Quantum phase transitions in antiferromagnets and *d*-wave superconductors

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Matthias Vojta (Augsburg)
Ying Zhang

Science **286**, 2479 (1999).

Transparencies online at
http://pantheon.yale.edu/~subir

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Outline

I. Theoretical Models of Quantum Phase Transitions

   A. Quantum Ising Chain

   B. Coupled Ladder Antiferromagnet

   C. Square Lattice Antiferromagnet

II. Magnetic transitions in a *d*-wave superconductor

   Survey of some recent experiments on the high temperature superconductors.

III. Damping of nodal quasiparticles.

   Non-magnetic quantum transitions in *d*-wave superconductors

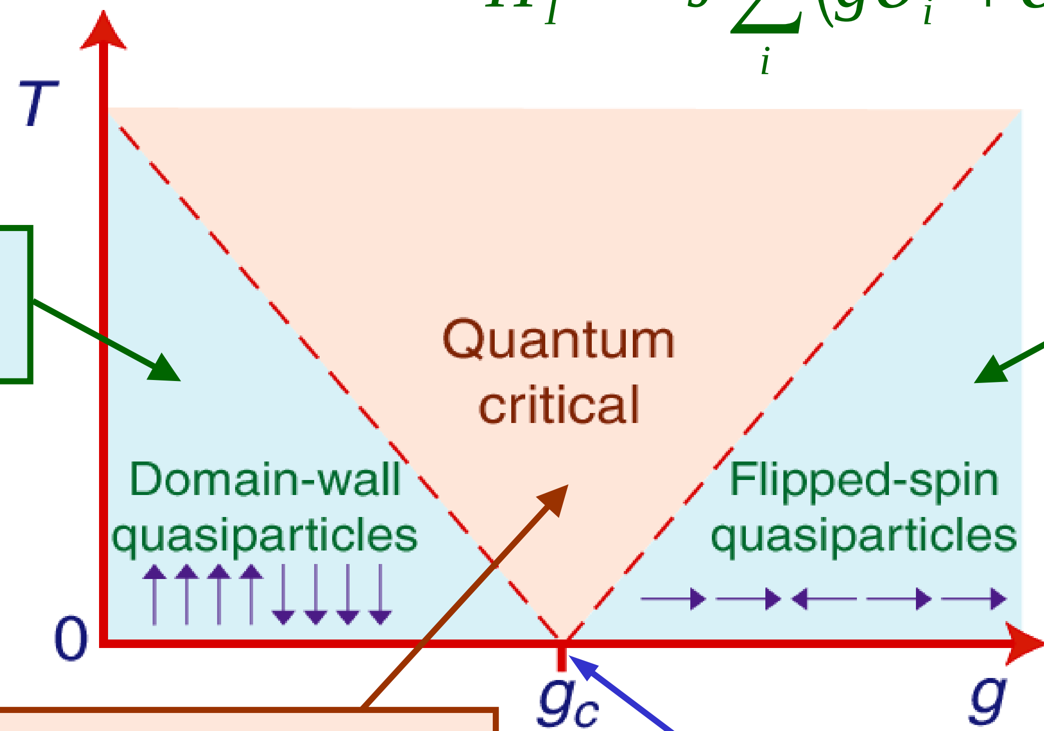
IV. Conclusions

I.A Quantum Ising Chain

$$H_I = -J\sum_i \left(g\sigma_i^x + \sigma_i^z \sigma_{i+1}^z \right)$$

Quasiclassical dynamics

Quasiclassical dynamics



$$|\rightarrow\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle \right)$$

$$|\leftarrow\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle - |\downarrow\rangle \right)$$

$$\chi(\omega) = \frac{i}{\hbar}\sum_k \int_0^\infty dt \left\langle \left[\sigma_j^z(t), \sigma_k^z(0) \right] \right\rangle e^{i\omega t}$$

$$= \frac{A}{T^{7/4}\left(1 - i\omega/\Gamma_R + \ldots\right)}$$

$$\Gamma_R = \left(2\tan\frac{\pi}{16} \right) \frac{k_B T}{\hbar}$$

$$\left\langle \sigma_j^z \sigma_k^z \right\rangle \sim \frac{1}{\left| j-k \right|^{1/4}}$$

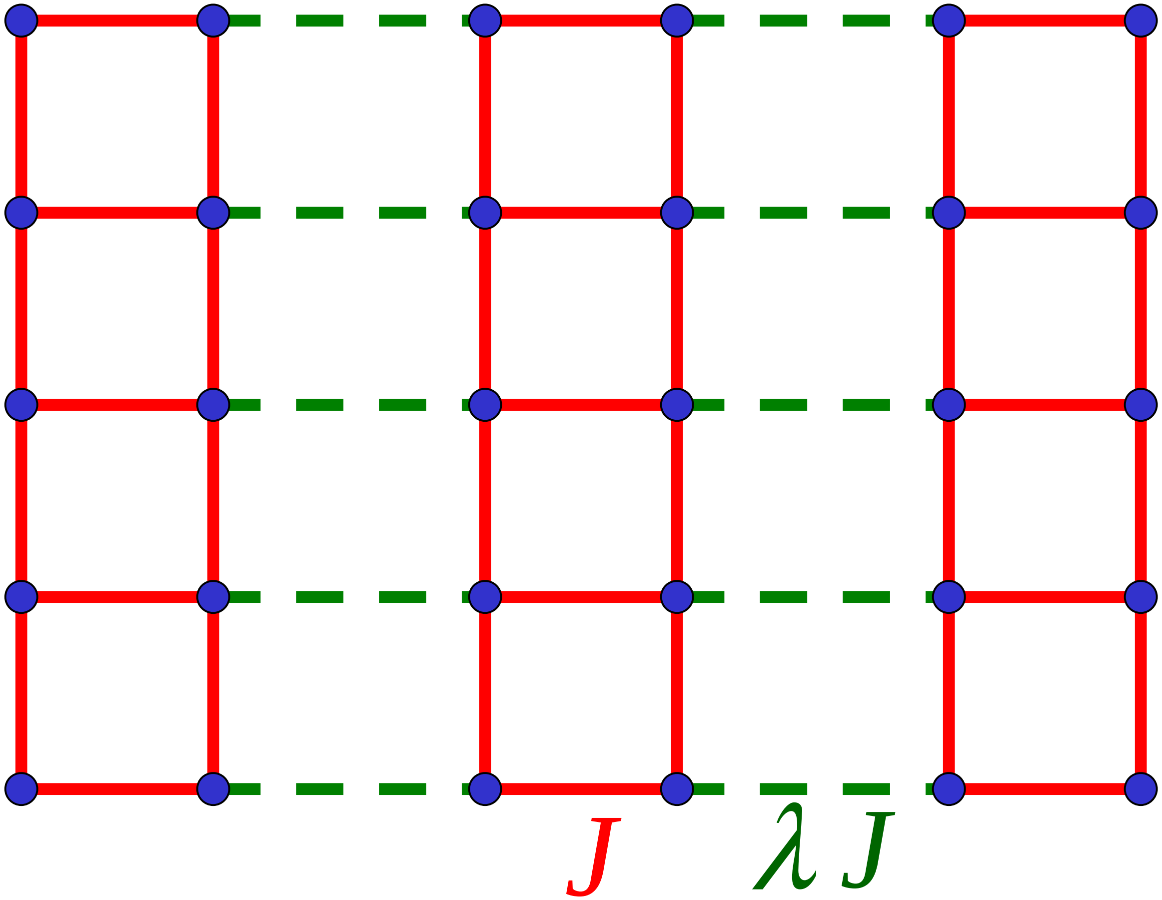
P. Pfeuty *Annals of Physics*, **57**, 79 (1970)

S. Sachdev and J. Ye, Phys. Rev. Lett. **69**, 2411 (1992).
S. Sachdev and A.P. Young, Phys. Rev. Lett. **78**, 2220 (1997).

I.B Coupled Ladder Antiferromagnet

N. Katoh and M. Imada, J. Phys. Soc. Jpn. **63**, 4529 (1994).
J. Tworzydlo, O. Y. Osman, C. N. A. van Duin, J. Zaanen, Phys. Rev. B **59**, 115 (1999).

S=1/2 spins on coupled 2-leg ladders



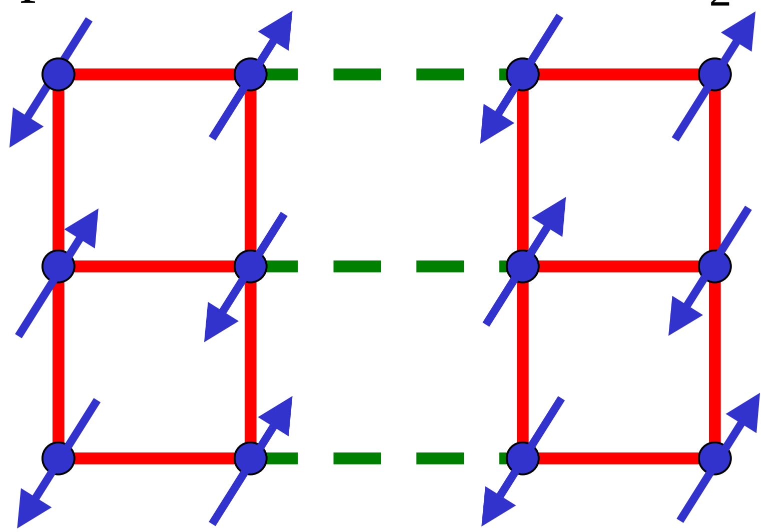
$$H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$0 \leq \lambda \leq 1$$

