Quantum matter without quasiparticles: SYK models, black holes, and the cuprate strange metal

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Subir Sachdev

Talk online: sachdev.physics.harvard.edu
Quantum matter with quasiparticles:

- Magnon
- Roton
- Plasmon
- Polaron
- Exciton
- Laughlin quasiparticle
- Composite fermion
- Bogoliubovon
- Anderson-Higgs mode
- Massless Dirac Fermions
- Weyl fermions
- ....
Quantum matter with quasiparticles:

Most generally, a quasiparticle is an “additive” excitation:

Quasiparticles can be combined to yield additional excitations, with energy determined by the energies and densities of the constituents. Such a procedure yields all the low-lying excitations. Then we can apply the Boltzmann-Landau theory to make predictions for dynamics.
Quantum matter without quasiparticles:

No quasiparticle structure to excitations.

But how can we be sure that no quasiparticles exist in a given system? Perhaps there are some exotic quasiparticles inaccessible to current experiments……..
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Consider how rapidly the system loses “phase coherence”, reaches local thermal equilibrium, or becomes “chaotic”
Local thermal equilibration or phase coherence time, $\tau_\varphi$:

- There is an *lower bound* on $\tau_\varphi$ in all many-body quantum systems of order $\hbar/(k_B T)$,

  $$\tau_\varphi > C \frac{\hbar}{k_B T},$$

  and the lower bound is realized by systems *without* quasiparticles.

- In systems *with* quasiparticles, $\tau_\varphi$ is parametrically larger at low $T$;
  *e.g.* in Fermi liquids $\tau_\varphi \sim 1/T^2$,
  and in gapped insulators $\tau_\varphi \sim e^{\Delta/(k_B T)}$ where $\Delta$ is the energy gap.

A bound on quantum chaos:

- The time over which a many-body quantum system becomes “chaotic” is given by $\tau_L = 1/\lambda_L$, where $\lambda_L$ is the “Lyapunov exponent” determining memory of initial conditions. This [LYAPUNOV TIME](#) obeys the rigorous lower bound

$$\tau_L \geq \frac{1}{2\pi} \frac{\hbar}{k_B T}$$

A. I. Larkin and Y. N. Ovchinnikov, JETP 28, 6 (1969)

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Quantum matter without quasiparticles

$\approx$ fastest possible many-body quantum chaos
Quantum matter without quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models

Black holes with AdS\(_2\) horizons

Fermi surface coupled to a gauge field

\[
\mathcal{L}[\Psi, a] = \Psi^\dagger \left( \partial_\tau - ia_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2
\]
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Black holes with AdS$_2$ horizons

Same low energy theory

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Quantum matter without quasiparticles:

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Black holes with AdS\(_2\) horizons

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\[ \tau_L: \text{ the Lyapunov time to reach quantum chaos} \]
**Quantum matter without quasiparticles:**

**The Sachdev-Ye-Kitaev (SYK) models**

\[ \tau_L = \frac{\hbar}{2\pi k_B T} \]

\[ v_B \sim T^{1/2} \]

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**Fermi surface coupled to a gauge field**

\[ \tau_L = \frac{\hbar}{2.48 k_B T} \]

\[ v_B \sim \frac{v_F^{5/3}}{e^{4/3} \gamma^{1/3}} T^{1/3} \]

\[ \mathcal{L}[\Psi, a] = \Psi^\dagger \left( \partial_\tau - ia_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2 \]

\( \tau_L \): the Lyapunov time to reach quantum chaos

\( v_B \): the “butterfly velocity” for the spatial propagation of chaos
Quantum matter without quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models

Black holes with AdS$_2$ horizons

Fermi surface coupled to a gauge field

Thermal diffusivity, $D_E$:

$$D_E = (\text{universal number}) \times v_B^2 \tau_L$$
in all three models

$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left( \partial_\tau - i a_\tau - \frac{(\nabla - i \vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

$\tau_L$: the Lyapunov time to reach quantum chaos

$v_B$: the “butterfly velocity” for the spatial propagation of chaos
The Sachdev-Ye-Kitaev (SYK) model:

- A theory of a strange metal
- Dual theory of gravity on AdS$_2$
- Fastest possible quantum chaos

with $\tau_L = \frac{\hbar}{2\pi k_B T}$
The SYK model is given by the Hamiltonian:

\[
H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;kl} c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i
\]

where \( c_i^\dagger c_j + c_j^\dagger c_i = 0 \), \( c_i^\dagger c_j^\dagger + c_j^\dagger c_i = \delta_{ij} \)

\[
Q = \frac{1}{N} \sum_i c_i^\dagger c_i
\]

\( J_{ij;kl} \) are independent random variables with \( \overline{J_{ij;kl}} = 0 \) and \( \overline{|J_{ij;kl}|^2} = J^2 \)

\( N \to \infty \) yields critical strange metal.

