Emergent “light” and the high temperature superconductors

Pennsylvania State University
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Maxwell's equations: 150 years of light

A century and a half ago, James Clerk Maxwell submitted a long paper to the Royal Society containing his famous equations. Inspired by Michael Faraday's experiments and insights, the equations unified electricity, magnetism and optics. Their far-reaching consequences for our civilisation, and our universe, are still being explored.

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Modern point-of-view:

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Electrons in crystals provide a new “vacuum”, and their interactions can naturally lead to quantum states which have long-range quantum entanglement, and require “emergent” gauge fields.
High temperature superconductors

CuO$_2$ plane

YBa$_2$Cu$_3$O$_{6+x}$
Figure: K. Fujita and J. C. Seamus Davis

$YBa_2Cu_3O_{6+x}$
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A conventional metal: the Fermi liquid
1. Emergent gauge fields and long-range entanglement in insulators
“Undoped” Anti-ferromagnet
Insulating spin liquid

\[ \psi = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2} \]

An insulator with emergent gauge fields: the first proposal of a quantum state with long-range entanglement

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obtain “topological” states nearly degenerate with the ground state: number of dimers crossing red line is conserved modulo 2

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Place insulator on a torus;
The sensitivity of the degeneracy to the global topology indicates long-range quantum entanglement

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Ground state degeneracy

Place insulator on a torus;
The degenerate states are conjugate to the flux of an emergent gauge field piercing the cycles of the torus.

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Emergent gauge fields

Local constraint on dimer number operators:

\[ \hat{n}_1 + \hat{n}_2 + \hat{n}_3 + \hat{n}_4 = 1. \]

Identify dimer number with an ‘electric’ field, \( \hat{E}_{i\alpha} = (-1)^{i_x+i_y} \hat{n}_{i\alpha} \), \( (\alpha = x, y) \); the constraint becomes ‘Gauss’s Law’:

\[ \Delta_\alpha \hat{E}_{i\alpha} = (-1)^{i_x+i_y}. \]

The theory of the dimers is compact U(1) quantum electrodynamics in the presence of static background charges. The compact theory allows the analog of Dirac’s magnetic monopoles as tunneling events/excitations.

Emergent gauge fields

Including dimers connecting the same sublattice leads to a $\mathbb{Z}_2$ gauge theory in the presence of Berry phases of static background charges. This has a stable deconfined phase in 2+1 dimensions. By varying parameters it can undergo a confinement transition to a valence bond solid, described by a frustrated Ising model.

I. Emergent gauge fields and long-range entanglement in insulators
A conventional metal: the Fermi liquid

1. Emergent gauge fields and long-range entanglement in insulators

2. Theory of ordinary metals: Fermi liquids (FL)
   (a) Quasiparticles
   (b) Luttinger theorem for volume enclosed by Fermi surface
Ordinary metals: the Fermi liquid

- Fermi surface separates empty and occupied states in momentum space.

\[ \text{Hall coefficient } R_H = \frac{1}{(\text{Fermi volume}) \times e} \]
Ordinary metals: the Fermi liquid

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- Fermi surface separates empty and occupied states in momentum space.
- *Luttinger Theorem*: volume (area) enclosed by Fermi surface = the electron density.
- Hall co-efficient \( R_H = -1/((\text{Fermi volume}) \times e) \).
“Undoped” Anti-ferromagnet
Anti-ferromagnet with $p$ holes per square
Anti-ferromagnet with $p$ holes per square

But relative to the band insulator, there are $1 + p$ holes per square, and so a Fermi liquid has a Fermi surface of size $1 + p$. 
Fermi liquid
Area enclosed by Fermi surface = $1 + \rho$

\[ R_H = +1/((1 + p)e) \]
2. Pseudogap metal at low $p$
The PG regime behaves in many respects like a Fermi liquid, but with a Fermi surface size of $p$ and not $1+p$. 
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3. The FL* phase:
   Quasiparticles with a non-Luttinger volume, and emergent gauge fields
Anti-ferromagnet with $p$ holes per square
Anti-ferromagnet with $p$ holes per square

Can we get a Fermi surface of size $p$?
(and full square lattice symmetry)
Spin liquid with density \( p \) of spinless, charge +e “holons”. These can form a Fermi surface of size \( p \), but this is not visible in electron photo-emission.

\[
\Psi = \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}
\]
Spin liquid with density $\rho$ of spinless, charge $+e$ “holons”. These can form a Fermi surface of size $\rho$, but this is not visible in electron photo-emission.

$$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$$

N. Read and B. Chakraborty, PRB 40, 7133 (1989)
Spin liquid with density $p$ of spinless, charge $+e$ "holons". These can form a Fermi surface of size $p$, but this is not visible in electron photo-emission.

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\[ \frac{1}{p^2} = \frac{(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)}{\sqrt{2}} \]
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FL*


\[
S = \frac{1}{2}, \text{charge} +e \text{fermionic dimers: form a Fermi surface of size } p \text{ visible in electron photo-emission}
\]

\[
= \frac{(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)}{\sqrt{2}}
\]

\[
= \frac{(|\uparrow\circ\rangle + |\circ\uparrow\rangle)}{\sqrt{2}}
\]


Mobile
S=1/2, charge +e fermionic dimers: form a Fermi surface of size \( p \) visible in electron photoemission

Mobile $S=1/2$, charge $+e$ fermionic dimers: form a Fermi surface of size $p$ visible in electron photoemission.

