Quantum criticality: where are we and where are we going ?

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Talk online at http://sachdev.physics.harvard.edu

Outline

1. Density-driven phase transitions
    A. Fermions with repulsive interactions
    B. Bosons with repulsive interactions
    C. Fermions with attractive interactions
2. Magnetic transitions of Mott insulators
    A. Dimerized Mott insulators – Landau-Ginzburg-Wilson theory
    B. S=1/2 per unit cell: deconfined quantum criticality
3. Transitions of the Kondo lattice
    A. Large Fermi surfaces – Hertz theory
    B. Fractional Fermi liquids and gauge theory

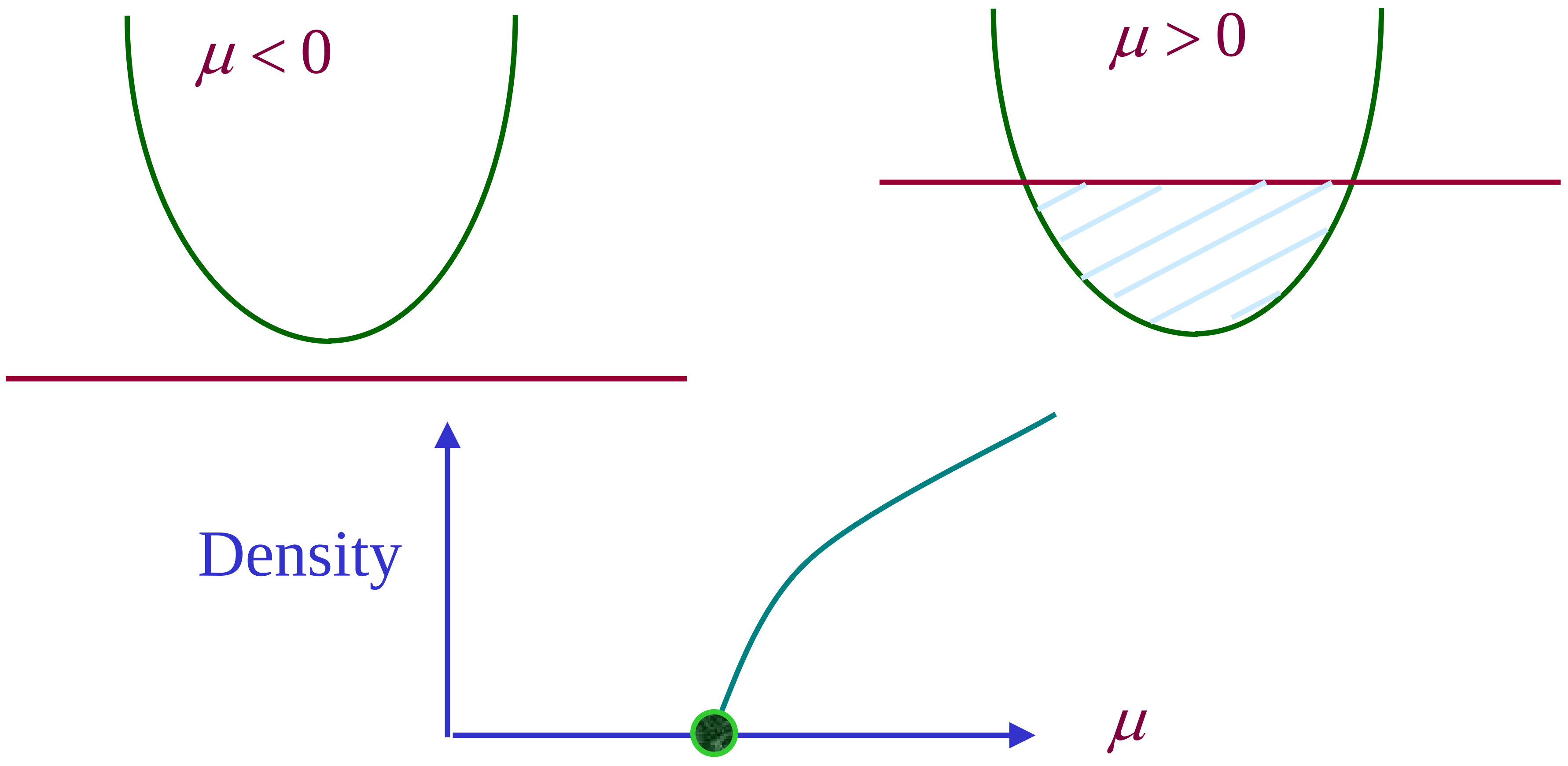
I. Density driven transitions

Non-analytic change in a conserved density (spin) driven by changes in chemical potential (magnetic field)

1.A Fermions with repulsive interactions

$$H = \sum_k \left(\varepsilon_k - \mu\right) c_{k\sigma}^{\dagger} c_{k\sigma}$$

+ short-range repulsive interactions of strength u



1.A Fermions with repulsive interactions

Characteristics of this 'trivial' quantum critical point:

- No "order parameter". "Topological" characterization in the existence of the Fermi surface in one state.

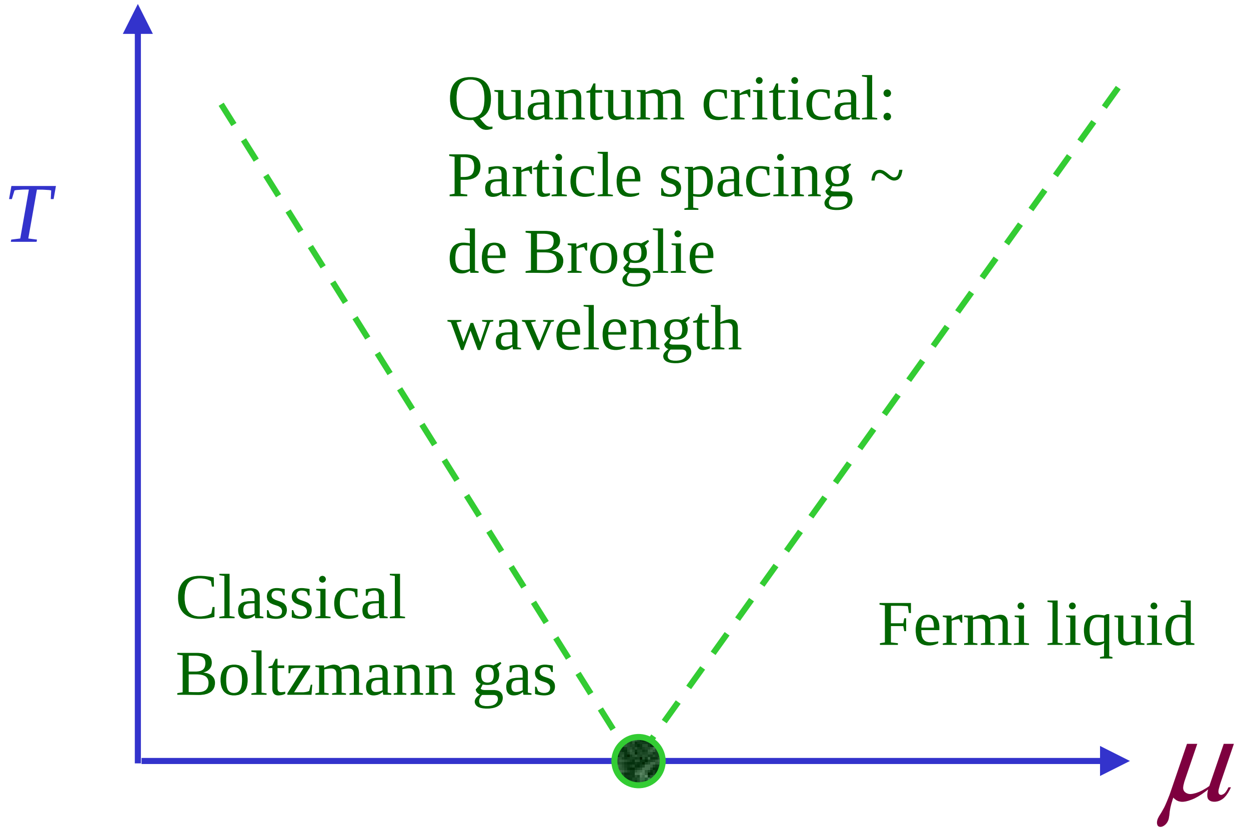
- No transition at $T > 0$.

- Characteristic crossovers at $T > 0$, between quantum criticality, and low T regimes.

- Strong T-dependent scaling in quantum critical regime, with response functions scaling universally as a function of k^z/T and ω/T, where z is the dynamic critical exponent.

1.A Fermions with repulsive interactions

Characteristics of this 'trivial' quantum critical point:

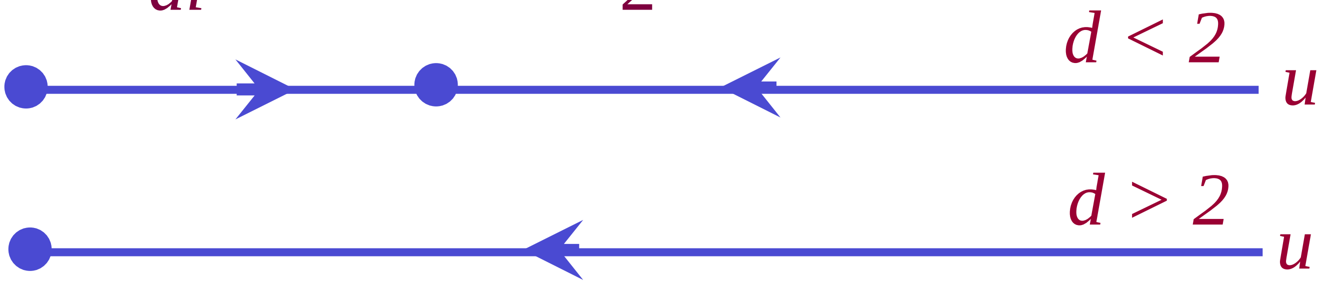


1.A Fermions with repulsive interactions

Characteristics of this 'trivial' quantum critical point:

RG flow characterizing quantum critical point:

$$\frac{du}{dl} = (2-d)u - \frac{u^2}{2}$$

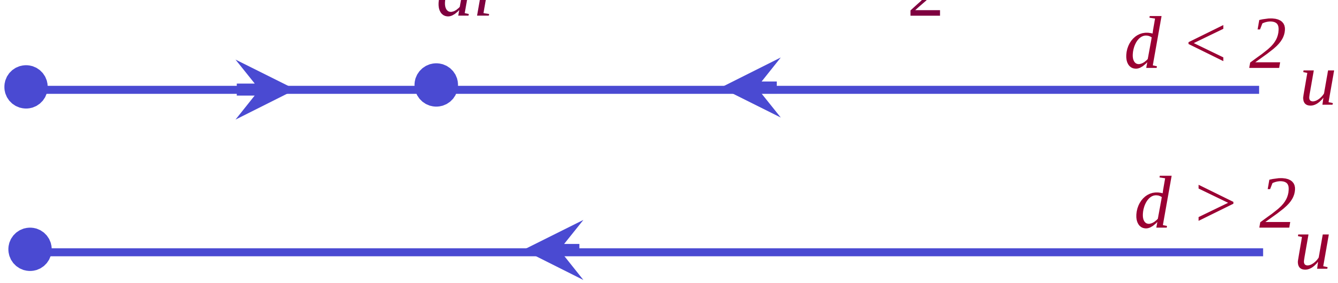


• *d > 2* – interactions are irrelevant. Critical theory is the *spinful* free Fermi gas.

• *d < 2* – universal fixed point interactions. In *d=1* critical theory is the *spinless* free Fermi gas

1.B Bosons with repulsive interactions

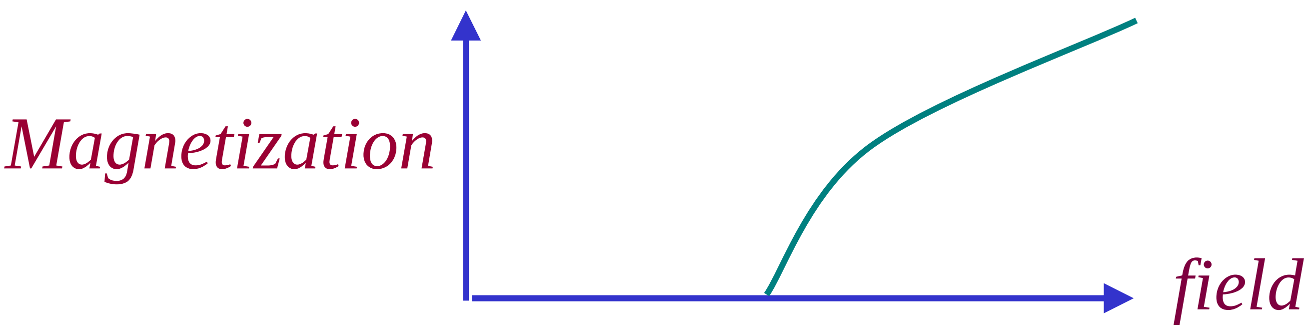
$$\frac{du}{dl} = (2-d)u - \frac{u^2}{2}$$



- Describes field-induced magnetization transitions in spin gap compounds

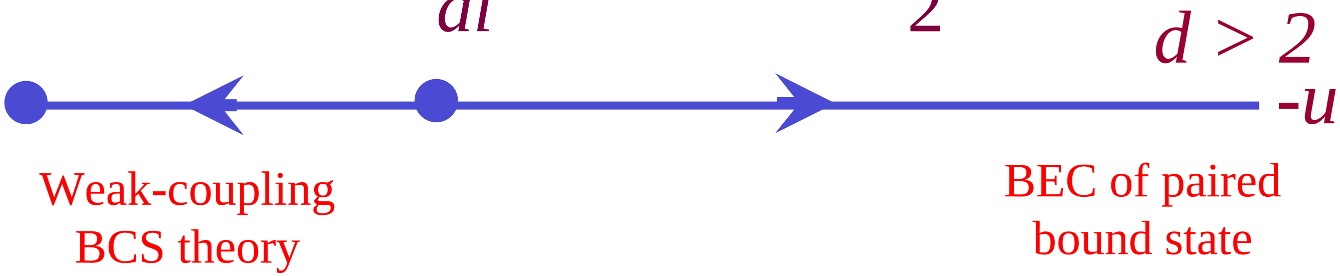
- Critical theory in *d =1* is also the *spinless* free *Fermi* gas.

- Properties of the dilute Bose gas in *d >2* violate hyperscaling and depend upon microscopic scattering length (Yang-Lee).



1.C Fermions with attractive interactions

$$\frac{du}{dl} = (2-d)u - \frac{u^2}{2}$$



• Universal fixed-point is accessed by *fine-tuning* to a Feshbach resonance.

• Density onset transition is described by free fermions for weak-coupling, and by (nearly) free bosons for strong coupling. The quantum-critical point between these behaviors is the Feshbach resonance.

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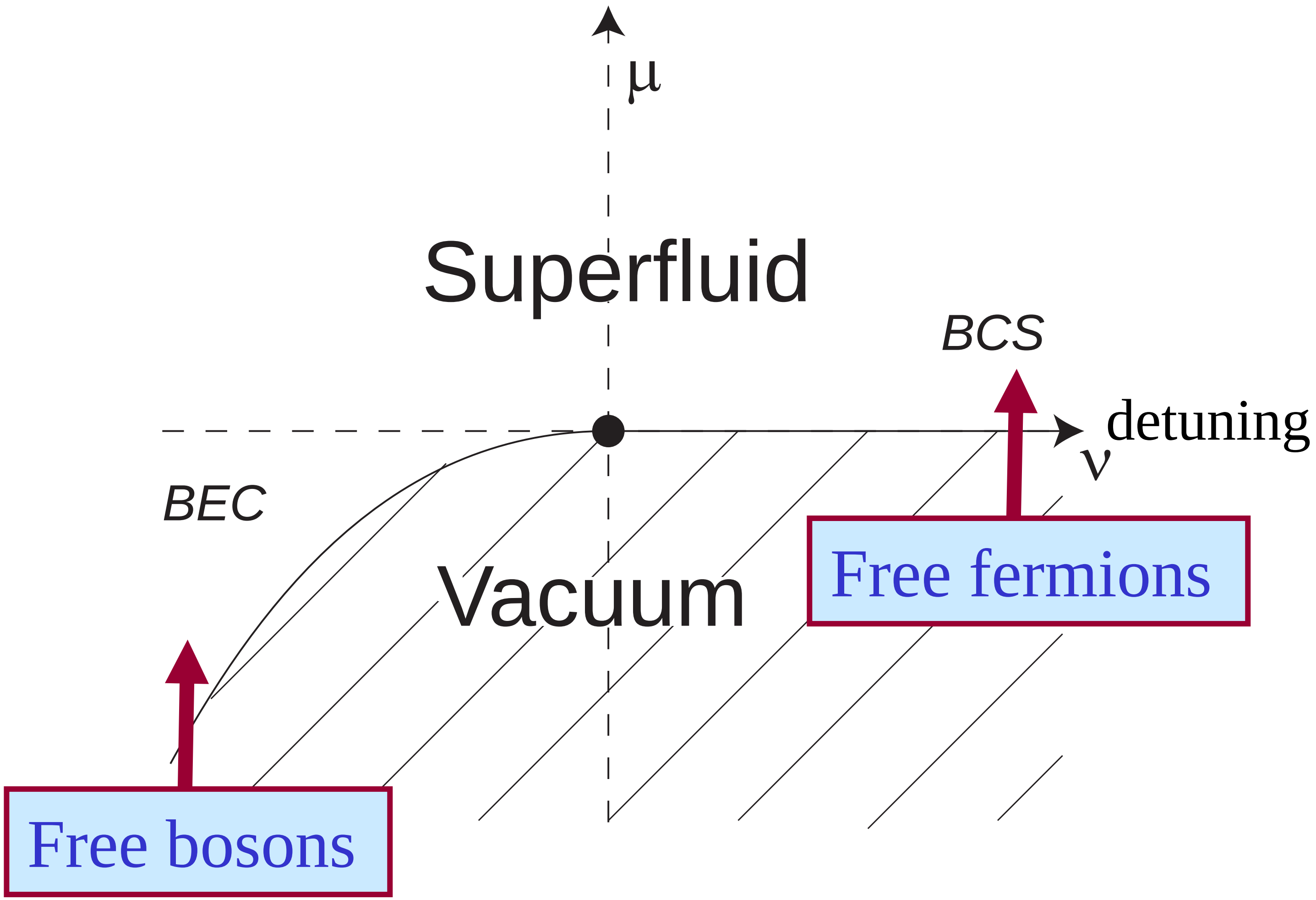
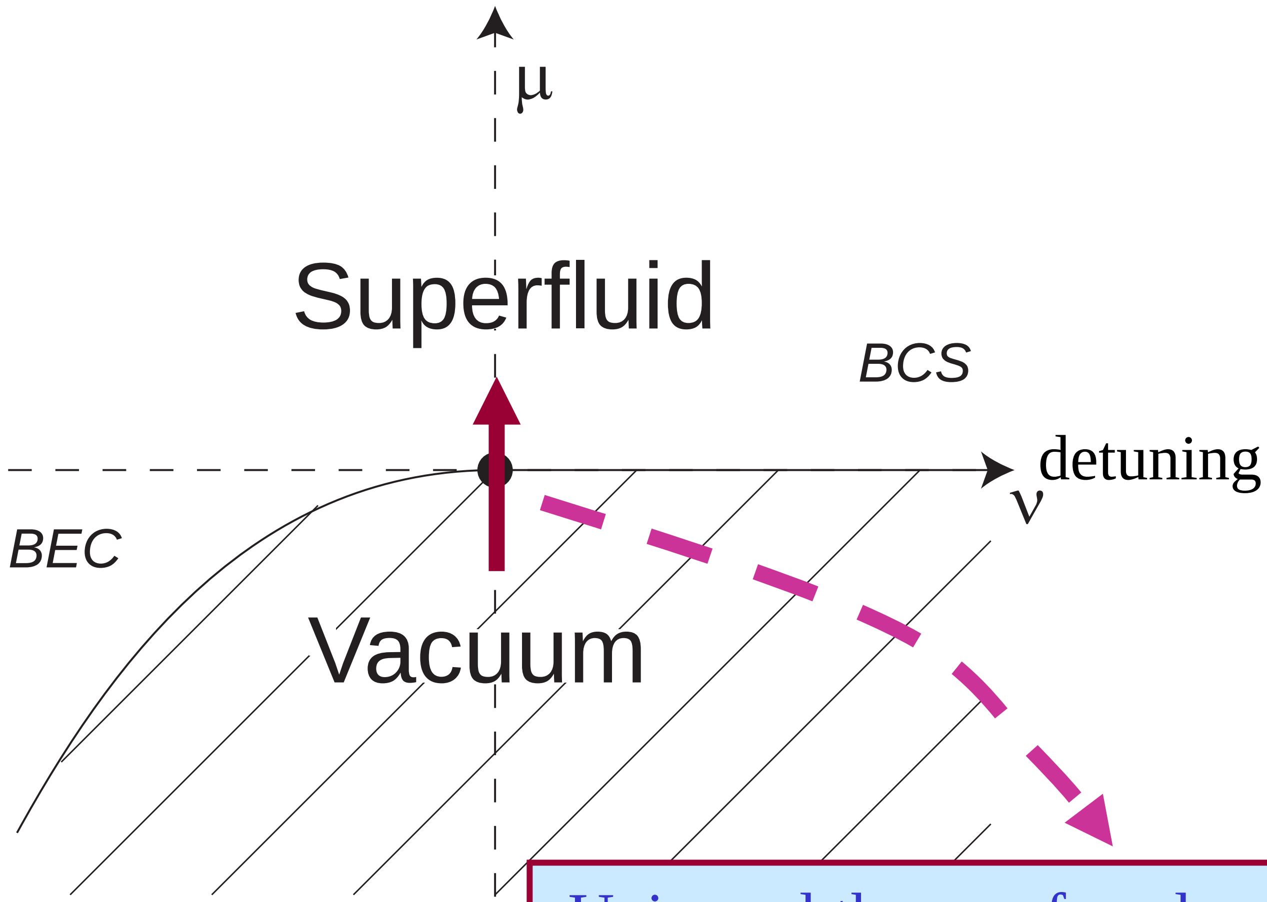
1.C Fermions with attractive interactions