S. Sachdev and J. Ye, PRL 70, 3339 (1993)
Feynman graph expansion in $J_{ij}$, and graph-by-graph average, yields exact equations in the large $N$ limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

$$G(\tau = 0^-) = Q.$$  

Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{1}{A} \sqrt{z} + \ldots , \quad G(z) = \frac{A}{\sqrt{z}}$$

for some complex $A$. The ground state is a non-Fermi liquid, with a continuously variable density $Q$.  

SYK and AdS$_2$

- Non-zero GPS entropy as $T \to 0$, $S(T \to 0) = N S_0 + \ldots$

Not a ground state degeneracy: due to an exponentially small (in $N$) many-body level spacing at all energies down to the ground state energy.

A. Georges, O. Parcollet, and S. Sachdev, PRB 63, 134406 (2001)
SYK and AdS$_2$

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- The correlators and thermodynamics of SYK match those of quantum matter holographically dual to AdS$_2$ black hole horizons (as computed by T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, PRD 83, 125002 (2011)). SYK models are “are states of matter at non-zero density realizing the near-horizon, AdS$_2 \times R^2$ physics of Reissner-Nördstrom black holes”. The Bekenstein-Hawking entropy is $NS_0$ ($\text{GPS} = \text{BH}$).

S. Sachdev, PRL 105, 151602 (2010)
Einstein-Maxwell theory
+ cosmological constant

AdS$_2 \times T^2$

\[ ds^2 = \left( \frac{d\zeta^2 - dt^2}{\zeta^2} \right) + d\vec{x}^2 \]

Gauge field: \( A = (\mathcal{E}/\zeta)dt \)

\( \zeta = \infty \)

Mapping to SYK applies when temperature \( \ll 1/(\text{size of } T^2) \)

S. Sachdev, PRL 105, 151602 (2010)
SYK and AdS$_2$

\[ G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau) \]

\[ \Sigma(z) = \mu - \frac{1}{A} \sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}} \]

SYK and AdS$_2$

\[
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\[
\Sigma(z) = -\frac{1}{A} \sqrt{z} + \ldots , \quad G(z) = \frac{A}{\sqrt{z}}
\]

At frequencies $\ll J$, the $i\omega + \mu$ can be dropped, and without it equations are invariant under the reparametrization and gauge transformations

\[
\tau = f(\sigma)
\]

\[
G(\tau_1, \tau_2) = [f'(\sigma_1)f'(\sigma_2)]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} G(\sigma_1, \sigma_2)
\]

\[
\Sigma(\tau_1, \tau_2) = [f'(\sigma_1)f'(\sigma_2)]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \Sigma(\sigma_1, \sigma_2)
\]

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

A. Kitaev, unpublished

S. Sachdev, PRX 5, 041025 (2015)
Let us write the large $N$ saddle point solutions of $S$ as

$$G_s(\tau_1 - \tau_2) \sim (\tau_1 - \tau_2)^{-1/2}$$
$$\Sigma_s(\tau_1 - \tau_2) \sim (\tau_1 - \tau_2)^{-3/2}.$$

These are not invariant under the reparametrization symmetry but are invariant only under a $\text{SL}(2,\mathbb{R})$ subgroup under which

$$f(\tau) = \frac{a\tau + b}{c\tau + d}, \quad ad - bc = 1.$$

So the (approximate) reparametrization symmetry is spontaneously broken.
Connections of SYK to gravity and AdS$_2$ horizons

- Reparameterization and gauge invariance are the ‘symmetries’ of the Einstein-Maxwell theory of gravity and electromagnetism.

- SL(2,R) is the isometry group of AdS$_2$. 
One can also derive the thermodynamic properties from the large-$N$ saddle point free energy:

$$F_N = \frac{1}{2} \log \mathcal{P} f (\tau) + \frac{1}{2} \int d \tau_1 d \tau_2 \mathcal{G} (\tau_1, \tau_2) J^2 \mathcal{G} (\tau_1, \tau_2) \mathcal{C} (8)$$

In the second line we write the free energy in a low temperature expansion, where $U \ll J_0$. $J_0$ is the ground state energy, $S_0 \ll 232$ is the zero temperature entropy [32, 4], and $T = c v = \frac{\pi \nu}{K 16 p} J_0 \ll 0.396$. $J$ is the specific heat [11]. The entropy term can be derived by inserting the conformal saddle point solution (2) in the effective action. The specific heat can be derived from knowledge of the leading (in $1/J$) correction to the conformal saddle, but the energy requires the exact (numerical) finite $J$ solution.

