Competing orders in the cuprate superconductors

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Talk online at
http://pantheon.yale.edu/~subir
Superconductivity in a doped Mott insulator

**Hypothesis**: cuprate superconductors are characterized by additional order parameters, associated with the proximate Mott insulator, along with the familiar order associated with the Bose condensation of Cooper pairs in BCS theory. These orders lead to new low energy excitations.

Superconductivity in a doped Mott insulator

Study physics in a generalized phase diagram which includes new phases (which need not be experimentally accessible) with long-range correlations in the additional order parameters. Expansion away from quantum critical points provides a systematic and controlled theory of the low energy excitations (including their behavior near imperfections such as impurities and vortices and their response to applied fields) and of crossovers into “incoherent” regimes at finite temperature.

Outline

I. Order in Mott insulators
   Magnetic order
      A. Collinear spins
      B. Non-collinear spins
   Paramagnetic states
      A. Bond order and confined spinons
      B. Topological order and deconfined spinons

II. Doping Mott insulators with collinear spins and bond order
    Global phase diagram

III. Spin density waves (SDW) in LSCO
    Tuning order and transitions by a magnetic field.

IV. Connection with LDOS modulations
    STM experiments on Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$

V. Conclusions
I. Order in Mott insulators

**Magnetic order** \( \langle S_j \rangle = N_1 \cos(\vec{K} \cdot \vec{r}_j) + N_2 \sin(\vec{K} \cdot \vec{r}_j) \)

A. Collinear spins

\[ \vec{K} = (\pi, \pi) ; \ N_2 = 0 \]

\[ \vec{K} = (3\pi/4, \pi) ; \ N_2 = 0 \]

\[ \vec{K} = (3\pi/4, \pi) ; \ N_2 = (\sqrt{2} - 1) N_1 \]
I. Order in Mott insulators

**Magnetic order**

\[ \langle S_j \rangle = N_1 \cos(\overline{K} \cdot r_j) + N_2 \sin(\overline{K} \cdot r_j) \]

A. Collinear spins

Key properties

States can also have **bond order**.

\[ Q_a(r_i) \equiv S_i \cdot S_{i+a} \]

Values of \( \langle Q_a(r_i) \rangle \) modulate with wavevector \( 2\overline{K} \);
\[ \langle Q_a(\theta) \rangle \] is a measure of site charge density
I. Order in Mott insulators

*Magnetic order* \[ \langle S_j \rangle = N_1 \cos(K \cdot \vec{r}_j) + N_2 \sin(K \cdot \vec{r}_j) \]

A. Collinear spins

Key properties

Order specified by a single vector \( \mathbf{N} \).

Quantum fluctuations leading to loss of magnetic order should produce a paramagnetic state with a vector \( S=1 \) quasiparticle excitation.
I. Order in Mott insulators

**Magnetic order**

\[
\langle S_j \rangle = N_1 \cos (\vec{K} \cdot \vec{r}_j) + N_2 \sin (\vec{K} \cdot \vec{r}_j)
\]

B. Noncollinear spins


\[
\vec{K} = (3\pi/4, \pi); \quad N_2^2 = N_1^2, \quad N_1 \cdot N_2 = 0
\]

Solve constraints by expressing \(N_{1,2}\) in terms of two complex numbers \(z_\uparrow, z_\downarrow\)

\[
N_1 + iN_2 = \begin{pmatrix} z_\downarrow^2 - z_\uparrow^2 \\ i(z_\downarrow^2 + z_\uparrow^2) \\ 2z_\uparrow z_\downarrow \end{pmatrix}
\]

Order in ground state specified by a spinor \((z_\uparrow, z_\downarrow)\) (modulo an overall sign)

Order parameter space: \(S_3/Z_2\)

Physical observables are invariant under the \(Z_2\) gauge transformation \(z_a \rightarrow \pm z_a\)
I. Order in Mott insulators

Magnetic order

\[ \langle S_j \rangle = N_1 \cos(\vec{K} \cdot \vec{r}_j) + N_2 \sin(\vec{K} \cdot \vec{r}_j) \]

B. Noncollinear spins

Vortices associated with \( \pi_1(S_3/Z_2) = Z_2 \) (visons)

Quantum fluctuations leading to loss of magnetic order produce a paramagnetic state with a spinor \( S = 1/2 \) quasiparticle excitation, \( (z_\uparrow, z_\downarrow) \), with a \( Z_2 \) gauge charge, a vison vortex gap, and topological order associated with vison suppression in the ground state.

