Quantum phases and critical points of correlated metals

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Outline

I. **Kondo lattice models**
   Doniach's phase diagram and its quantum critical point

II. Paramagnetic states of quantum antiferromagnets:
   (A) Confinement of spinons and bond order
   (B) Spin liquids with deconfined spinons: Z_2 and U(1) gauge theories

III. A new phase: a fractionalized Fermi liquid (FL*)

IV. Extended phase diagram and its critical points

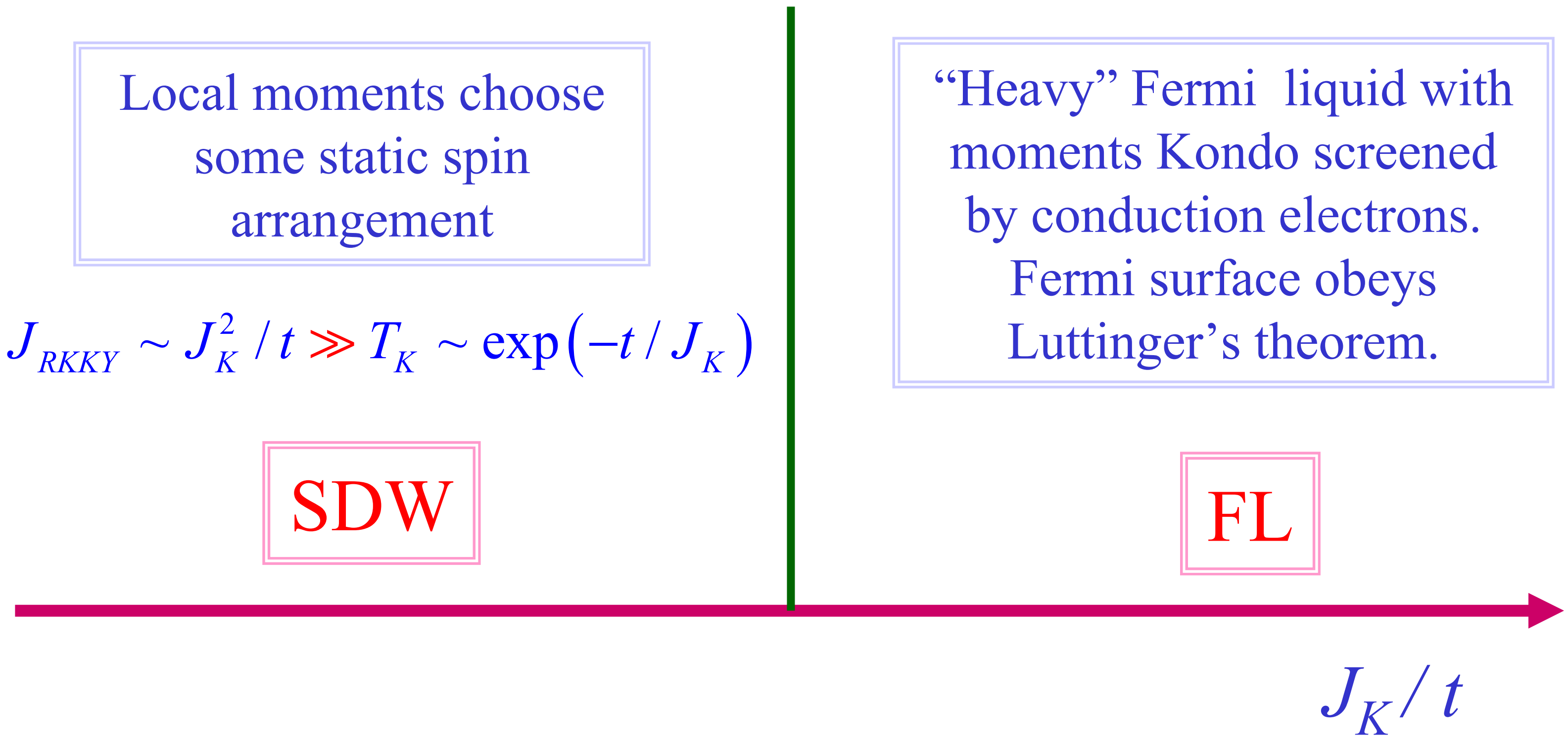
V. Conclusions

I. Doniach's $T=0$ phase diagram for the Kondo lattice

$$H = \sum_{i<j} t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + \sum_{i} \left(J_K c_{i\sigma}^{\dagger} \vec{\tau}_{\sigma\sigma'} c_{i\sigma} \cdot \vec{S}_{fi} \right)$$

$c_{i\sigma} \rightarrow$ Conduction electrons;

$\vec{S}_{fi} \rightarrow$ localized $f_{i\sigma}$ moments (assumed S=1/2, for specificity)



Luttinger's theorem on a d-dimensional lattice for the FL phase

Let v_0 be the volume of the unit cell of the ground state,
n_T be the total number density of electrons per volume v_0.
(need not be an integer)

$$n_T = n_f + n_c = 1 + n_c$$

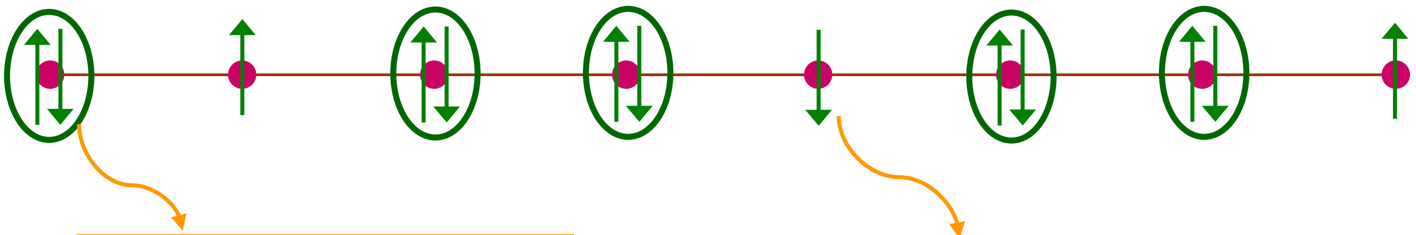
$$2 \times \frac{v_0}{(2\pi)^d} \left(\text{Volume enclosed by Fermi surface} \right)$$

$$= n_T \,(\mathrm{mod}\ 2)$$

A "large" Fermi surface

Arguments for the Fermi surface volume of the FL phase

Single ion Kondo effect implies $J_K \to \infty$ at low energies



$$\left(c_{i\uparrow}^{\dagger} f_{i\downarrow}^{\dagger} - c_{i\downarrow}^{\dagger} f_{i\uparrow}^{\dagger}\right)\left|0\right\rangle$$

