Collective spin dynamics of low-dimensional paramagnets and d-wave superconductors

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•K. Damle
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Phys. Rev. Lett. **78**, 943 (1997).
Phys. Rev. B **57**, 8307 (1998).
Science **286**, 2479 (1999).
cond-mat/9912020
Quantum Phase Transitions,
Cambridge University Press (January 1, 2000)

Yale University

Transparencies available online at
http://pantheon.yale.edu/~subir/qimptalk.pdf

Thermal and quantum damping of collective spin-1 mode – analogy with rotons in ^{4}He and magnetorotons in QH liquids

1. Paramagnetic and Neel ground states in low dimensions --- coupled-ladder antiferromagnet

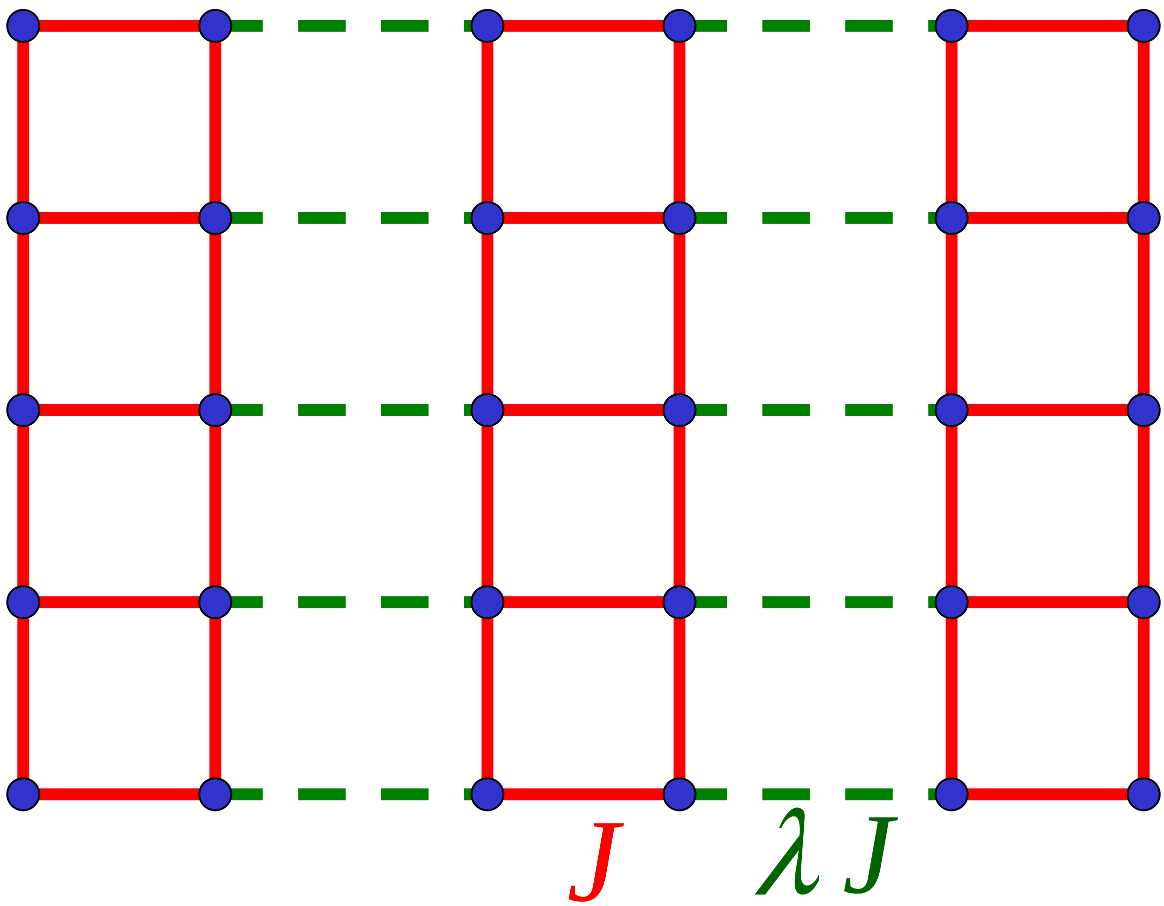
2. Thermal damping of the collective spin-1 excitation of the paramagnet:
   Quasiclassical particle model
   Exact results in one dimension and comparison with expts.

3. Nearly-critical paramagnets with spin gap $\Delta << J$
   Quantum field theory: low T spin dynamics is universally determined by Δ and $c = (c_x c_y)^{1/2}$

4. Non-magnetic impurities (Zn or Li) in nearly-critical paramagnets and d-wave superconductors:
   Boundary quantum field theory:
   T=0 damping of spin-1 collective mode - universal line-shape with
   no additional parameters needed:
   only need n_{imp}
   Comparison with, and predictions for, expts

1. Paramagnetic and Neel states

S=1/2 spins on coupled 2-leg ladders



$$H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

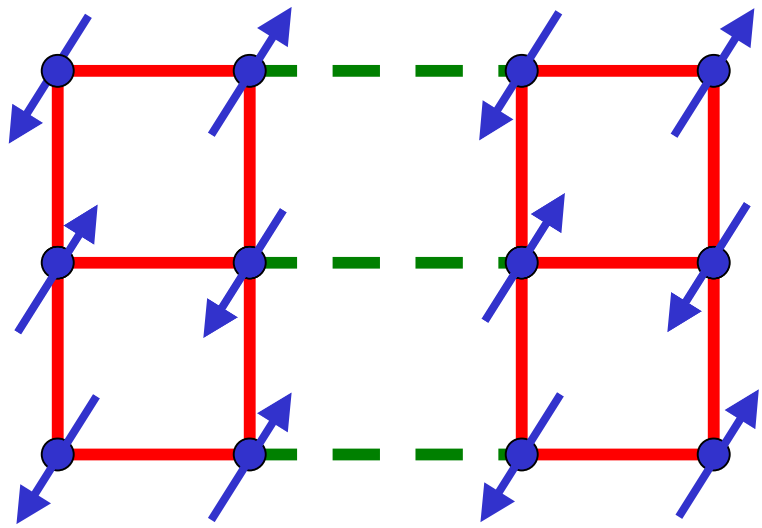
Follow ground state as a function of λ

$$0 \leq \lambda \leq 1$$

$\lambda = 1$

Square lattice antiferromagnet

Experimental realization: La_2CuO_4



Ground state has long-range magnetic (Neel) order

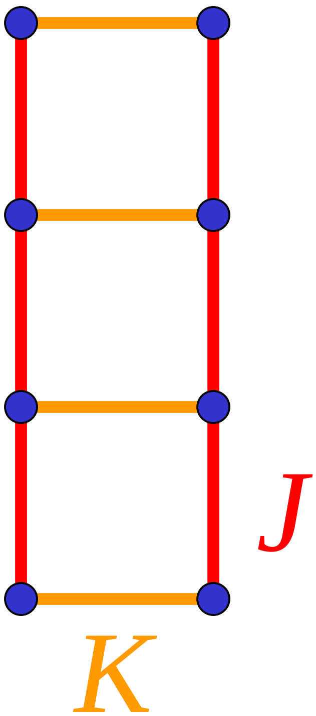
$$\left\langle \vec{S}_i \right\rangle = \left(-1\right)^{i_x + i_y} N_0 \neq 0$$

