Quantum phases and critical points of correlated metals

T. Senthil (MIT)
Subir Sachdev
Matthias Vojta (Karlsruhe)

cond-mat/0209144

paper rejected by cond-mat

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Outline

I. Kondo lattice models

  Doniach’s phase diagram and its quantum critical point

II. Paramagnetic states of quantum antiferromagnets:

  (A) Confinement of spinons and bond order
  (B) Spin liquids with deconfined spinons: $Z_2$ and $U(1)$ gauge theories

III. A new phase: a fractionalized Fermi liquid (FL*)

IV. Extended phase diagram and its critical points

V. Conclusions
I. Doniach’s $T=0$ phase diagram for the Kondo lattice

\[ H = \sum_{i<j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_i \left( J_K c_{i\sigma}^\dagger \vec{\tau}_{\sigma\sigma} c_{i\sigma} \cdot \vec{S}_{fi} \right) \]

$c_{i\sigma} \rightarrow$ Conduction electrons;

$\vec{S}_{fi} \rightarrow$ localized $f_{i\sigma}$ moments (assumed $S=1/2$, for specificity)

Local moments choose some static spin arrangement

$J_{RKKY} \sim J_K^2 / t \gg T_K \sim \exp\left(-t / J_K\right)$

“Heavy” Fermi liquid with moments Kondo screened by conduction electrons. Fermi surface obeys Luttinger’s theorem.
Luttinger’s theorem on a $d$-dimensional lattice for the FL phase

Let $\nu_0$ be the volume of the unit cell of the ground state, $n_T$ be the total number density of electrons per volume $\nu_0$. (need not be an integer)

$$n_T = n_f + n_c = 1 + n_c$$

$$2 \times \frac{\nu_0}{(2\pi)^d} \text{(Volume enclosed by Fermi surface)} = n_T \text{ (mod 2)}$$

A “large” Fermi surface
Arguments for the Fermi surface volume of the FL phase

Single ion Kondo effect implies $J_K \to \infty$ at low energies

\[
\left( c_{i\uparrow}^+ f_{i\uparrow}^+ - c_{i\downarrow}^+ f_{i\uparrow}^+ \right) |0\rangle
\]

\[
f_{i\downarrow}^+ |0\rangle, \ S=1/2 \ hole
\]

Fermi liquid of $S=1/2$ holes with hard-core repulsion

Fermi surface volume $= - (\text{density of holes}) \mod 2$

$= - (1 - n_c) = (1 + n_c) \mod 2$
Arguments for the Fermi surface volume of the FL phase

Alternatively:

Formulate Kondo lattice as the large $U$ limit of the Anderson model

$$H = \sum_{i<j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_i \left( V c_{i\sigma}^\dagger f_{i\sigma} + V f_{i\sigma}^\dagger c_{i\sigma} + \varepsilon_f \left( n_{fi\uparrow} + n_{fi\downarrow} \right) + Un_{fi\uparrow} n_{fi\downarrow} \right) + \cdots$$

$$n_T = n_f + n_c$$

For small $U$, Fermi surface volume = $(n_f + n_c) \mod 2$. This is adiabatically connected to the large $U$ limit where $n_f = 1$
Quantum critical point between SDW and FL phases

Spin fluctuations of renormalized $S=1/2$ fermionic quasiparticles, $h_\sigma$ (loosely speaking, $T_K$ remains finite at the quantum critical point)

**Gaussian** theory of paramagnon fluctuations: $\vec{\phi} \sim h_\sigma^+ \tau_{\sigma\sigma} h_\sigma$.

Action: $S = \int \frac{d^dq d\omega}{(2\pi)^{d+1}} |\vec{\phi}(q,\omega)|^2 \left( q^2 + |\omega| + \Gamma(\delta,T) \right)$


Characteristic paramagnon energy at finite temperature $\Gamma(0,T) \sim T^p$ with $p > 1$.

Arises from non-universal *corrections* to scaling, generated by $\vec{\phi}^4$ term.