P. Nikolic and S. Sachdev cond-mat/0609106

1.C Fermions with attractive interactions



Universal theory of gapless bosons and fermions, with decay of boson into 2 fermions relevant for $d < 4$

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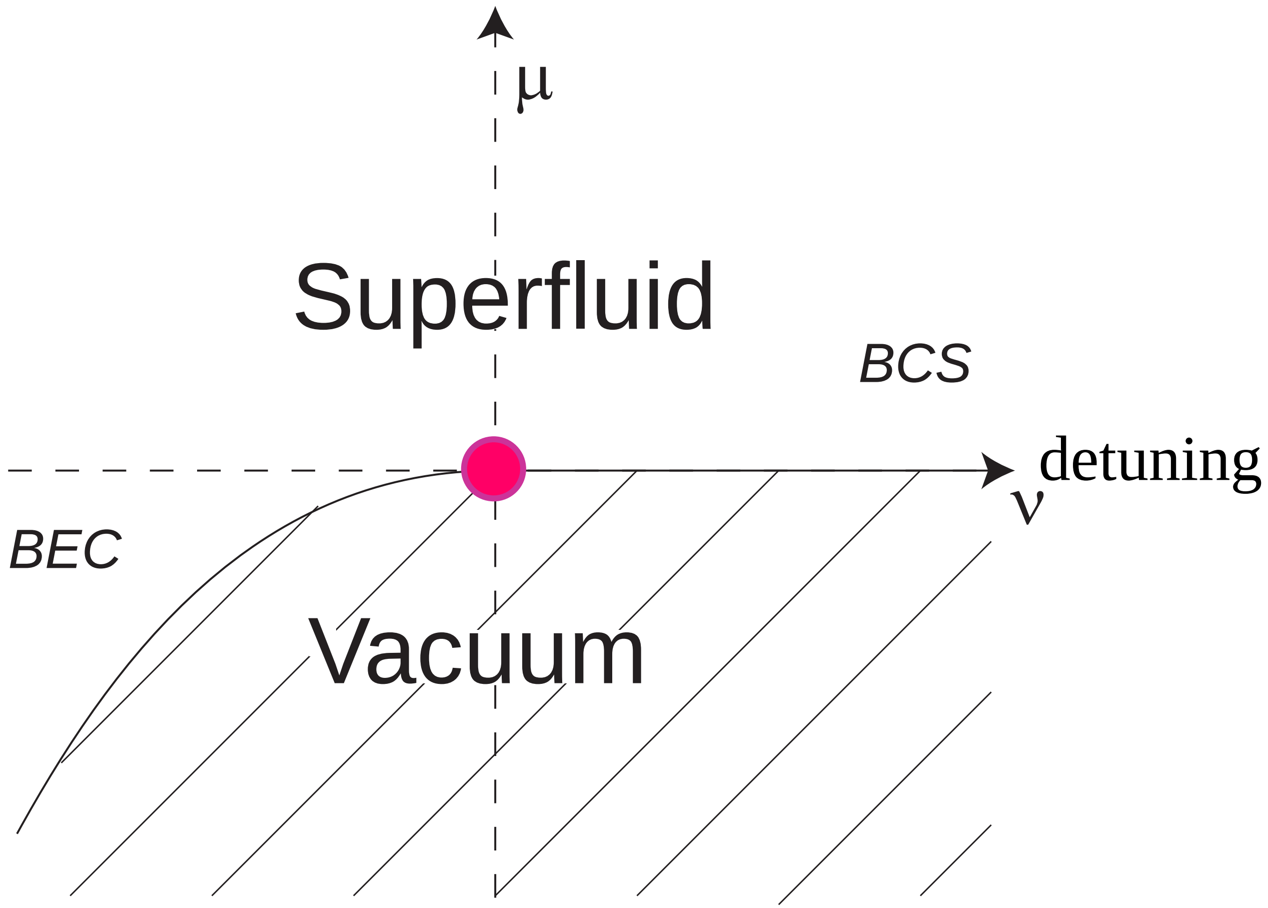
1.C Fermions with attractive interactions



Quantum critical point at $\mu=0$, $\nu=0$, forms the basis of a theory which describes ultracold atom experiments, including the transitions to FFLO and normal states with unbalanced densities

P. Nikolic and S. Sachdev cond-mat/0609106

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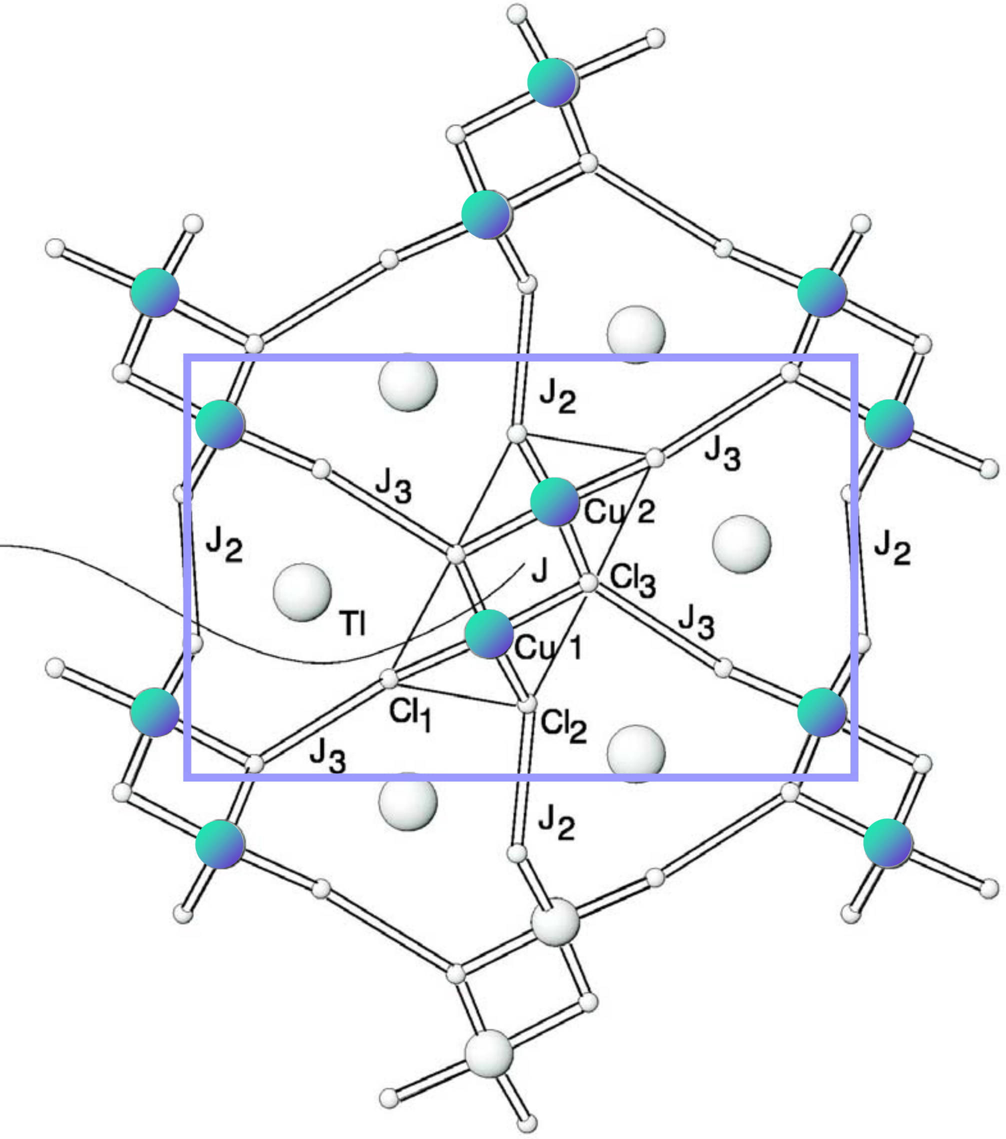
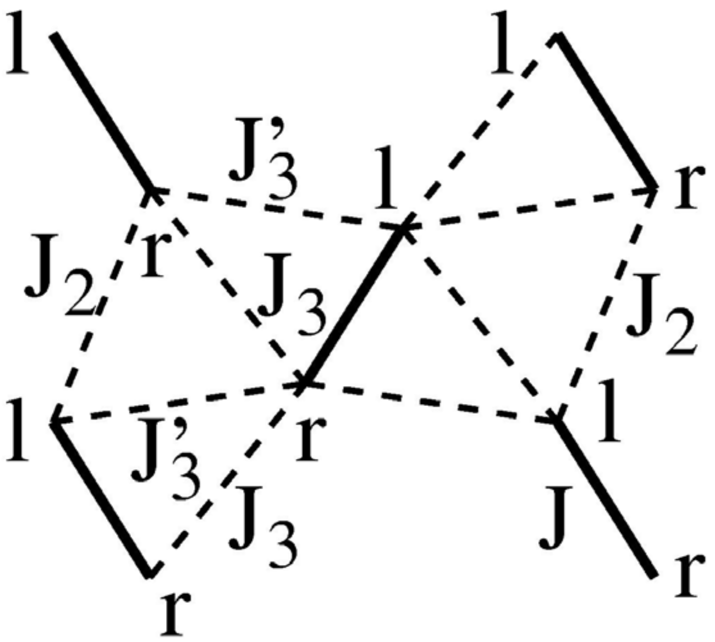
2.A. Magnetic quantum phase transitions in "dimerized" Mott insulators:

Landau-Ginzburg-Wilson (LGW) theory:

Second-order phase transitions described by fluctuations of an ***order parameter*** *associated with a* ***broken symmetry***

$TlCuCl_3$



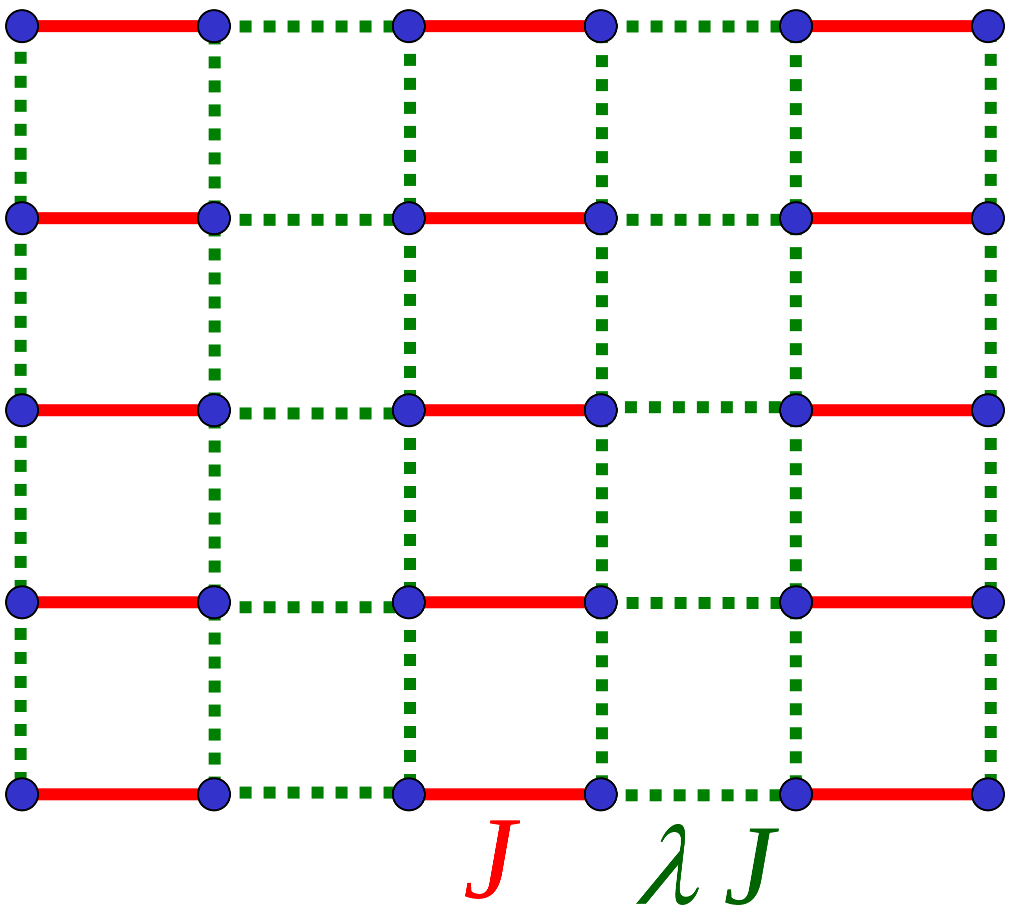


M. Matsumoto, B. Normand, T.M. Rice, and M. Sigrist, cond-mat/0309440.

Coupled Dimer Antiferromagnet

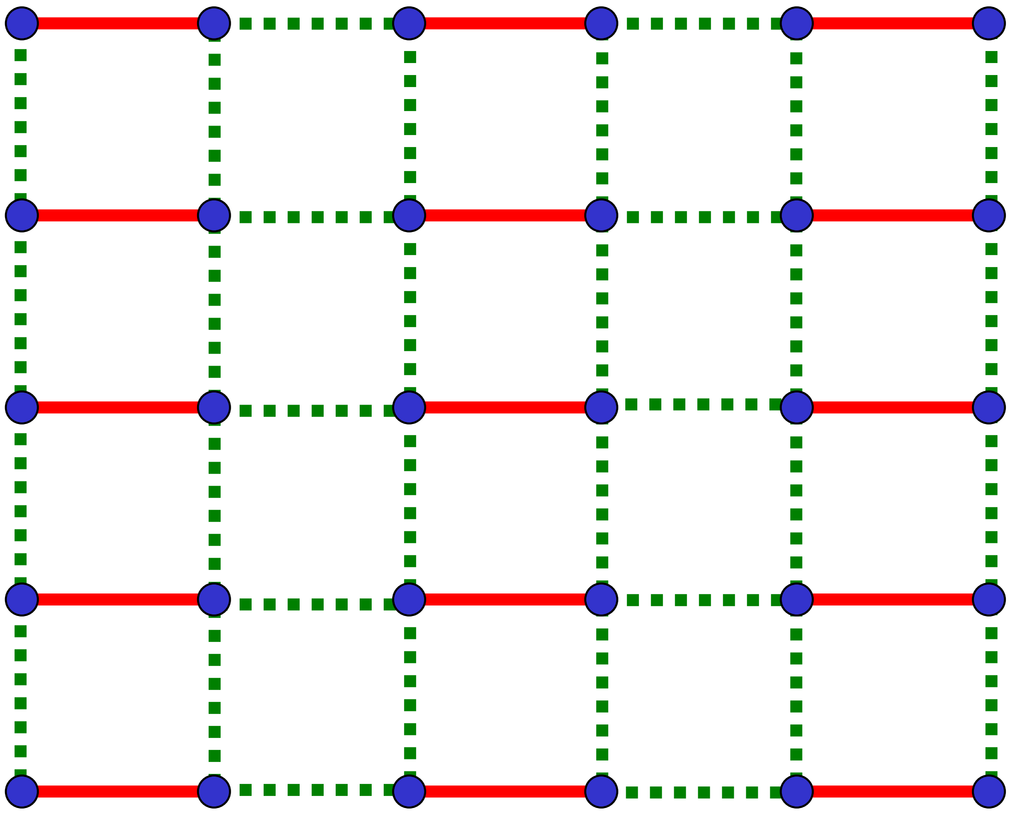
M. P. Gelfand, R. R. P. Singh, and D. A. Huse, *Phys. Rev. B* **40**, 10801-10809 (1989).
N. Katoh and M. Imada, *J. Phys. Soc. Jpn.* **63**, 4529 (1994).
J. Tworzydlo, O. Y. Osman, C. N. A. van Duin, J. Zaanen, *Phys. Rev.* B **59**, 115 (1999).
M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev.* B **65**, 014407 (2002).

S=1/2 spins on coupled dimers



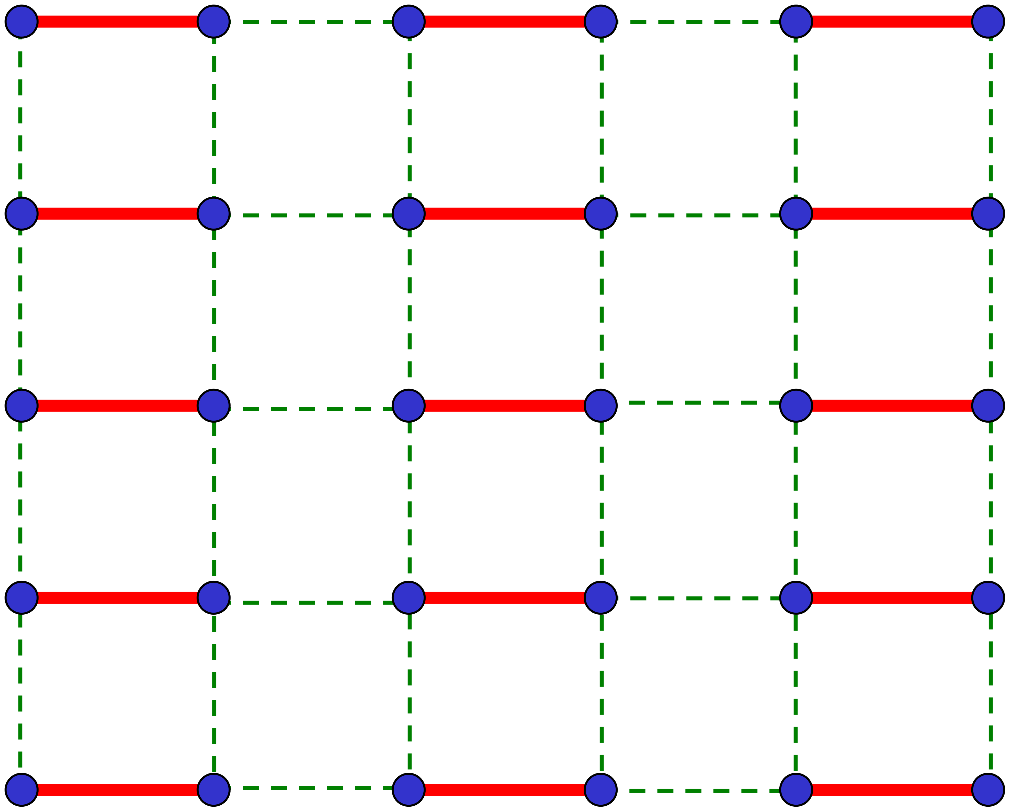
$$H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$0 \le \lambda \le 1$$



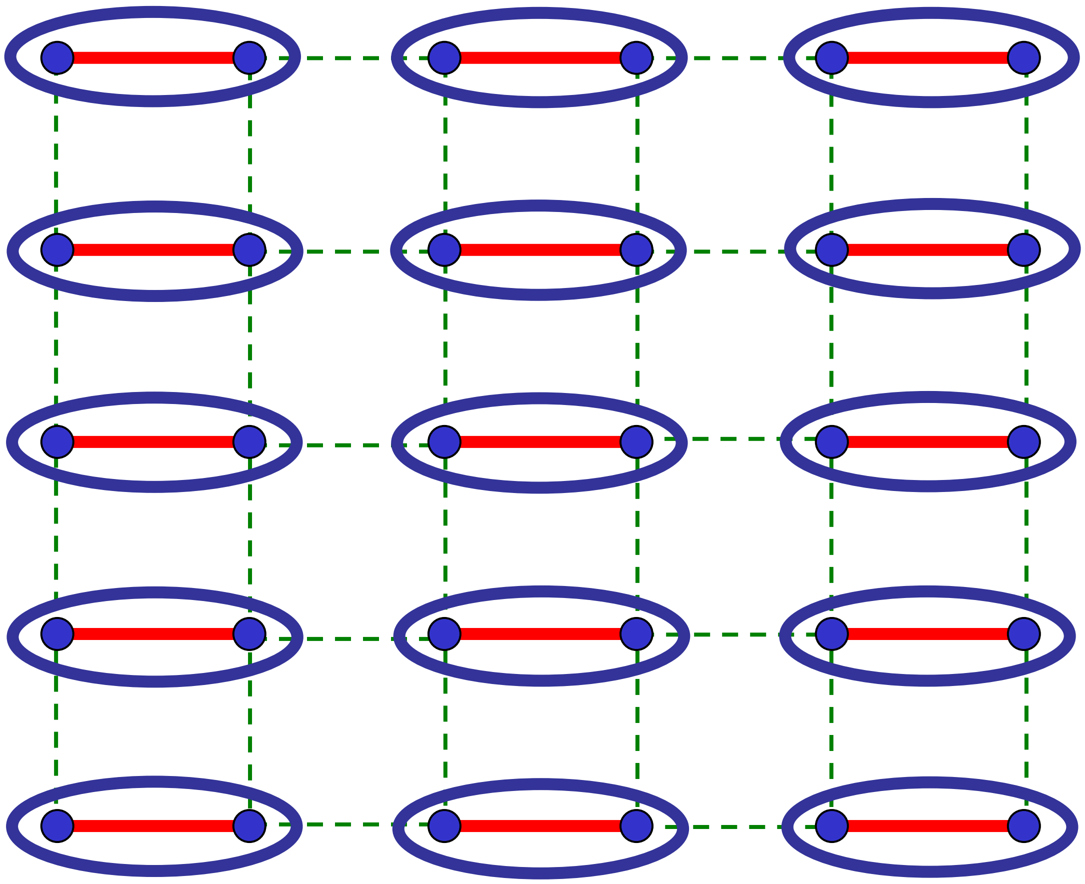
λ close to 0

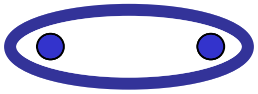
Weakly coupled dimers



λ close to 0

Weakly coupled dimers



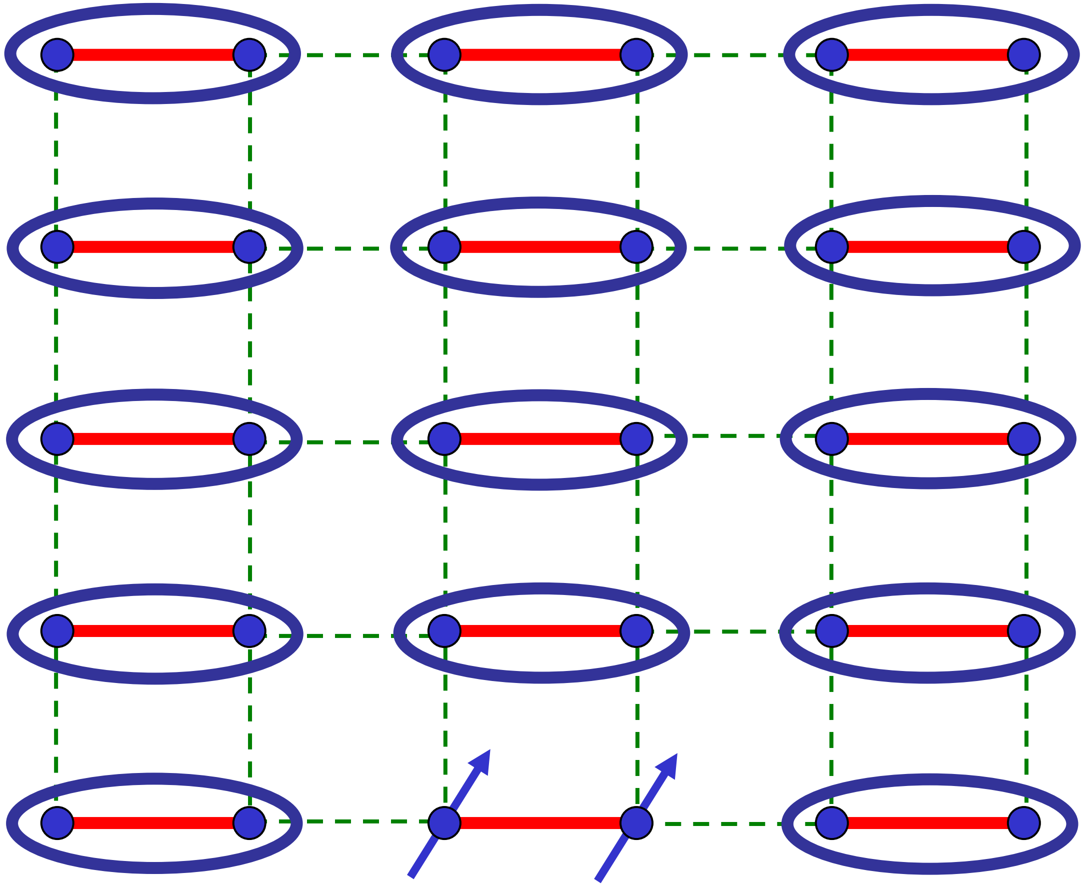

$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$

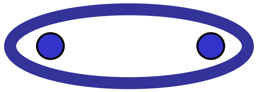
Paramagnetic ground state

$$\left\langle \vec{S}_i \right\rangle = 0, \left\langle \vec{\varphi} \right\rangle = 0$$