$f_{i\downarrow}^{\dagger}\left|0\right\rangle$, S=1/2 hole

Fermi liquid of S=1/2 holes with hard-core repulsion

$$\begin{aligned} \text{Fermi surface volume } &= -\left(\text{density of holes}\right) \bmod 2 \\ &= -\left(1 - n_c\right) = \left(1 + n_c\right) \bmod 2 \end{aligned}$$

Arguments for the Fermi surface volume of the FL phase

Alternatively:

Formulate Kondo lattice as the large U limit of the Anderson model

$$H = \sum_{i<j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_i \left(V c_{i\sigma}^\dagger f_{i\sigma} + V f_{i\sigma}^\dagger c_{i\sigma} + \varepsilon_f \left(n_{fi\uparrow} + n_{fi\downarrow} \right) + U n_{fi\uparrow} n_{fi\downarrow} \right) + \cdots$$

$$n_T = n_f + n_c$$

For small U, Fermi surface volume $= \left(n_f + n_c \right) \bmod 2.$

This is adiabatically connected to the large U limit where $n_f = 1$

Quantum critical point between SDW and FL phases

Spin fluctuations of renormalized S=1/2 fermionic quasiparticles, h_σ
(*loosely speaking*, T_K remains finite at the quantum critical point)

Gaussian theory of paramagnon fluctuations: $\vec{\phi} \sim h_\sigma^\dagger \vec{\tau}_{\sigma\sigma'} h_{\sigma'}$

Action: $$S = \int \frac{d^d q d\omega}{(2\pi)^{d+1}} \left|\vec{\phi}(q,\omega)\right|^2 \left(q^2 + |\omega| + \Gamma(\delta,T)\right)$$

J.A. Hertz, *Phys. Rev.* B **14**, 1165 (1976).

Characteristic paramagnon energy at finite temperature $\Gamma(0,T) \sim T^p$ with $p > 1$.

Arises from non-universal *corrections* to scaling, generated by $\vec{\phi}^4$ term.

J. Mathon, *Proc. R. Soc. London* A, **306**, 355 (1968);
T.V. Ramakrishnan, *Phys. Rev.* B **10**, 4014 (1974);
T. Moriya, *Spin Fluctuations in Itinerant Electron Magnetism*, Springer-Verlag, Berlin (1985)
G. G. Lonzarich and L. Taillefer, *J. Phys*. C **18**, 4339 (1985);
A.J. Millis, *Phys. Rev.* B **48**, 7183 (1993).

Quantum critical point between SDW and FL phases

Additional singular corrections to quasiparticle self energy in d=2

Ar. Abanov and A. V. Chubukov *Phys. Rev. Lett*. **84**, 5608 (2000);
A. Rosch *Phys. Rev*. B **64**, 174407 (2001).

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Additional corrections in dynamic mean field theory:
Q. Si, S. Rabello, K. Ingersent, and J. L. Smith, *Nature* **413**, 804 (2001)

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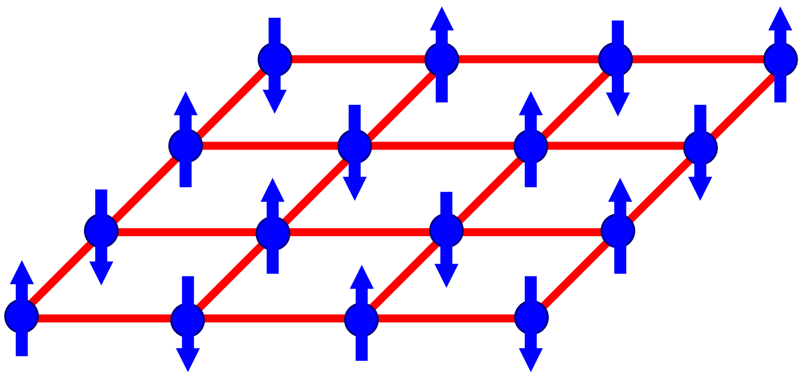
Ground states of quantum antiferromagnets

Begin with magnetically ordered states, and consider quantum transitions which restore spin rotation invariance

Two classes of ordered states:

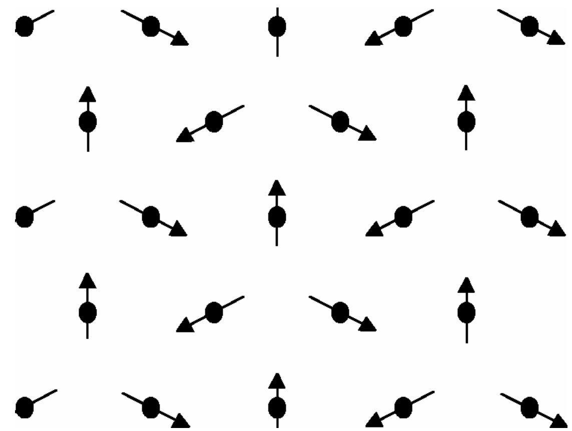
(α) Collinear spins



$$\left\langle \vec{S}(\boldsymbol{r}) \right\rangle \propto \vec{N} \cos(\boldsymbol{Q} \bullet \boldsymbol{r})$$

$$\boldsymbol{Q} = (\pi, \pi); \ \vec{N}^2 = 1$$

(β) Non-collinear spins



$$\left\langle \vec{S}(\boldsymbol{r}) \right\rangle \propto \vec{N}_1 \cos(\boldsymbol{Q} \bullet \boldsymbol{r}) + \vec{N}_2 \sin(\boldsymbol{Q} \bullet \boldsymbol{r})$$

$$\boldsymbol{Q} = \left(\frac{4\pi}{3}, \frac{4\pi}{\sqrt{3}} \right); \ \vec{N}_1^2 = \vec{N}_2^2 = 1 \, ; \ \vec{N}_1 \bullet \vec{N}_2 = 0$$