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Mobile S=1/2, charge +e fermionic dimers: form a Fermi surface of size $p$ visible in electron photo-emission

$\begin{align*}
\text{FL}^* &= \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2} \\
\text{closed bond} &= \left( |\uparrow\circ\rangle + |\circ\uparrow\rangle \right) / \sqrt{2}
\end{align*}$

FL*


Mobile
S = 1/2, charge +e fermionic dimers: form a Fermi surface of size $p$ visible in electron photo-emission

$| \uparrow \downarrow \rangle - | \downarrow \uparrow \rangle |_{/ \sqrt{2}}$

$| \uparrow \circ \rangle + | \circ \uparrow \rangle |_{/ \sqrt{2}}$

Mobile $S=1/2$, charge $+e$ fermionic dimers: form a Fermi surface of size $p$ visible in electron photo-emission

$\frac{1}{\sqrt{2}} = (\left| \uparrow \downarrow \right\rangle - \left| \downarrow \uparrow \right\rangle) / \sqrt{2}$

$\frac{1}{\sqrt{2}} = (\left| \uparrow \circ \right\rangle + \left| \circ \uparrow \right\rangle) / \sqrt{2}$


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$\psi = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

$\psi' = (|\uparrow\circ\rangle + |\circ\uparrow\rangle)/\sqrt{2}$

Ground state degeneracy

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\[ \langle \uparrow \downarrow \rangle - \langle \downarrow \uparrow \rangle \approx \sqrt{2} \]

\[ \langle \uparrow \circ \rangle + \langle \circ \uparrow \rangle \approx \sqrt{2} \]

Place FL* on a torus:

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\[ \frac{\left| \uparrow \downarrow \right> - \left| \downarrow \uparrow \right>}{\sqrt{2}} \]

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\[ \text{FL}^* \]

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We have described a metal with:

- A Fermi surface of electrons enclosing volume $p$, and not the Luttinger volume of $1+p$
- Additional low energy quantum states on a torus not associated with quasiparticle excitations i.e. emergent gauge fields
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There is a general and fundamental relationship between these two characteristics.

Following the evolution of the quantum state under adiabatic insertion of a flux quantum leads to a non-perturbative argument for the volume enclosed by the Fermi surface.

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   Quasiparticles with a non-Luttinger volume, and emergent gauge fields
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4. The pseudogap metal of the cuprate superconductors
2. **Pseudogap metal at low $p$**

A new metal - FL*:
with electron-like quasiparticles on a Fermi surface of size $p$ and emergent gauge fields

Recent evidence for pseudogap metal as FL*

- Density wave instabilities of FL* have wave vector and form-factors which agree with STM/X-ray observations in DW region (D. Chowdhury and S. Sachdev, PRB 90, 245136 (2014)).
Density wave (DW) order at low $T$ and $p$
\[ Q = (\pi/2, 0) \]

**Legend**:
- **PG**: Phase Goldstone
- **SM**: Superconducting Macroscopic
- **FL**: Fermi liquid
- **AF**: Antiferromagnet
- **DW**: Dimerized Wigner
- **dSC**: Dimerized Superconducting

**Diagram**: A color gradient map showing temperature and pressure phases, with region labels for PG, SM, FL, DW, and dSC + DW. A vector arrow indicates the transition from AF to dSC + DW at a specific temperature \( T^* \).
$Q = (\pi/2, 0)$
The high $T$ FL* can help explain the “d-form factor density wave” observed at low $T$. 

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- $T$-independent positive Hall co-efficient, $R_H$, corresponding to carrier density $p$ in the higher temperature pseudogap (Ando et al., PRL 92, 197001 (2004)) and in recent measurements at high fields, low $T$, and around $p \approx 0.16$ in YBCO (Proust-Taillefer-UBC collaboration, Badoux et al., arXiv:1511.08162).
Figure: K. Fujita and J. C. Seamus Davis

$\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$
High field, low $T$ measurements show a positive Hall co-efficient corresponding to carriers of density $1 + p$.

High field, low $T$ measurements show a positive Hall co-efficient corresponding to carriers of density $p$. This is likely due to the presence of antiferromagnetic order.
High field, low $T$ measurements show a positive Hall co-efficient corresponding to carriers of density $p$


Figure: K. Fujita and J. C. Seamus Davis
Recent evidence for pseudogap metal as FL*

Fig. 2 | Field dependence of the Hall coefficient in YBCO.

Hall coefficient of YBCO at various fixed temperatures, as indicated, plotted as \( R_H \) vs \( H / H_{vs} \), where \( H_{vs}(T) \) is the vortex-lattice melting field above which \( R_H \) becomes non-zero, for two dopings: \( p = 0.15 \) (top panel) and \( p = 0.16 \) (bottom panel).

Upon cooling, we see that \( R_H \) decreases and eventually becomes negative at \( p = 0.15 \), while it never drops at \( p = 0.16 \).
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