λ close to 0

Weakly coupled dimers



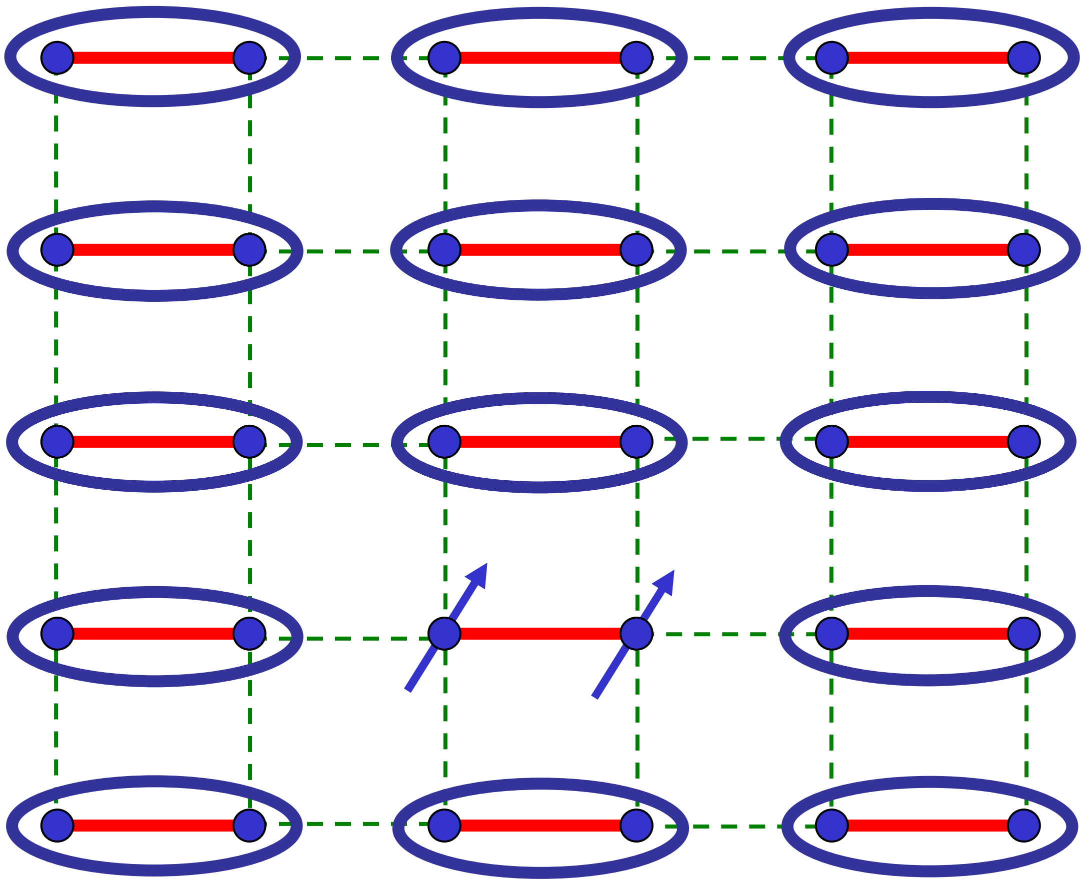


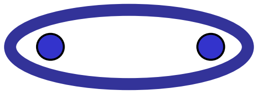
$$= \frac{1}{\sqrt{2}} \left(\left| \uparrow \downarrow \right\rangle - \left| \downarrow \uparrow \right\rangle \right)$$

Excitation: S=1 ***triplon***

λ close to 0

Weakly coupled dimers





$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

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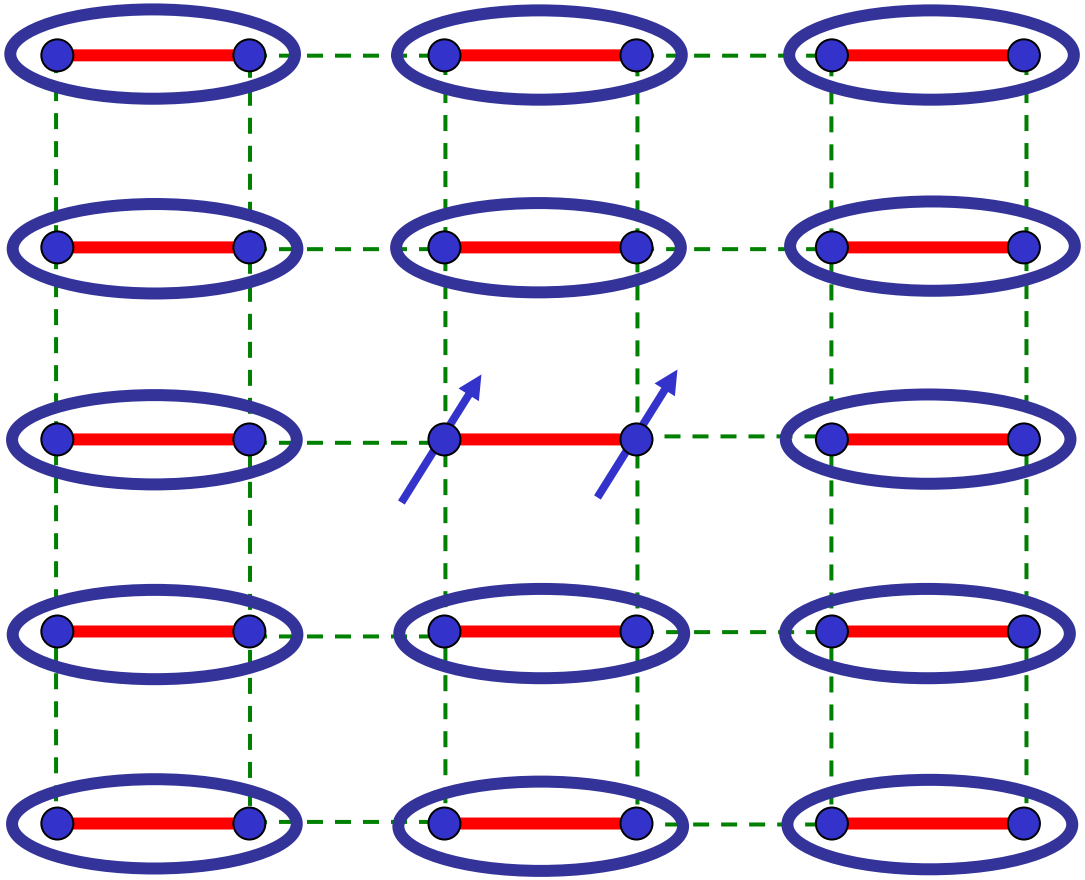
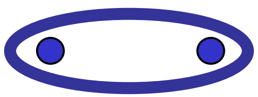
Weakly coupled dimers



 $= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$

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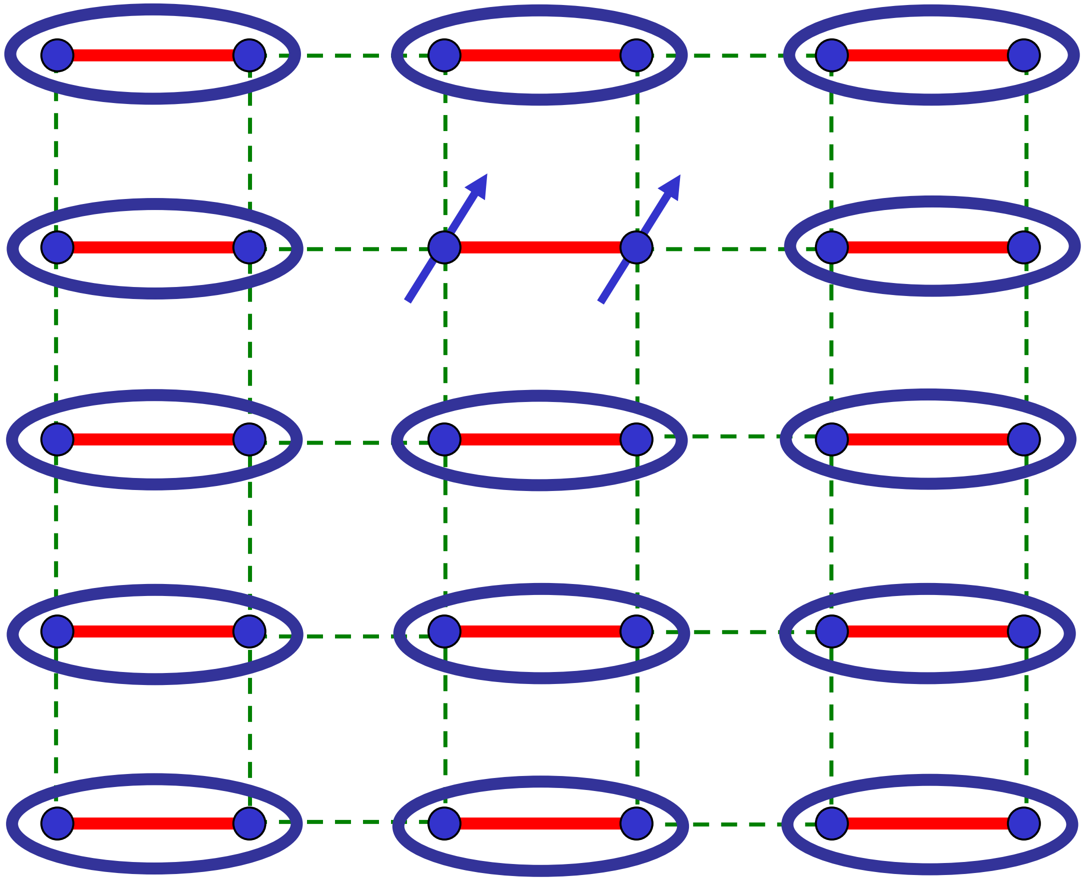
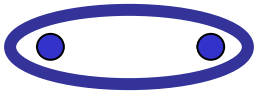
Weakly coupled dimers



 $= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$

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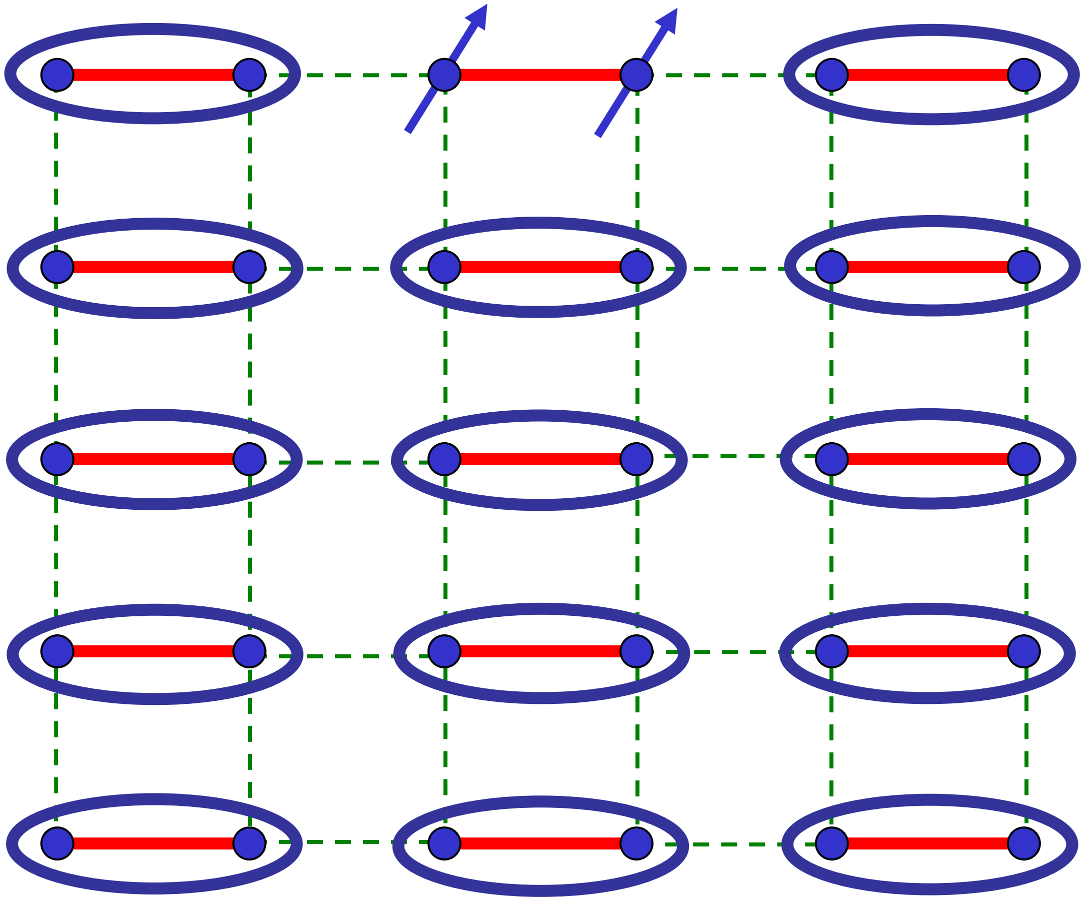
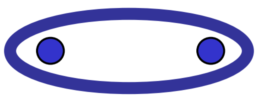
Weakly coupled dimers

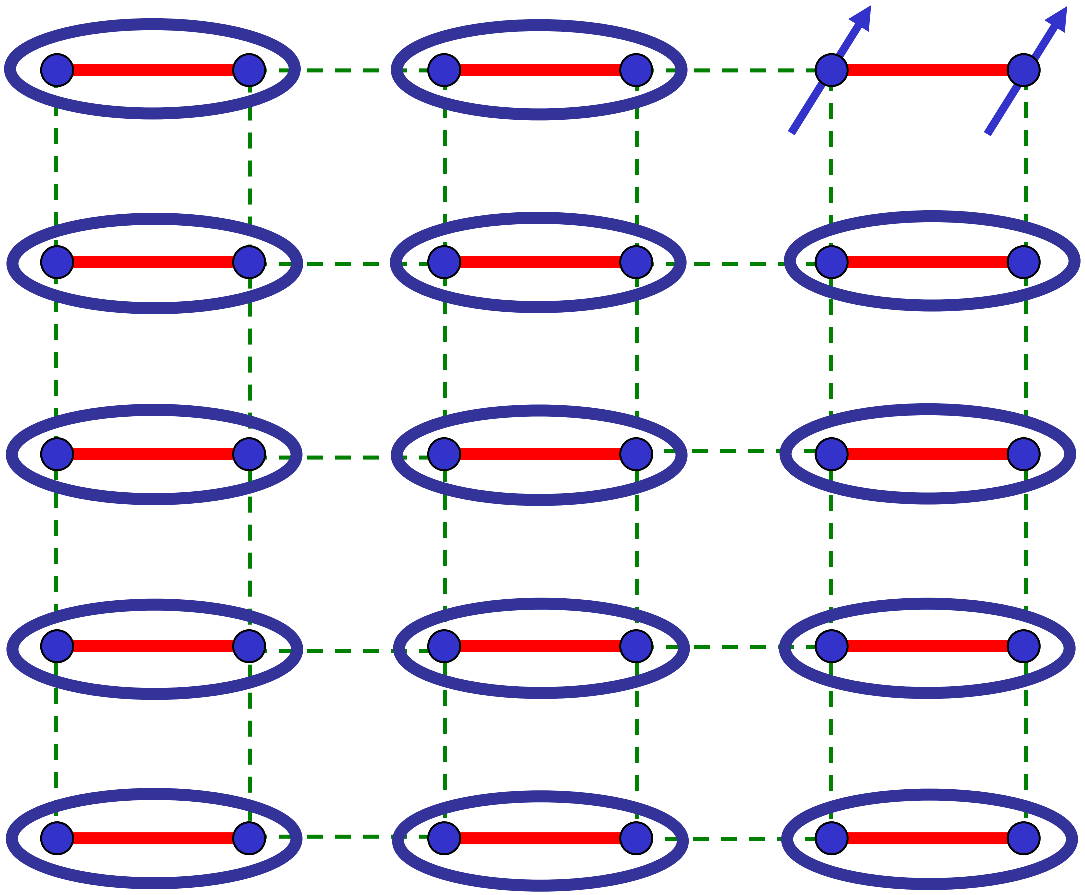


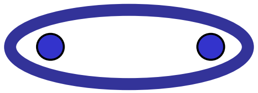
 $= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$

Excitation: S=1 ***triplon***

λ close to 0

Weakly coupled dimers



 $= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$

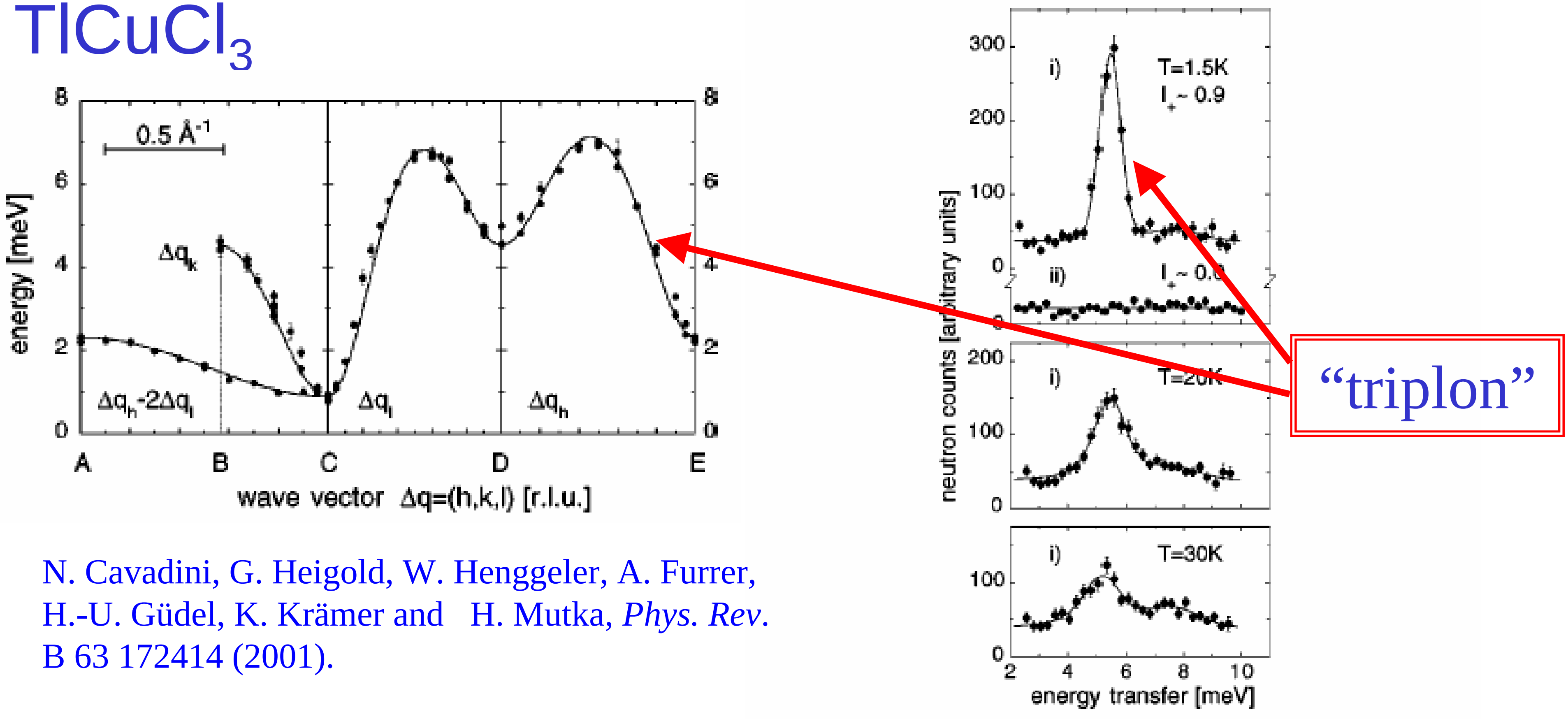
Excitation: S=1 ***triplon***
(***exciton***, spin collective mode)