λ close to 1

Square lattice antiferromagnet

Experimental realization: La_2CuO_4



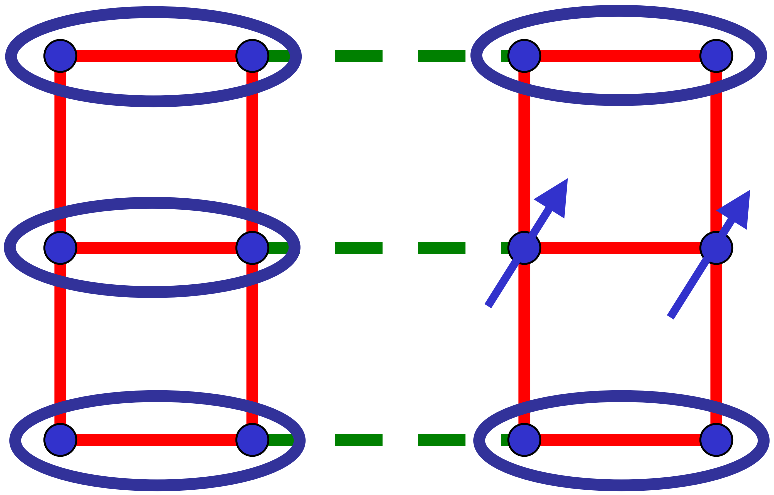
Ground state has long-range magnetic (Neel) order

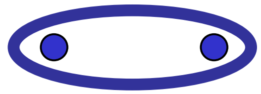
$$\left\langle \vec{S}_i \right\rangle = (-1)^{i_x+i_y} N_0 \neq 0$$

Excitations: 2 spin waves

λ close to 0

Weakly coupled ladders





$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

Paramagnetic ground state

$$\left\langle \vec{S}_i \right\rangle = 0$$

Excitation: S=1 ***exciton*** (spin collective mode)

Energy dispersion away from antiferromagnetic wavevector

$$\varepsilon = \Delta + \frac{c^2 k^2}{2\Delta}$$

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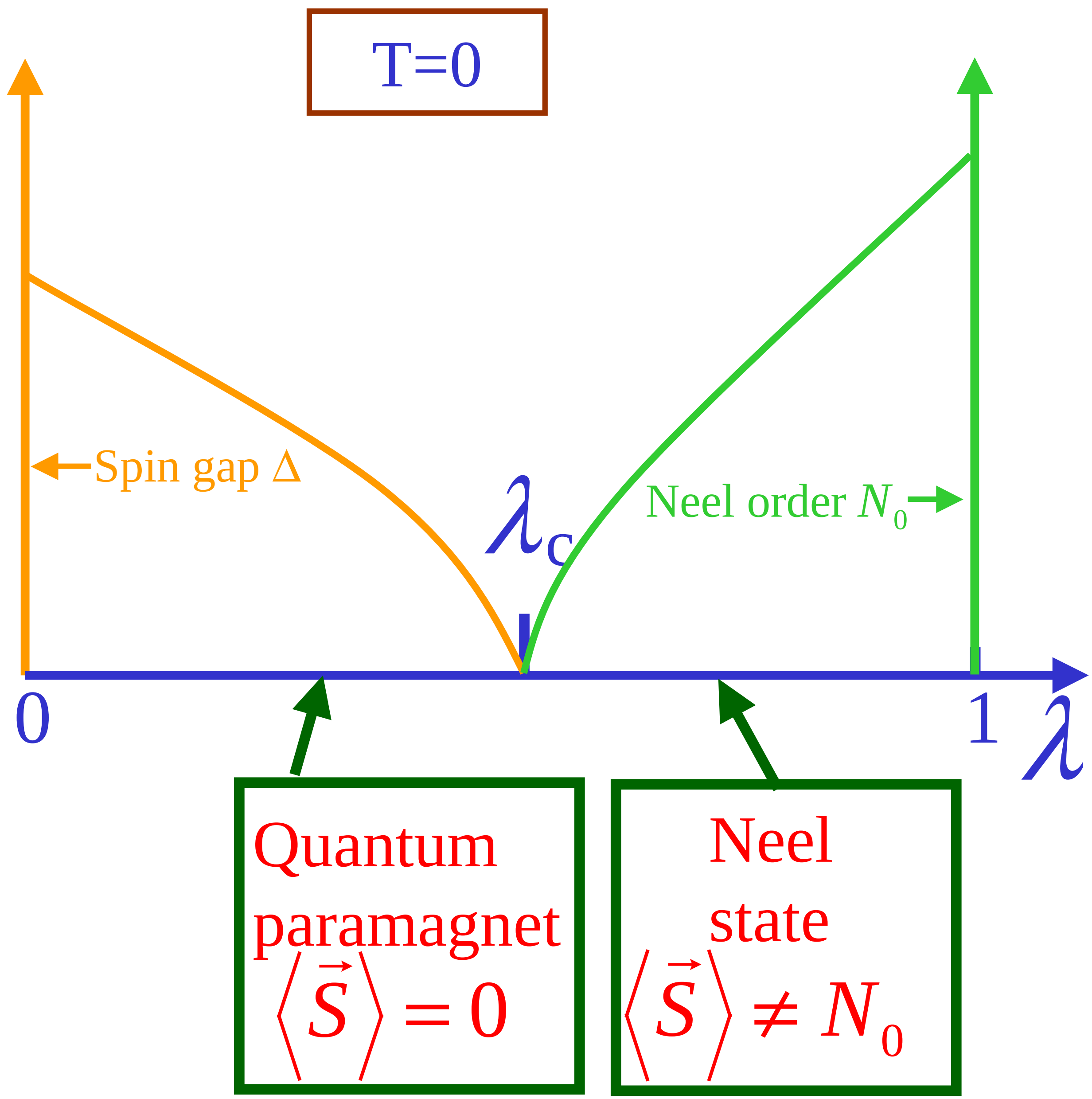

T=0
Spin gap Δ
λ_c
Neel order N_0
0
1
λ
Quantum paramagnet
$\langle \vec{S} \rangle = 0$
Neel state
$\langle \vec{S} \rangle \neq N_0$

Quantum field theory:

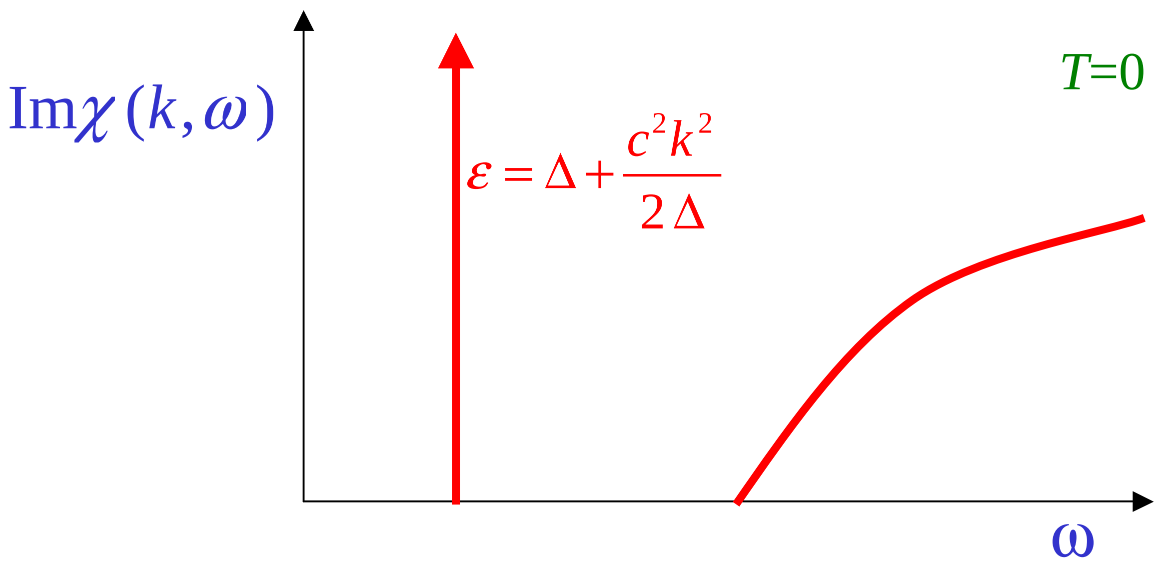
λ close to λ_c

$$S_b = \int d^2x d\tau \left[\frac{1}{2}\left(\left(\nabla_x \phi_\alpha \right)^2 + c^2 \left(\partial_\tau \phi_\alpha \right)^2 + r\phi_\alpha^2 \right) + \frac{g}{4!}\left(\phi_\alpha^2 \right)^2 \right]$$

ϕ_α → 3-component antiferromagnetic order parameter

$r > 0$	→	$\lambda < \lambda_c$
$r < 0$	→	$\lambda > \lambda_c$

Oscillations of ϕ_α about zero (for $r > 0$)
→ spin-1 collective mode



T=0 spectrum

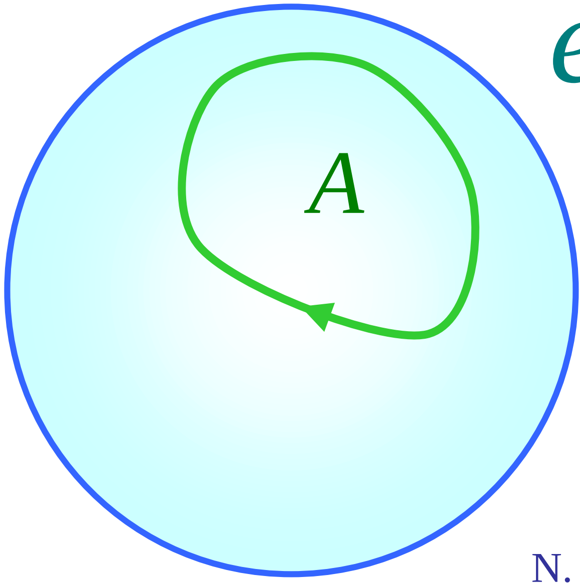
I.C Square Lattice Antiferromagnet

$$H = \sum_{i<j} J_{ij} \; \vec{S}_i \cdot \vec{S}_j$$

Action: $$S_b = \int d^2x d\tau \left[\frac{1}{2}\left(\left(\nabla_x \phi_\alpha\right)^2 + c^2 \left(\partial_\tau \phi_\alpha\right)^2 \right) + V\left(\phi_\alpha^2\right) \right]$$