Excitations: 2 spin waves

Quasiclassical wave dynamics at low T

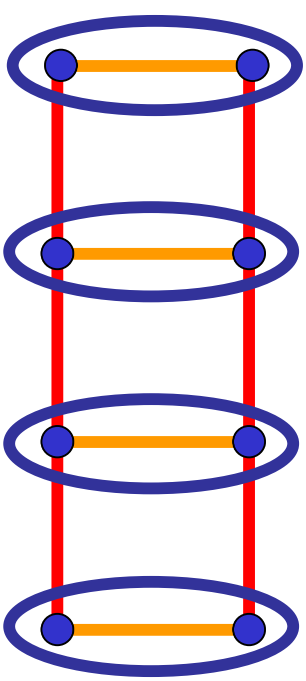
(Chakravarty et al, 1989;
Tyc et al, 1989)

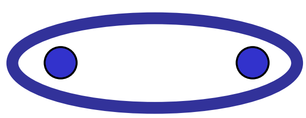
$\lambda = 0$



Decoupled 2-leg ladders

Allow $J \neq K$





$$= \frac{1}{\sqrt{2}}\left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$$

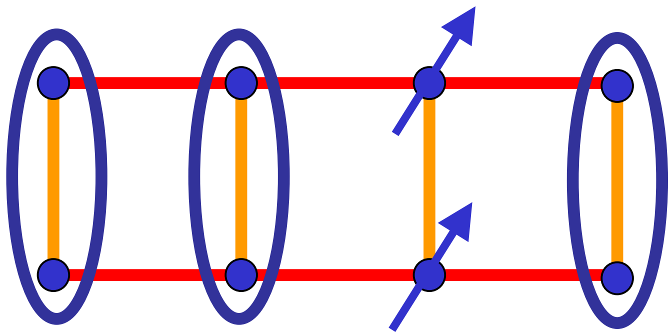
Quantum paramagnet ground state for

$J \ll K$

Qualitatively similar ground state for all

J / K

Excited states



Triplet (S=1) particle (collective mode)

Energy dispersion away from antiferromagnetic wavevector

$$\varepsilon = \Delta + \frac{c^2 k^2}{2\Delta}$$

$\Delta \rightarrow$ Spin gap

In one dimension,
all low T dynamic correlations
are universal functions of Δ and c

2. Thermal damping of spin-1 collective mode

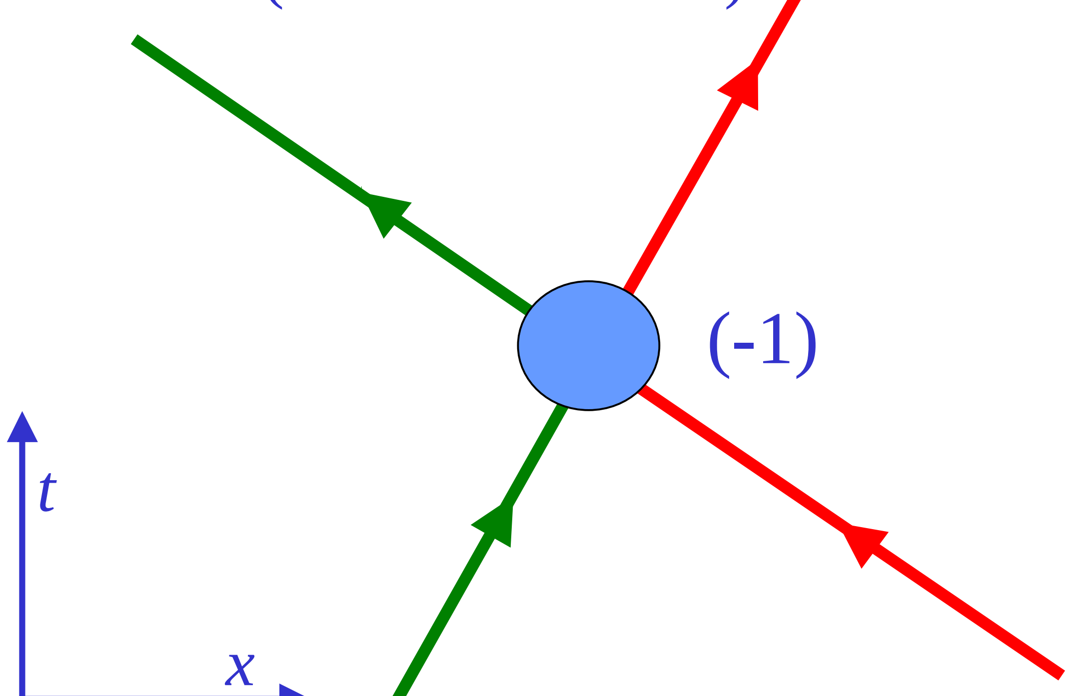
Key Observations

1. De Broglie wavelength $\sim \dfrac{\hbar c}{\sqrt{\Delta k_B T}}$

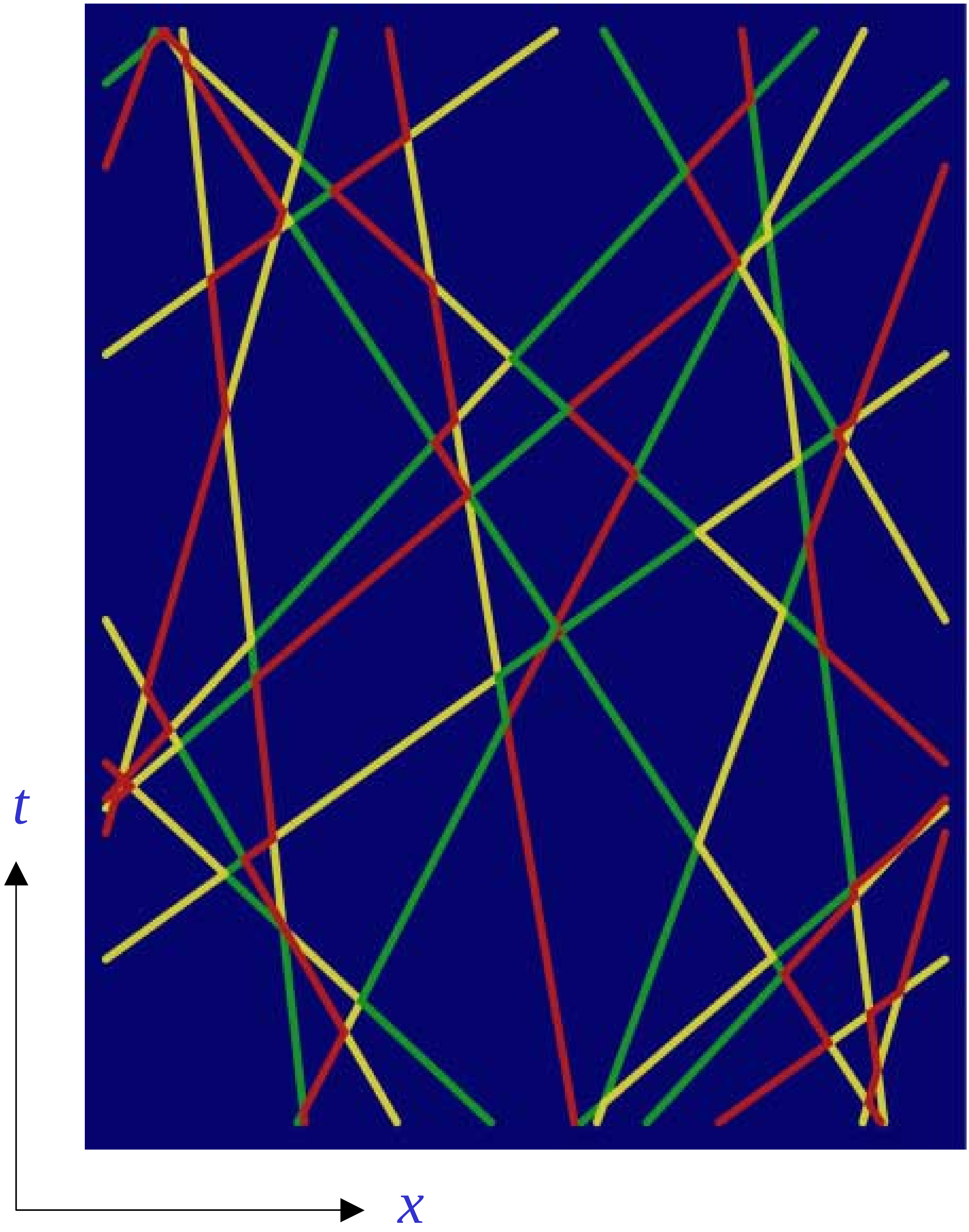
   << interparticle spacing $\sim e^{\Delta / k_B T}$