Energy dispersion away from antiferromagnetic wavevector

$$\varepsilon_p = \Delta + \frac{c_x^2 p_x^2 + c_y^2 p_y^2}{2\Delta}$$

$\Delta \rightarrow$ spin gap

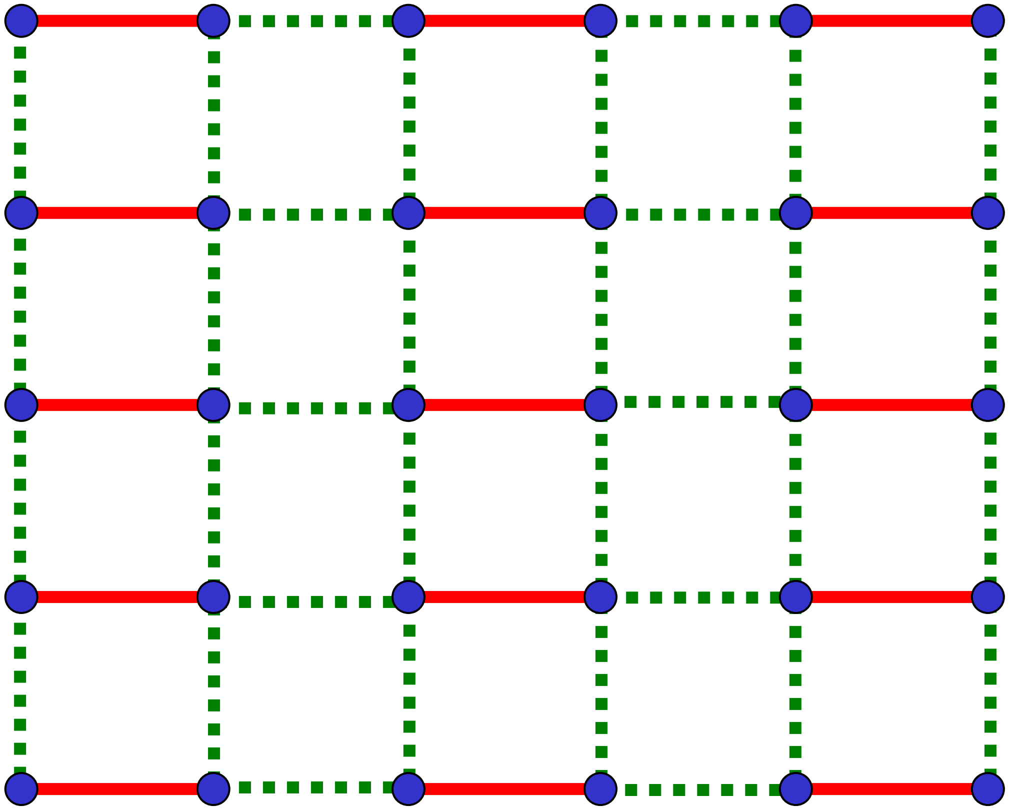
$TlCuCl_3$



N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer and H. Mutka, *Phys. Rev.* B 63 172414 (2001).

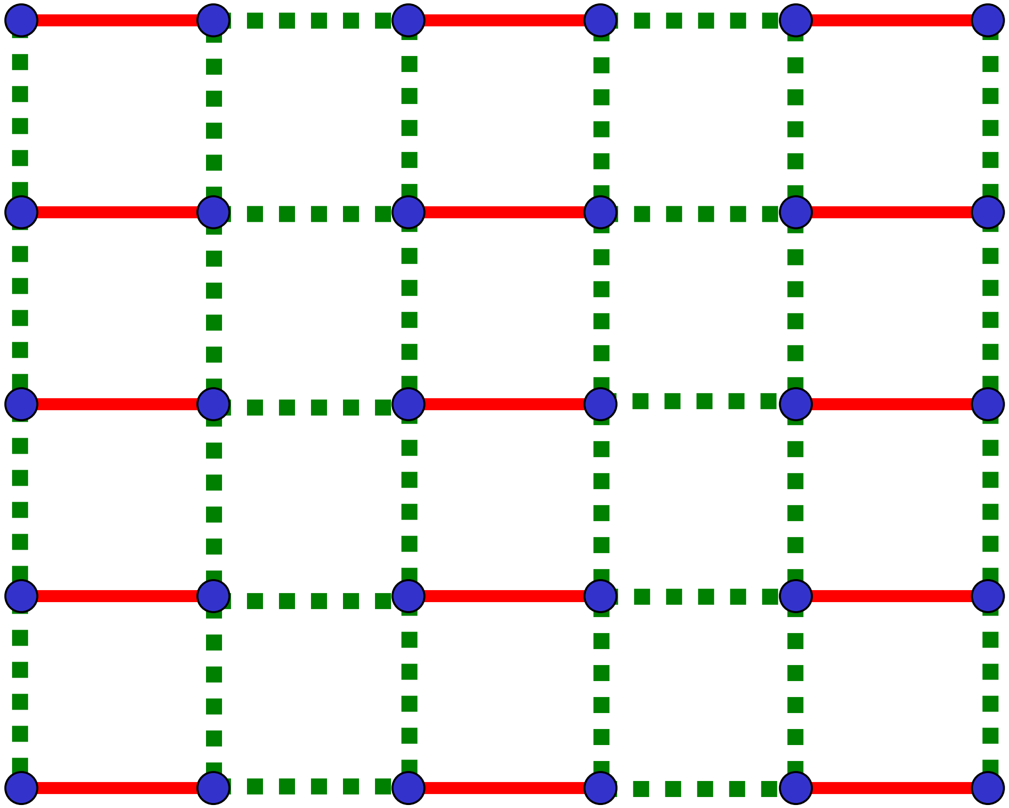
FIG. 1. Measured neutron profiles in the a^*c^* plane of $TlCuCl_3$ for $i=(1.35,0,0)$, $ii=(0,0,3.15)$ [r.l.u]. The spectrum at $T=1.5$ K

Coupled Dimer Antiferromagnet



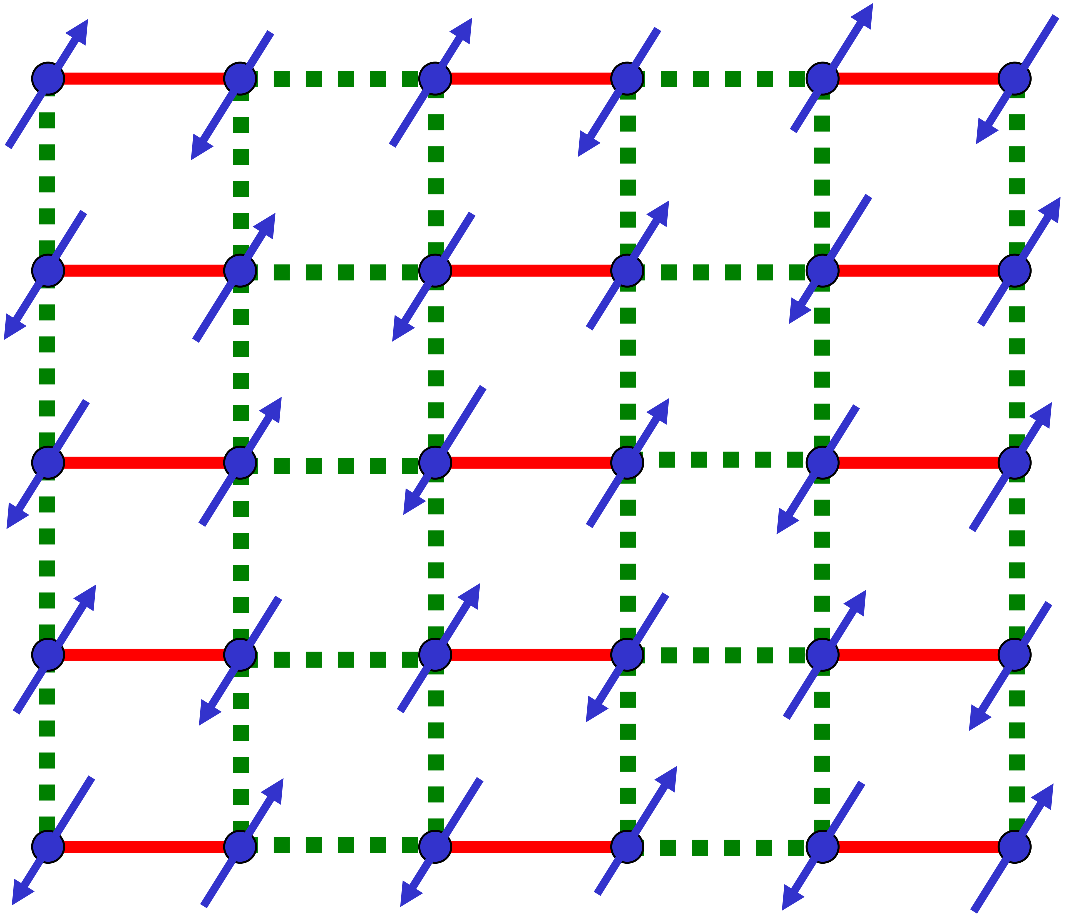
λ close to 1

Weakly dimerized square lattice



λ close to 1

Weakly dimerized square lattice



Excitations:

2 spin waves (***magnons***)

$$\varepsilon_p = \sqrt{c_x^2 p_x^2 + c_y^2 p_y^2}$$

Ground state has long-range spin density wave (Néel) order at wavevector ***K***= (π,π)

$$\langle \vec{\varphi} \rangle \neq 0$$

spin density wave order parameter: $\vec{\varphi} = \eta_i \dfrac{\vec{S}_i}{S}$; $\eta_i = \pm 1$ on two sublattices

$TlCuCl_3$

Neutron Diffraction Study of the Pressure-Induced Magnetic Ordering in the Spin Gap System $TlCuCl_3$

Akira OOSAWA*, Masashi FUJISAWA[1], Toyotaka OSAKABE, Kazuhisa KAKURAI and Hidekazu TANAKA[2]

Advanced Science Research Center, Japan Atomic Energy Research Institute, Tokai, Ibaraki 319-1195
[1]*Department of Physics, Tokyo Institute of Technology, Oh-okayama, Meguro-ku, Tokyo 152-8551*
[2]*Research Center for Low Temperature Physics, Tokyo Institute of Technology, Oh-okayama, Meguro-ku, Tokyo 152-8551*

(Received February 3, 2003)

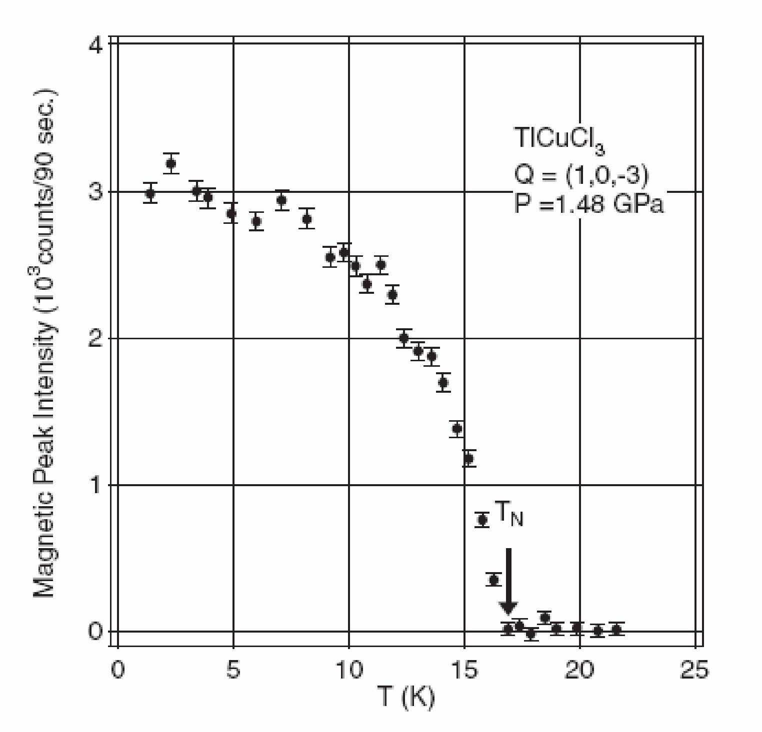


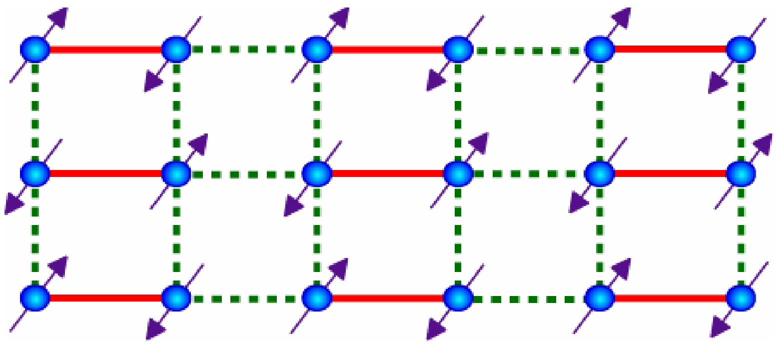
Fig. 3. Temperature dependence of the magnetic Bragg peak intensity for $Q = (1, 0, -3)$ reflection measured at $P = 1.48$ GPa in $TlCuCl_3$.

J. Phys. Soc. Jpn **72**, 1026 (2003)

T=0

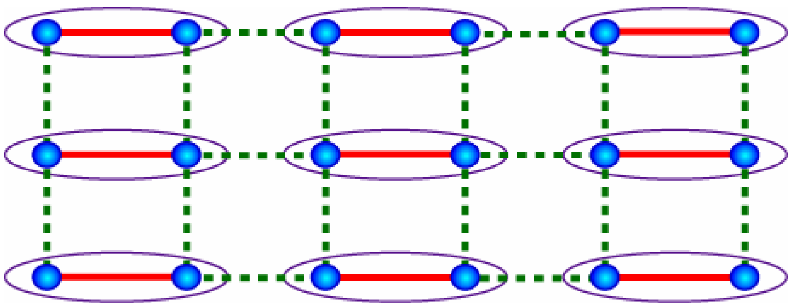
$\lambda_c = 0.52337(3)$

M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev.* B **65**, 014407 (2002)



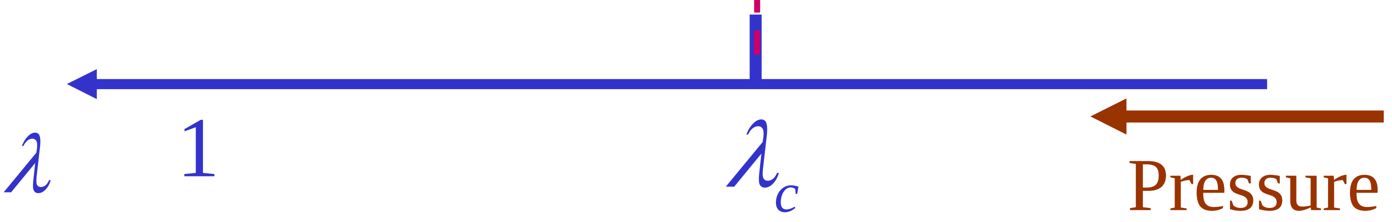
Néel state

$\langle \vec{\varphi} \rangle \neq 0$



Quantum paramagnet

$\langle \vec{\varphi} \rangle = 0$



LGW theory for quantum criticality

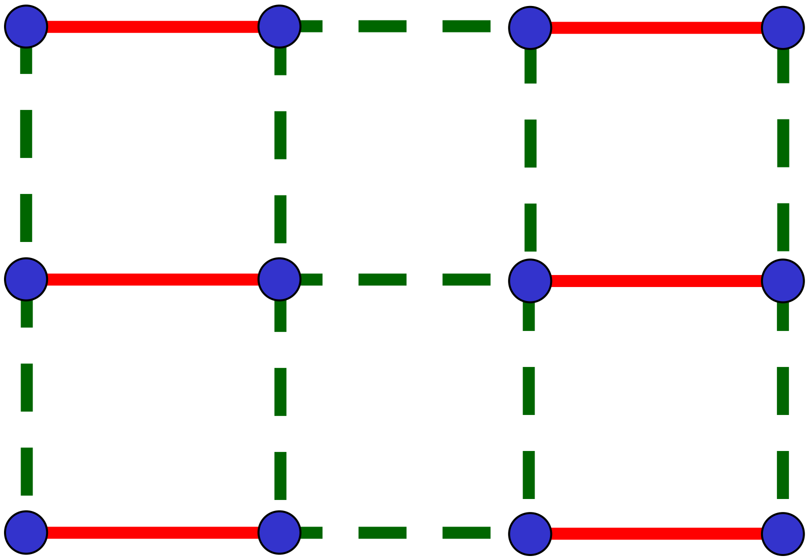
Landau-Ginzburg-Wilson theory: write down an effective action for the antiferromagnetic order parameter $\vec{\varphi}$ by expanding in powers of $\vec{\varphi}$ and its spatial and temporal derivatives, while preserving all symmetries of the microscopic Hamiltonian

$$S_{\varphi} = \int d^2x d\tau \left[\frac{1}{2}\left(\left(\nabla_x \vec{\varphi}\right)^2 + \frac{1}{c^2}\left(\partial_\tau \vec{\varphi}\right)^2 + \left(\lambda_c - \lambda\right)\vec{\varphi}^2 \right) + \frac{u}{4!}\left(\vec{\varphi}^2\right)^2 \right]$$

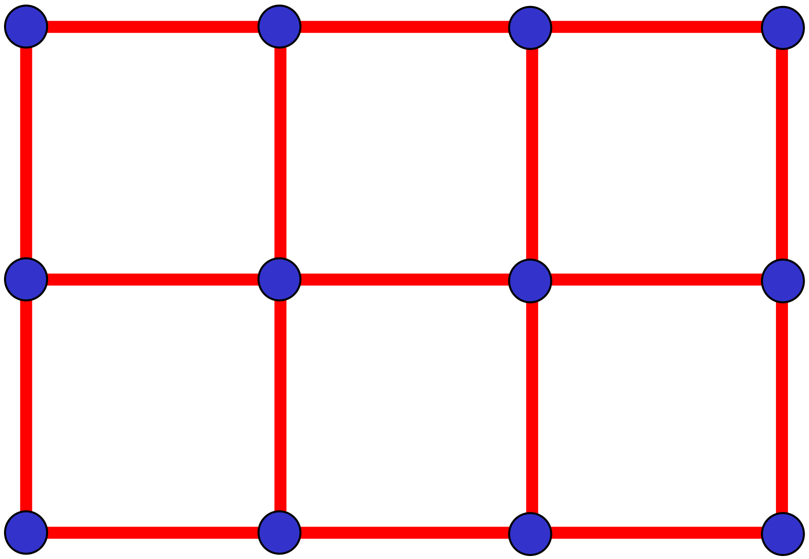
2.A. Magnetic quantum phase transitions in Mott insulators with S=1/2 per unit cell

Deconfined quantum criticality

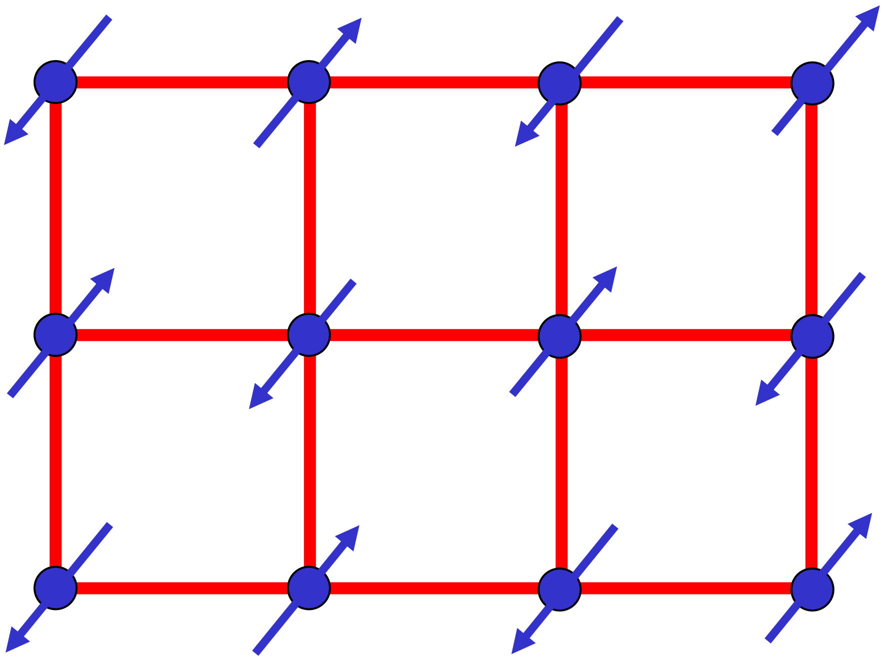
Mott insulator with two S=1/2 spins per unit cell



Mott insulator with one S=1/2 spin per unit cell

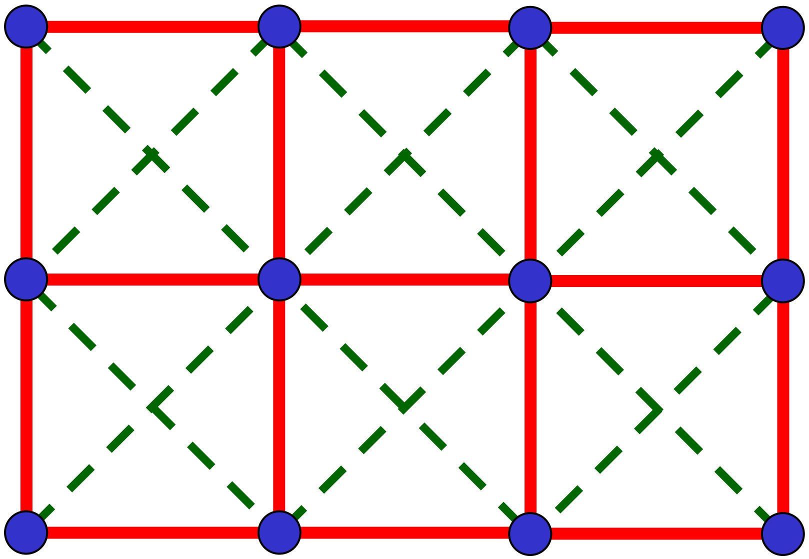


Mott insulator with one S=1/2 spin per unit cell



Ground state has Neel order with $\vec{\varphi} \neq 0$

Mott insulator with one S=1/2 spin per unit cell

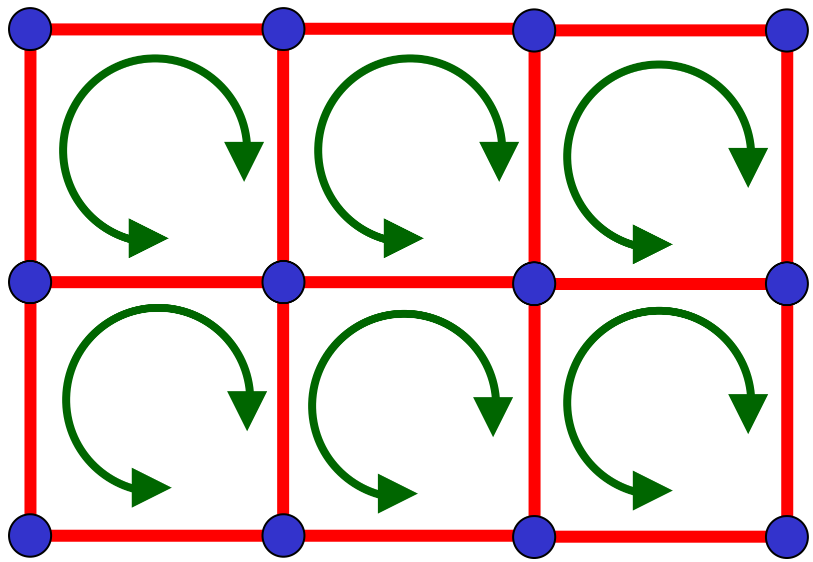


Destroy Neel order by perturbations which preserve full square lattice symmetry *e.g.* second-neighbor or ring exchange. The strength of this perturbation is measured by a coupling g.