S. Chakravarty, B.I. Halperin, and D.R. Nelson, Phys. Rev. B **39**, 2344 (1989).

Missing: Spin Berry Phases



$$e^{iSA}$$

Berry phases induce
bond charge order
in quantum "disordered"
phase with $\langle \phi_\alpha \rangle = 0$;
"Dual order parameter"

N. Read and S. Sachdev, Phys. Rev. Lett. **62**, 1694 (1989).

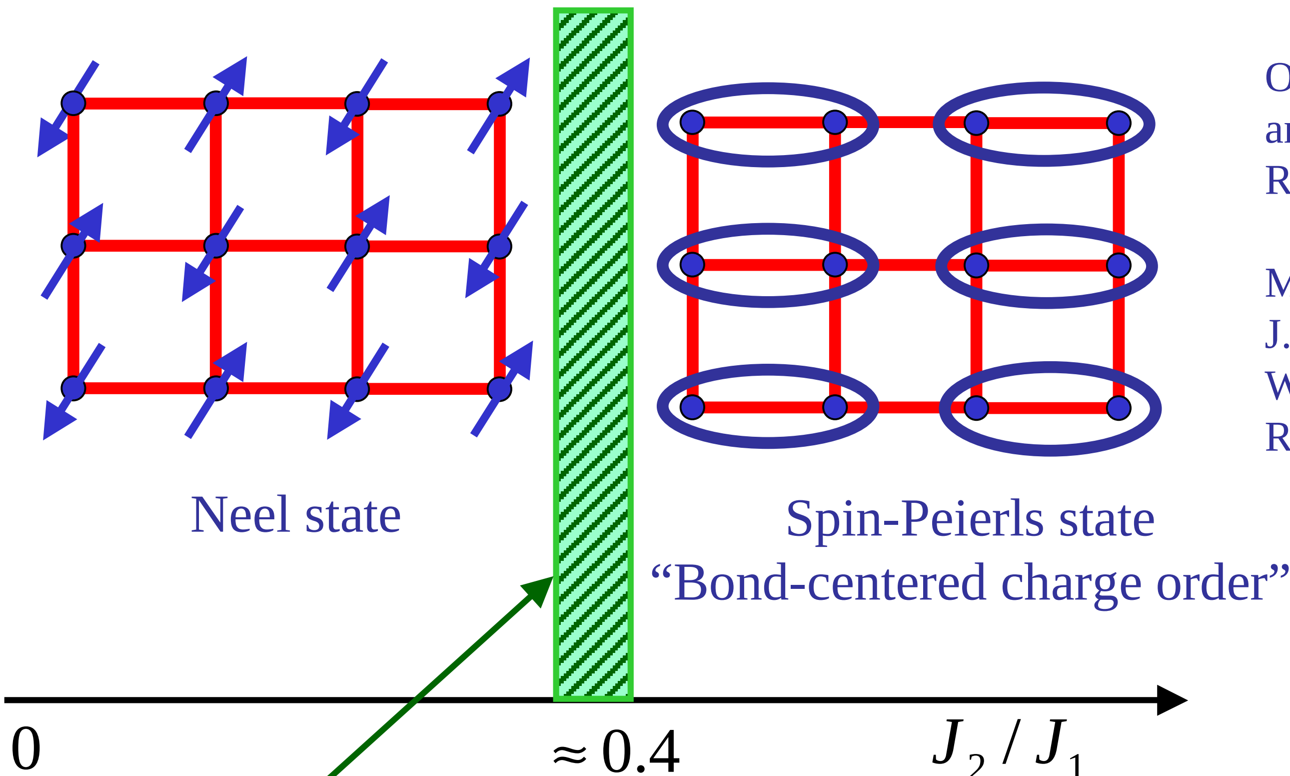
$$H = \sum_{i<j} J_{ij} \; \vec{S}_i \cdot \vec{S}_j$$

Square lattice with first (J_1) and second (J_2) neighbor exchange interactions

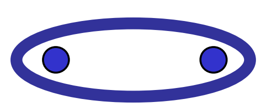
N. Read and S. Sachdev, Phys. Rev. Lett. **62**, 1694 (1989).

O. P. Sushkov, J. Oitmaa, and Z. Weihong, Phys. Rev. B **63**, 104420 (2001).

M.S.L. du Croo de Jongh, J.M.J. van Leeuwen, W. van Saarloos, Phys. Rev. B **62**, 14844 (2000).



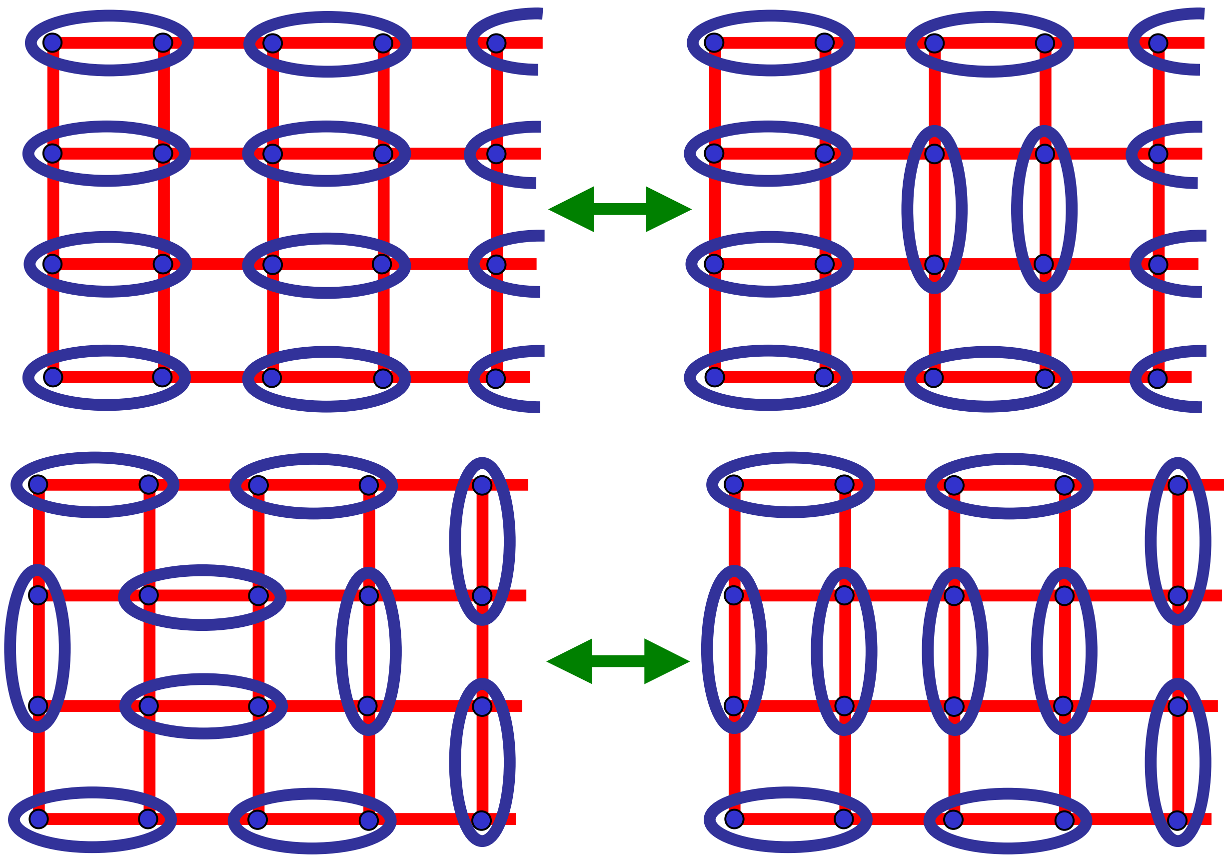
Co-existence



$$= \frac{1}{\sqrt{2}} \left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$$

Quantum dimer model –

D. Rokhsar and S. Kivelson Phys. Rev. Lett. **61**, 2376 (1988)



Quantum "entropic" effects prefer one-dimensional striped structures in which the largest number of singlet pairs can resonate. The state on the upper left has more flippable pairs of singlets than the one on the lower left.

These effects lead to a broken square lattice symmetry near the transition to the Neel state.