(α) Collinear spins, Berry phases, and bond-order

S=1/2 antiferromagnet on a bipartitie lattice

$$H = \sum_{i<j} J_{ij} \; \vec{S}_i \cdot \vec{S}_j$$

Include Berry phases after discretizing coherent state path integral on a cubic lattice in spacetime

$$Z = \prod_a \int d\boldsymbol{n}_a \delta\left(\boldsymbol{n}_a^2 - 1\right) \exp\left(\frac{1}{g} \sum_{a,\mu} \boldsymbol{n}_a \cdot \boldsymbol{n}_{a+\mu} - \frac{i}{2} \sum_a \eta_a A_{a\tau} \right)$$

$\eta_{\mathrm{a}} \rightarrow \pm 1$ on two sublattices ;

$\boldsymbol{n}_a \sim \eta_a \vec{S}_a \rightarrow$ Neel order parameter;

$A_{a\mu} \rightarrow$ oriented area of spherical triangle

formed by $\boldsymbol{n}_a$, $\boldsymbol{n}_{a+\mu}$, and an arbitrary reference point $\boldsymbol{n}_0$

Small g $\rightarrow$ Spin-wave theory about Neel state receives minor modifications from Berry phases.

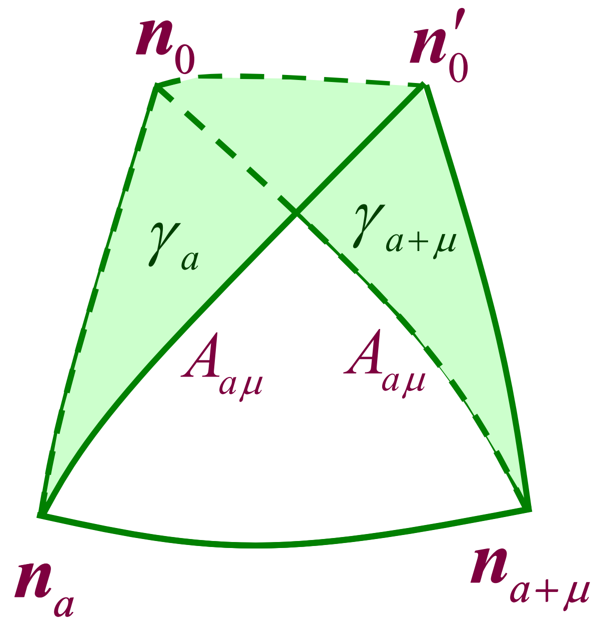
Large g $\rightarrow$ Berry phases are crucial in determining structure of "quantum-disordered" phase with $\langle \boldsymbol{n}_a \rangle = 0$

Integrate out $\boldsymbol{n}_a$ to obtain effective action for $A_{a\mu}$

Change in choice of $\boldsymbol{n}_0$ is like a "gauge transformation"

$$A_{a\mu} \rightarrow A_{a\mu} - \gamma_{a+\mu} + \gamma_a$$

(γ_a is the oriented area of the spherical triangle formed by $\boldsymbol{n}_a$ and the two choices for $\boldsymbol{n}_0$).



The area of the triangle is uncertain modulo 4π, and the action is invariant under

$$A_{a\mu} \rightarrow A_{a\mu} + 4\pi$$

These principles strongly constrain the effective action for $A_{a\mu}$

Simplest large g effective action for the $A_{a\mu}$

$$Z = \prod_{a,\mu} \int dA_{a\mu} \exp\left(-\frac{1}{2e^2} \sum_{\square} \cos\left(\frac{1}{2} \varepsilon_{\mu\nu\lambda} \Delta_\nu A_{a\lambda} \right) - \frac{i}{2} \sum_a \eta_a A_{a\tau} \right)$$

with $e^2 \sim g^2$

This is compact QED in $d+1$ dimensions with Berry phases.

This theory can be reliably analyzed by a duality mapping.

(I) $d=2$:

The gauge theory is always in a confining phase. There is an energy gap and the ground state has bond order (induced by the Berry phases).

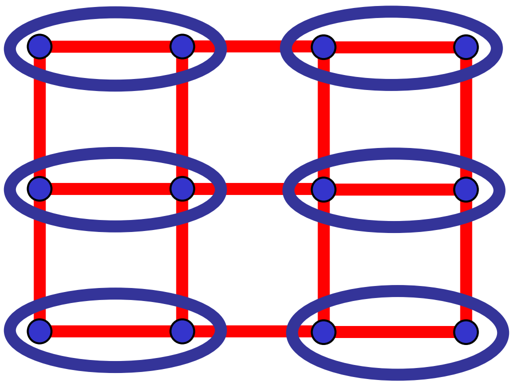
(II) $d=3$:

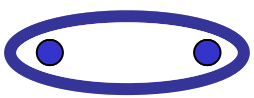
An additional Coulomb phase is also possible. There are deconfined spinons which are minimally coupled to a gapless U(1) photon.

N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).
S. Sachdev and R. Jalabert, *Mod. Phys. Lett*. B **4**, 1043 (1990).
K. Park and S. Sachdev, *Phys. Rev*. B **65**, 220405 (2002).