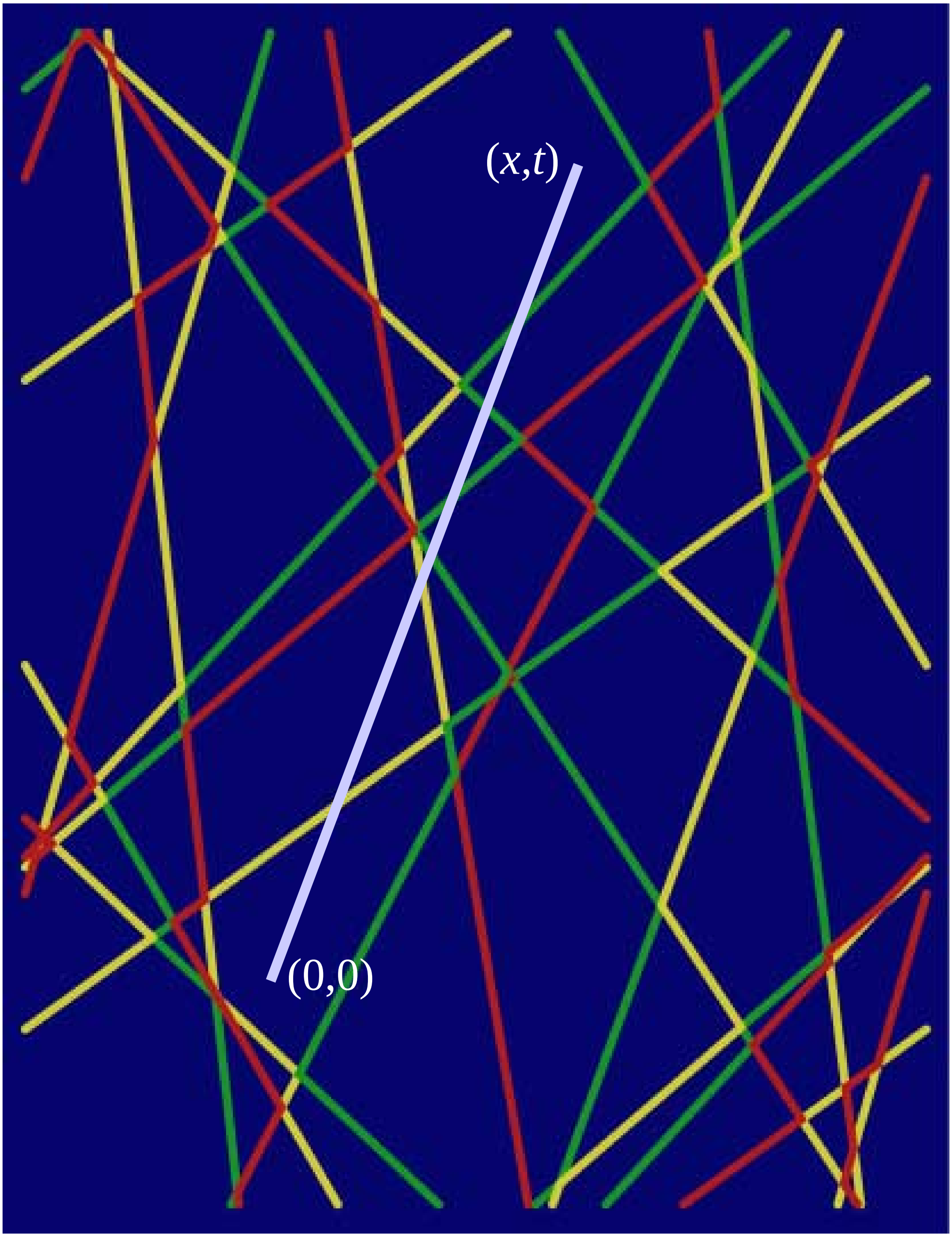
   (Quasiclassical particles)

