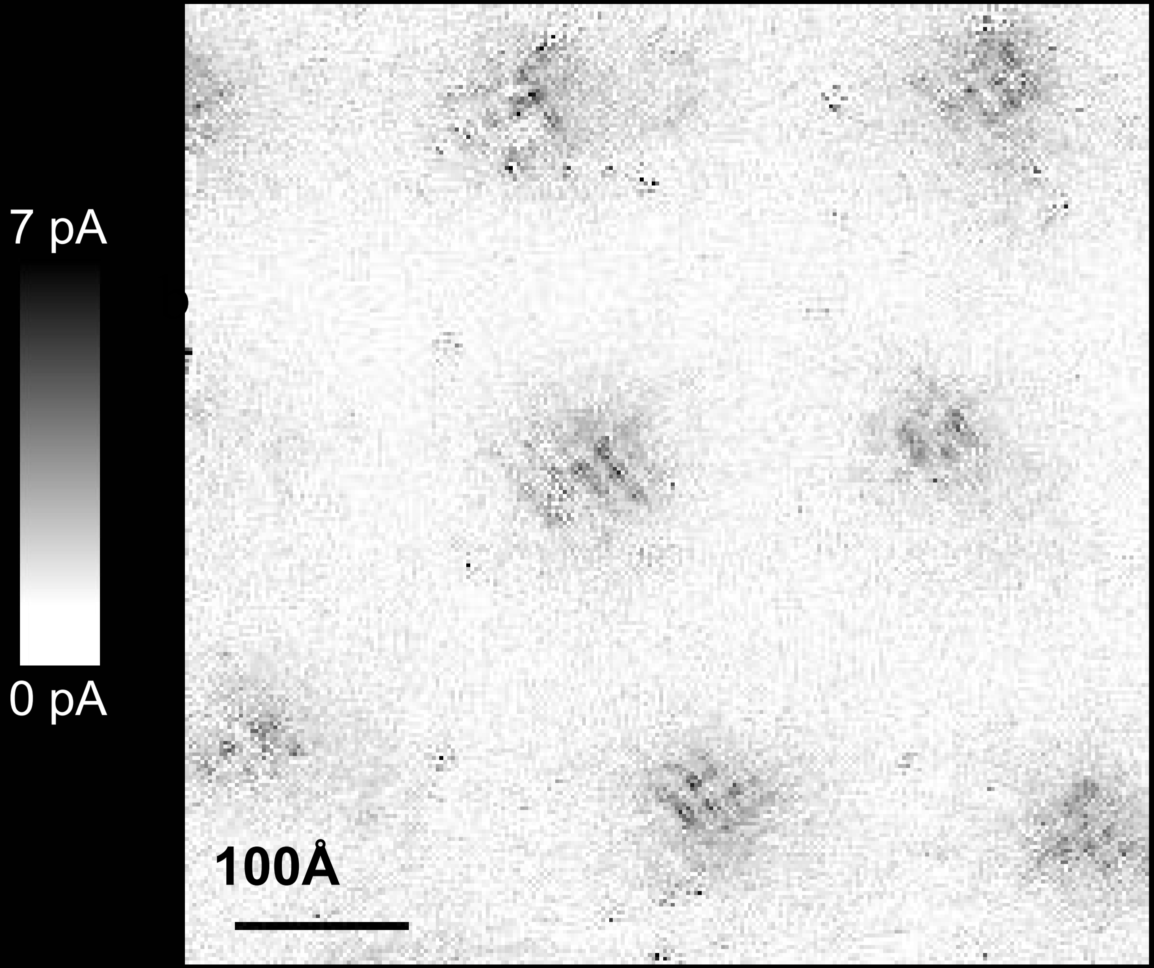
E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Superflow kinetic energy $\langle v_s^2 \rangle \propto \dfrac{H}{H_{c2}} \ln \dfrac{3H_{c2}}{H} \Rightarrow \delta_{\text{eff}}(H) = \delta - C\dfrac{H}{H_{c2}} \ln\left(\dfrac{3H_{c2}}{H}\right)$