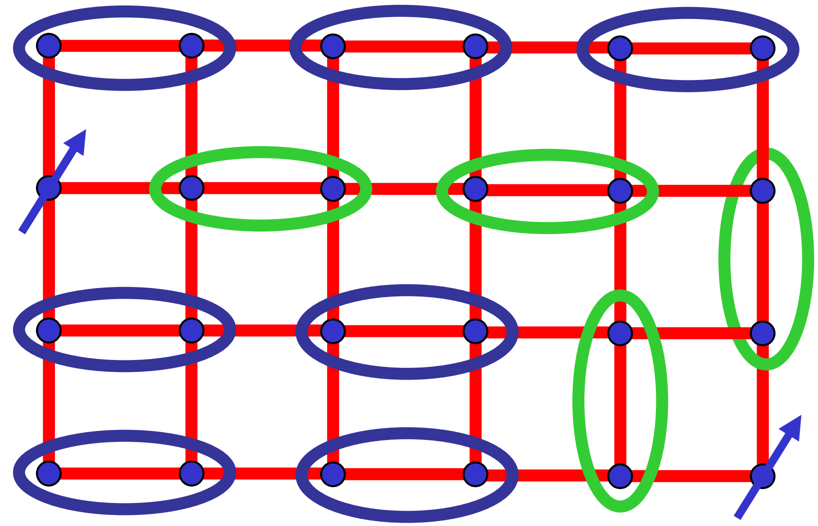
Paramagnetic states with $\left\langle \mathbf{S}_j \right\rangle = 0$

Bond order and confined spinons





$$= \frac{1}{\sqrt{2}}\left(\left| \uparrow \downarrow \right\rangle - \left| \downarrow \uparrow \right\rangle \right)$$

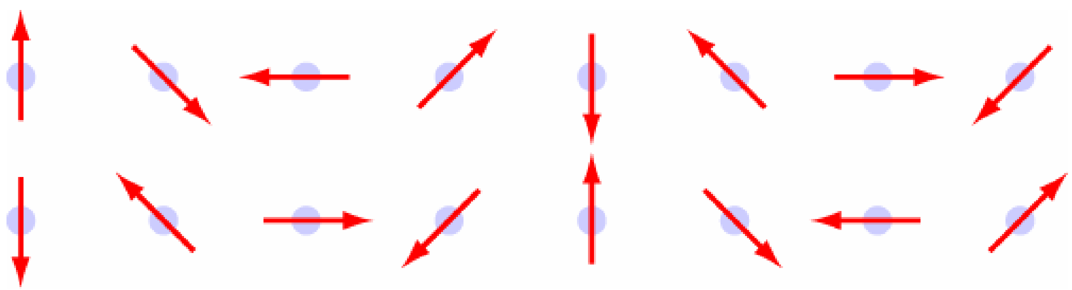


S=1/2 spinons are ***confined***
by a linear potential into a
S=1 spin exciton

Confinement is required U(1) paramagnets in d=2

β. Noncollinear spins

Magnetic order $\langle \mathbf{S}_j \rangle = \mathbf{N}_1 \cos\left(\vec{K} \, . \, \vec{r}_j\right) + \mathbf{N}_2 \sin\left(\vec{K} \, . \, \vec{r}_j\right)$



$$\vec{K} = (3\pi/4, \pi) \; ;$$

$$\mathbf{N}_2^2 = \mathbf{N}_1^2 \, , \, \mathbf{N}_1 . \mathbf{N}_2 = 0$$

Solve constraints by expressing $\mathbf{N}_{1,2}$ in terms of two complex numbers $z_\uparrow$, $z_\downarrow$

$$\mathbf{N}_1 + i\mathbf{N}_2 = \begin{pmatrix} z_\downarrow^2 - z_\uparrow^2 \\ i\left(z_\downarrow^2 + z_\uparrow^2\right) \\ 2z_\uparrow z_\downarrow \end{pmatrix}$$

Order in ground state specified by a spinor $\left(z_\uparrow, z_\downarrow\right)$ (modulo an overall sign).

This spinor could become a S=1/2 spinon in a quantum "disordered" state.

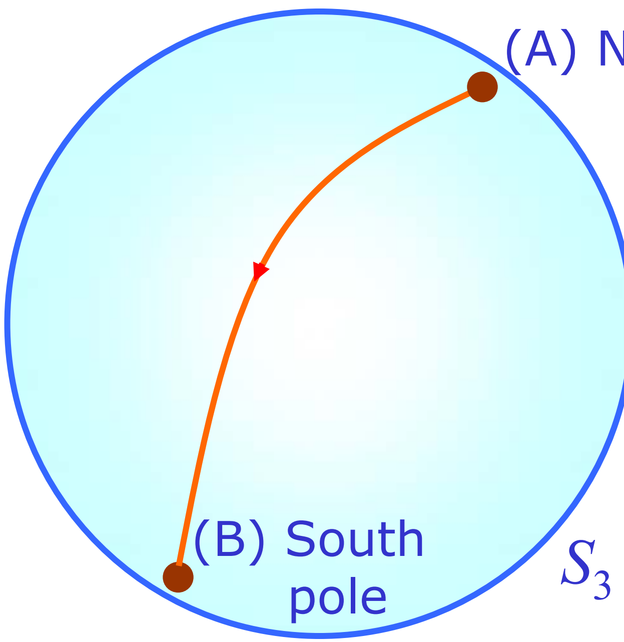
Order parameter space: S_3/Z_2

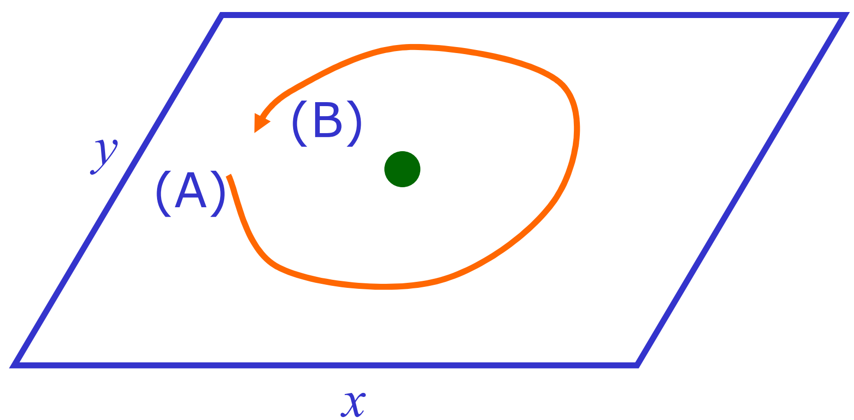
Physical observables are invariant under the Z_2 gauge transformation $z_a \rightarrow \pm z_a$