2. Quantum mechanical S-matrix has a super-universal form at low momenta (in one dimension)



Spins bounce while momenta exchange


t
x

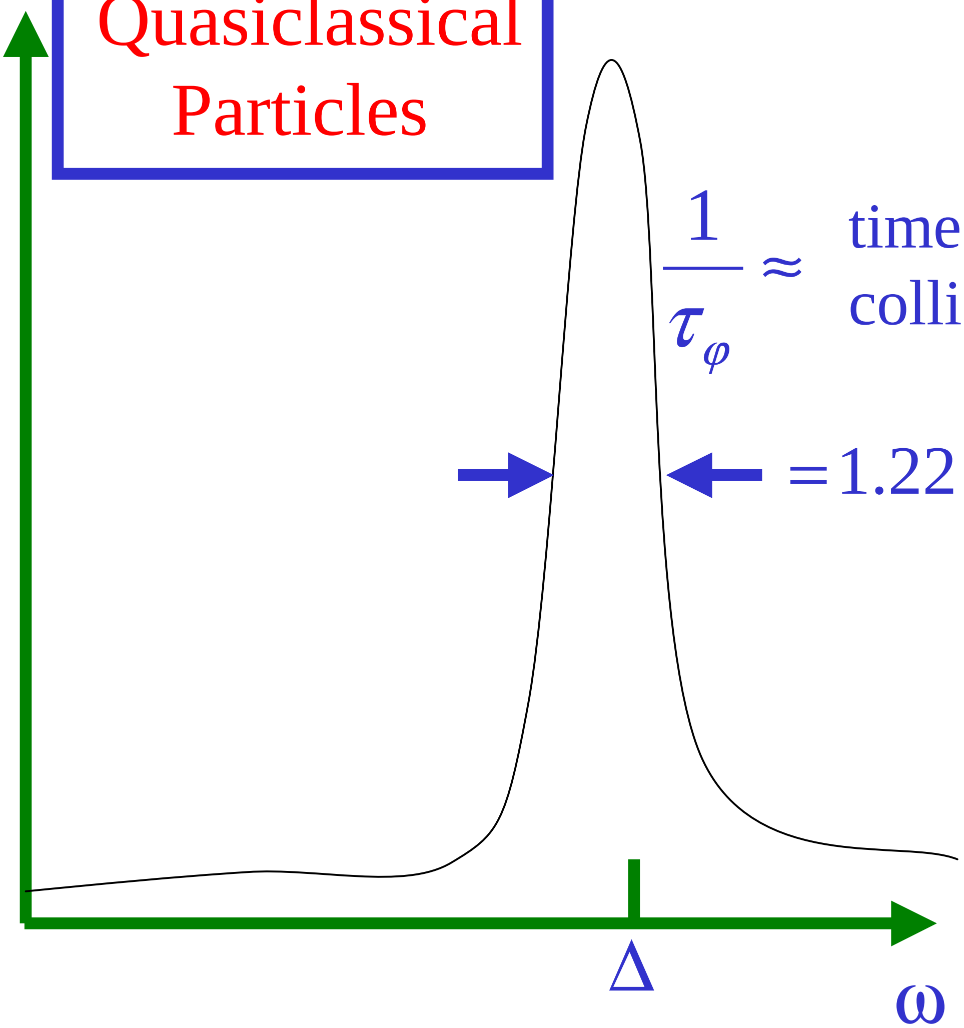


$$G(x,t) = G(x,t)\Big|_{T=0} (-1)^{\text{number of collisions}}$$

Dynamic Structure Factor at antiferromagnetic wavevector

$S(\pi,\omega)$

Quasiclassical Particles



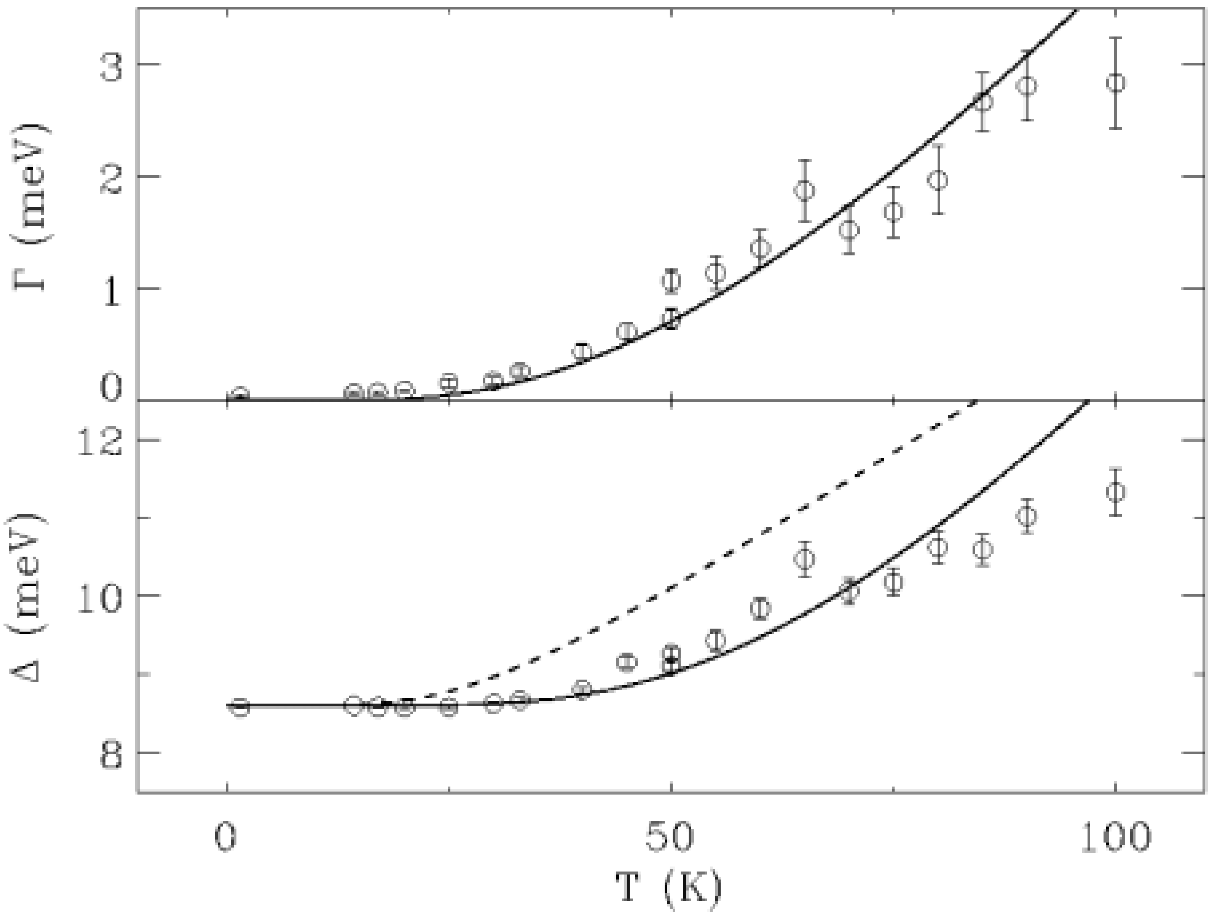
$\frac{1}{\tau_\varphi} \approx$ time between collisions

$$= 1.22 \frac{k_B T}{\hbar} e^{-\Delta / k_B T}$$

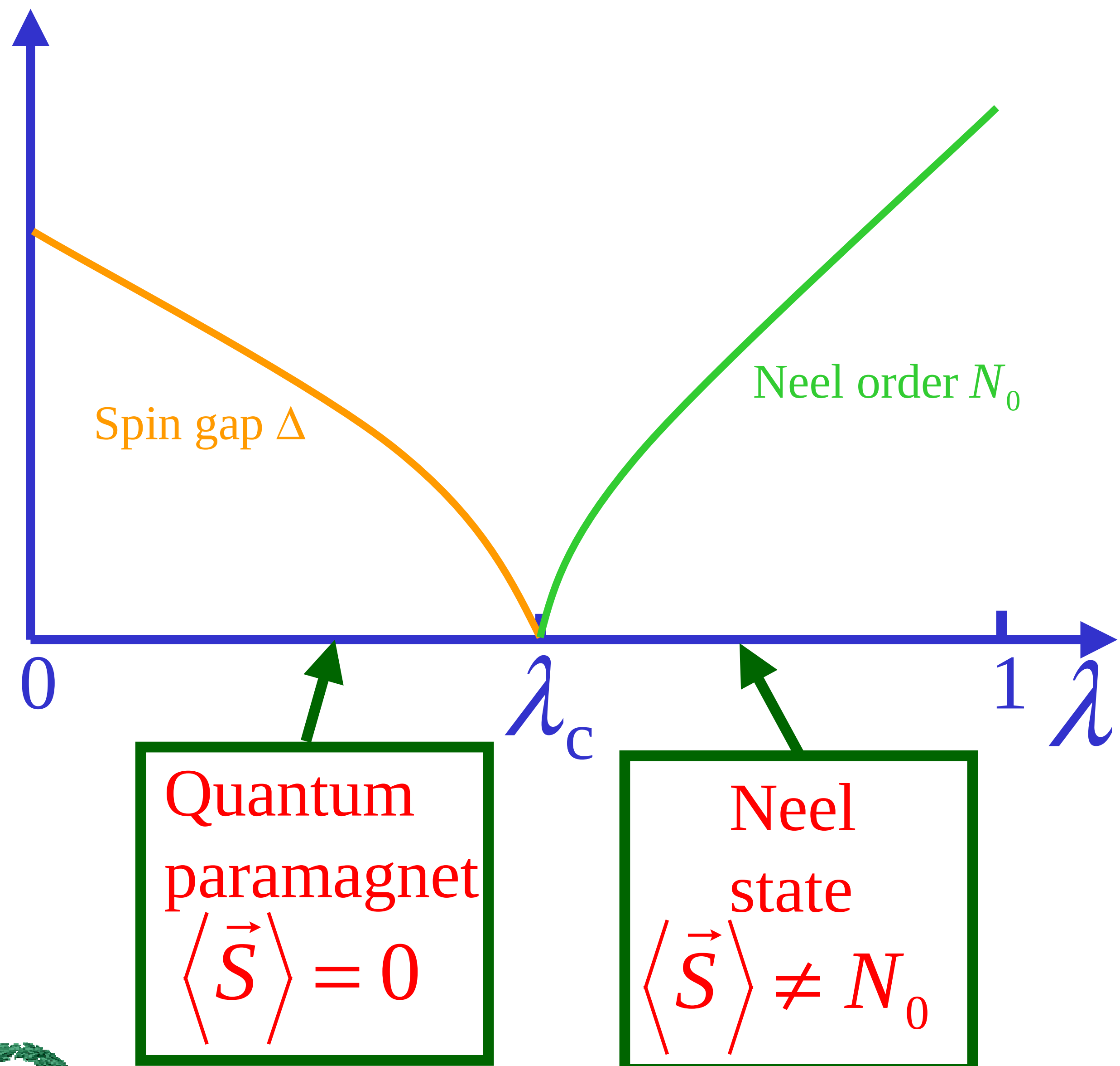
Particles are quantum lumps of "amplitude oscillation" of anti-ferromagnetic order parameter