N. Read and S. Sachdev Phys. Rev. B **42**, 4568 (1990).

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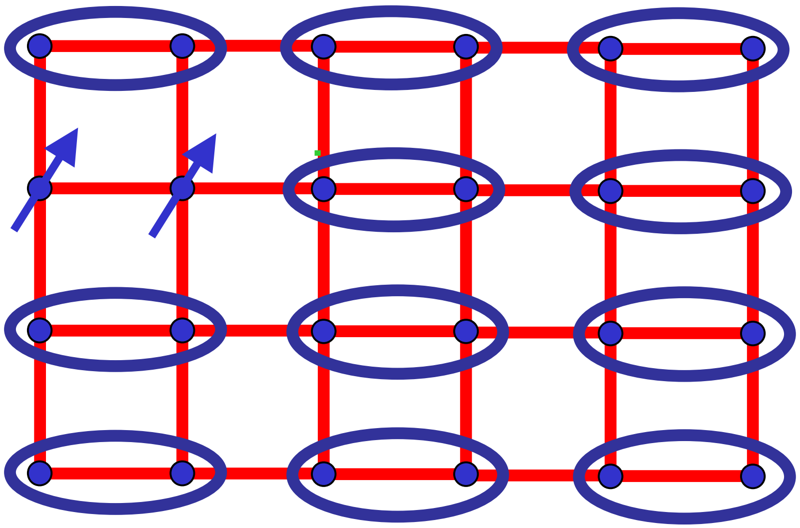
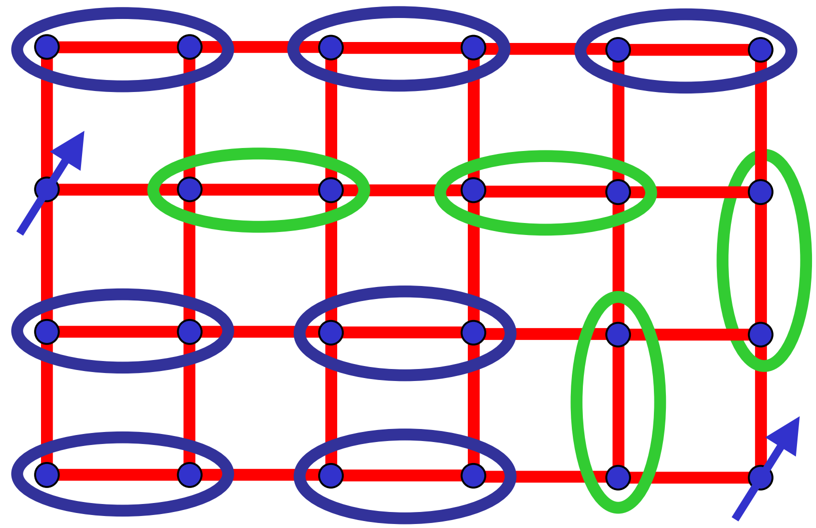
Properties of paramagnet with bond-charge-order

Stable S=1 spin exciton – quanta of 3-component ϕ_α

$$\varepsilon_k = \Delta + \frac{c_x^2 k_x^2 + c_y^2 k_y^2}{2\Delta}$$

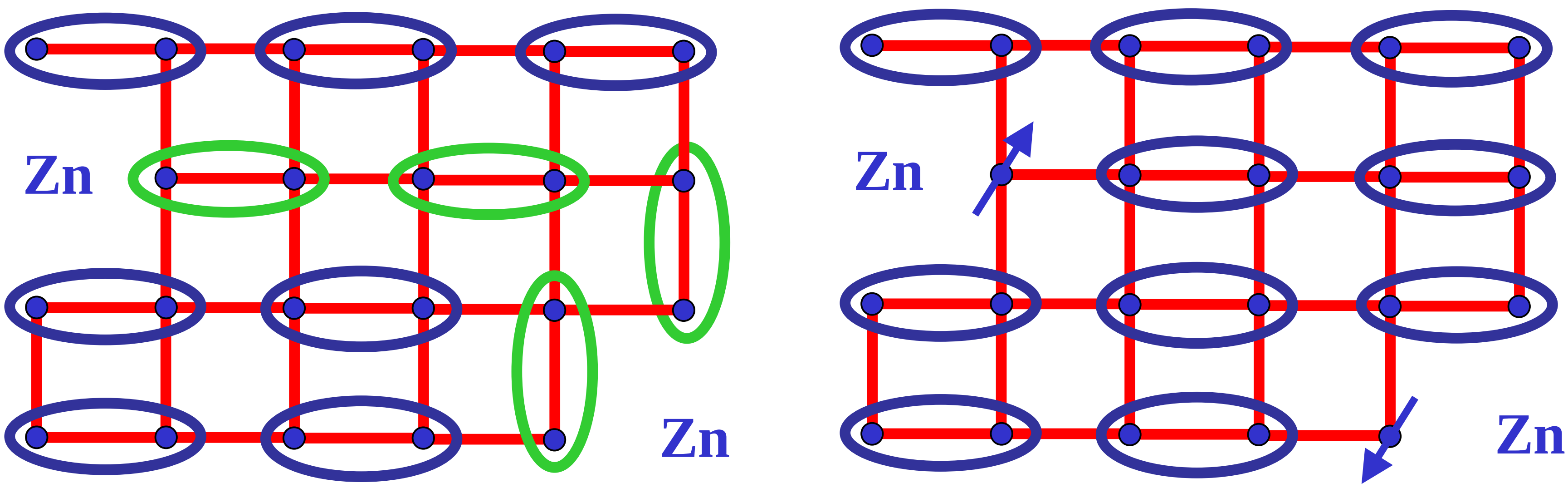
$\Delta \longrightarrow$ Spin gap





S=1/2 spinons are ***confined***
by a linear potential.

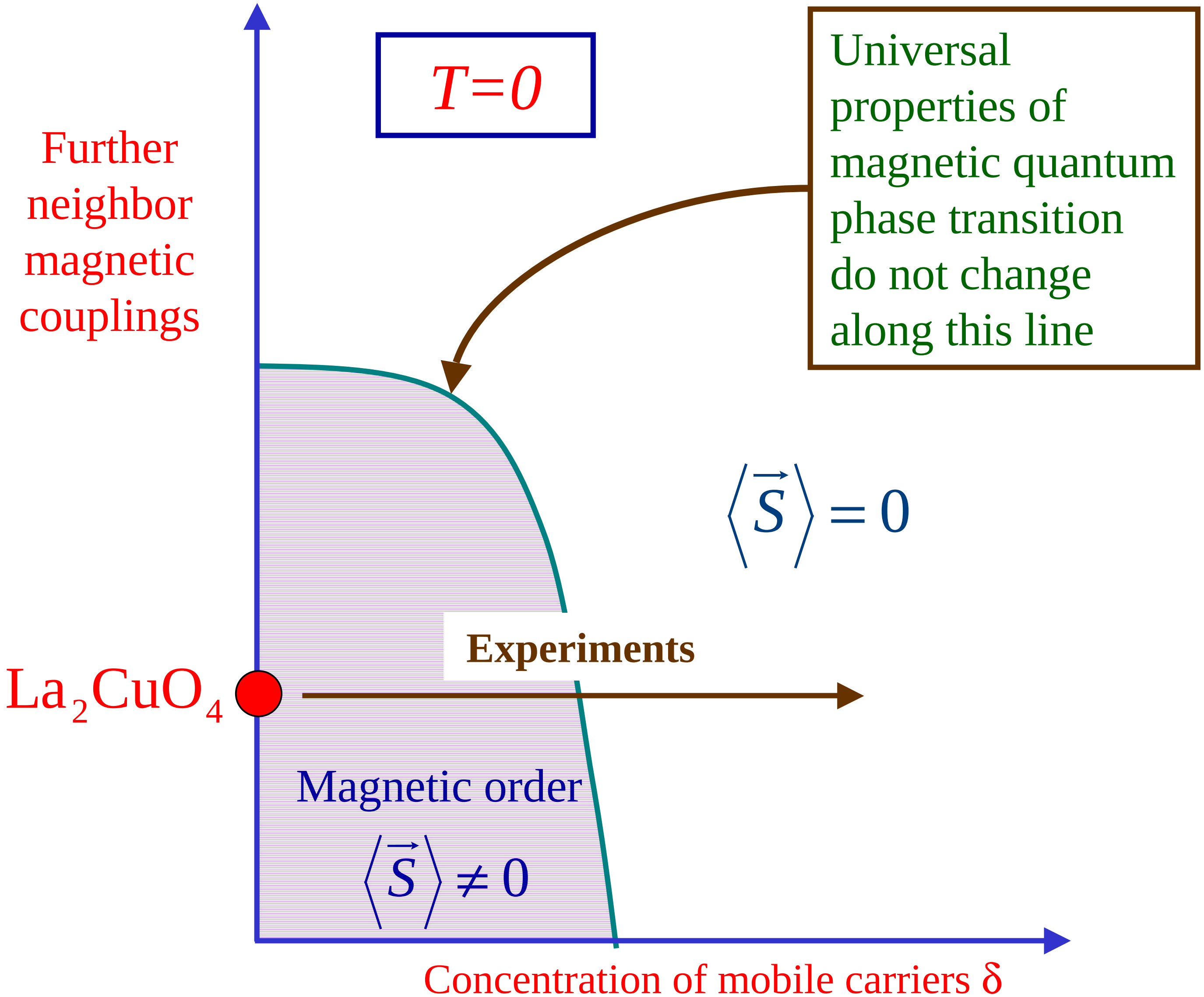
Effect of static non-magnetic impurities (Zn or Li)



Spinon confinement implies that free S=1/2 moments form near each impurity

$$\chi_{\text{impurity}}(T \to 0) = \frac{S(S+1)}{3k_B T}$$

A.M Finkelstein, V.E. Kataev, E.F. Kukovitskii, G.B. Teitel'baum, Physica C **168**, 370 (1990).



in *e.g.* $La_{2-\delta}Sr_{\delta}CuO_4$

S. Sachdev and J. Ye, Phys. Rev. Lett. **69**, 2411 (1992).
A.V. Chubukov, S. Sachdev, and J. Ye, Phys. Rev. B **49**, 11919 (1994)