β. Noncollinear spins

Paramagnetic state $\left\langle \boldsymbol{S}_j \right\rangle = 0$

Vortices associated with $\pi_1(S_3/Z_2)=Z_2$ (*visons*)





Such vortices (visons) can also be defined in the phase in which spins are "quantum disordered". A Z_2 spin liquid with deconfined spinons must have ***visons supressed***

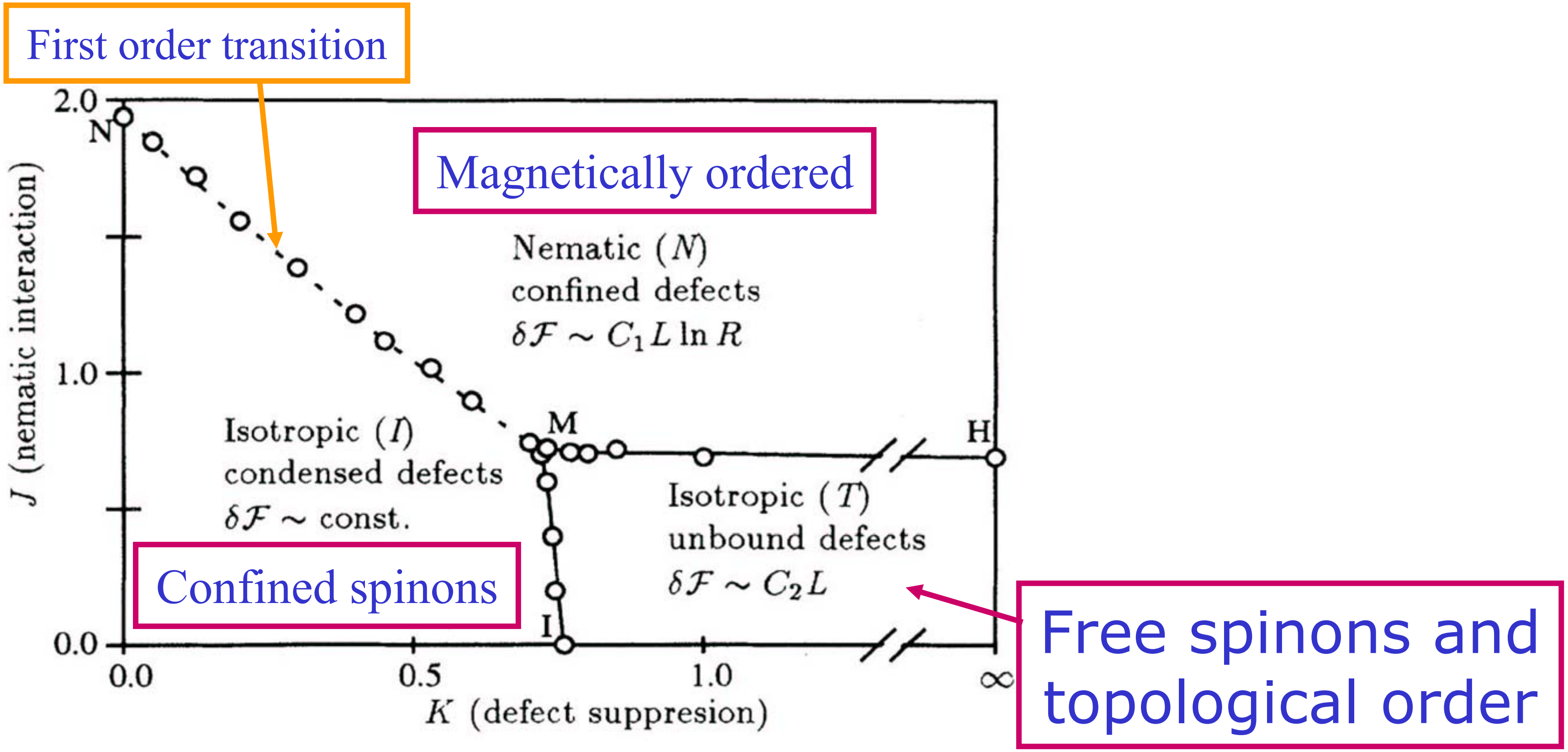
N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991)

Model effective action and phase diagram

$$S = -J \sum_{\langle ij \rangle} \sigma_{ij} z^*_{\alpha i} z_{\alpha j} + \text{h.c.} - K \sum_{\square} \prod_{\square} \sigma_{ij}$$

(Derivation using Schwinger bosons on a quantum antiferromagnet: S. Sachdev and N. Read, *Int. J. Mod. Phys*. B **5**, 219 (1991)).

$$\sigma_{ij} \to Z_2 \text{ gauge field}$$



P. E. Lammert, D. S. Rokhsar, and J. Toner, *Phys. Rev. Lett*. **70**, 1650 (1993) ; *Phys. Rev.* E **52**, 1778 (1995). (For nematic liquid crystals)

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III. Doping spin liquids

Reconsider Doniach phase diagram

It is more convenient to analyze the Kondo-Heiseberg model:

$$H = \sum_{i<j} t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + \sum_{i} \left(J_K c_{i\sigma}^{\dagger} \vec{\tau}_{\sigma\sigma'} c_{i\sigma} \cdot \vec{S}_{fi} \right) + \sum_{i<j} J_H \left(i, j \right) \vec{S}_{fi} \cdot \vec{S}_{fj}$$

Work in the regime $J_H > J_K$

Determine the ground state of the quantum antiferromagnet defined by J_H, and then couple to conduction electrons by J_K

Choose J_H so that ground state of antiferromagnet is a spin liquid

State of conduction electrons

At $J_K = 0$ the conduction electrons form a Fermi surface on their own with volume determined by n_c

Perturbation theory in J_K is regular, and topological order is robust, and so this state will be stable for finite J_K

So volume of Fermi surface is determined by **$(n_T - 1) = n_c (\text{mod } 2)$,** and Luttinger's theorem is violated.