E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV

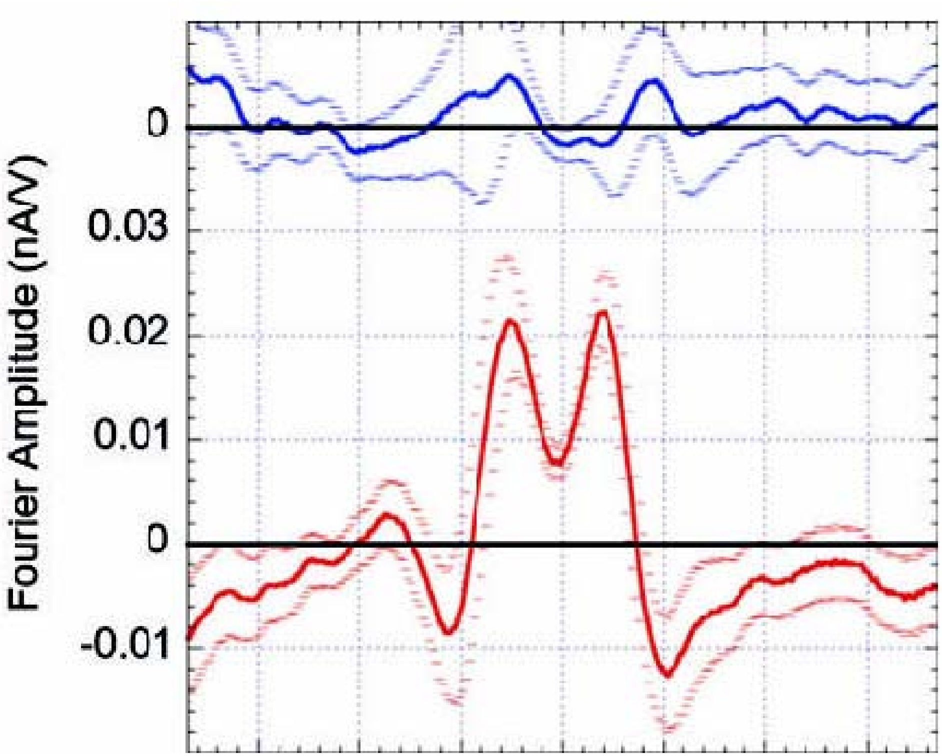


Our interpretation:
LDOS modulations are signals of bond order of period 4 revealed in vortex halo

See also:
S. A. Kivelson, E. Fradkin, V. Oganesyan, I. P. Bindloss, J. M. Tranquada, A. Kapitulnik, and C. Howald, cond-mat/0210683.

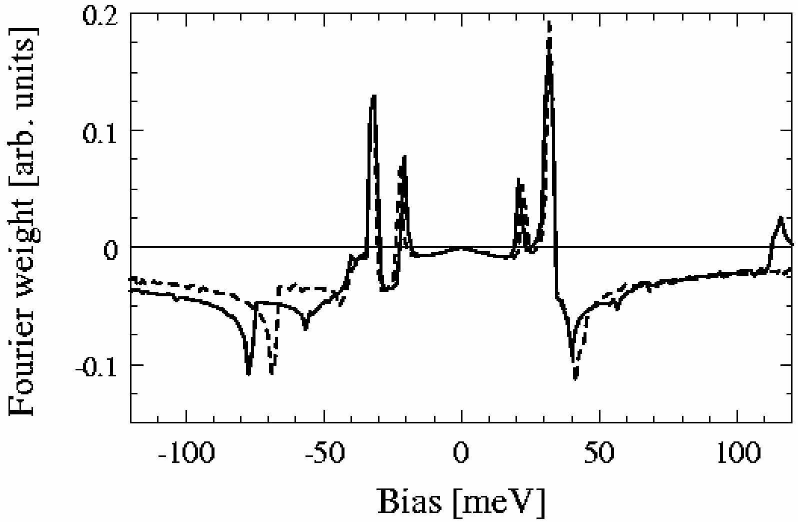
J. Hoffman E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* 295, 466 (2002).

Spectral properties of the STM signal are sensitive to the microstructure of the charge order



Measured energy dependence of the Fourier component of the density of states which modulates with a period of 4 lattice spacings

C. Howald, H. Eisaki, N. Kaneko, and A. Kapitulnik, *Phys. Rev.* B **67**, 014533 (2003).



Theoretical modeling shows that this spectrum is best obtained by a modulation of bond variables, such as the exchange, kinetic or pairing energies.

M. Vojta, *Phys. Rev.* B **66**, 104505 (2002); D. Podolsky, E. Demler, K. Damle, and B.I. Halperin, *Phys. Rev.* B in press, cond-mat/0204011

Conclusions

I. Two classes of Mott insulators:

(A) Collinear spins, compact U(1) gauge theory;
bond order and confinements of spinons in d=2

(B) Non-collinear spins, Z_2 gauge theory

II. Doping Class A in d=2

Magnetic/bond order co-exist with superconductivity at low doping

Cuprates most likely in this class.

Theory of quantum phase transitions provides a description of "fluctuating order" in the superconductor.

III. Class A in d=3

Deconfined spinons and quantum criticality in heavy fermion compounds (cond-mat/0209144 and cond-mat/0305193)