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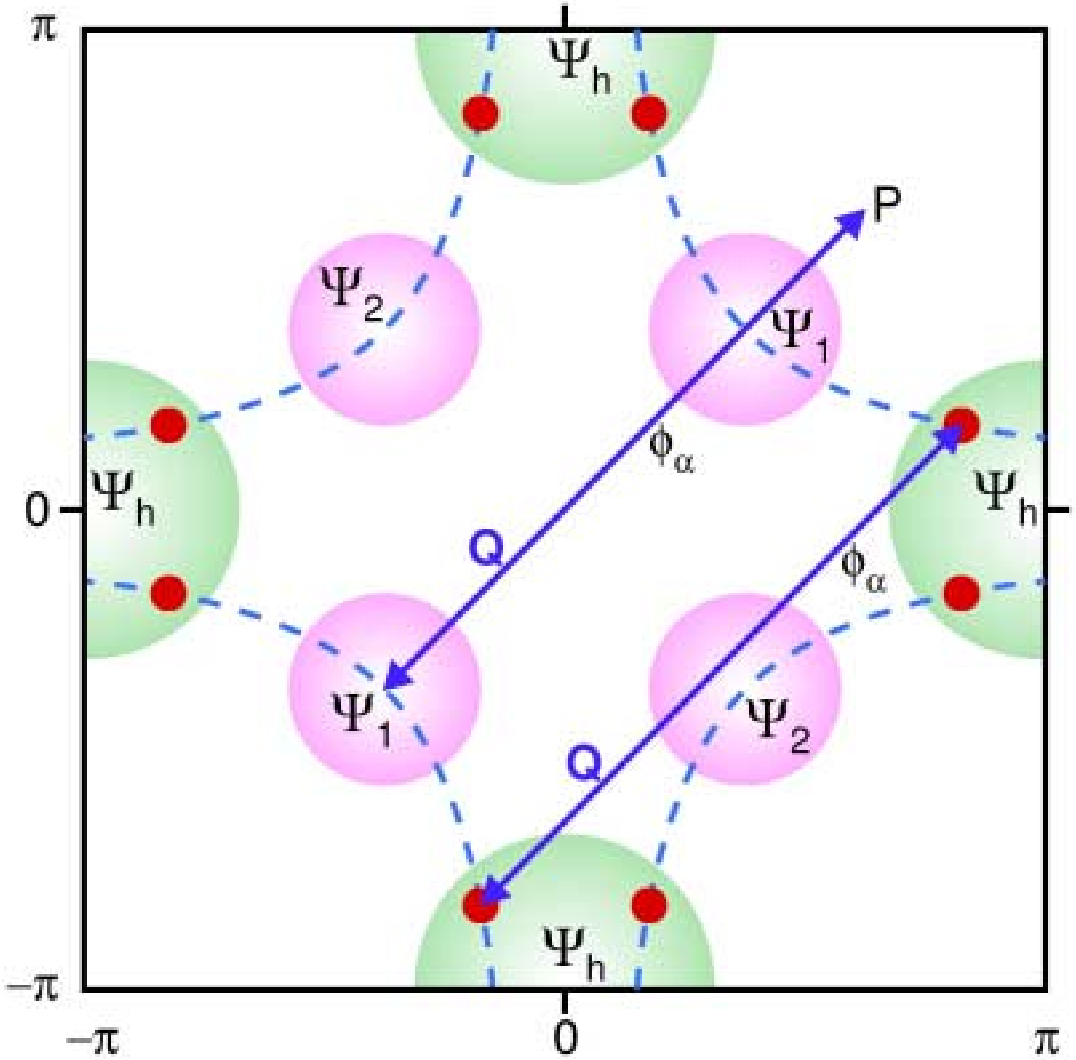
II. Magnetic transitions in a d-wave superconductor

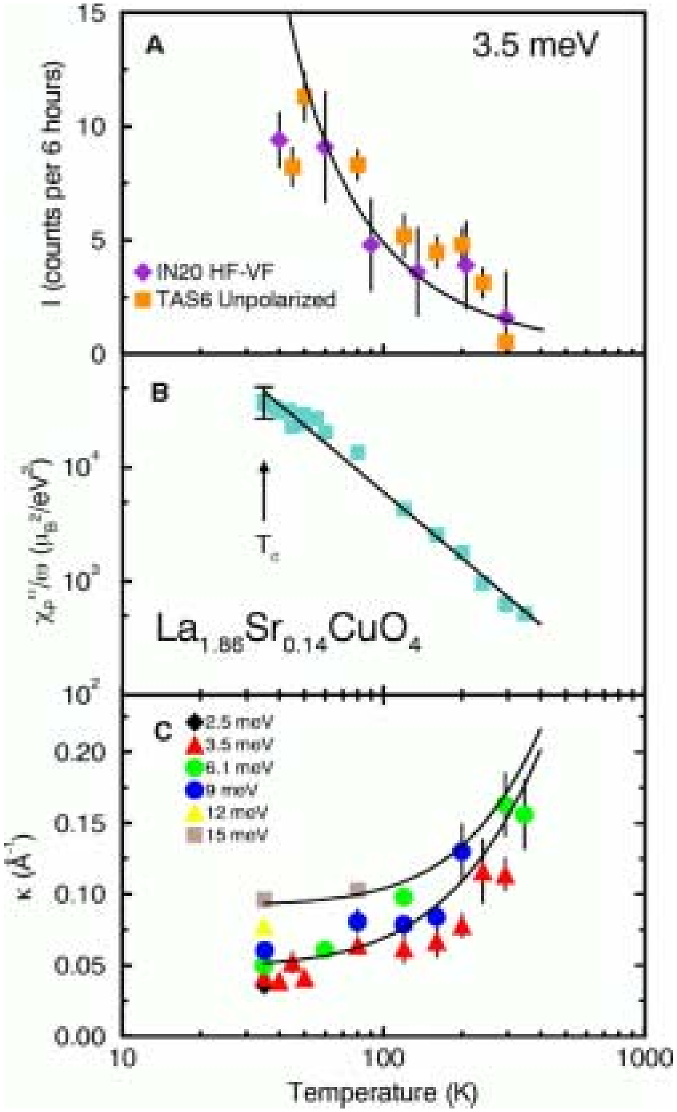


Ψ_h : *strongly coupled* to ϕ_α, but do not damp ϕ_α as long as $\Delta < 2\,\Delta_h$

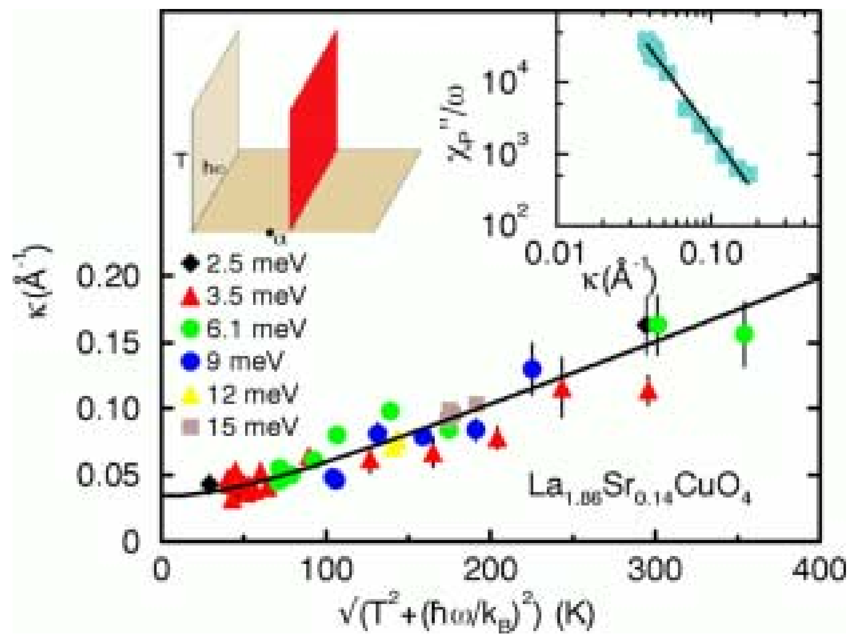
$\Psi_{1,2}$: *decoupled* from ϕ_α

Leading universal properties of transition are identical to that in an insulator

Neutron scattering measurements of dynamic spin susceptibility at an incommensurate wavevector: T and ω dependent divergence scaling as a function of $\hbar\omega/k_B T$

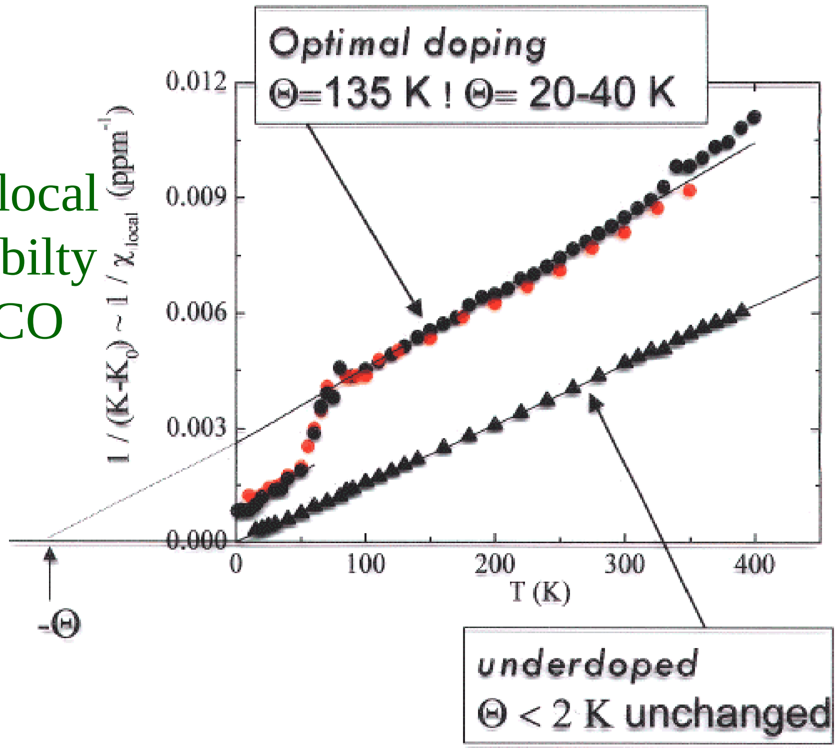


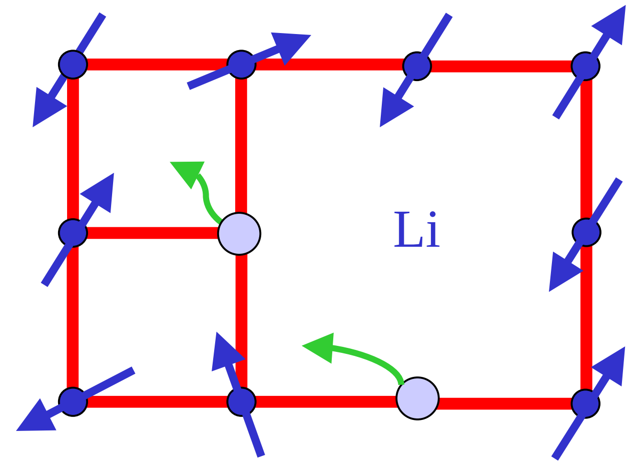
G. Aeppli, T.E. Mason, S,M. Hayden, H.A. Mook, and J. Kulda, *Science* **278**, 1432 (1998).