The FL* state

III. Doping spin liquids

A likely possibility:

Added electrons do *not* fractionalize, but retain their bare quantum numbers.
Spinon, photon, and vison states of the insulator survive unscathed.
There is a Fermi surface of *sharp electron-like* quasiparticles, enclosing a volume determined by the dopant electron alone.

This is a "Fermi liquid" state which violates Luttinger's theorem

A Fractionalized Fermi Liquid (FL*)

T. Senthil, S. Sachdev, and M. Vojta, cond-mat/0209144

III. A new phase: FL*

This phase preserves spin rotation invariance, and has a Fermi surface of *sharp* electron-like quasiparticles.

The state has "*topological order*" and associated neutral excitations. The topological order can be easily detected by the violation of Luttinger's theorem. It can only appear in dimensions $d > 1$

$$2 \times \frac{v_0}{(2\pi)^d} \left(\text{Volume enclosed by Fermi surface} \right)$$

$$= \left(n_T - 1 \right) \left(\text{mod } 2 \right)$$

Precursors: L. Balents and M. P. A. Fisher and C. Nayak, *Phys. Rev.* B **60**, 1654, (1999);
T. Senthil and M.P.A. Fisher, Phys. Rev. B **62**, 7850 (2000);
S. Burdin, D. R. Grempel, and A. Georges, *Phys. Rev.* B **66**, 045111 (2002).

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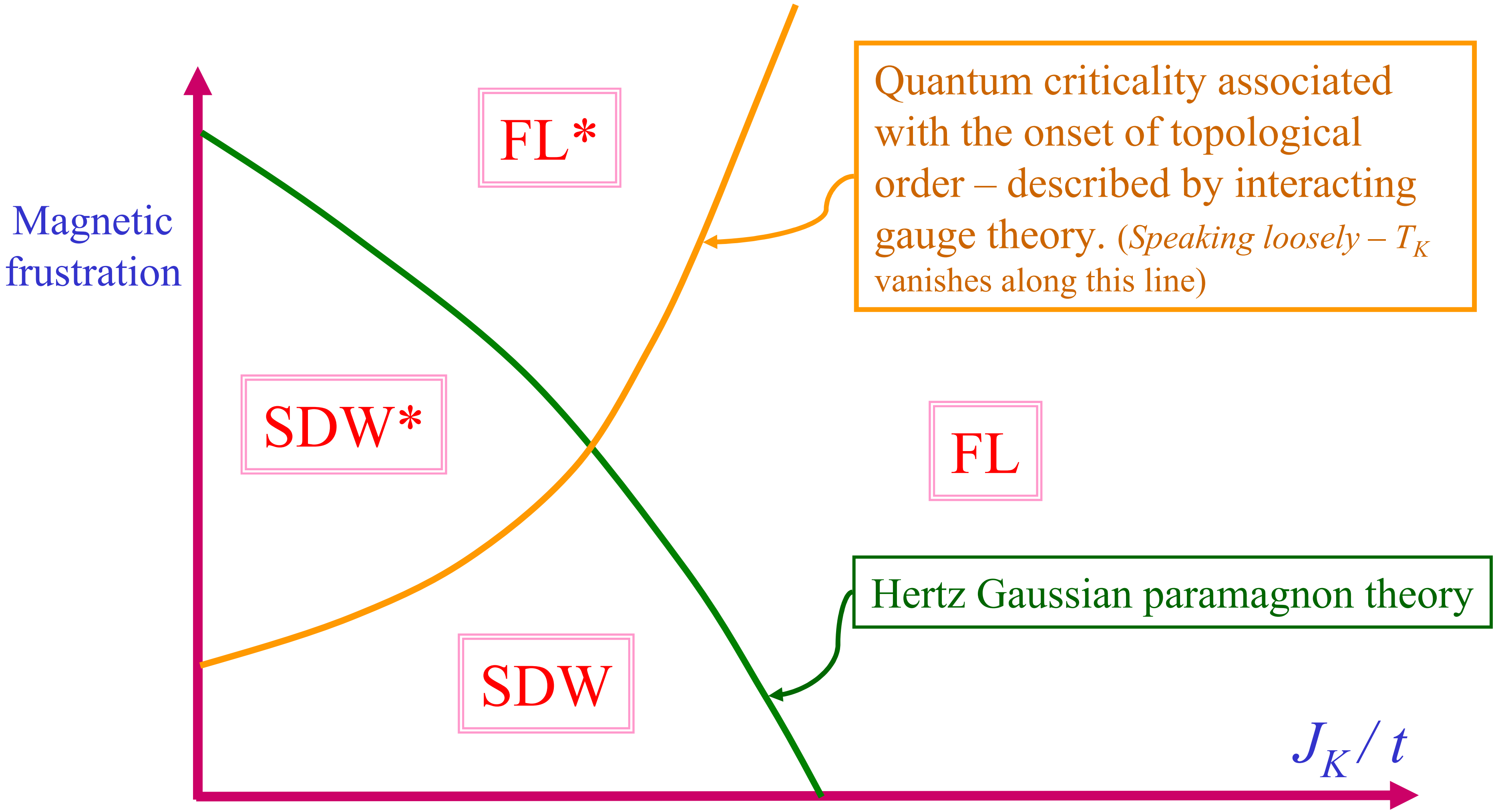
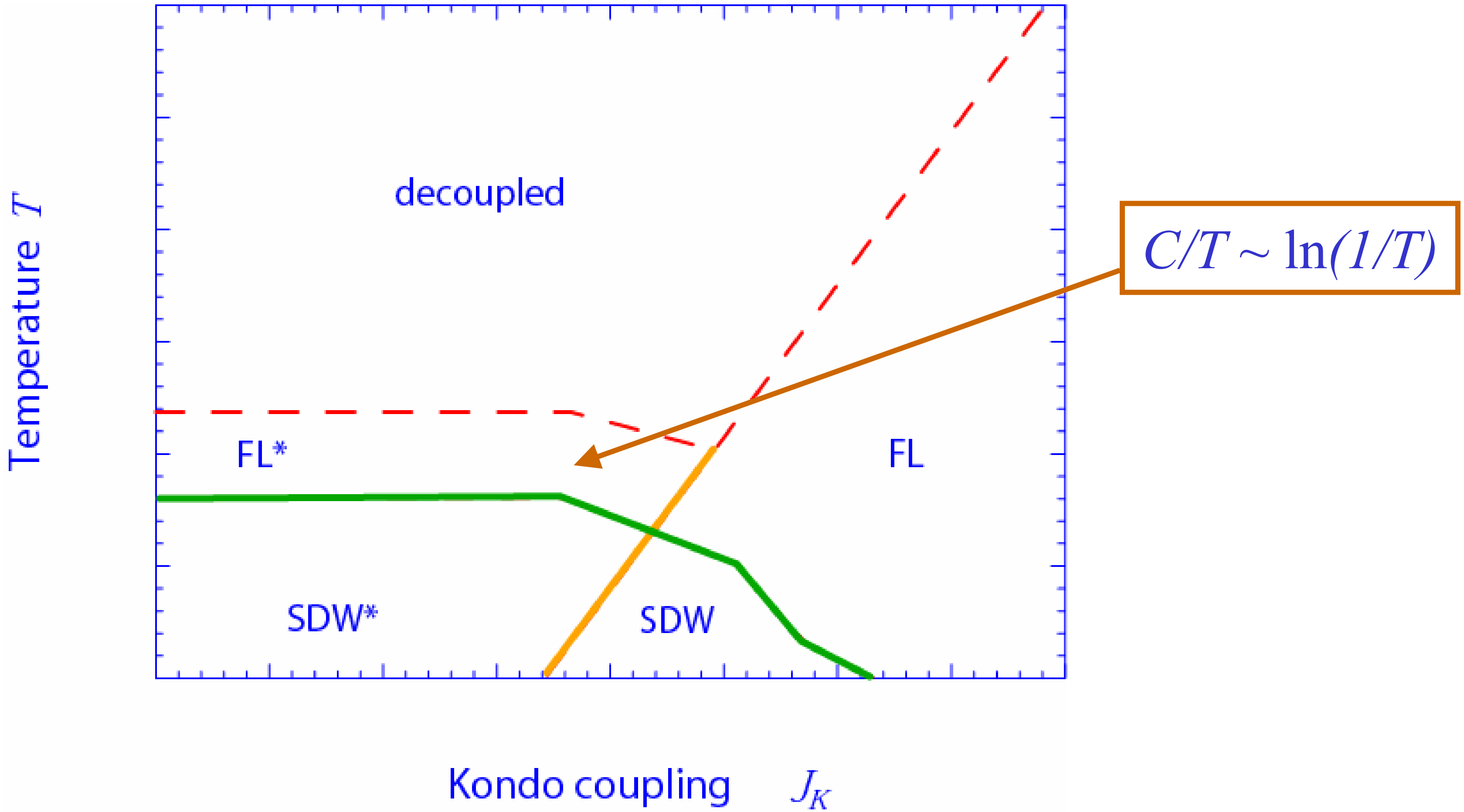
IV. Extended *T=0* phase diagram for the Kondo lattice