Small $g \Rightarrow$ ground state has Neel order with $\langle \vec{\varphi} \rangle \neq 0$

Large $g \Rightarrow$ paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Mott insulator with one S=1/2 spin per unit cell

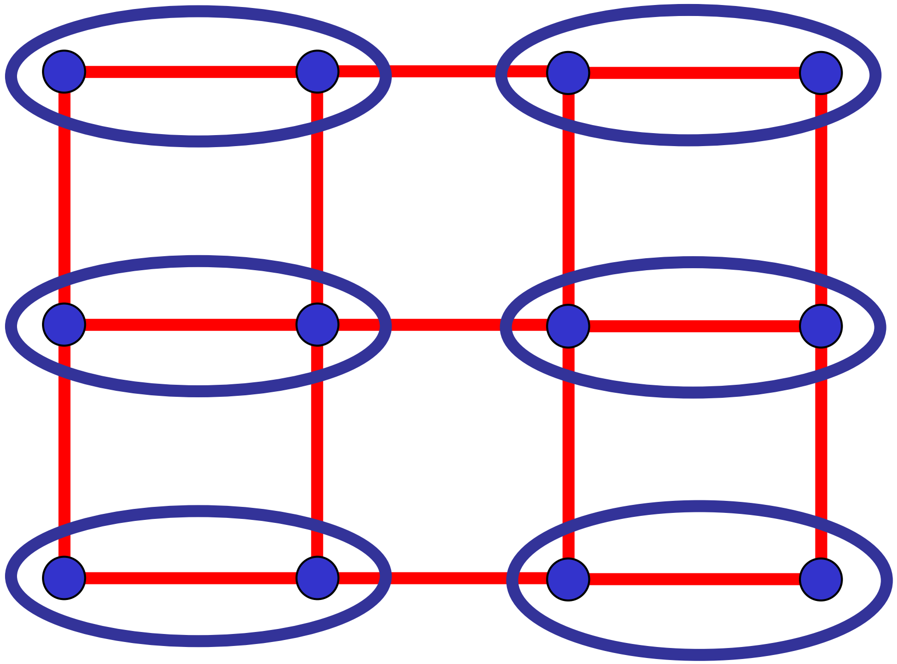


Destroy Neel order by perturbations which preserve full square lattice symmetry *e.g.* second-neighbor or ring exchange.
The strength of this perturbation is measured by a coupling g.

Small $g \Rightarrow$ ground state has Neel order with $\langle \vec{\varphi} \rangle \neq 0$

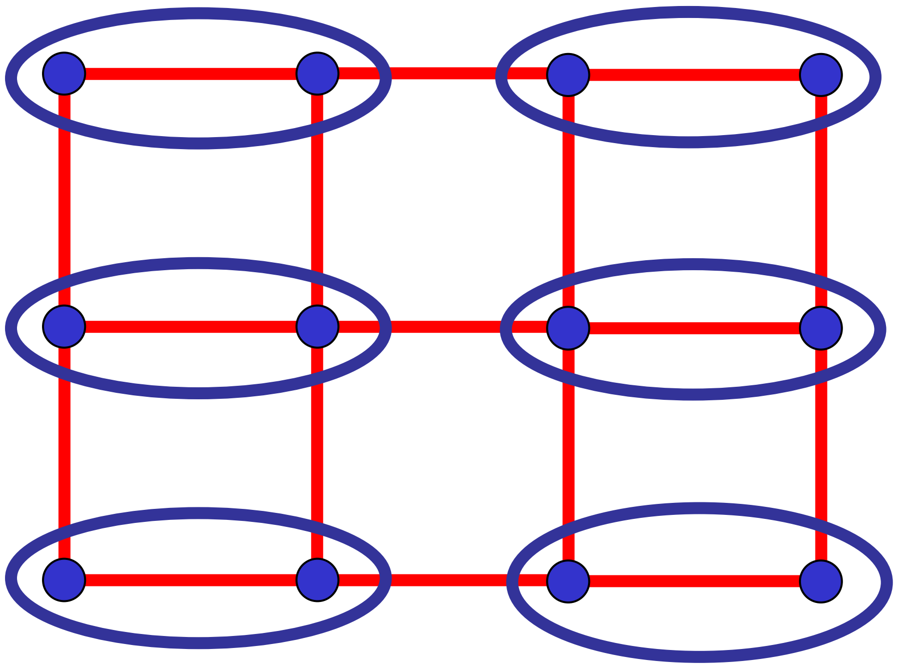
Large $g \Rightarrow$ paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

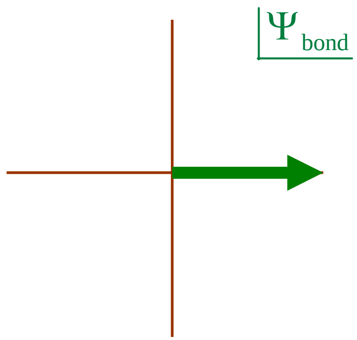
Mott insulator with one *S*=1/2 spin per unit cell



Possible large g paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Mott insulator with one S=1/2 spin per unit cell



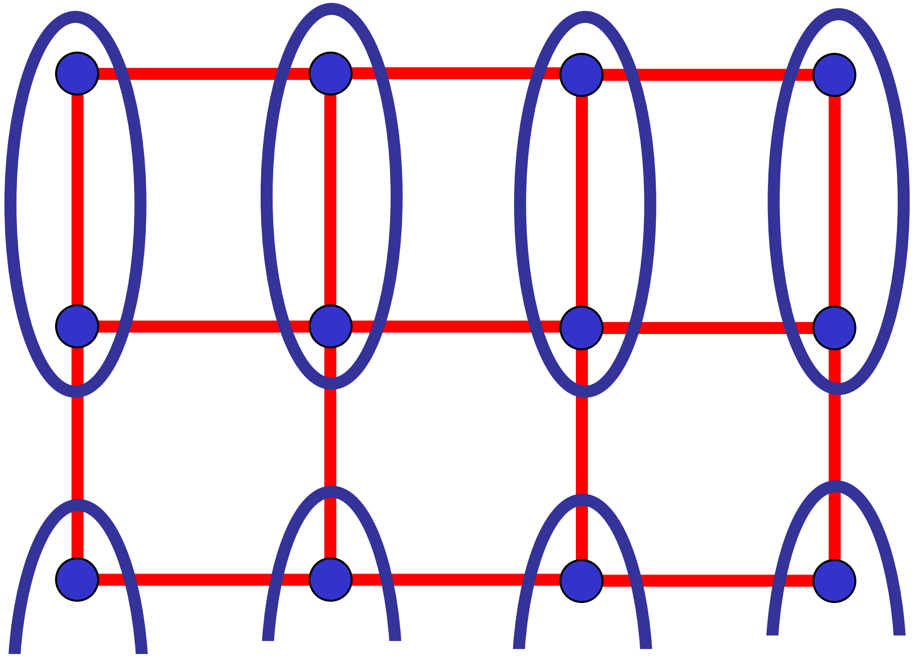


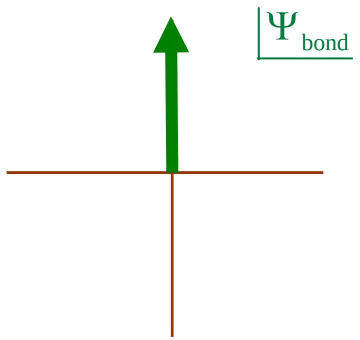
Possible large g paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Such a state breaks the symmetry of rotations by $n\pi/2$ about lattice sites, and has $\langle \Psi_{\text{bond}} \rangle \neq 0$, where Ψ_{bond} is the ***bond order parameter***

$$\Psi_{\text{bond}}(i) = \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \, e^{i \arctan(\boldsymbol{r}_j - \boldsymbol{r}_i)}$$

Mott insulator with one S=1/2 spin per unit cell



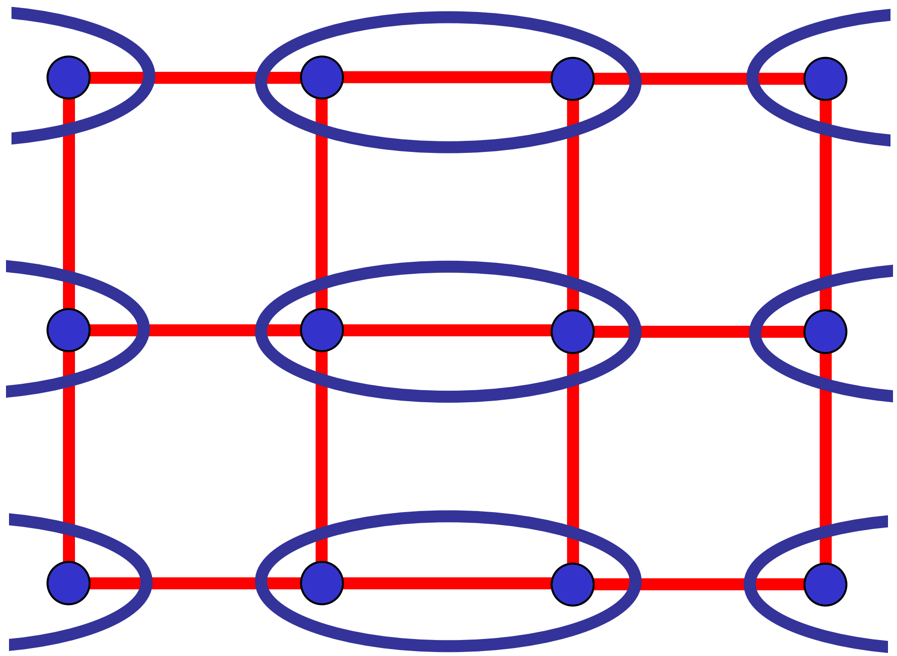


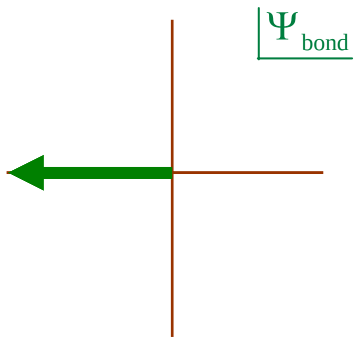
Possible large g paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Such a state breaks the symmetry of rotations by $n\pi/2$ about lattice sites, and has $\langle \Psi_{\text{bond}} \rangle \neq 0$, where Ψ_{bond} is the ***bond order parameter***

$$\Psi_{\text{bond}}(i) = \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \, e^{i \arctan(\mathbf{r}_j - \mathbf{r}_i)}$$

Mott insulator with one S=1/2 spin per unit cell





Possible large g paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Such a state breaks the symmetry of rotations by $n\pi/2$ about lattice sites, and has $\langle \Psi_{\text{bond}} \rangle \neq 0$, where Ψ_{bond} is the ***bond order parameter***

$$\Psi_{\text{bond}}(i) = \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j e^{i \arctan(\mathbf{r}_j - \mathbf{r}_i)}$$

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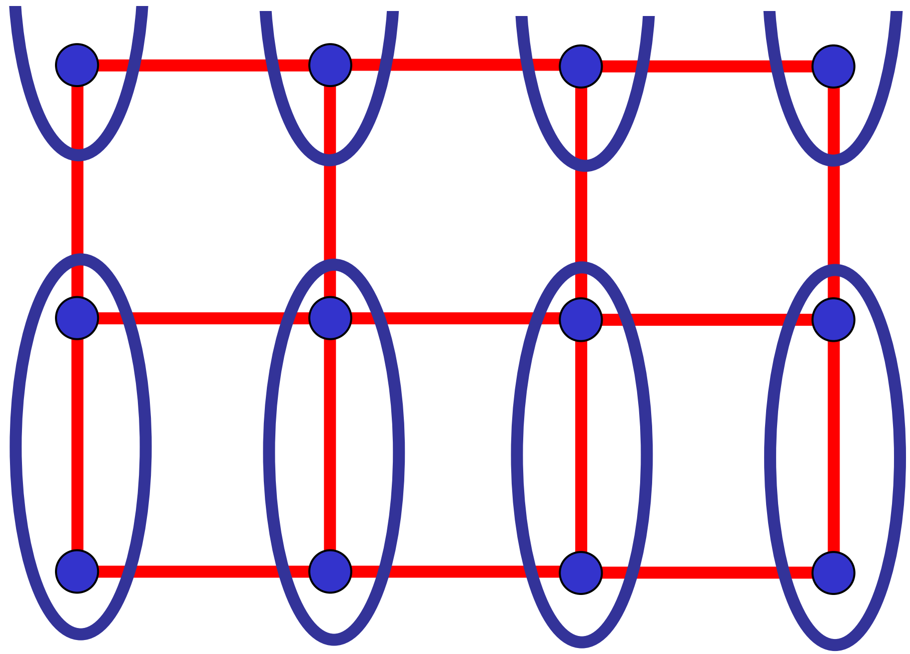

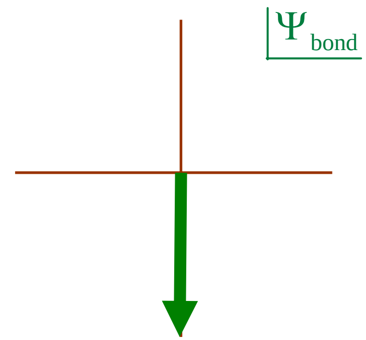

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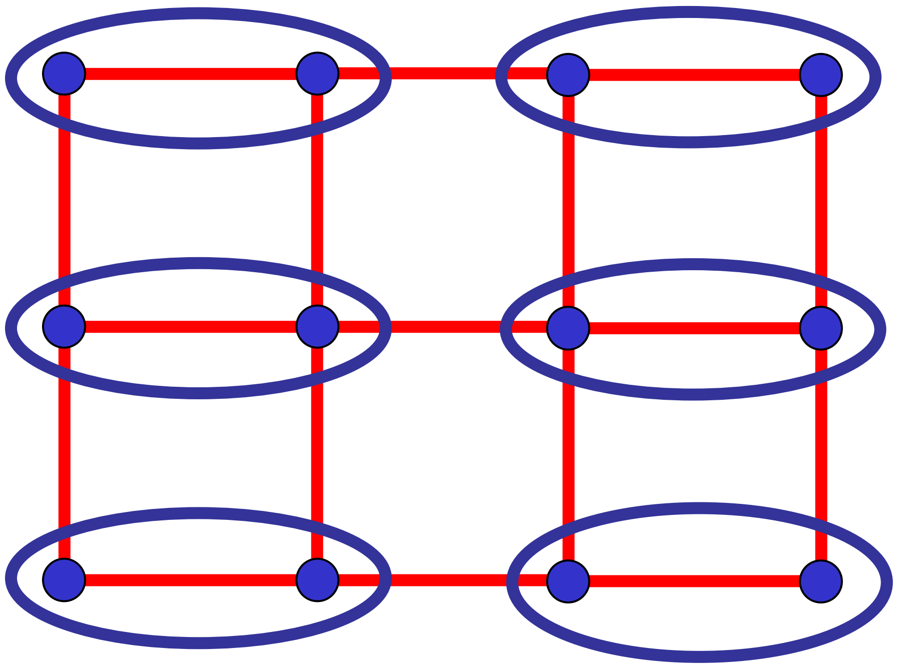
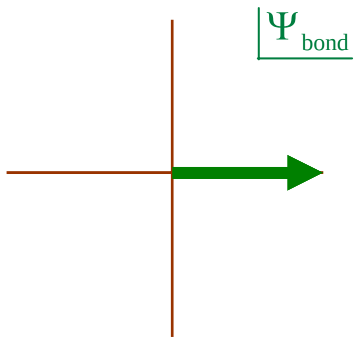


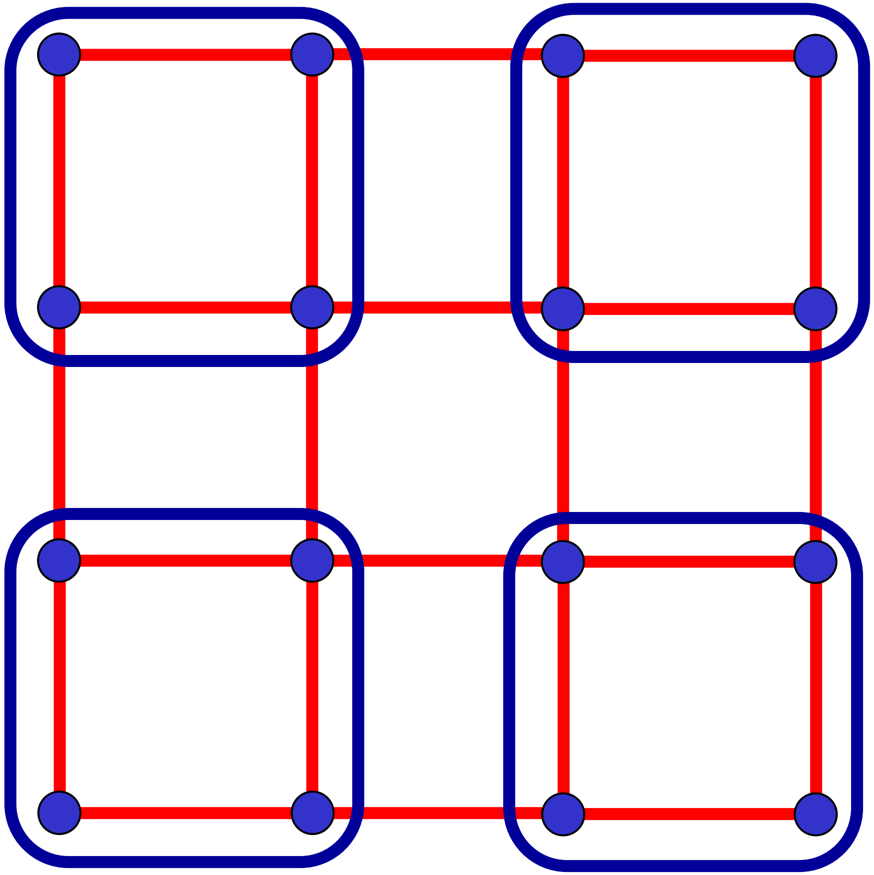


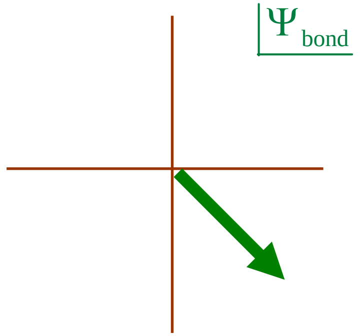
Possible large g paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Such a state breaks the symmetry of rotations by $n\pi / 2$ about lattice sites, and has $\langle \Psi_{\text{bond}} \rangle \neq 0$, where Ψ_{bond} is the ***bond order parameter***

$$\Psi_{\text{bond}}(i) = \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \, e^{i \arctan(\mathbf{r}_j - \mathbf{r}_i)}$$