Measurement of spin susceptibility near non-magnetic (Zn/Li) impurities

Inverse local susceptibilty in YBCO





^{7}Li NMR below T_c

J. Bobroff, H. Alloul, W.A. MacFarlane, P. Mendels, N. Blanchard, G. Collin, and J.-F. Marucco, Phys. Rev. Lett. **86**, 4116 (2001)

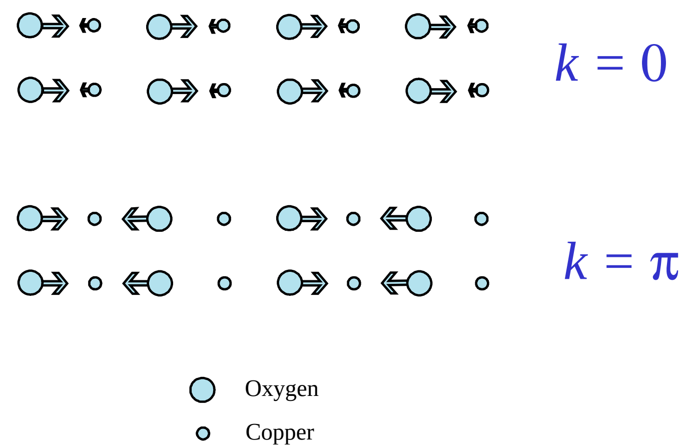
See also D. L. Sisson, S. G. Doettinger, A. Kapitulnik, R. Liang, D. A. Bonn, and W. N. Hardy, Phys. Rev. B **61**, 3604 (2000).

Measured $\chi_{\text{impurity}}(T \to 0) = \dfrac{S(S+1)}{3k_B T}$ with $S = 1/2$ in underdoped sample.

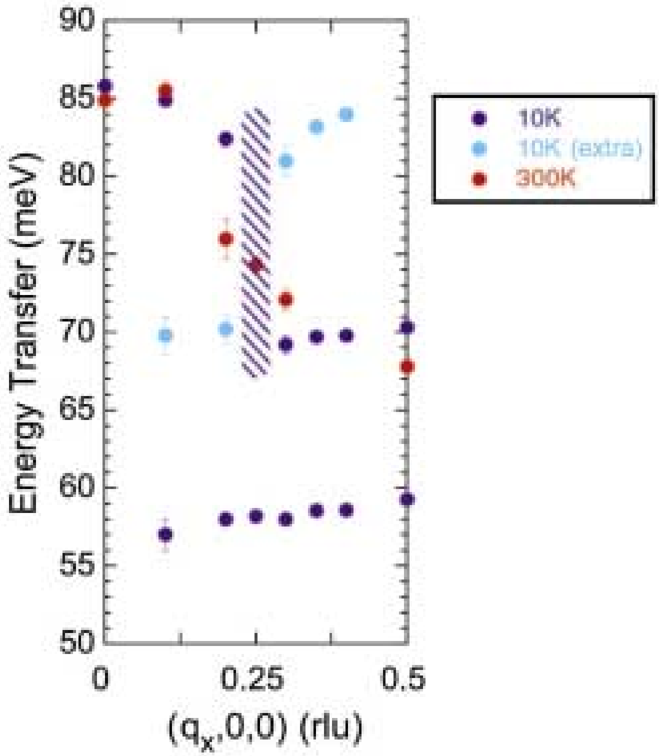
Not expected from BCS theory, which predicts $\chi_{\text{impurity}}(T \to 0) \neq \infty$ for a non-magnetic impurity with strong potential scattering.

Neutron scattering measurements of phonon spectra

Discontinuity in the dispersion of a LO phonon of the O ions at wavevector $k = \pi/2$: evidence for bond-charge order with period *2a*

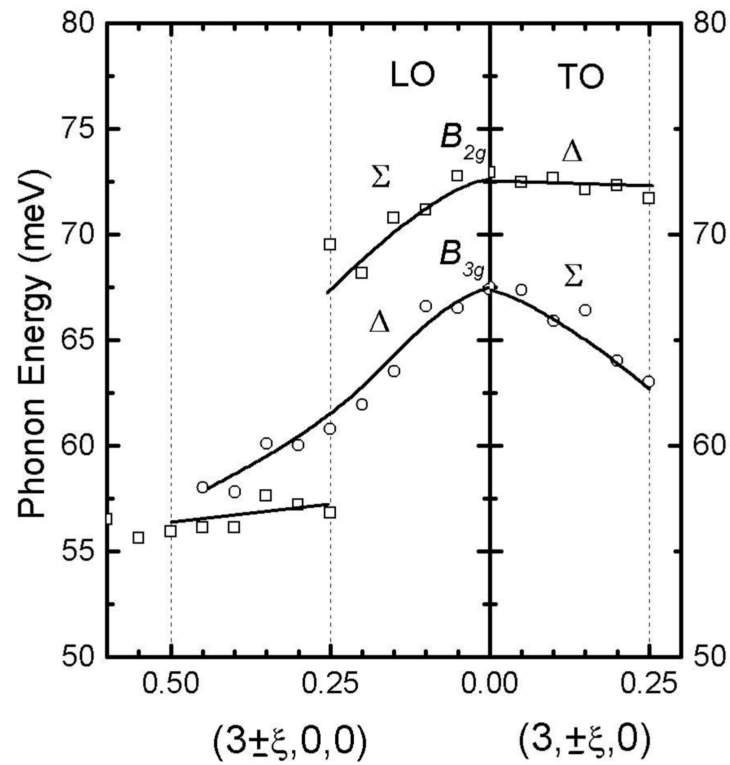


$La_{1.85}Sr_{0.15}CuO_4$



R. J. McQueeney, Y. Petrov, T. Egami, M. Yethiraj, G. Shirane, and Y. Endoh, Phys. Rev. Lett. **82**, 628 (
L. Pintschovius and M. Braden, Phys. Rev. B **60**, R15039 (1999).

$YBa_2Cu_3CuO_{6.95}$

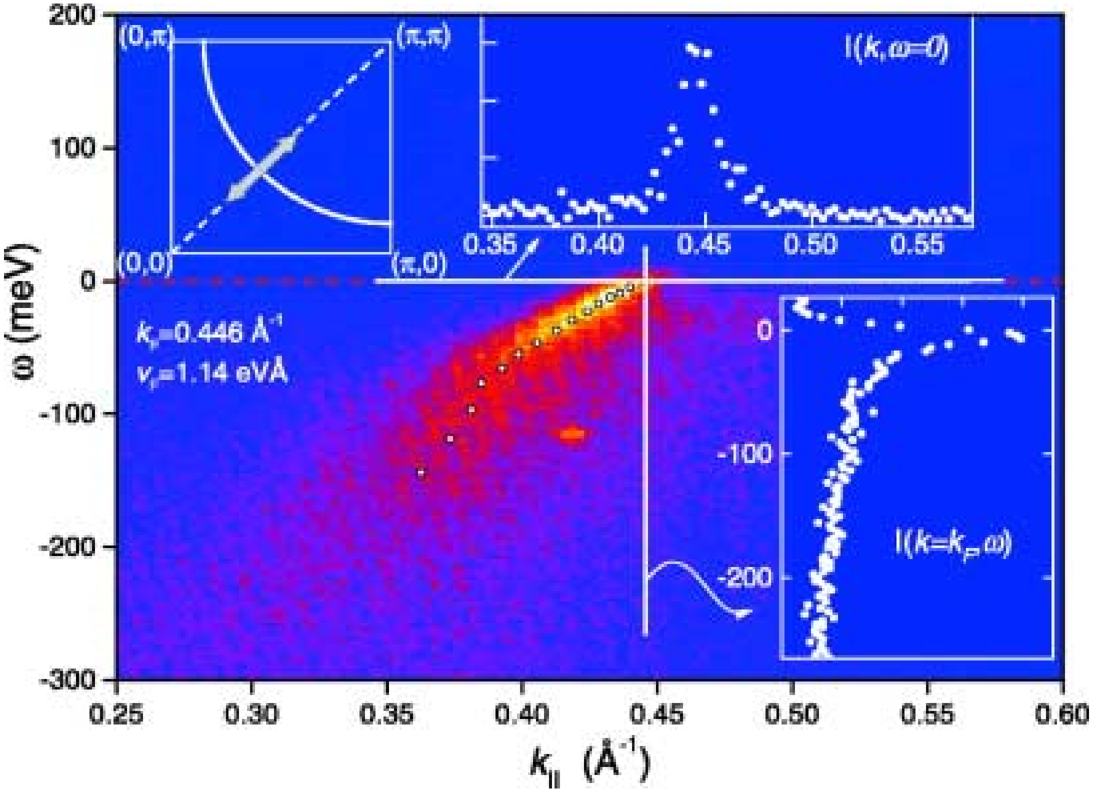


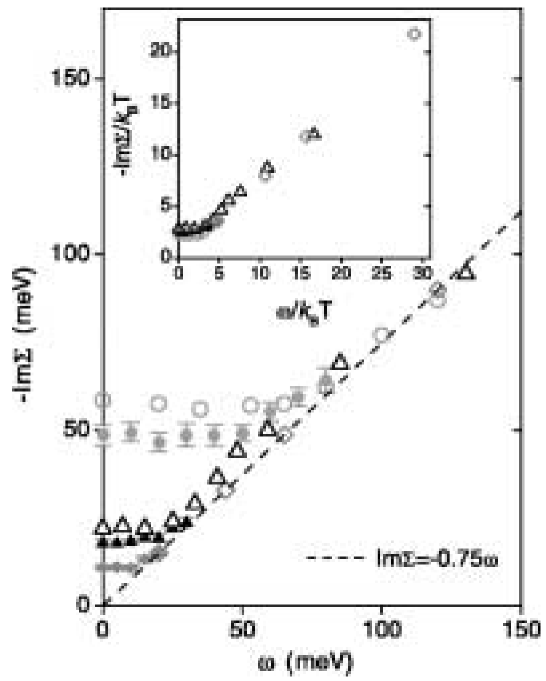
R. J. McQueeney, T. Egami, J.-H. Chung, Y. Petrov, M. Yethiraj, M. Arai, Y. Inamura, Y. Endoh, C. Frost and F. Dogan, cond-mat/0105593.