- $*$ phases have spinons with Z_2 (d=2,3) or U(1) (d=3) gauge charges, and associated gauge fields.
- Fermi surface volume does *not* distinguish SDW and SDW* phases.

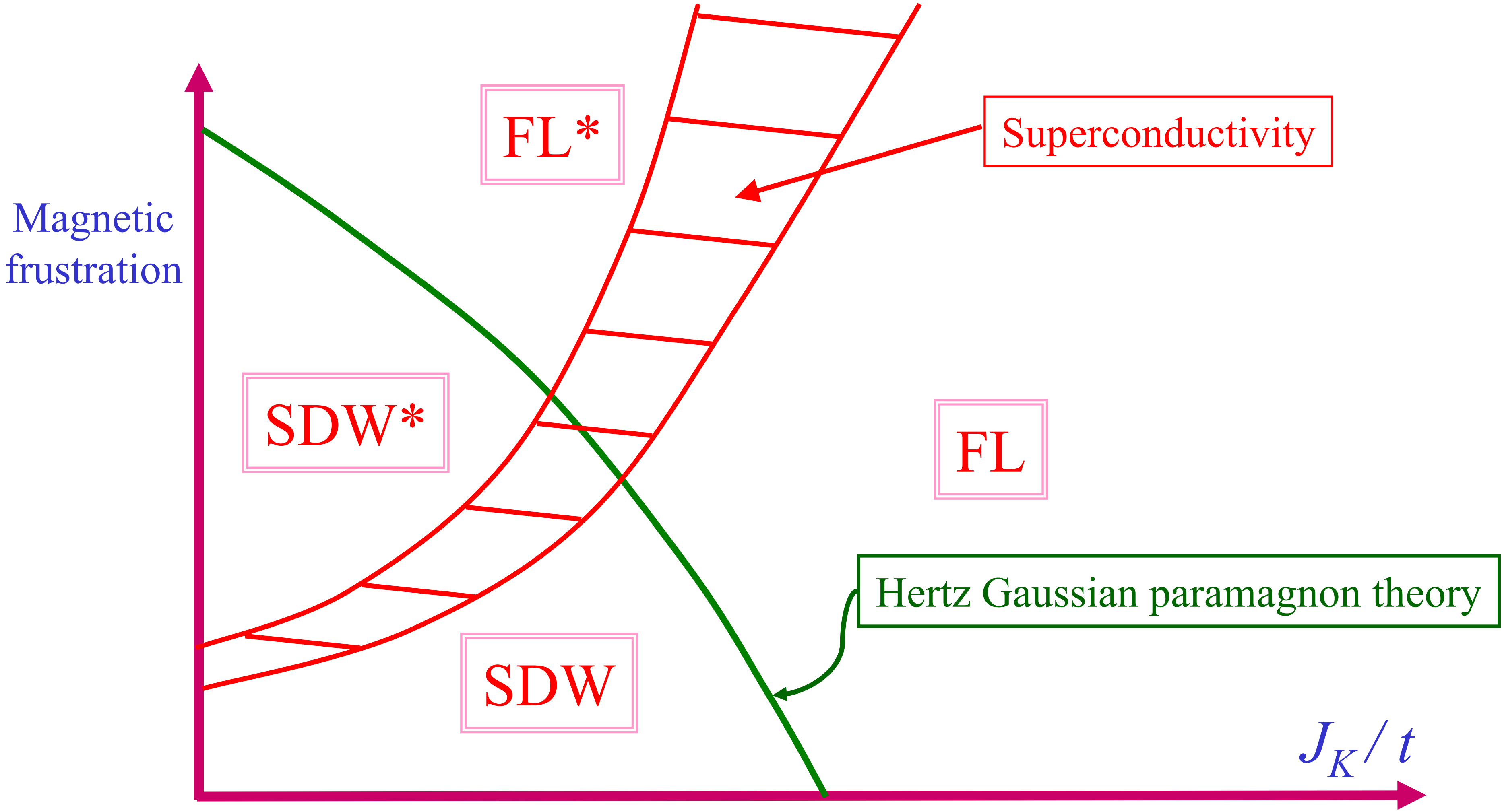
U(1) fractionalization (d=3)