Quantum critical point between SDW and FL phases

Additional singular corrections to quasiparticle self energy in $d=2$

Ar. Abanov and A. V. Chubukov *Phys. Rev. Lett.* **84**, 5608 (2000);

Additional corrections in dynamic mean field theory:
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Ground states of quantum antiferromagnets

Begin with magnetically ordered states, and consider quantum transitions which restore spin rotation invariance

Two classes of ordered states:

(α) Collinear spins

\[
\langle \vec{S}(r) \rangle \propto \overline{N} \cos(\vec{Q} \cdot \vec{r})
\]

\[
\vec{Q} = (\pi, \pi); \; \overline{N}^2 = 1
\]

(β) Non-collinear spins

\[
\langle \vec{S}(r) \rangle \propto \overline{N}_1 \cos(\vec{Q} \cdot \vec{r}) + \overline{N}_2 \sin(\vec{Q} \cdot \vec{r})
\]

\[
\vec{Q} = \left( \frac{4\pi}{3}, \frac{4\pi}{\sqrt{3}} \right); \; \overline{N}_1^2 = \overline{N}_2^2 = 1; \; \overline{N}_1 \cdot \overline{N}_2 = 0
\]
(α) **Collinear spins, Berry phases, and bond-order**

\[ S=1/2 \text{ antiferromagnet on a bipartite lattice} \]

\[ H = \sum_{i<j} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Include Berry phases after discretizing coherent state path integral on a cubic lattice in spacetime

\[ Z = \prod_a \int dn_a \delta \left( n_a^2 - 1 \right) \exp \left( \frac{1}{g} \sum_{a,\mu} n_a \cdot n_{a+\mu} - \frac{i}{2} \sum_a \eta_a A_{a\tau} \right) \]

\( \eta_a \rightarrow \pm 1 \) on two sublattices;

\( n_a \sim \eta_a \hat{S}_a \rightarrow \text{Neel order parameter;} \]

\( A_{a\mu} \rightarrow \text{oriented area of spherical triangle} \)

formed by \( n_a, n_{a+\mu} \), and an arbitrary reference point \( n_0 \).
Small $g \rightarrow$ Spin-wave theory about Neel state receives minor modifications from Berry phases.

Large $g \rightarrow$ Berry phases are crucial in determining structure of "quantum-disordered" phase with $\langle n_a \rangle = 0$

Integrate out $n_a$ to obtain effective action for $A_{a\mu}$

Change in choice of $n_0$ is like a "gauge transformation"

$$A_{a\mu} \rightarrow A_{a\mu} - \gamma_{a+\mu} + \gamma_a$$

($\gamma_a$ is the oriented area of the spherical triangle formed by $n_a$ and the two choices for $n_0$).

The area of the triangle is uncertain modulo $4\pi$, and the action is invariant under

$$A_{a\mu} \rightarrow A_{a\mu} + 4\pi$$

These principles strongly constrain the effective action for $A_{a\mu}$
Simplest large \( g \) effective action for the \( A_{a\mu} \)

\[
Z = \prod_{a,\mu} \int dA_{a\mu} \exp \left( -\frac{1}{2e^2} \sum \cos \left( \frac{1}{2} \varepsilon_{\mu\nu\lambda} \Delta_{\nu} A_{a\lambda} \right) - \frac{i}{2} \sum_{a} \eta_{a} A_{a\tau} \right)
\]

with \( e^2 \sim g^2 \)

This is compact QED in \( d+1 \) dimensions with Berry phases.

This theory can be reliably analyzed by a duality mapping.

(I) \( d=2 \):
The gauge theory is always in a confining phase. There is an energy gap and the ground state has bond order (induced by the Berry phases).

(II) \( d=3 \):
An additional Coulomb phase is also possible. There are deconfined spinons which are minimally coupled to a gapless U(1) photon.

Paramagnetic states with $\langle S_j \rangle = 0$

Bond order and confined spinons

\[ = \frac{1}{\sqrt{2}} \left( \left| \uparrow \downarrow \right\rangle - \left| \downarrow \uparrow \right\rangle \right) \]

$S=1/2$ spinons are confined by a linear potential into a $S=1$ spin exciton

Confinement is required $U(1)$ paramagnets in $d=2$
**β. Noncollinear spins**

\[
\langle S_j \rangle = N_1 \cos(\vec{K} \cdot \vec{r}_j) + N_2 \sin(\vec{K} \cdot \vec{r}_j)
\]

\[
\vec{K} = \left( \frac{3\pi}{4}, \pi \right) ;
\]

\[
N_2^2 = N_1^2 , N_1 \cdot N_2 = 0
\]

Solve constraints by expressing \( N_{1,2} \) in terms of two complex numbers \( z_\uparrow, z_\downarrow \)

\[
N_1 + iN_2 = \begin{pmatrix}
  z_\downarrow^2 - z_\uparrow^2 \\
  i(z_\downarrow^2 + z_\uparrow^2) \\
  2z_\uparrow z_\downarrow
\end{pmatrix}
\]

Order in ground state specified by a spinor \( (z_\uparrow, z_\downarrow) \) (modulo an overall sign).