Excitation energy Δ and linewidth Γ for Y_2BaNiO_5

C. Broholm et al. (unpublished)



Solid line for Γ – theory with no free parameters



Spin gap Δ
Neel order N_0
0
λ_c
1
λ
Quantum paramagnet
$\langle \vec{S} \rangle = 0$
Neel state
$\langle \vec{S} \rangle \neq N_0$

3. Nearly-critical paramagnets

λ is close to λ_c

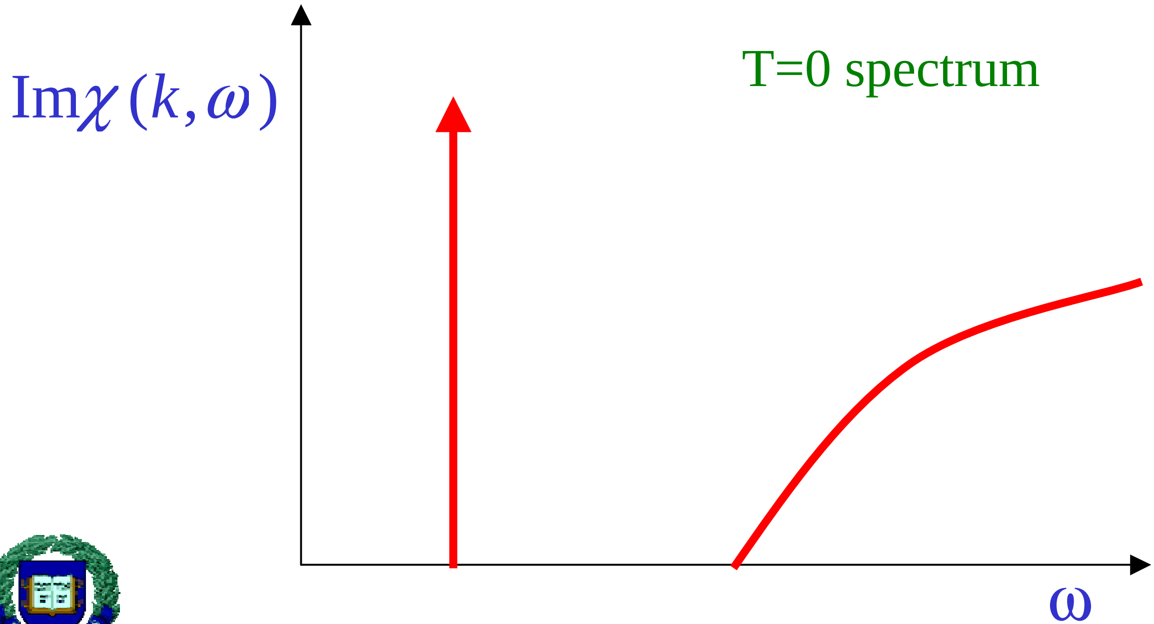
Quantum field theory:

$$\mathcal{S}_b = \int d^d x d\tau \left[\frac{1}{2}\left((\nabla_x \phi_\alpha)^2 + c^2(\partial_\tau \phi_\alpha)^2 + r\phi_\alpha^2\right) + \frac{g}{4!}(\phi_\alpha^2)^2\right]$$

ϕ_α → 3-component antiferromagnetic order parameter

$r > 0$	→	$\lambda < \lambda_c$
$r < 0$	→	$\lambda > \lambda_c$

Oscillations of ϕ_α about zero (for $r > 0$)
→ spin-1 collective mode



Coupling *g* approaches fixed-point value under renormalization group flow: beta function ($\varepsilon = 3-d$) :

$$\beta(g) = -\epsilon g + \frac{11g^2}{6} - \frac{23g^3}{12} + \mathcal{O}(g^4)$$

Only relevant perturbation – ***r***
strength is measured by the spin gap Δ

Δ and *c* completely determine entire spectrum of quasi-particle peak and multiparticle continua, the S matrices for scattering between the excitations, and $T > 0$ modifications.

H. F. Fong,
B. Keimer,
D. Reznik,
D. L. Milius,
and
I. A. Aksay,
Phys. Rev.
B **54**, 6708
(1996)

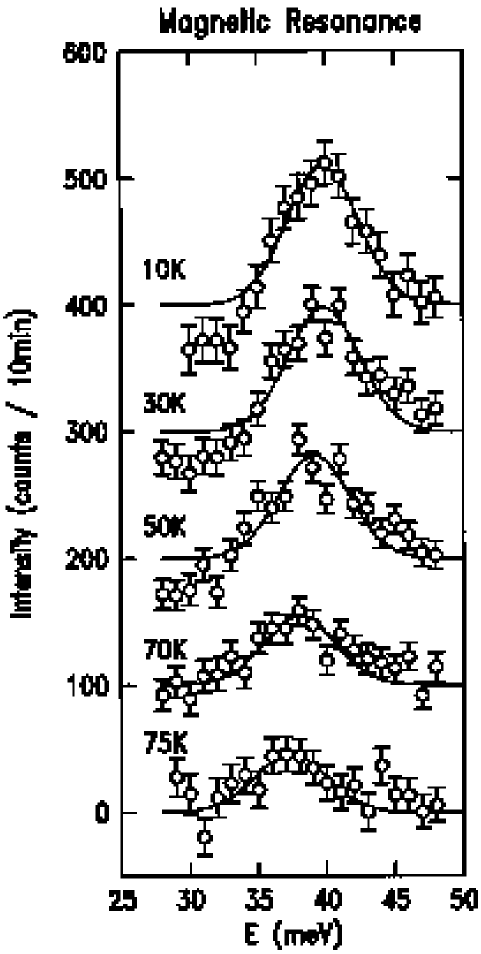
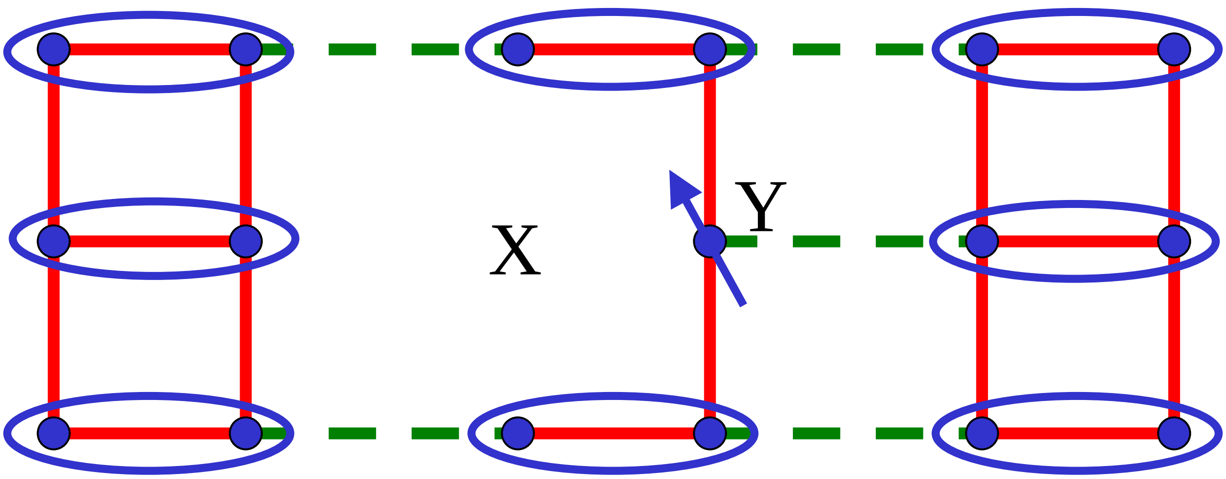


