Collective spin dynamics of low-dimensional paramagnets and d-wave superconductors

• C. Buragohain
• K. Damle
• M. Vojta

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Transparencies available online at http://pantheon.yale.edu/~subir/qimptalk.pdf
1. Paramagnetic and Neel ground states in low dimensions --- coupled-ladder antiferromagnet

2. Thermal damping of the collective spin-1 excitation of the paramagnet:
   Quasiclassical particle model
   Exact results in one dimension and comparison with expts.

3. Nearly-critical paramagnets with spin gap $\Delta \ll J$
   Quantum field theory: low T spin dynamics is universally determined by $\Delta$ and $c = (c_x c_y)^{1/2}$

4. Non-magnetic impurities (Zn or Li) in nearly-critical paramagnets and d-wave superconductors:
   Boundary quantum field theory:
   T=0 damping of spin-1 collective mode - universal line-shape with
   no additional parameters needed:
   only need $n_{\text{imp}}$
   Comparison with, and predictions for, expts
1. Paramagnetic and Neel states

$S=1/2$ spins on coupled 2-leg ladders

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

Follow ground state as a function of $\lambda$

$$0 \leq \lambda \leq 1$$
\[ \lambda = 1 \]

**Square lattice antiferromagnet**

Experimental realization: \( La_2CuO_4 \)

Ground state has long-range magnetic (Neel) order

\[
\langle \vec{S}_i \rangle = (-1)^{i_x + i_y} N_0 \neq 0
\]

Excitations: 2 spin waves

Quasiclassical wave dynamics at low T

(Chakravarty et al, 1989; Tyc et al, 1989)
\[ \lambda = 0 \]

Decoupled 2-leg ladders

Allow \( J \neq K \)

Quantum paramagnet ground state for \( J \ll K \)

Qualitatively similar ground state for all \( J/K \)
Excited states

Triplet \((S=1)\) particle (collective mode)

Energy dispersion away from antiferromagnetic wavevector

\[ \varepsilon = \Delta + \frac{c^2 k^2}{2 \Delta} \]

\(\Delta \rightarrow \) Spin gap

In one dimension, all low T dynamic correlations are universal functions of \(\Delta\) and \(c\)
2. Thermal damping of spin-1 collective mode

Key Observations

1. De Broglie wavelength
   \[ \frac{\hbar c}{\sqrt{\Delta k_B T}} \ll \text{interparticle spacing} \sim e^{\Delta/\kappa_B T} \]
   (Quasiclassical particles)

2. Quantum mechanical S-matrix
   has a super-universal form at low momenta
   (in one dimension)

Spins bounce while momenta exchange
\[ G(x, t) = G(x, t) \bigg|_{T=0} (-1)^{\text{number of collisions}} \]
Dynamic Structure Factor at antiferromagnetic wavevector $S(\pi, \omega)$

Quasiclassical Particles

\[ \frac{1}{\tau_\varphi} \approx \text{time between collisions} \]

\[ = 1.22 \frac{k_B T}{\hbar} e^{-\Delta/k_B T} \]

Particles are quantum lumps of “amplitude oscillation” of anti-ferromagnetic order parameter
Excitation energy $\Delta$ and linewidth $\Gamma$

for Y$_2$BaNiO$_5$

C. Broholm et al. (unpublished)

Solid line for $\Gamma$ – theory with no free parameters
Quantum paramagnet $\langle \vec{S} \rangle = 0$

Neel state $\langle \vec{S} \rangle \neq N_0$

Spin gap $\Delta$

Neel order $N_0$
3. Nearly-critical paramagnets

\( \lambda \) is close to \( \lambda_c \)

Quantum field theory:

\[
S_b = \int d^d x d\tau \left[ \frac{1}{2} (\nabla_x \phi_\alpha)^2 + c^2 (\partial_t \phi_\alpha)^2 + r \phi_\alpha^2 \right] + \frac{g}{4!} (\phi_\alpha^2)^2
\]

\( \phi_\alpha \rightarrow \) 3-component antiferromagnetic order parameter

\( r > 0 \rightarrow \lambda < \lambda_c \)

\( r < 0 \rightarrow \lambda > \lambda_c \)

Oscillations of \( \phi_\alpha \) about zero (for \( r > 0 \))

\( \rightarrow \) spin-1 collective mode

\[ \text{Im} \chi (k, \omega) \rightarrow T=0 \text{ spectrum} \]
Coupling $g$ approaches fixed-point value under renormalization group flow: beta function ($\epsilon = 3-d$):

$$\beta(g) = -\epsilon g + \frac{11g^2}{6} - \frac{23g^3}{12} + \mathcal{O}(g^4)$$

Only relevant perturbation – $r$
strength is measured by the spin gap $\Delta$

$\Delta$ and $c$ completely determine entire spectrum of quasi-particle peak and multiparticle continua, the $S$ matrices for scattering between the excitations, and $T > 0$ modifications.
Spin-1 collective mode in YBa$_2$Cu$_3$O$_7$ - little observable damping at low T $\rightarrow$ coupling to superconducting quasiparticles unimportant, and spin correlations in some regions of phase space are like those of a (nearly-critical ?) paramagnet
4. Quantum impurities in nearly-critical paramagnets and d-wave superconductors

Make *any* localized deformation of antiferromagnet; e.g. remove a spin

Susceptibility \( \chi = A \chi_b + \chi_{imp} \)

\( A = \text{area of system} \)

In paramagnetic phase as \( T \to 0 \)

\[ \chi_b = \left( \frac{\Delta}{\hbar^2 c^2 \pi} \right) e^{-\Delta/k_B T} \quad ; \quad \chi_{imp} = \frac{S(S+1)}{3k_B T} \]