Mott insulator with one S=1/2 spin per unit cell



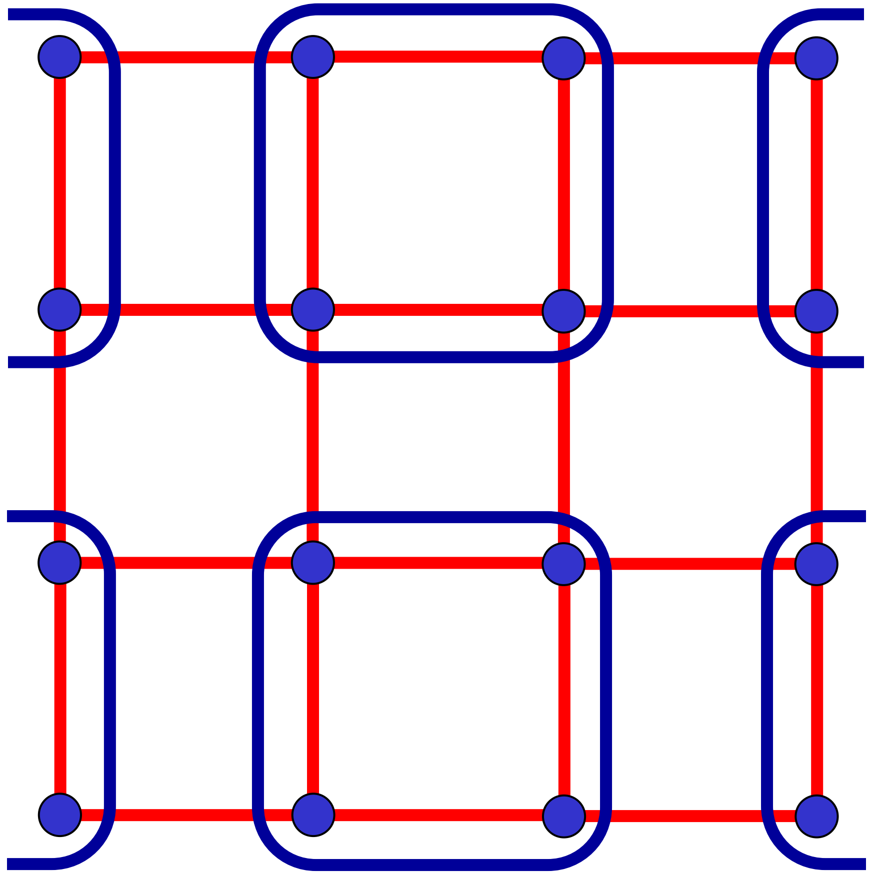


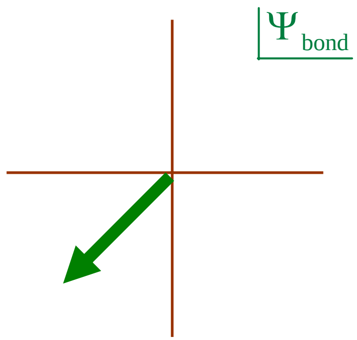
Possible large g paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Another state breaking the symmetry of rotations by $n\pi / 2$ about lattice sites, which also has $\langle \Psi_{\text{bond}} \rangle \neq 0$, where Ψ_{bond} is the ***bond order parameter***

$$\Psi_{\text{bond}}(i) = \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j e^{i \arctan(\boldsymbol{r}_j - \boldsymbol{r}_i)}$$

Mott insulator with one S=1/2 spin per unit cell





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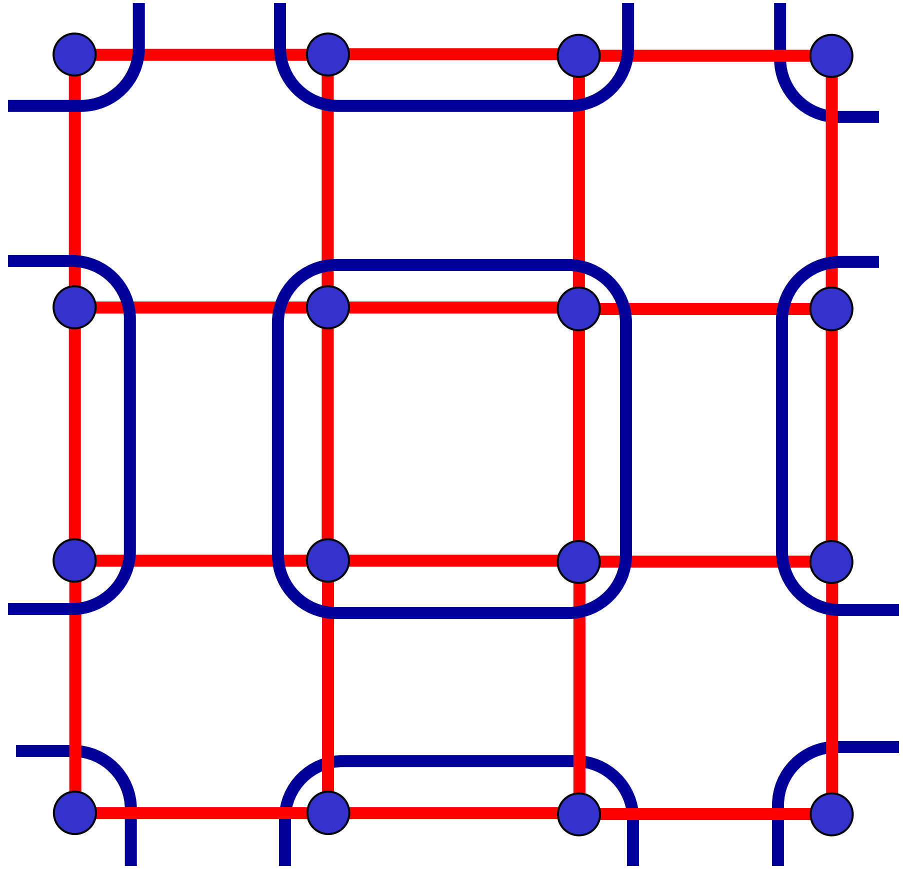

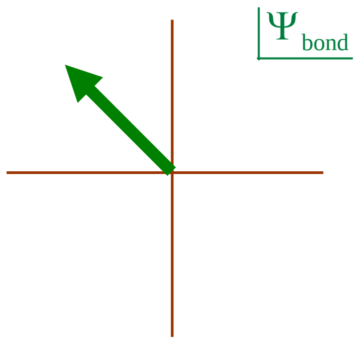

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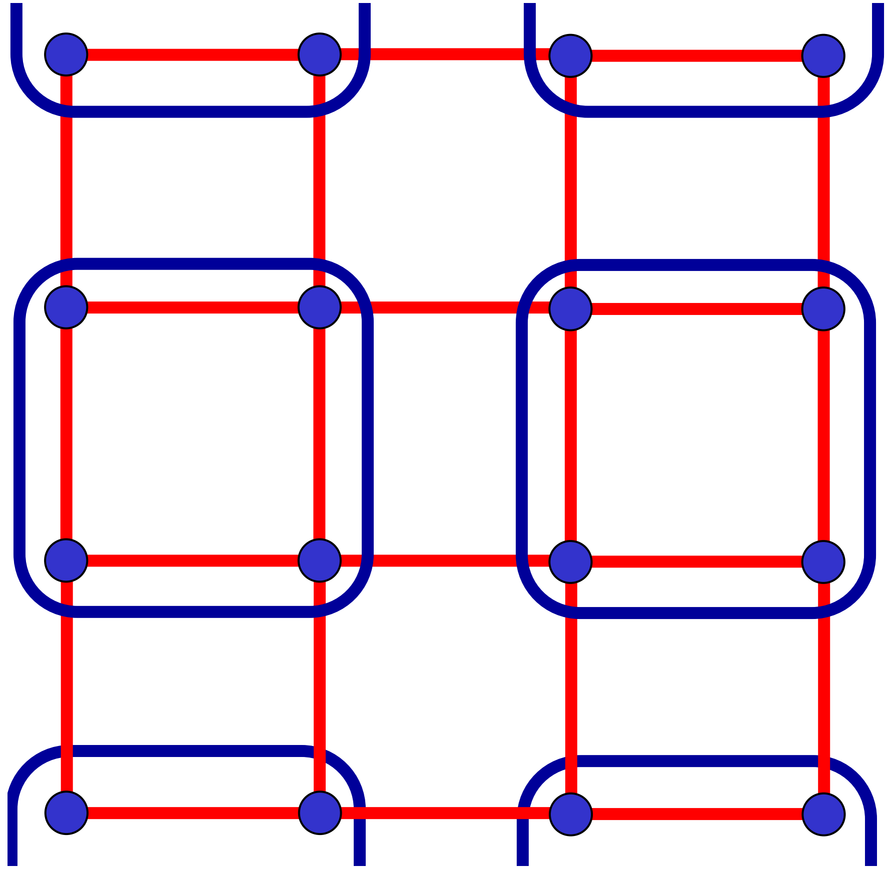

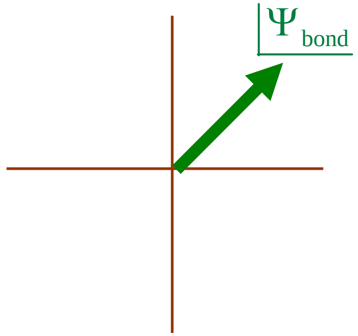

Possible large g paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

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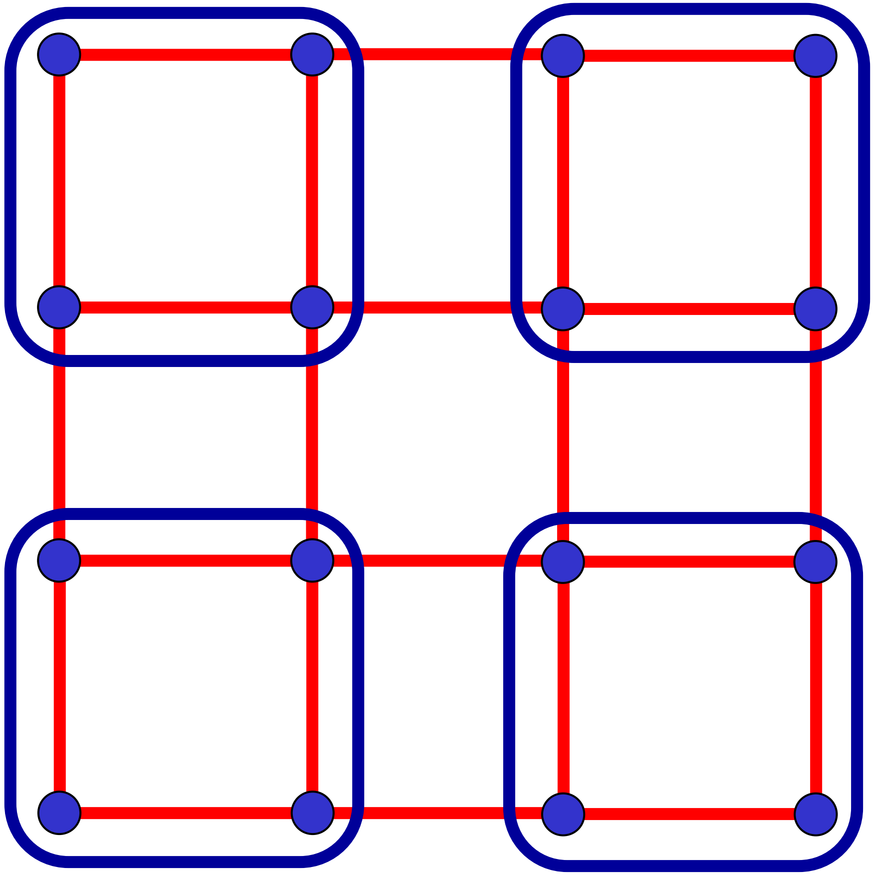
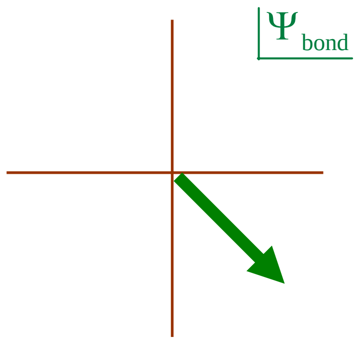
Mott insulator with one S=1/2 spin per unit cell





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$$\Psi_{\text{bond}}(i) = \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \, e^{i \arctan(\mathbf{r}_j - \mathbf{r}_i)}$$

LGW theory of multiple order parameters

$$F = F_{\mathrm{vbs}}\left[\Psi_{\mathrm{vbs}}\right] + F_{\varphi}\left[\vec{\varphi}\right] + F_{\mathrm{int}}$$

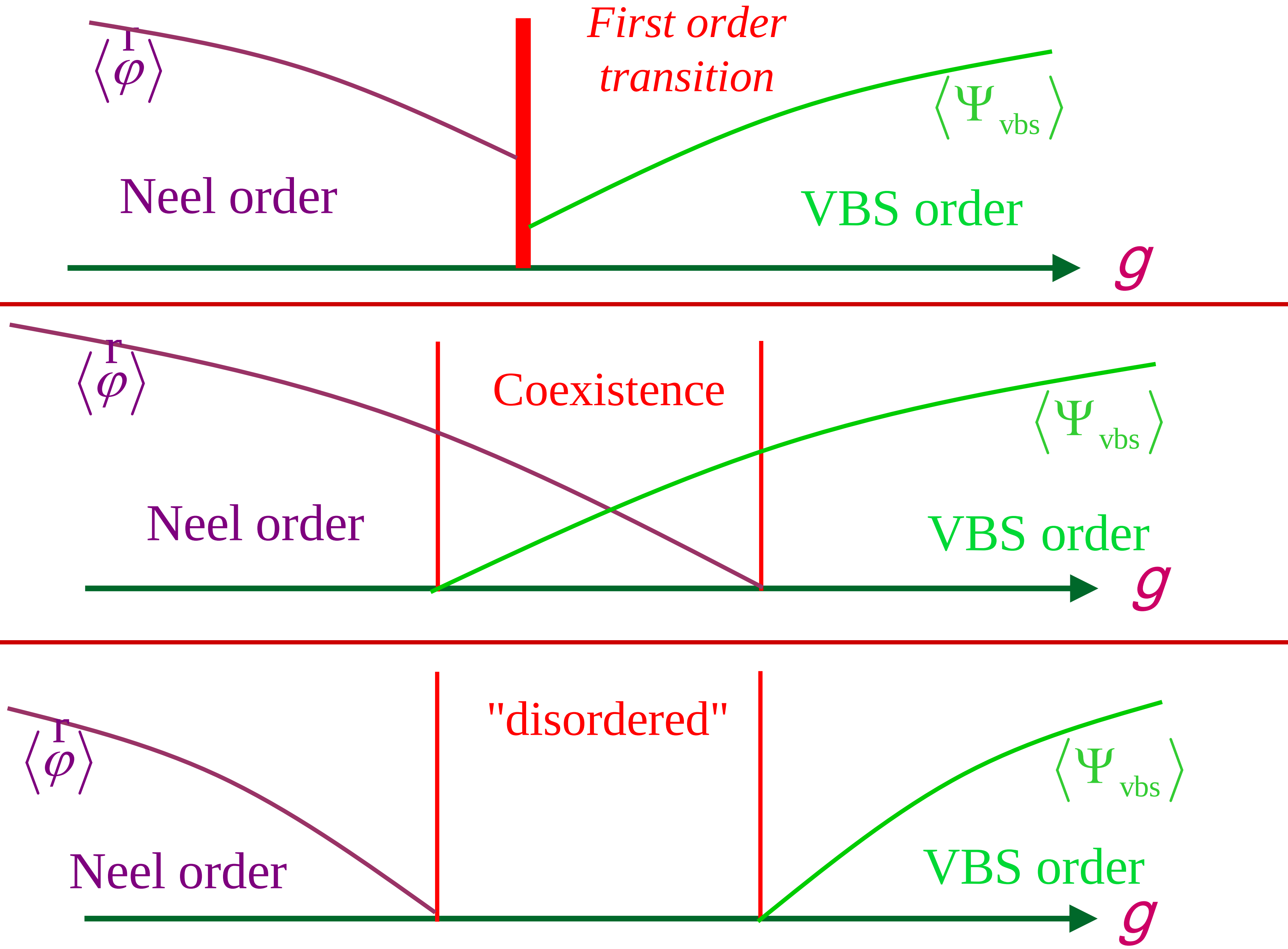
$$F_{\mathrm{vbs}}\left[\Psi_{\mathrm{vbs}}\right] = r_1 \left|\Psi_{\mathrm{vbs}}\right|^2 + u_1 \left|\Psi_{\mathrm{vbs}}\right|^4 + \ldots$$

$$F_{\varphi}\left[\vec{\varphi}\right] = r_2 \left|\vec{\varphi}\right|^2 + u_2 \left|\vec{\varphi}\right|^4 + \ldots$$

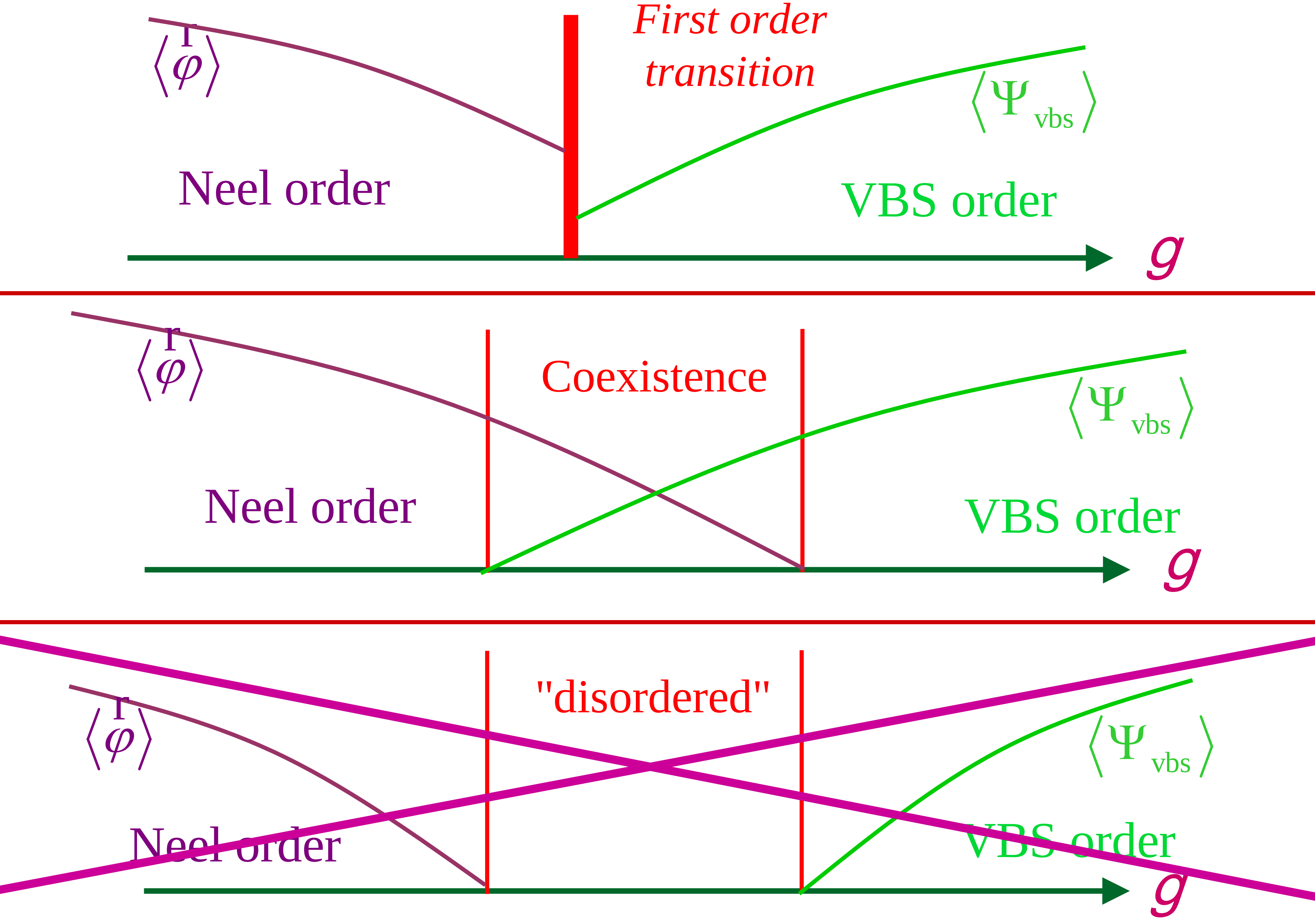
$$F_{\mathrm{int}} = v \left|\Psi_{\mathrm{vbs}}\right|^2 \left|\vec{\varphi}\right|^2 + \ldots$$

Distinct symmetries of order parameters permit couplings only between their energy densities