I. Order in Mott insulators

**Paramagnetic states** \( \langle S_j \rangle = 0 \)

A. Bond order and spin excitons

\[
\frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)
\]

Such a state is obtained by quantum-``disordering" collinear state with \( \vec{K} = (\pi, \pi) \): fluctuating \( N \) becomes the \( S=1 \) spin exciton and Berry phases induce bond order

I. Order in Mott insulators

*Paramagnetic states* \[ \langle S_j \rangle = 0 \]

B. Topological order and deconfined spinons

Number of valence bonds cutting line is conserved modulo 2 – this is described by the same $Z_2$ gauge theory as non-collinear spins


RVB state with free spinons

**Orders of Mott insulators in two dimensions**


**A. Collinear spins, Berry phases, and bond order**

Néel ordered state

Bond order and $S=1$ spin exciton

**B. Non-collinear spins and deconfined spinons.**

Non-collinear ordered antiferromagnet

Topological order: RVB state with $Z_2$ gauge visons, $S=1/2$ spinons
Bond order in a frustrated $S=1/2$ XY magnet

A. W. Sandvik, S. Daul, R. R. P. Singh, and D. J. Scalapino, cond-mat/0205270

First large scale numerical study of the destruction of Neel order in a $S=1/2$ antiferromagnet with full square lattice symmetry

\[
H = 2J \sum_{\langle ij \rangle} \left( S_i^x S_j^x + S_i^y S_j^y \right) - K \sum_{\langle ijk \rangle \subset \square} \left( S_i^+ S_j^- S_k^+ S_{l}^- + S_i^- S_j^+ S_k^- S_{l}^+ \right)
\]
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Doping a paramagnetic bond-ordered Mott insulator

systematic Sp(N) theory of translational symmetry breaking, while preserving spin rotation invariance.

II. Global phase diagram

Include long-range Coulomb interactions: frustrated phase separation


M. Vojta, cond-mat/0204284.

See also J. Zaanen, *Physica C* 217, 317 (1999),

Further neighbor exchange with square lattice symmetry (Sandvik axis)

Anisotropic "d-wave" superconductor

Hatched region --- static spin order
Shaded region ---- static bond/charge order

J. Zaanen
II. Global phase diagram

Include long-range Coulomb interactions: frustrated phase separation


M. Vojta, cond-mat/0204284.

Hatched region --- static spin order
Shaded region ---- static bond/charge order

Non-magnetic “d-wave” superconductor with even period bond order.

See also J. Zaanen, Physica C 217, 317 (1999),
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T=0 phases of LSCO

(additional commensurability effects near $\delta=0.125$)

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III. Tuning magnetic order in LSCO by a magnetic field

T=0 phases of LSCO

Superconductor with $T_{c,\text{min}} = 10$ K

(additional commensurability effects near $\delta = 0.125$)

III. Tuning magnetic order in LSCO by a magnetic field

T=0 phases of LSCO

Use simplest assumption of a direct second-order quantum phase transition between SC and SC+SDW phases

Follow intensity of elastic Bragg spots in a magnetic field
Effect of the Zeeman term: precession of SDW order about the magnetic field

Characteristic field $g\mu_B H = \Delta$, the spin gap

$1 \text{ Tesla} = 0.116 \text{ meV}$

Elastic scattering intensity

$I(H) = I(0) + a \left( \frac{H}{J} \right)^2$

Effect is negligible over experimental field scales
Dominant effect: uniform softening of spin excitations by superflow kinetic energy

Spatially averaged superflow kinetic energy

\[
\sim \langle v_s^2 \rangle \sim \frac{H}{H_{c2}} \ln \frac{3H_{c2}}{H}
\]

The suppression of SC order appears to the SDW order as an effective \( \delta \):

\[
\delta_{\text{eff}}(H) = \delta - C \frac{H}{H_{c2}} \ln \left(\frac{3H_{c2}}{H}\right)
\]


Competing order is enhanced in a “halo” around each vortex
Elastic scattering intensity

\[ I[H, \delta] \approx I[0, \delta_{\text{eff}}] \]

\[ \approx I[0, \delta] + a \frac{H}{H_{c2}} \ln \left( \frac{3H_{c2}}{H} \right) \]

\[ \delta_{\text{eff}}(H) = \delta_c \quad \Rightarrow \]

\[ H \sim \frac{(\delta - \delta_c)}{\ln \left( \frac{1}{(\delta - \delta_c)} \right)} \]

Main results

"Normal" (Bond order)

T=0


Main results


Neutron scattering of La$_{2-x}$Sr$_x$CuO$_4$ at $x=0.1$


Neutron scattering measurements of static spin correlations of the superconductor+spin-density-wave (SC+SDW) in a magnetic field

Elastic neutron scattering off $\text{La}_2\text{CuO}_{4+y}$
B. Khaykovich, Y. S. Lee, S. Wakimoto,
K. J. Thomas, M. A. Kastner,

Solid line --- fit to: \[ \frac{I(H)}{I(0)} = 1 + a \frac{H}{H_{c2}} \ln \left( \frac{3.0H_{c2}}{H} \right) \]