FIG. 8. Unpolarized beam, constant-Q data [Q=(3/2,1/2,−1.7)] of the 40 meV magnetic resonance obtained by subtracting the signal below T_c from the $T=100$ K background. The lines are fits to Gaussians, as described in the text. For clarity successive scans are offset by 100.

Spin-1 collective mode in $YBa_2Cu_3O_7$ - little observable damping at low T → coupling to superconducting quasiparticles unimportant, and spin correlations in some regions of phase space are like those of a (nearly-critical ?) paramagnet

4. Quantum impurities in nearly-critical paramagnets and d-wave superconductors

Make *any* localized deformation of antiferromagnet; e.g. remove a spin



Susceptibility $\chi = A\chi_b + \chi_{imp}$

(A = area of system)

In paramagnetic phase as $T \to 0$

$$\chi_b = \left(\frac{\Delta}{\hbar^2 c^2 \pi}\right) e^{-\Delta/k_B T} \quad ; \chi_{imp} = \frac{S(S+1)}{3k_B T}$$

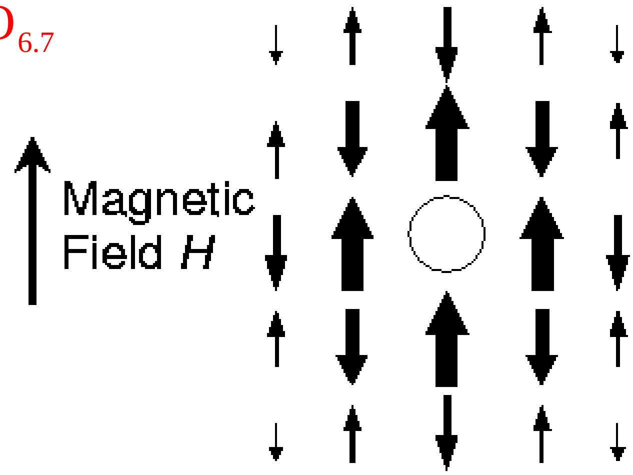
For a general impurity χ_{imp} *defines* the value of S

$$\lim_{\tau\to\infty} \left\langle \vec{S}_Y(\tau) \cdot \vec{S}_Y(0) \right\rangle = m^2 \neq 0$$

Zn impurity in $YBa_2Cu_3O_{6.7}$

Moments measured by
analysis of Knight shifts

M.-H. Julien, T. Feher,
M. Horvatic, C. Berthier,
O. N. Bakharev, P. Segransan,
G. Collin, and J.-F. Marucco,
cond-mat/9911194.



Berry phases of precessing spins do not cancel between the sublattices in the vicinity of the impurity

Orientation of "impurity" spin -- $n_\alpha(\tau)$ (unit vector)

Action of "impurity" spin

$$\mathcal{S}_{\rm imp} = \int d\tau \left[iSA_\alpha(n)\frac{dn_\alpha}{d\tau} - \gamma S n_\alpha(\tau)\phi_\alpha(x=0,\tau)\right]$$

$A_\alpha(n)$ → Dirac monopole function

Boundary quantum field theory: $\mathcal{S}_b + \mathcal{S}_{\rm imp}$

Recall -

$$\mathcal{S}_b = \int d^d x d\tau \left[\frac{1}{2}\left((\nabla_x\phi_\alpha)^2 + c^2(\partial_\tau\phi_\alpha)^2 + r\phi_\alpha^2\right) + \frac{g}{4!}(\phi_\alpha^2)^2\right]$$

Coupling γ approaches *also* approaches a fixed-point value under the renormalization group flow

(Sengupta, 97
Sachdev+Ye, 93
Smith+Si 99)

Beta function:

$$\beta(\gamma) = -\frac{\epsilon\gamma}{2} + \gamma^3 - \gamma^5 + \frac{5g^2\gamma}{144} + \pi^2\left(S(S+1) - \frac{1}{3}\right) g\gamma^3 + \mathcal{O}\left((\gamma, \sqrt{g})^7\right)$$

No new relevant perturbations on the boundary;
All other boundary perturbations are irrelevant –
e.g.

$$\lambda \int d\tau \phi_\alpha^2 (x=0, \tau)$$

Δ and c completely determine spin dynamics near an impurity –
No new parameters are necessary !

Universal properties at the critical point $\lambda = \lambda_c$

$$\left\langle \vec{S}_Y(\tau) \cdot \vec{S}_Y(0) \right\rangle = \frac{1}{\tau^{\eta'}}$$