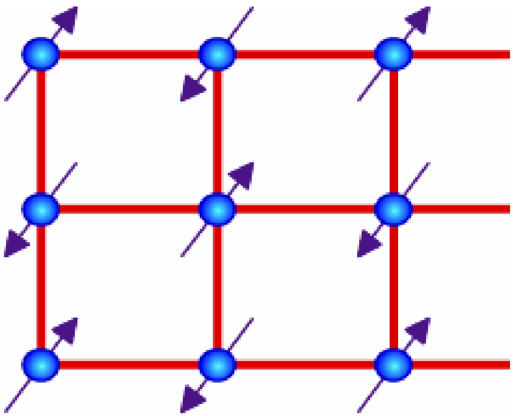
LGW theory of multiple order parameters



LGW theory of multiple order parameters

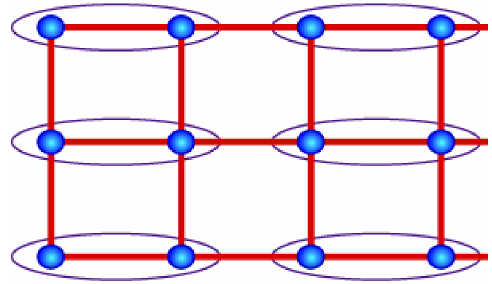


Proposal of deconfined quantum criticality

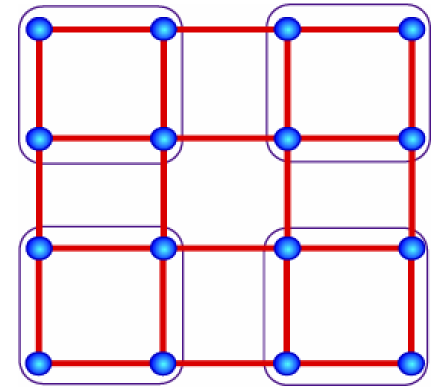


Neel order

$$\langle \vec{\varphi} \rangle \sim \langle z^*_\alpha \vec{\sigma}_{\alpha\beta} z_\beta \rangle \neq 0$$



or



VBS order $\langle \Psi_{\text{vbs}} \rangle \neq 0$

(associated with condensation of monopoles in A_μ),

$S = 1/2$ spinons z_α confined,

$S = 1$ triplon excitations

g

Second-order critical point described by

$$\mathcal{S}_{\text{critical}} = \int d^2x d\tau \left[|(\partial_\mu - iA_\mu) z_\alpha|^2 + r\,|z_\alpha|^2 + \frac{u}{2}\left(|z_\alpha|^2\right)^2 + \frac{1}{4e^2}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 \right]$$

at its critical point $r = r_c$, where A_μ is *non-compact*

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

Theory of a second-order quantum phase transition between Neel and VBS phases

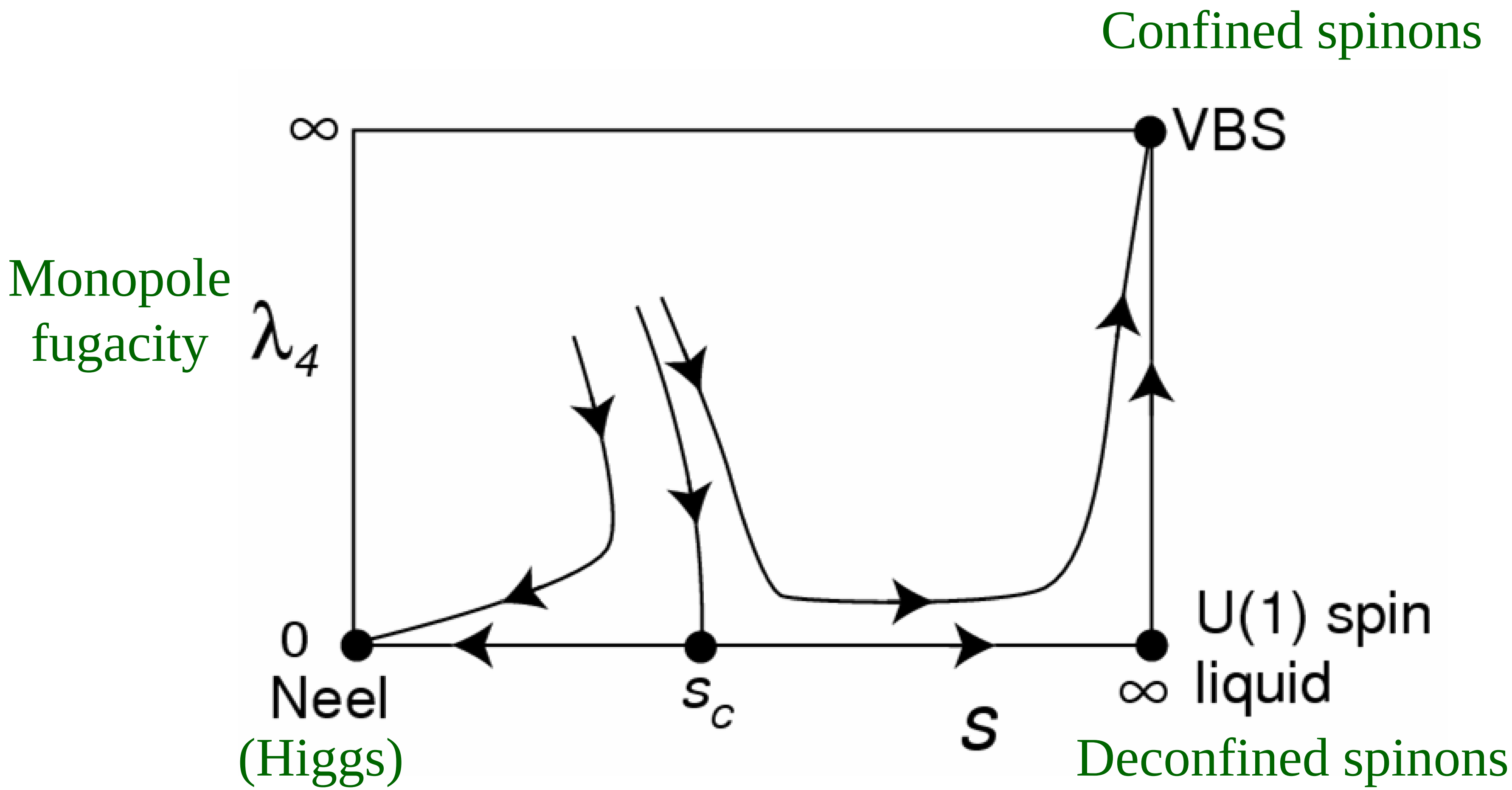
At the quantum critical point:

- $A_\mu \rightarrow A_\mu + 2\pi$ periodicity can be ignored

(Monopoles interfere destructively and are dangerously irrelevant).

- $S=1/2$ *spinons* z_α, with $\vec{\varphi} \sim z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$, are globally propagating degrees of freedom.

Second-order critical point described by emergent fractionalized degrees of freedom (A_μ and z_α); Order parameters (φ and Ψ_{vbs}) are "composites" and of secondary importance



N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).

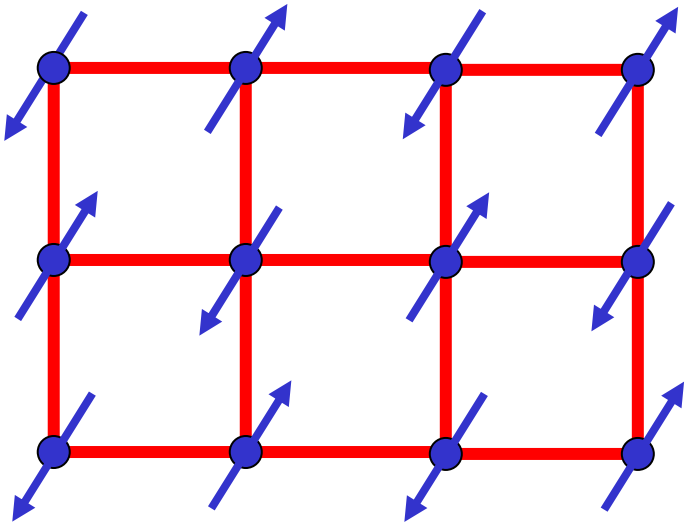
A. V. Chubukov, S. Sachdev, and J. Ye, *Phys. Rev. B* **49**, 11919 (1994).

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

Outline

1. Density-driven phase transitions
   A. Fermions with repulsive interactions
   B. Bosons with repulsive interactions
   C. Fermions with attractive interactions
2. Magnetic transitions of Mott insulators
   A. Dimerized Mott insulators – Landau-Ginzburg-Wilson theory
   B. S=1/2 per unit cell: deconfined quantum criticality
3. Transitions of the Kondo lattice
   A. Large Fermi surfaces – Hertz theory
   B. Fractional Fermi liquids and gauge theory

The Kondo lattice



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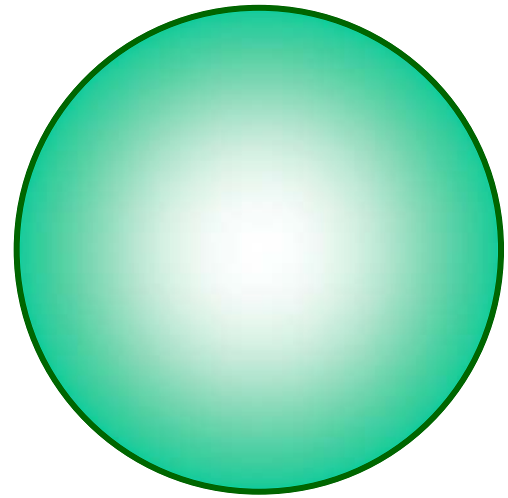


Local moments f_σ

Conduction electrons c_σ

$$H_K = \sum_{i<j} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + J_K \sum_i c^\dagger_{i\sigma} \vec{\tau}_{\sigma\sigma'} c_{i\sigma} \cdot \vec{S}_{fi} + J \sum_{\langle ij \rangle} \vec{S}_{fi} \cdot \vec{S}_{fj}$$

Number of f electrons per unit cell = $n_f = 1$

Number of c electrons per unit cell = n_c

3.A. The heavy Fermi liquid (FL)

Hertz theory for the onset of spin density wave order

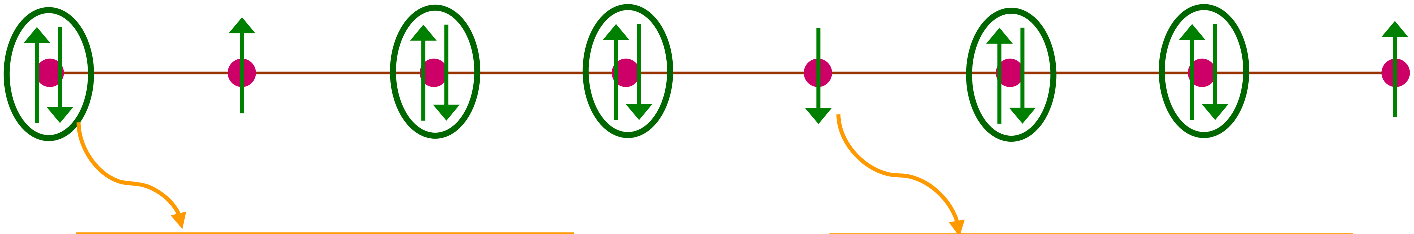
The “large” Fermi surface is obtained in the limit of large J_K

The Fermi surface of heavy quasiparticles encloses a volume which counts *all* electrons.

$$\text{Fermi volume} = 1 + n_c$$

Argument for the Fermi surface volume of the FL phase

Single ion Kondo effect implies $J_K \to \infty$ at low energies



$$\left(c_{i\uparrow}^{\dagger} f_{i\downarrow}^{\dagger} - c_{i\downarrow}^{\dagger} f_{i\uparrow}^{\dagger}\right)|0\rangle$$

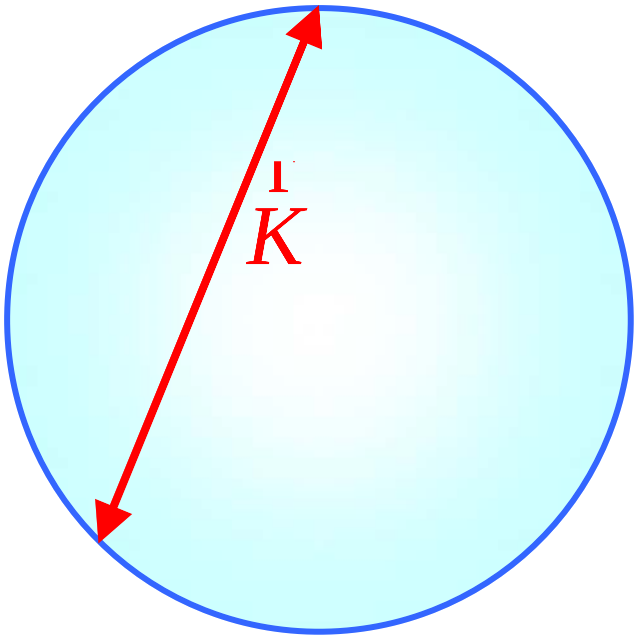
$$f_{i\downarrow}^{\dagger}|0\rangle, \; S = 1/2 \text{ hole}$$

Fermi liquid of S=1/2 holes with hard-core repulsion

$$\text{Fermi surface volume } = -\left(\text{density of holes}\right) \bmod 2$$
$$= -\left(1 - n_c\right) = \left(1 + n_c\right) \bmod 2$$

LGW (Hertz) theory for QCP to SDW order

Write down effective action for SDW order parameter $\vec{\varphi}$



$\vec{\varphi}$ fluctuations are damped by mixing with fermionic quasiparticles near the Fermi surface

$$S_\varphi = \int \frac{d^d q\, d\omega}{(2\pi)^{d+1}} \left|\vec{\varphi}(q,\omega)\right|^2 \left(q^2 + |\omega| + \left(J_K - J_{Kc}\right)\right) + \frac{u}{4}\int d^d r\, d\tau \left(\vec{\varphi}^2\right)^2$$

Fluctuations of $\vec{\varphi}$ about $\vec{\varphi} = 0 \Rightarrow$ paramagnons

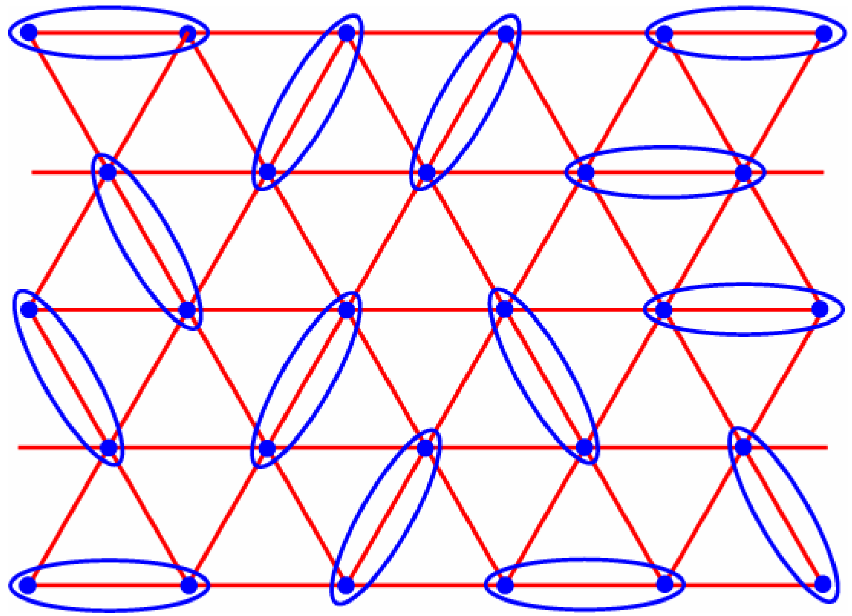
J. Mathon, *Proc. R. Soc. London* A, **306**, 355 (1968); T.V. Ramakrishnan, *Phys. Rev.* B **10**, 4014 (1974);
M. T. Bea… **4**, 1165 (1976).
T. Moriya…);
G. G. Lonzarich and L. Taillefer, *J. Phys.* C **18**, 4339 (1985); A.J. Millis, *Phys. Rev.* B **48**, 7183 (1993).

No Mottness

3.B. The Fractionalized Fermi liquid (FL*)

Phases and quantum critical points characterized by gauge theory and "topological" excitations

Work in the regime with small J_K, and consider destruction of magnetic order by frustrating (RKKY) exchange interactions between *f* moments

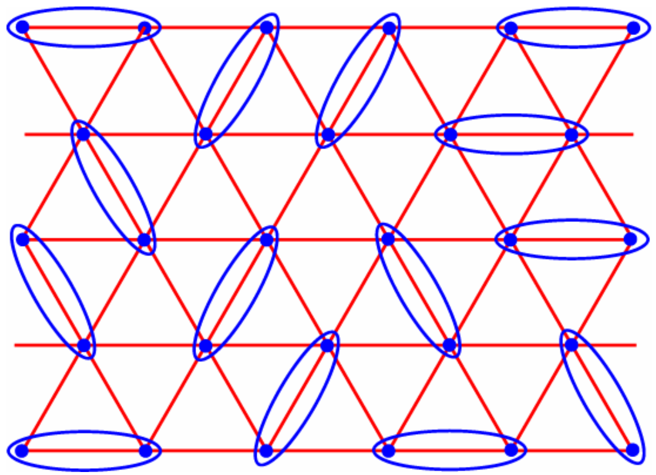


A *spin liquid* ground state with $\langle \vec{\varphi} \rangle = 0$ and no broken lattice symmetries.

Such a state has emergent excitations described by a Z_2 or U(1) gauge theory

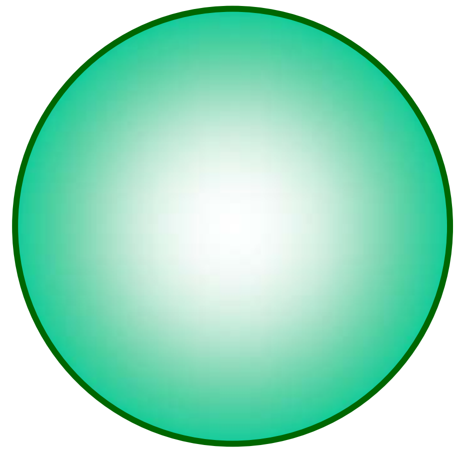
P. Fazekas and P.W. Anderson, *Phil Mag* **30**, 23 (1974).
N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991);
X. G. Wen, *Phys. Rev*. B **44**, 2664 (1991).

Influence of conduction electrons



Local moments f_σ

+



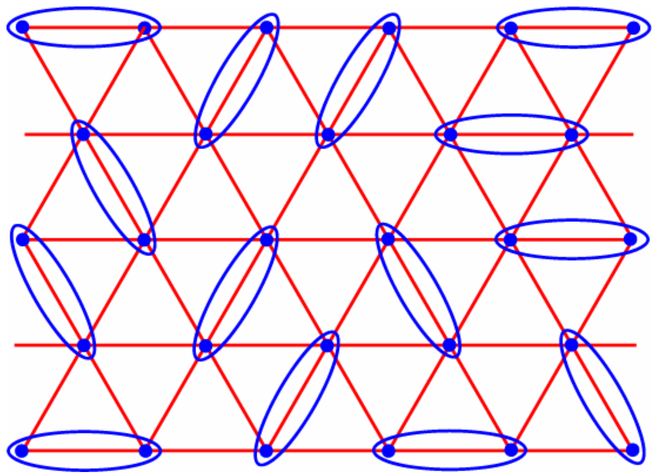
Conduction electrons c_σ

$$H = \sum_{i<j} t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + \sum_{i} \left(J_K c_{i\sigma}^{\dagger} \vec{\tau}_{\sigma\sigma'} c_{i\sigma} \cdot \vec{S}_{fi} \right) + \sum_{i<j} J_H(i,j) \vec{S}_{fi} \cdot \vec{S}_{fj}$$

Determine the ground state of the quantum antiferromagnet defined by J_H, and then couple to conduction electrons by J_K

Choose J_H so that ground state of antiferromagnet is a Z_2 or U(1) spin liquid

Influence of conduction electrons



Local moments f_σ

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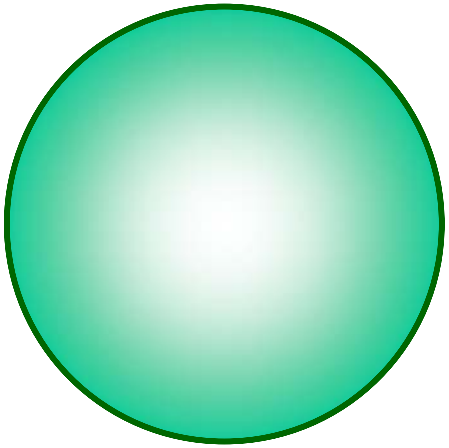


Conduction electrons c_σ

At $J_K = 0$ the conduction electrons form a Fermi surface on their own with volume determined by n_c.

Perturbation theory in J_K is regular, and so this state will be stable for finite J_K.

So volume of Fermi surface is determined by **$(n_c + n_f - 1) = n_c (\text{mod } 2)$,** and does not equal the Luttinger value.

The (U(1) or Z_2) FL* state

A new phase: FL*

This phase preserves spin rotation invariance, and has a Fermi surface of *sharp* electron-like quasiparticles.