This spinor could become a \( S=1/2 \) spinon in a quantum "disordered" state.

Order parameter space: \( S_3/Z_2 \)

Physical observables are invariant under the \( Z_2 \) gauge transformation \( z_a \rightarrow \pm z_a \)
β. Noncollinear spins

Paramagnetic state \( \langle \vec{S}_j \rangle = 0 \)

Vortices associated with \( \pi_1(S_3/Z_2) = \mathbb{Z}_2 \) \((\text{visons})\)

Such vortices (visons) can also be defined in the phase in which spins are "quantum disordered". A \( \mathbb{Z}_2 \) spin liquid with deconfined spinons must have \textit{visons suppressed}

Model effective action and phase diagram

\[ S = -J \sum_{\langle ij \rangle} \sigma_{ij} \bar{z}_\alpha z_{\alpha j} + \text{h.c.} - K \prod \sigma_{ij} \]


\[ \sigma_{ij} \rightarrow Z_2 \text{ gauge field} \]

First order transition

Magnetically ordered

Confined spinons

Free spinons and topological order

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III. **Doping spin liquids**

Reconsider Doniach phase diagram

It is more convenient to analyze the Kondo-Heiseberg model:

\[
H = \sum_{i<j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_i \left( J_K c_{i\sigma}^\dagger \tau_{\sigma\sigma} c_{i\sigma} \cdot \mathbf{S}_i \right) + \sum_{i<j} J_H (i, j) \mathbf{S}_i \cdot \mathbf{S}_j
\]

**Work in the regime** \( J_H > J_K \)

Determine the ground state of the quantum antiferromagnet defined by \( J_H \), and then couple to conduction electrons by \( J_K \)

Choose \( J_H \) so that ground state of antiferromagnet is a spin liquid
State of conduction electrons

At $J_K = 0$ the conduction electrons form a Fermi surface on their own with volume determined by $n_c$

Perturbation theory in $J_K$ is regular, and topological order is robust, and so this state will be stable for finite $J_K$

So volume of Fermi surface is determined by $(n_T - 1) = n_c \pmod{2}$, and Luttinger’s theorem is violated.

The FL* state
III. Doping spin liquids

A likely possibility:

Added electrons do not fractionalize, but retain their bare quantum numbers. Spinon, photon, and vison states of the insulator survive unscathed.

There is a Fermi surface of sharp electron-like quasiparticles, enclosing a volume determined by the dopant electron alone.

This is a “Fermi liquid” state which violates Luttinger’s theorem

A Fractionalized Fermi Liquid (FL*)

T. Senthil, S. Sachdev, and M. Vojta, cond-mat/0209144
III. A new phase: FL*

This phase preserves spin rotation invariance, and has a Fermi surface of *sharp* electron-like quasiparticles.

The state has “*topological order*” and associated neutral excitations. The topological order can be easily detected by the violation of Luttinger’s theorem. It can only appear in dimensions $d > 1$

$$2 \times \frac{\nu_0}{(2\pi)^d} \left( \text{Volume enclosed by Fermi surface} \right) = (n_T - 1) \pmod{2}$$

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IV. Extended $T=0$ phase diagram for the Kondo lattice

Quantum criticality associated with the onset of topological order – described by interacting gauge theory. (Speaking loosely – $T_K$ vanishes along this line)

- * phases have spinons with $Z_2$ ($d=2,3$) or U(1) ($d=3$) gauge charges, and associated gauge fields.
- Fermi surface volume does not distinguish SDW and SDW* phases.
Because of strong gauge fluctuations, U(1)-FL* may be unstable to U(1)-SDW* at low temperatures on certain lattices. Quantum criticality dominated by a $T=0$ FL-FL* transition.
Superconductivity is generic between FL and \( Z_2 \) FL* phases.

- Magnetic frustration

\( J_K / t \)

\( Z_2 \) fractionalization

Hertz Gaussian paramagnon theory

- Superconductivity is generic between FL and \( Z_2 \) FL* phases.
Pairing of spinons in small Fermi surface state induces superconductivity at the confinement transition.

Small Fermi surface state can also exhibit a second-order metamagnetic transition in an applied magnetic field, associated with vanishing of a spinon gap.
Conclusions

• New phase diagram as a paradigm for clean metals with local moments.
• Topologically ordered (*)& phases lead to novel quantum criticality.
• New FL* allows easy detection of topological order by Fermi surface volume

![Phase Diagram]

- FL*
- SDW*
- FL
- SDW

Magnetic frustration

$J_K/t$