3 The generalized SYK model

In this section, we will present a simple way to generalize the SYK model to higher dimensions while keeping the solvable properties of the model in the large-$N$ limit. For concreteness of the presentation, in this section we focus on a (1 + 1)-dimensional example, which describes a one-dimensional array of SYK models with coupling between neighboring sites. It should be clear how to generalize, and we will discuss more details of the generalization to arbitrary dimensions and generic graphs in section 6.

3.1 Definition of the chain model

Figure 1: A chain of coupled SYK sites: each site contains $N \gg 1$ fermion with SYK interaction. The coupling between nearest neighbor sites are four fermion interaction with two from each site.

Yingfei Gu, Xiao-Liang Qi, and D. Stanford, arXiv:1609.07832
Coupled SYK models

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Can compute butterfly velocity, and thermo-electric transport correlators, and they precisely match those of momentum-dissipating charged black holes with $\text{AdS}_2 \times R^d$ horizons.

R. Davison, Wenbo Fu, Yingfei Gu, K. Jensen. S. Sachdev, unpublished
Quantum matter without quasiparticles:

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Black holes with AdS$_2$ horizons

Fermi surface coupled to a gauge field

\[
\mathcal{L}[\Psi, a] = \Psi^\dagger \left( \partial_\tau - ia_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2
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\[ \mathcal{L}[\Psi, a] = \Psi^{\dagger} \left( \partial_{\tau} - i a_{\tau} - \frac{(\nabla - i \vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2 \]
Fermi surface coupled to a gauge field

\[ \mathcal{L}[\psi_{\pm}, a] = \]
\[ \psi_+^\dagger \left( \partial_\tau - i \partial_x - \partial_y^2 \right) \psi_+ + \psi_-^\dagger \left( \partial_\tau + i \partial_x - \partial_y^2 \right) \psi_- \]
\[ -a \left( \psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_- \right) + \frac{1}{2g^2} \left( \partial_y a \right)^2 \]

Fermi surface coupled to a gauge field

Compute out-of-time-order correlator to diagnose quantum chaos

\[ f(t) = \frac{1}{N^2} \theta(t) \sum_{i,j=1}^{N} \int d^2x \ \text{Tr} \left[ e^{-\beta H/2} \{ \psi_i(x,t), \psi_j^\dagger(0) \} \right. \]

\[ \times e^{-\beta H/2} \{ \psi_i(x,t), \psi_j^\dagger(0) \}^\dagger \left. \right] \]

\[ \sim \exp \left( \frac{(t - x/v_B)}{\tau_L} \right) \]

A. A. Patel and S. Sachdev, arXiv:1611.00003
Fermi surface coupled to a gauge field

Compute out-of-time-order correlator to diagnose quantum chaos

Strongly-coupled theory with no quasiparticles and fast scrambling:

\[ \tau_L \approx \frac{\hbar}{2.48 k_B T} \]
\[ v_B \approx 4.1 \frac{NT^{1/3}}{e^{4/3}} \frac{v_F^{5/3}}{\gamma^{1/3}} \]
\[ D_E \approx 0.42 v_B^2 \tau_L \]

\( N \) is the number of fermion flavors, \( v_F \) is the Fermi velocity, \( \gamma \) is the Fermi surface curvature, \( e \) is the gauge coupling constant.
Figure: K. Fujita and J. C. Seamus Davis

YBa$_2$Cu$_3$O$_{6+x}$
A conventional metal: the Fermi liquid with Fermi surface of size $1+p$. 

Many indications that this metal behaves like a Fermi liquid, but with Fermi surface size $p$ and not $1+p$.

Many indications that this metal behaves like a Fermi liquid, but with Fermi surface size \( p \) and not \( 1+p \).

If present at \( T=0 \), a metal with a size \( p \) Fermi surface (and translational symmetry preserved) must have topological order.
Hall effect measurements in YBCO

Badoux, Proust, Taillefer et al., Nature 531, 210 (2016)

\[ n_H = \frac{V}{e R_H} \]

Fermi liquid (FL) with carrier density \(1 + p\)
Spin density wave (SDW) breaks translational invariance, and the Fermi liquid then has carrier density $p$. 

Badoux, Proust, Taillefer et al., Nature 531, 210 (2016)
Charge density wave (CDW) leads to complex Fermi surface reconstruction and negative Hall resistance.
Hall effect measurements in YBCO

Evidence for a metal with topological order: Fermi surface of size $p$!
Metal with topological order?
Gauge theory for a topological phase transition, and for the strange metal (SM)

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$\tau_L$: the Lyapunov time to reach quantum chaos

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