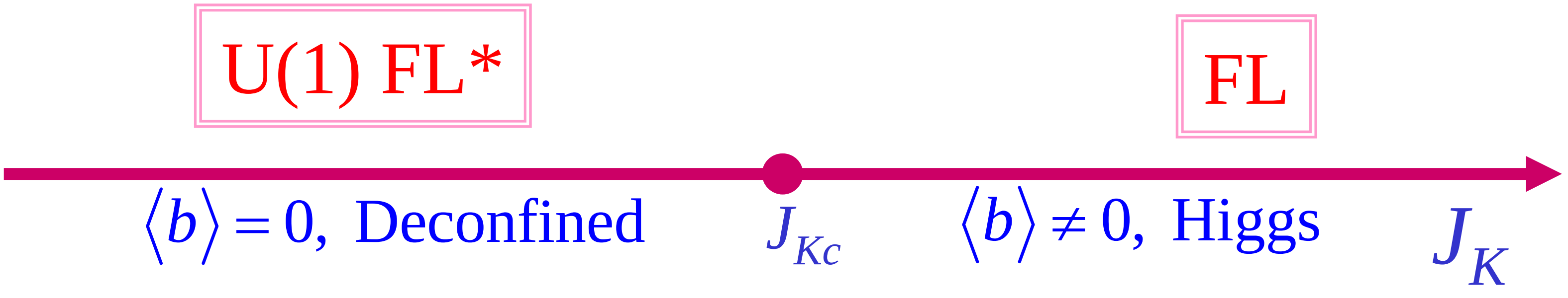
The state has "*topological order*" and associated neutral excitations. The topological order can be detected by the violation of Luttinger's Fermi surface volume. It can only appear in dimensions $d > 1$

$$2 \times \frac{v_0}{(2\pi)^d}\left(\text{Volume enclosed by Fermi surface}\right)$$

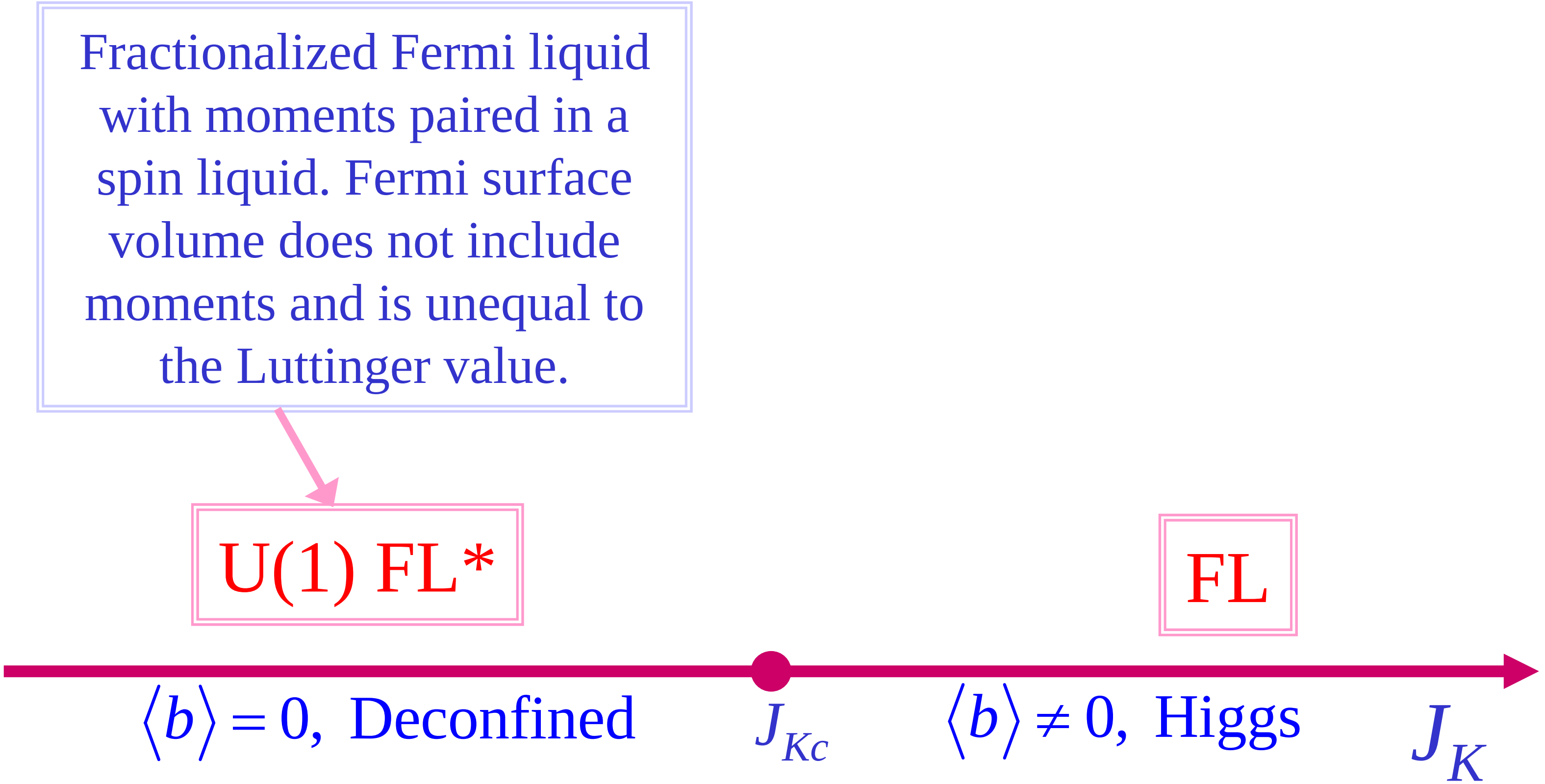
$$= \left(n_f + n_c - 1\right)\left(\text{mod } 2\right)$$

Precursors: N. Andrei and P. Coleman, *Phys. Rev. Lett*. **62**, 595 (1989).
Yu. Kagan, K. A. Kikoin, and N. V. Prokof'ev, *Physica* B **182**, 201 (1992).
Q. Si, S. Rabello, K. Ingersent, and L. Smith, *Nature* **413**, 804 (2001).
S. Burdin, D. R. Grempel, and A. Georges, *Phys. Rev*. B 66, 045111 (2002).
L. Balents and M. P. A. Fisher and C. Nayak, *Phys. Rev*. B **60**, 1654, (1999);
T. Senthil and M.P.A. Fisher, *Phys. Rev*. B **62**, 7850 (2000).

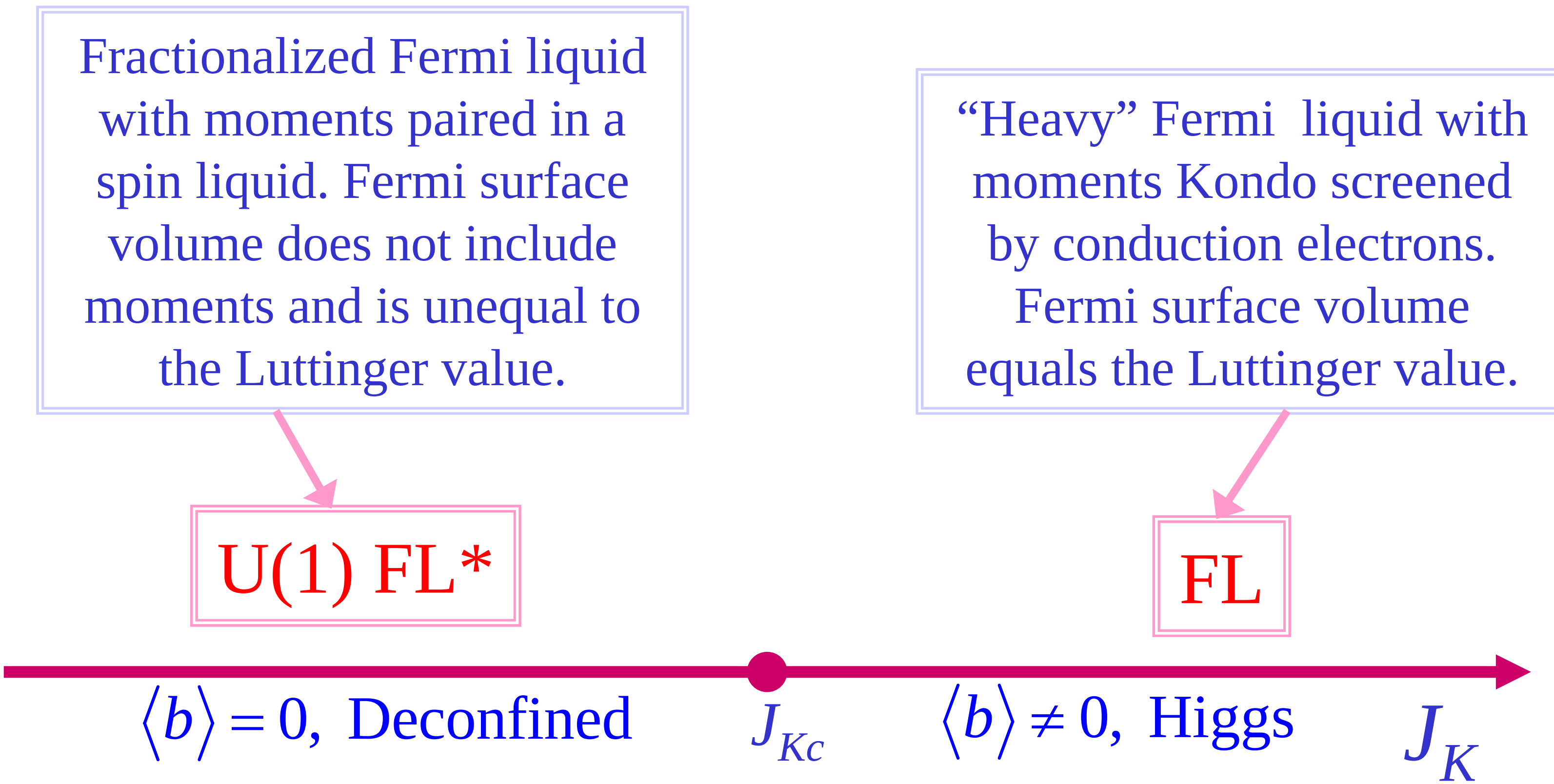
Phase diagram



Phase diagram



Phase diagram



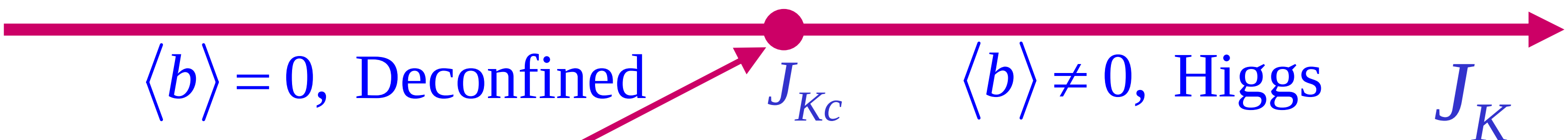
Phase diagram

Fractionalized Fermi liquid with moments paired in a spin liquid. Fermi surface volume does not include moments and is unequal to the Luttinger value.

U(1) FL*

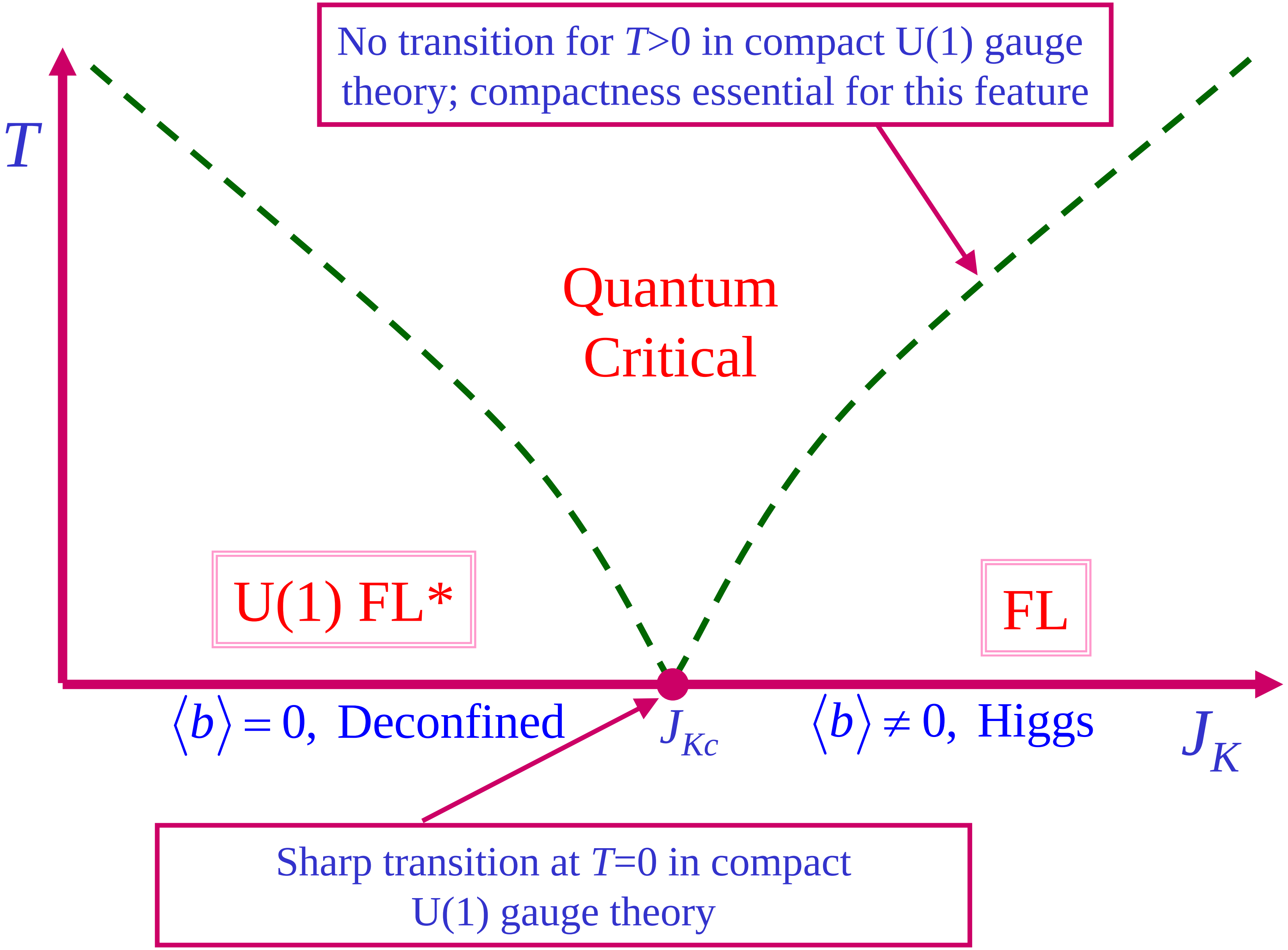
"Heavy" Fermi liquid with moments Kondo screened by conduction electrons. Fermi surface volume equals the Luttinger value.

FL

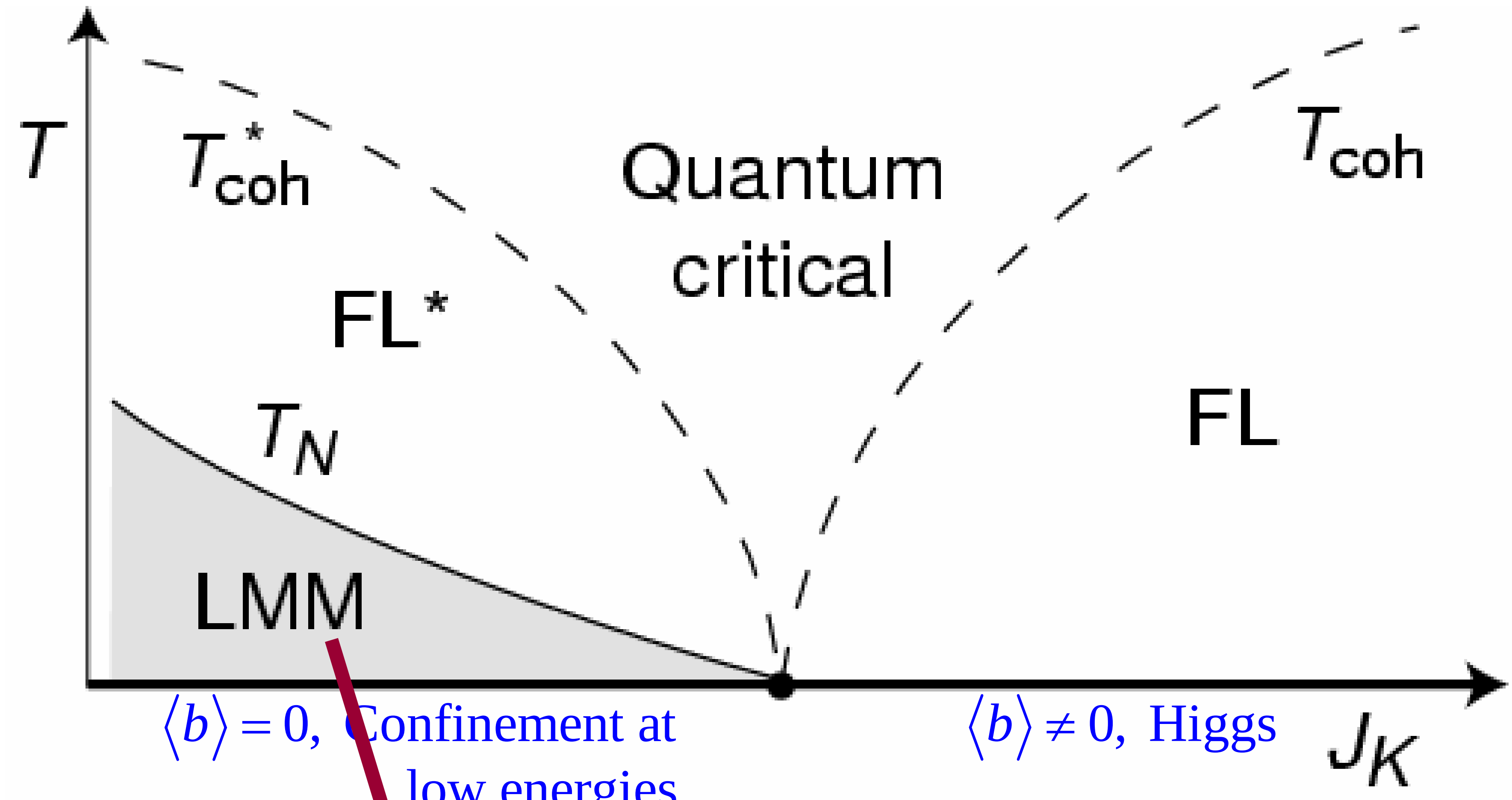


Sharp transition at T=0 in compact U(1) gauge theory

Phase diagram



Deconfined criticality in the Kondo lattice ?

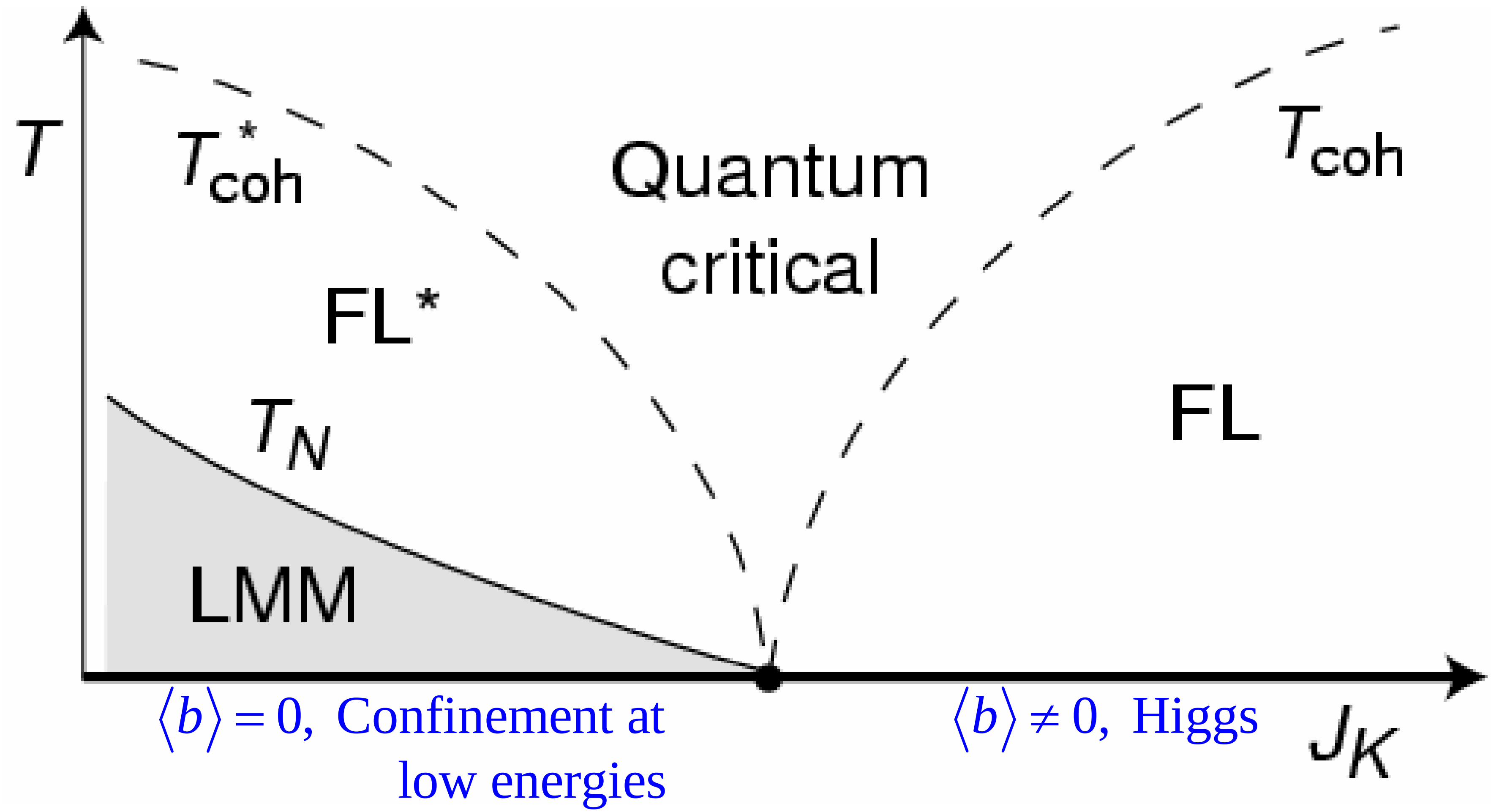


$\langle b \rangle = 0$, Confinement at low energies

$\langle b \rangle \neq 0$, Higgs

Local moment magnetism: magnetism appears by spontaneous polarization of f moments (c electrons remain spectators). Distinct from SDW order in FL state. **Includes Mottness**

Deconfined criticality in the Kondo lattice ?



$\langle b \rangle = 0$, Confinement at low energies

$\langle b \rangle \neq 0$, Higgs

U(1) FL* phase generates magnetism at energies much lower than the critical energy of the FL to FL* transition

Conclusions

1. Good experimental and theoretical progress in understanding density-driven and LGW quantum phase transitions.

2. Many interesting transitions of strongly correlated materials associated with gauge or "topological" order parameters. Intimate connection with Luttinger theorem and lattice commensuration effects. Classification scheme ?

3. Many experiments on heavy fermions compounds and cuprates remain mysterious – effects of disorder ?

4. Ultracold atoms offer new regime for studying many quantum phase transitions.