Mean-field phase diagram



- Because of strong gauge fluctuations, U(1)-FL* may be unstable to U(1)-SDW* at low temperatures on certain lattices.
- Quantum criticality dominated by a T=0 FL-FL* transition.

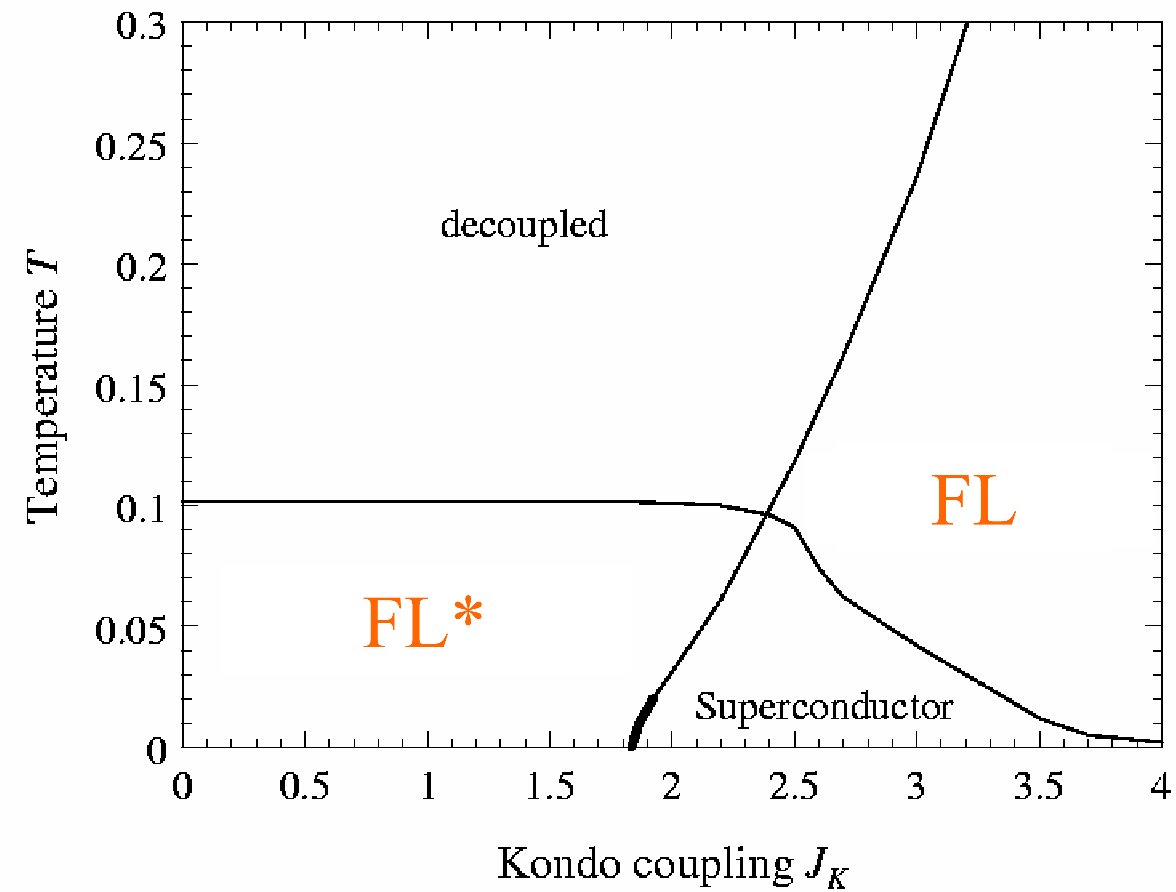
Z_2 fractionalization



• **Superconductivity is generic between FL and Z_2 FL* phases.**

Z_2 fractionalization

Mean-field phase diagram



Pairing of spinons in small Fermi surface state induces superconductivity at the confinement transition

Small Fermi surface state can also exhibit a second-order metamagnetic transition in an applied magnetic field, associated with vanishing of a spinon gap.

Conclusions

- New phase diagram as a paradigm for clean metals with local moments.
- Topologically ordered (*) phases lead to novel quantum criticality.
- New FL* allows easy detection of topological order by Fermi surface volume