For a general impurity \( \chi_{imp} \) *defines* the value of \( S \)

\[ \lim_{\tau \to \infty} \langle \mathbf{S}_Y(\tau) \cdot \mathbf{S}_Y(0) \rangle = m^2 \neq 0 \]
Zn impurity in YBa$_2$Cu$_3$O$_{6.7}$

Moments measured by analysis of Knight shifts


Berry phases of precessing spins do not cancel between the sublattices in the vicinity of the impurity
Orientation of “impurity” spin -- \( n_\alpha (\tau) \) (unit vector)

**Action of “impurity” spin**

\[
S_{\text{imp}} = \int d\tau \left[ iS A_\alpha (n) \frac{dn_\alpha}{d\tau} - \gamma S n_\alpha (\tau) \phi_\alpha (x = 0, \tau) \right]
\]

\( A_\alpha (n) \rightarrow \) Dirac monopole function

**Boundary quantum field theory:** \( S_b + S_{\text{imp}} \)

Recall -

\[
S_b = \int d^d x d\tau \left[ \frac{1}{2} \left( (\nabla_x \phi_\alpha)^2 + c^2 (\partial_\tau \phi_\alpha)^2 + r \phi_\alpha^2 \right) + \frac{g}{4!} (\phi_\alpha^2)^2 \right]
\]
Coupling $\gamma$ approaches also approaches a fixed-point value under the renormalization group flow

(Sengupta, 97  
Sachdev+Ye, 93  
Smith+Si 99)

Beta function:

$$\beta(\gamma) = -\frac{\epsilon \gamma}{2} + \gamma^3 - \gamma^5 + \frac{5g^2 \gamma}{144}$$

$$+ \frac{\pi^2}{2} \left(S(S+1) - \frac{1}{3}\right) g \gamma^3 + O((\gamma, \sqrt{g})^7)$$

No new relevant perturbations on the boundary; 
All other boundary perturbations are irrelevant –  

e.g. $$\lambda \int d\tau \phi^2_\alpha(x = 0, \tau)$$

$\Delta$ and $c$ completely determine spin dynamics near an impurity –  
No new parameters are necessary!
Universal properties at the critical point $\lambda = \lambda_c$

$$\langle \vec{S}_Y(\tau) \cdot \vec{S}_Y(0) \rangle = \frac{1}{\tau^\eta'}$$

(and $m = |\lambda - \lambda_c|^{\eta''}$)

However $\chi_{imp} \neq \frac{1}{T^{1-\eta'}}$

This last relationship holds in the multi-channel Kondo problem because the magnetic response of the screening cloud is negligible due to an exact "compensation" property. There is no such property here, and naïve scaling applies. This leads to

$$\chi_{imp} = \frac{\text{Universal number}}{k_B T}$$

Curie response of an irrational spin
In the Neel phase

\[ \chi_{\text{imp}} = \frac{C_1}{(k_B T)} \]

\[ \chi_b = C_2 k_B T / (\hbar^2 c^2) \]

\[ \chi_{\text{imp}} = S(S + 1) / (3 k_B T) \]

\[ \chi_{\text{imp}} = \rho s \]

\[ \chi_{\text{imp}} = \rho s / (\hbar^2 c^2) \]

\[ \chi_{b\perp} = \rho s / (\hbar^2 c^2) \]

\[ \chi_{b\perp} = C_3 / \rho s \]

\[ \chi_{b} = (\Delta / (\pi \hbar^2 c^2)) e^{-\Delta / k_B T} \]

Bulk susceptibility vanishes while impurity susceptibility diverges as \( \rho s \to 0 \)

At \( T > 0 \), thermal averaging leads to

\[ \chi_{\text{imp}} = \frac{S^2}{3 k_B T} + \frac{2}{3} \chi_{\text{imp}\perp} \]
Finite density of impurities \( n_{\text{imp}} \)

Relevant perturbation – strength determined by only energy scale that is linear in \( n_{\text{imp}} \) and contains only bulk parameters

\[
\Gamma \equiv \frac{n_{\text{imp}} (\hbar c)^2}{\Delta}
\]

Two possible phase diagrams

\[
\Gamma / \Delta = 0^+ \\
\Gamma / \Delta = \text{constant}
\]
Fate of collective mode peak

Without impurities \( \chi(G, \omega) = \frac{A}{\Delta^2 - \omega^2} \)

With impurities \( \chi(G, \omega) = \frac{A}{\Delta^2} \Phi\left(\frac{\hbar \omega}{\Delta}, \frac{\Gamma}{\Delta}\right) \)

\( \Phi \rightarrow \text{Universal scaling function. We computed it in a "self-consistent, non-crossing" approximation} \)

Predictions: Half-width of line \( \approx \Gamma \)
Universal asymmetric lineshape
YBa$_2$Cu$_3$O$_y$ + 0.5% Zn


\[ n_{\text{imp}} = 0.005 \]
\[ \Delta = 40 \text{ meV} \]
\[ \hbar c = 0.2 \text{ eV} \]
\[ \Rightarrow \Gamma = 5 \text{ meV}, \Gamma/\Delta = 0.125 \]

Quoted half-width = 4.25 meV
Conclusions

1. Briefly described T>0 crossovers near a simple magnetic quantum phase transition: relaxation rates and transport coefficients are universal functions of fundamental constants and thermodynamic variables ("universal incoherent conductance").

2. Theory of T>0 spin dynamics in gapped quasi-one-dimensional antiferromagnets-quantitative comparisons with experiments.