\( a \) is the only fitting parameter
Best fit value: \( a = 2.4 \) with \( H_{c2} = 60 \text{ T} \)
Aside: Topological order with collinear spins (J. Zaanen)

SDW order: \( S_\alpha (r) = \Phi_\alpha (r) e^{iK \cdot r} + \text{c.c.} \)

Collinear spins: \( \Rightarrow \Phi_\alpha = n_\alpha e^{i\theta} \) with \( n_\alpha \) real

\( Z_2 \) gauge symmetry: \( n_\alpha \rightarrow -n_\alpha \) and \( \theta \rightarrow \theta + \pi \)

Effective action

\[
S = -J \sum_{\langle ij \rangle} \sigma_{ij} n_{\alpha i} n_{\alpha j} - J \sum_{\langle ij \rangle} \sigma_{ij} \cos(\theta_i - \theta_j) - K \sum \prod \sigma_{ij}
\]

\( \sigma_{ij} \rightarrow Z_2 \) gauge field

Can obtain a topologically ordered state with

\( \langle n_\alpha \rangle = 0 \); \( \langle e^{i\theta} \rangle = 0 \)

but \( Z_2 \) gauge flux suppressed

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STM observation of LDOS modulations. Our prediction: SDW fluctuations enhanced by superflow and bond order pinned by vortex cores.

\[ H \sim \frac{\delta - \delta_c}{\ln\left(\frac{1}{\delta - \delta_c}\right)} \]

IV. Connections with LDOS modulations

SDW order: \( S_\alpha (r) = \Phi_\alpha (r) e^{iK \cdot r} + \text{c.c.} \)

Bond order: \( Q_\alpha (r) = \sum_\alpha S_\alpha (r) S_\alpha (r + a) \approx \sum_\alpha \Phi_\alpha^2 (r) e^{iK \cdot a} e^{2iK \cdot r} + \text{c.c.} \)

Superflow reduces energy of dynamic spin exciton, but action so far does not lead to static charge order because all terms are invariant under the “sliding” symmetry:

\[ \Phi_\alpha (r) \rightarrow \Phi_\alpha (r) e^{i\theta} \]

Small vortex cores break this sliding symmetry on the lattice scale, and lead to a pinning term, which picks particular phase of the local bond order

\[ S_{\text{pin}} = \zeta \sum_{\text{All } r_v \text{ where } \psi(r_v) = 0} \int_0^{1/T} d\tau \left[ \sum_\alpha \Phi_\alpha^2 (r_v) e^{i\varphi} + \text{c.c.} \right] \]

With this term, SC phase has bond order but dynamic SDW i.e. there is no static spin order (no “spins in vortices”)

\[ \langle \Phi_\alpha^2 (r) \rangle \neq 0 \quad ; \quad \langle \Phi_\alpha (r) \rangle = 0 \]
Pinning of static bond order by vortex cores in SC phase, with dynamic SDW correlations

\[ \langle \Phi^2_{\alpha}(r, \tau) \rangle \propto \zeta \int d\tau_1 \langle \Phi_{\alpha}(r, \tau) \Phi^*_{\alpha}(r_v, \tau_1) \rangle^2 ; \quad \langle \Phi_{\alpha}(r, \tau) \rangle = 0 \]

Vortex-induced LDOS of Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ integrated from 1meV to 12meV

IV. STM image of LDOS modulations in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ in zero magnetic field

Period = 4 lattice spacings

C. Howald, H. Eisaki, N. Kaneko, and A. Kapitulnik, cond-mat/0201546
Spectral properties of the STM signal are sensitive to the microstructure of the charge order

Theoretical modeling shows that this spectrum is best obtained by a modulation of bond variables, such as the exchange, kinetic or pairing energies.

Measured energy dependence of the Fourier component of the density of states which modulates with a period of 4 lattice spacings

C. Howald, H. Eisaki, N. Kaneko, and A. Kapitulnik, cond-mat/0201546

Global phase diagram

Hatched region --- static spin order
Shaded region ---- static bond/charge order

Non-magnetic “d-wave” superconductor with even period bond order.

M. Vojta and S. Sachdev,
M. Vojta, Y. Zhang, and S. Sachdev,
M. Vojta, cond-mat/0204284.

Conclusions

I. Cuprate superconductivity is associated with doping Mott insulators with charge carriers. The correct paramagnetic Mott insulator has bond-order and confinement of spinons (collinear spins in magnetically ordered state.

II. Theory of quantum phase transitions provides semi-quantitative predictions for neutron scattering measurements of spin-density-wave order in superconductors; theory also proposes a connection to STM experiments.

III. Future experiments should search for SC+SDW to SC quantum transition driven by a magnetic field.

IV. Major open question: how does understanding of low temperature order parameters help explain anomalous behavior at high temperatures?