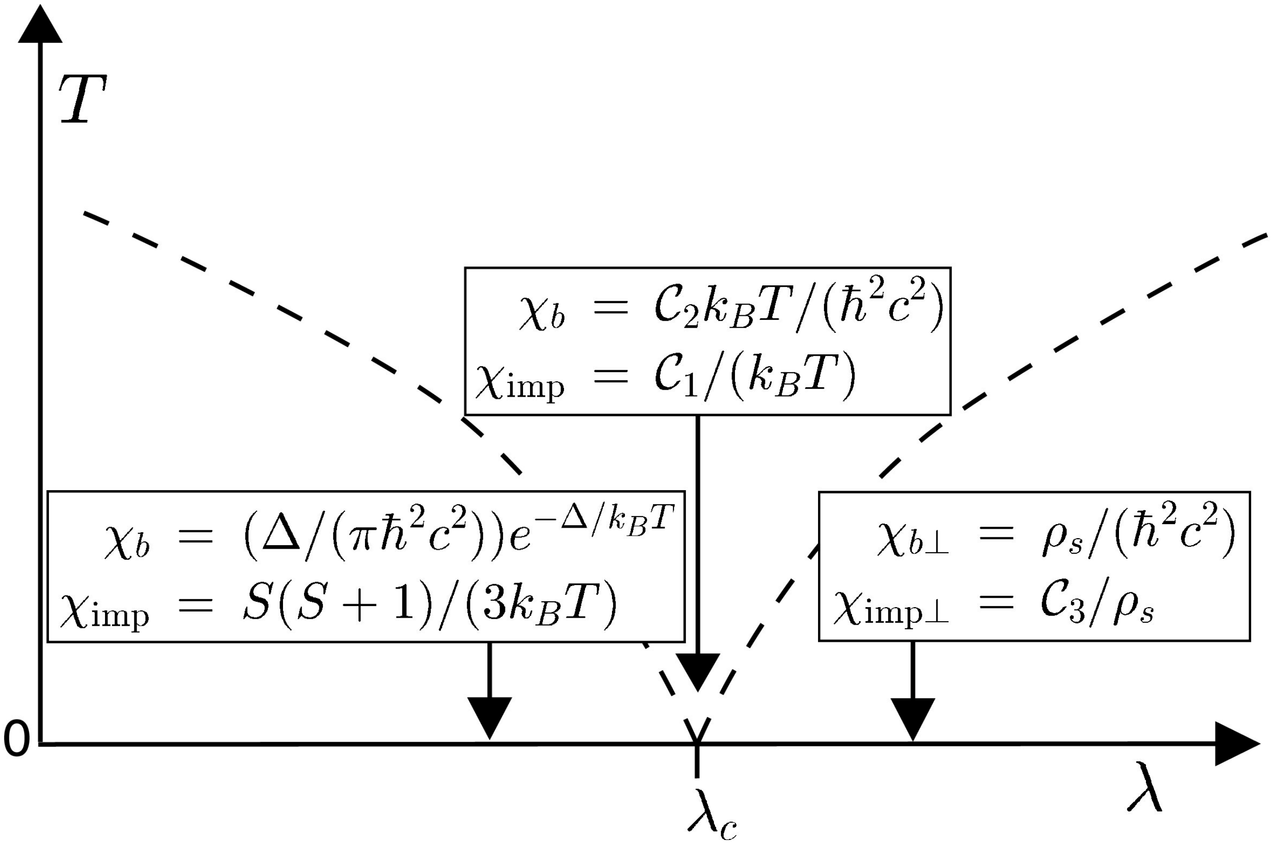
$$\text{(and } m = \left|\lambda - \lambda_c\right|^{\eta'\nu} \text{)}$$

However $$\chi_{imp} \neq \frac{1}{T^{1-\eta'}}$$

This last relationship holds in the multi-channel Kondo problem because the magnetic response of the screening cloud is negligible due to an exact "compensation" property. There is no such property here, and naïve scaling applies. This leads to

$$\chi_{imp} = \frac{\text{Universal number}}{k_B T}$$

Curie response of an irrational spin



In the Neel phase

$$\chi_{imp\perp} = \frac{\text{Universal number}}{\text{spin stiffness}}$$

spin stiffness $\rho_s = \left(\rho_{sx}\rho_{sy}\right)^{1/2}$

Bulk susceptibility vanishes while impurity susceptibility diverges as $\rho_s \to 0$

At $T > 0$, thermal averaging leads to

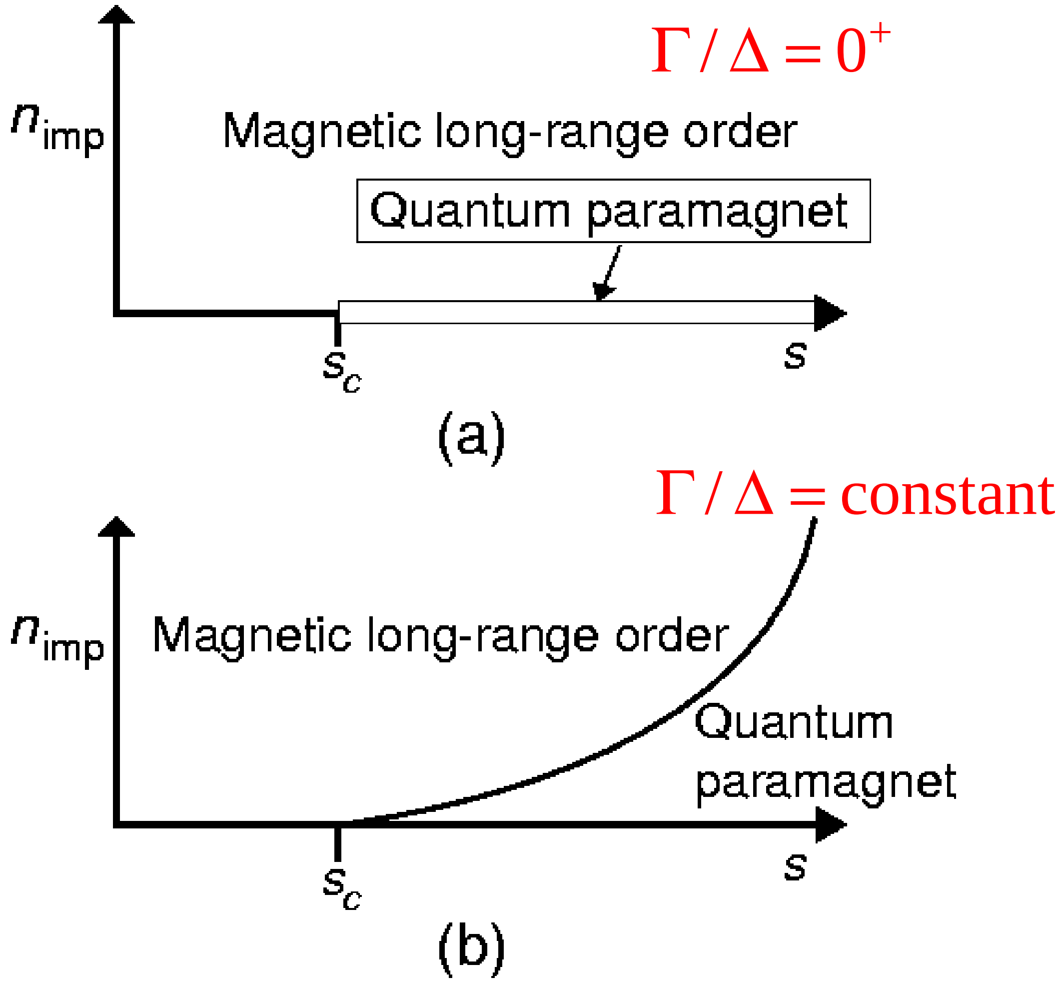
$$\chi_{\rm imp} = \frac{S^2}{3k_B T} + \frac{2}{3}\chi_{\rm imp\perp}$$