III. Damping of Nodal Quasiparticles

Photoemission on BSSCO

(Valla et al Science **285**, 2110 (1999))





$$\mathrm{Im}\,\Sigma \sim k_B T$$

Goal: Classify theories in which, with minimal fine tuning, a *d*-wave superconductor has a fermionic quasiparticle momentum distribution curve (MDC), at the nodal points, with a width proportional to $k_B T$

In a Fermi liquid, MDC width $\sim T^2$

In a BCS d-wave superconductor, MDC width $\sim T^3$

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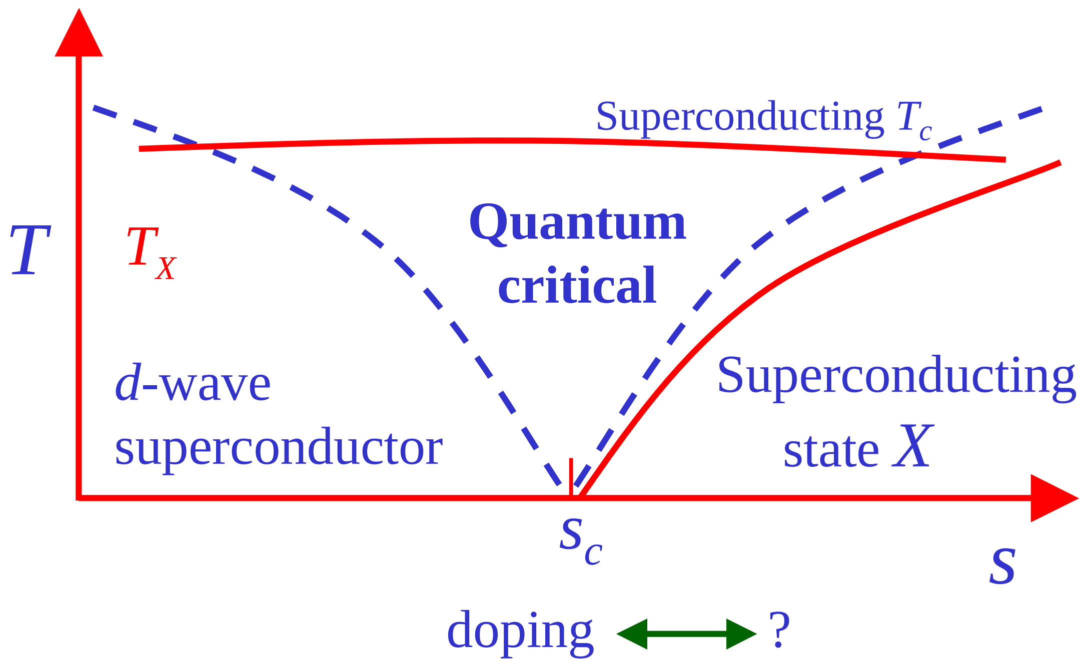
Proximity to a quantum-critical point



S. Sachdev and J. Ye, Phys. Rev. Lett. **69**, 2411 (1992).
M. Vojta, Y. Zhang, and S. Sachdev, Phys. Rev. Lett. **85**, 4940 (2000).

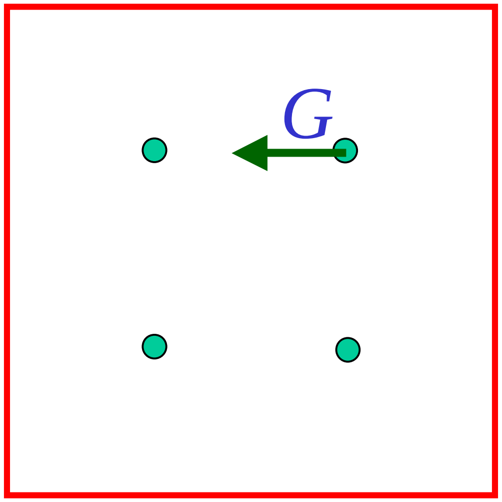
Necessary conditions

1. Quantum-critical point should be below its upper-critical dimension and obey hyperscaling.
2. Nodal quasi-particles should be part of the critical-field theory.
3. Critical field theory should not be free – required to obtain damping in the scaling limit.

A spin-singlet, fermion bilinear, zero momentum order parameter for *X* is preferred.

An order parameter with momentum *G*:
Charge (or spin) density-wave order

$$\delta\rho \sim \mathrm{Re}\left[\Phi_x e^{iGx} + \Phi_y e^{iGy}\right]$$



If *G* does not connect two nodal points, fermions are not part of the critical theory

Order parameter for X should be a component of

$$\Delta_k = \langle c_{k\uparrow} c_{-k\downarrow} \rangle \text{ (fermion pairing)}$$

or

$$A_k = \langle c^{\dagger}_{k\alpha} c_{k\alpha} \rangle \text{ (excitonic order)}$$

Complete group-theoretic classification

X has $d_{x^2-y^2}$ pairing plus

(A) is pairing

(B) id_{xy} pairing

(C) ig pairing

→ fermion spectrum fully gapped

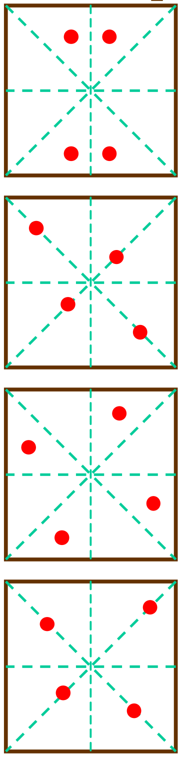
Nodal points

(D) s pairing

(E) d_{xy} excitons

(F) d_{xy} pairing

(G) p excitons



(D), (E) → superconducting nematics

Main results

Only cases

(A) $d_{x^2-y^2} \Leftrightarrow d_{x^2-y^2} + is$ pairing and

(B) $d_{x^2-y^2} \Leftrightarrow d_{x^2-y^2} + id_{xy}$ pairing

have renormalization group fixed points with a non-zero interaction strength between the bosonic order parameter mode and the nodal fermions.

Only cases (A) and (B) satisfy conditions 1,2,3

Transition to d_{xy} pairing is expected with increasing J_2

M. Vojta, Y. Zhang, and S. Sachdev, Phys. Rev. Lett. **85**, 4940 (2000).

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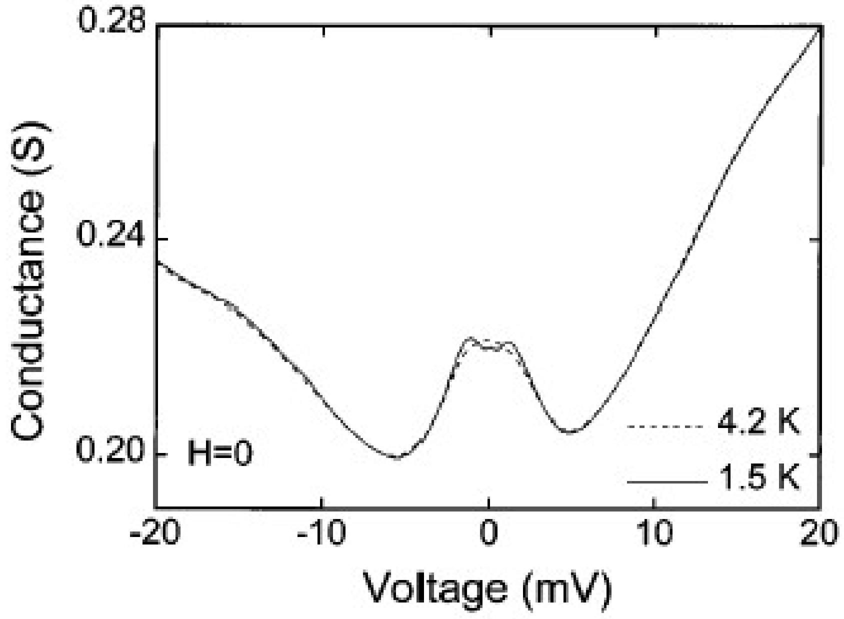
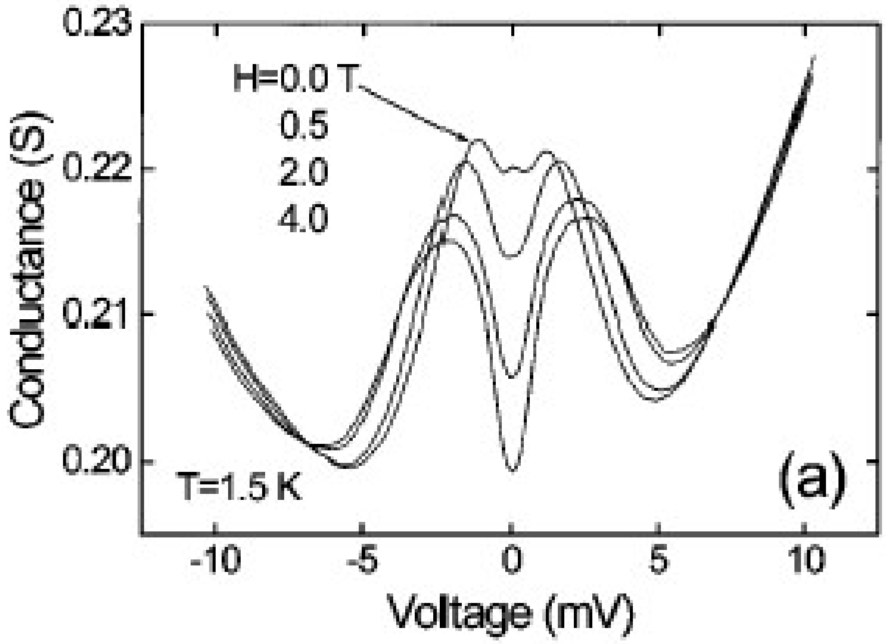
Yoram Dagan and Guy Deutscher, cond-mat/0106128