Finite density of impurities n_{imp}

Relevant perturbation – strength determined by only energy scale that is linear in n_{imp} and contains only bulk parameters

$$\Gamma \equiv \frac{n_{\text{imp}}(\hbar c)^2}{\Delta}$$

Two possible phase diagrams

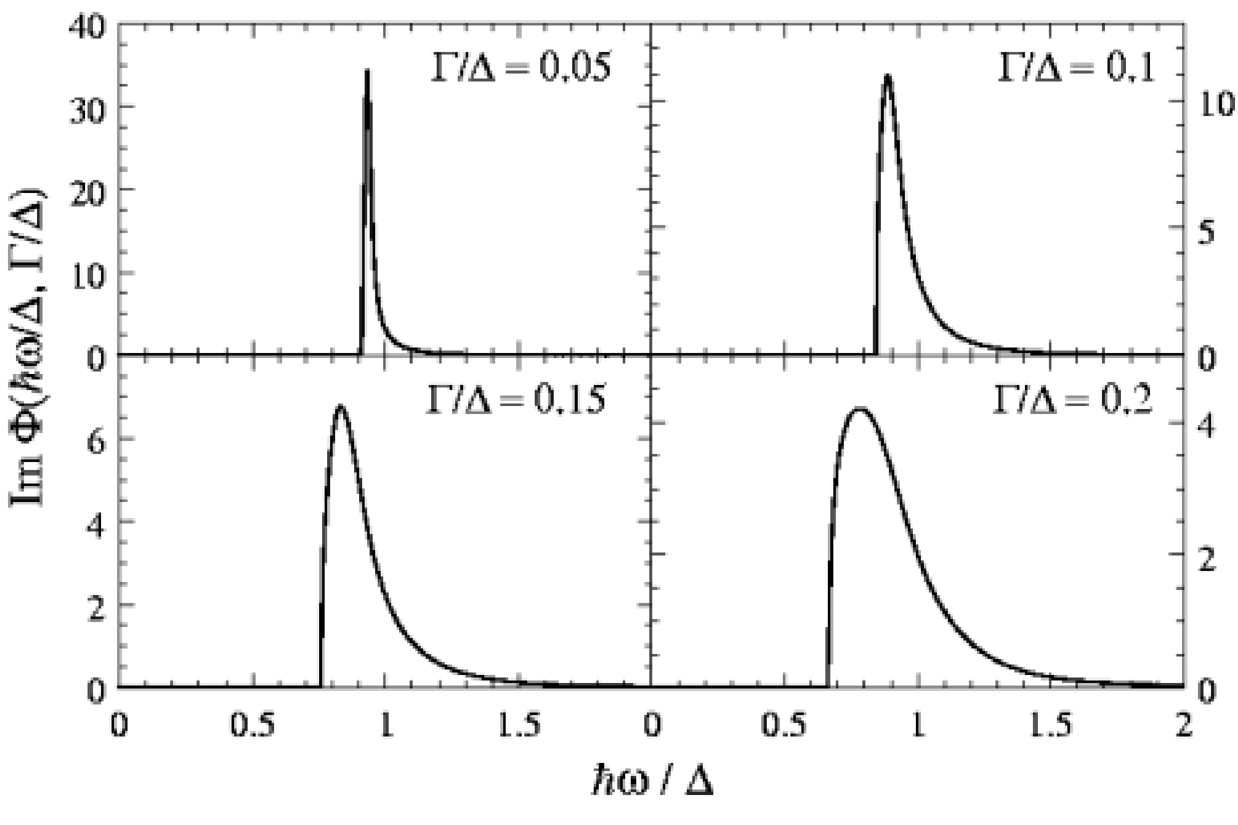


Fate of collective mode peak

Without impurities $\chi(G,\omega) = \dfrac{A}{\Delta^2 - \omega^2}$

With impurities $\chi(G,\omega) = \dfrac{A}{\Delta^2}\Phi\left(\dfrac{\hbar\omega}{\Delta}, \dfrac{\Gamma}{\Delta}\right)$

Φ → *Universal* scaling function. We computed it in a "self-consistent, non-crossing" approximation

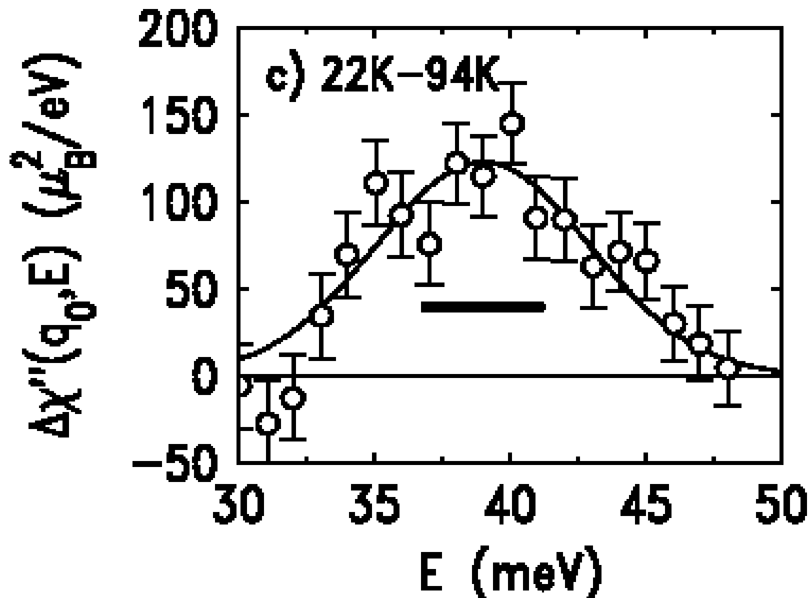


Predictions: Half-width of line $\approx \Gamma$

Universal asymmetric lineshape

$YBa_2Cu_3O_7 + 0.5\%$ Zn

H. F. Fong, P. Bourges, Y. Sidis, L. P. Regnault, J. Bossy, A. Ivanov, D.L. Milius, I. A. Aksay, and B. Keimer, Phys. Rev. Lett. **82**, 1939 (1999)



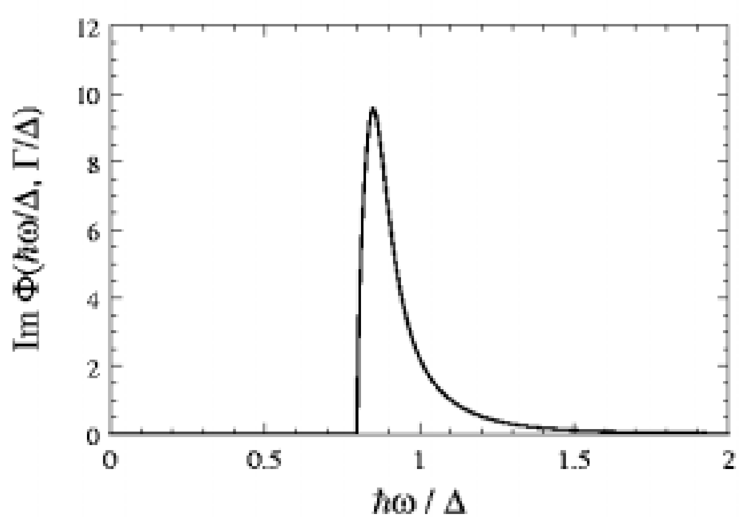
$$n_{\mathrm{imp}} = 0.005$$

$$\Delta = 40\ \mathrm{meV}$$

$$\hbar c = 0.2\ \mathrm{eV}$$

$$\Rightarrow\quad \Gamma = 5\ \mathrm{meV},\ \Gamma/\Delta = 0.125$$

Quoted half-width = 4.25 meV



Conclusions

1. Briefly described T>0 crossovers near a simple magnetic quantum phase transition: relaxation rates and transport coefficients are universal functions of fundamental constants and thermodynamic variables (“universal incoherent conductance”).

2. Theory of T>0 spin dynamics in gapped quasi-one-dimensional antiferromagnets- quantitative comparisons with experiments.

3. Theory of quantum impurities in a nearly critical antiferromagnet – irrational spin excitations and a new boundary quantum field theory. Prediction of neutron scattering linewidth and universal asymmetric lineshapes.