Observations of splitting of the ZBCP

Spontaneous splitting (zero field)

Magnetic field splitting





Covington, M. *et al.* Observation of Surface-Induced Broken Time-Reversal Symmetry in $YBa_2Cu_3O_{7-\delta}$ Tunnel Junctions, Phys. Rev. Lett. **79**, 277-281 (1997)

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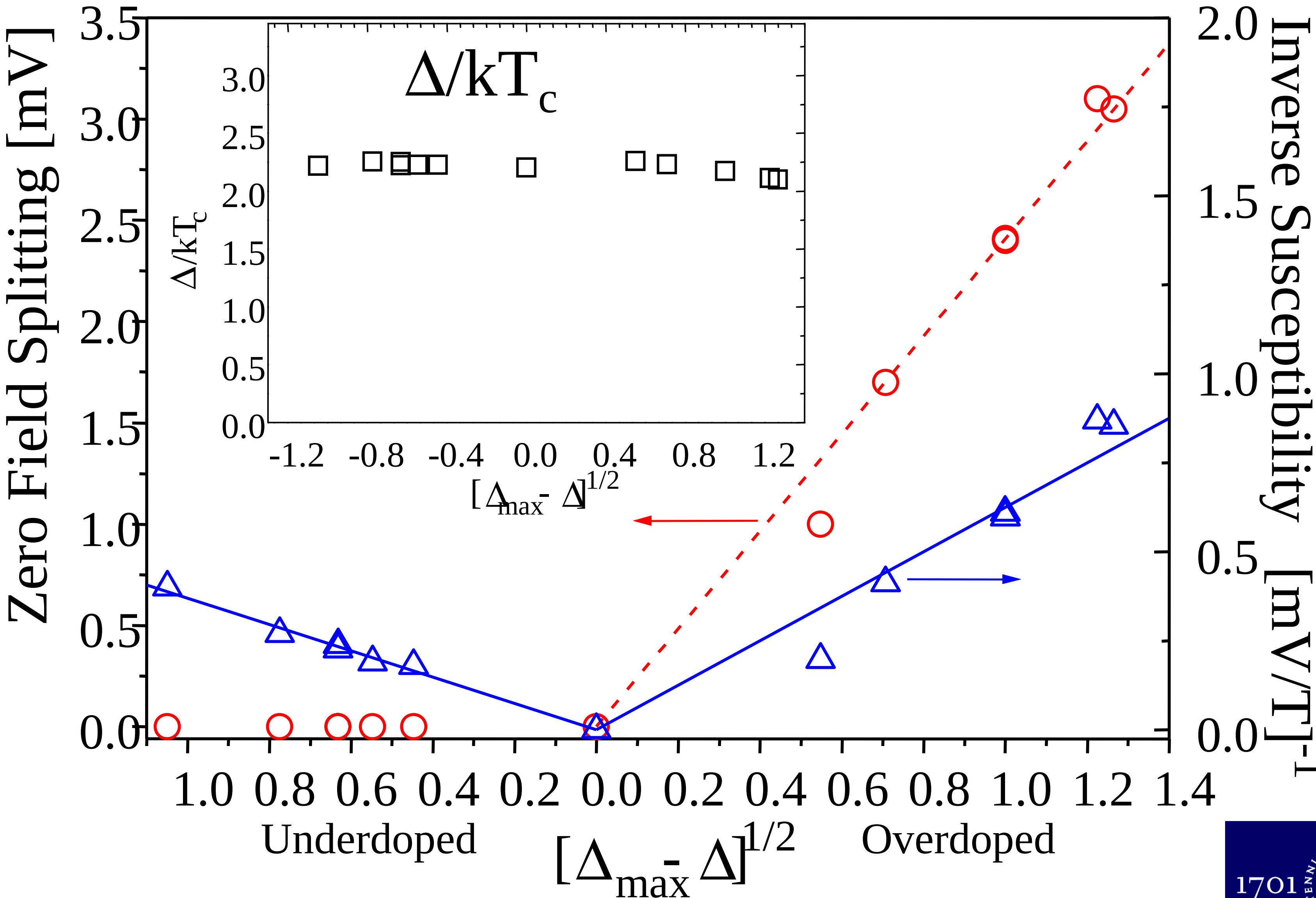
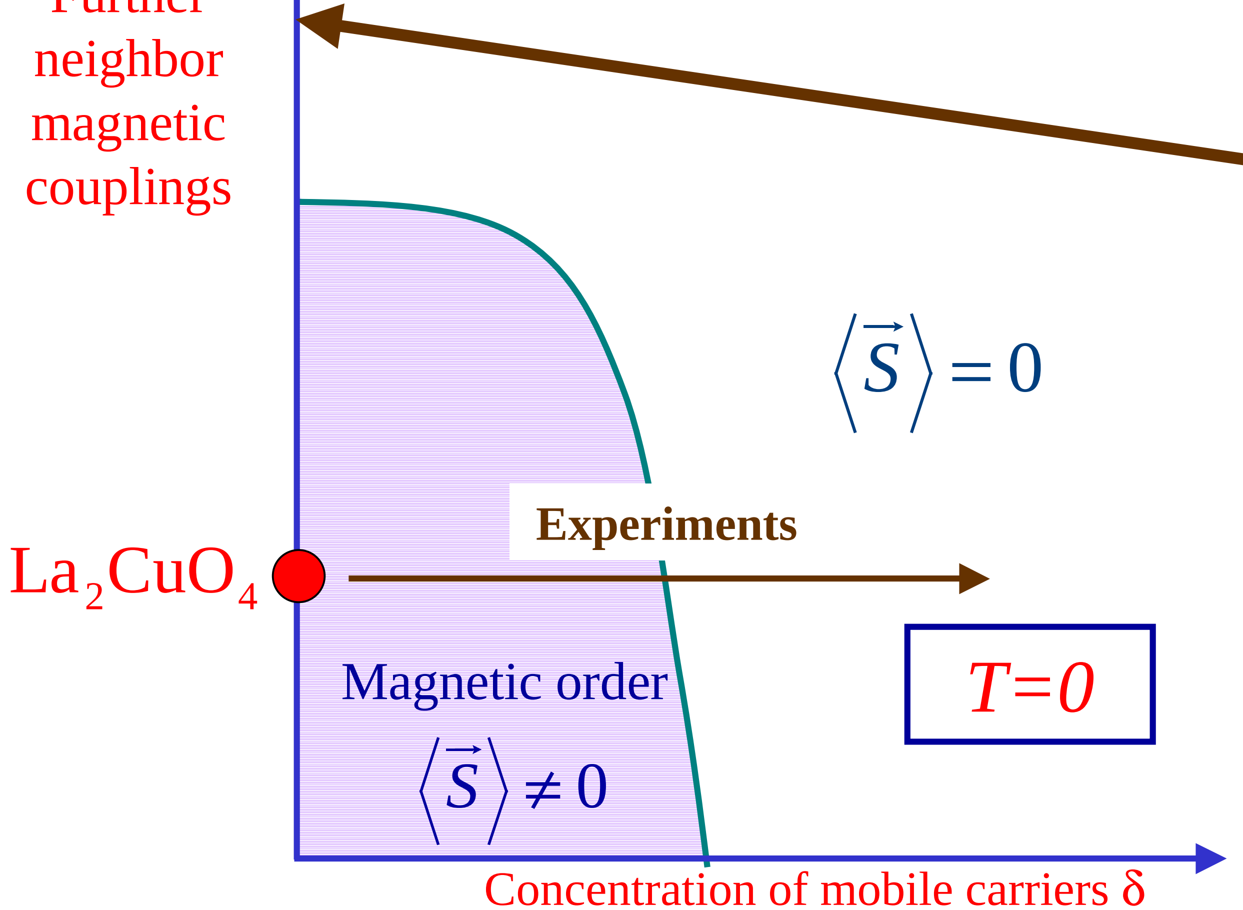
Zero Field splitting and χ^{-1} versus $[\Delta_{max}-\Delta]^{1/2}$ All YBCO samples



Conclusions



Confined, paramagnetic Mott insulator has

1. Stable *S*=1 spin exciton ϕ_α .
2. Broken translational symmetry:- bond-centered charge order.
3. Pairing of holes.
4. *S*=1/2 moments near non-magnetic impurities

Theory of magnetic ordering quantum transitions in antiferromagnets and superconductors leads to quantitative theories for

- Spin correlations in a magnetic field
- Effect of Zn/Li impurities on collective spin excitations