Exploring quantum matter in the high temperature superconductors

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Flavors of Quantum Matter
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A. Ordinary quantum matter

*Independent electrons, or pairs of electrons*
Flavors of Quantum Matter

A. Ordinary quantum matter
   Independent electrons, or pairs of electrons

B. Topological quantum matter
   Long-range quantum entanglement leads to sensitivity to spatial topology
Flavors of Quantum Matter

A. Ordinary quantum matter
   Independent electrons, or pairs of electrons

B. Topological quantum matter
   Long-range quantum entanglement leads to sensitivity to spatial topology

C. Quantum matter without quasiparticles
   Strange metals: infinite-range model maps to extremal charged black holes and yields Bekenstein-Hawking entropy
High temperature superconductors

\[ \text{CuO}_2 \text{ plane} \]

\[ \text{YBa}_2\text{Cu}_3\text{O}_{6+x} \]
“Undoped” Anti-ferromagnet
“Undoped” Anti-ferromagnet
Anti-ferromagnet with $p$ holes per square
Anti-ferromagnet with $p$ holes per square

But relative to the band insulator, there are $1 + p$ holes per square
Antiferromagnet

Figure: K. Fujita and J. C. Seamus Davis
High temperature Superconductor
Figure: K. Fujita and J. C. Seamus Davis

Conventional metal
Area enclosed by Fermi surface = $1 + p$
Ordinary quantum matter: the Fermi liquid (FL)

- Fermi surface separates empty and occupied states in momentum space.

\[
\text{Area enclosed by Fermi surface} = \text{total density of electrons (mod 2)} = 1 + \sqrt{p}.
\]

- Density of electrons can be continuously varied at zero temperature.

- Long-lived electron-like quasiparticle excitations near the Fermi surface: lifetime of quasiparticles \( \sim 1/T^2 \).
**Ordinary quantum matter: the Fermi liquid (FL)**

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- Area enclosed by Fermi surface = total density of electrons (mod 2) = 1 + p.
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- Fermi surface separates empty and occupied states in momentum space.
- Area enclosed by Fermi surface = total density of electrons (mod 2) = $1+p$.
- Density of electrons can be continuously varied at zero temperature.
- Long-lived electron-like quasiparticle excitations near the Fermi surface: lifetime of quasiparticles $\sim 1/T^2$. 
Figure: K. Fujita and J. C. Seamus Davis

Conventional metal
Area enclosed by Fermi surface = \(1 + \rho\)

“Fermi arcs” at low $p$
Strange metal

Metal without quasi-particles
Outline

1. The pseudogap metal
   Fermi liquid co-existing with topological order

2. The strange metal
   Metal without quasiparticles
   Infinite-range model: dual to extremal charged black holes and yields Bekenstein-Hawking entropy

“Fermi arcs” at low \( p \)
A new metal — a fractionalized Fermi liquid (FL*) — with electron-like quasiparticles on a Fermi surface of size $p$ coexisting with topological order.

M. Punk, A. Allais, and S. Sachdev, arXiv:1501.00978
Evidence for Fermi surface of long-lived quasiparticles of density $p$

- Hall effect (Ando PRL 2004)
- Optical conductivity (van der Marel PNAS 2013)
- Magnetoresistance (Greven PRL 2014)
- Scanning Tunneling Microscopy (Seamus Davis, PNAS 2014):
  
  $d$-form factor density wave
Density wave (DW) order at low $T$ and $p$
Identified as a predicted “d-form factor density wave”

Q = (π/2, 0)
Anti-ferromagnet with $p$ holes per square

Note: relative to the fully-filled band insulator, there are $1+p$ holes per square
Fractionalized Fermi liquid (FL*)

\[ | \uparrow \downarrow \rangle - | \downarrow \uparrow \rangle \]

Realizes a metal with a Fermi surface of area \( p \) co-existing with “topological order”

Realizes a metal with a Fermi surface of area $\rho$ co-existing with "topological order"
Fractionalized Fermi liquid (FL*)

Realizes a metal with a Fermi surface of area $p$ co-existing with “topological order”

A fermionic “dimer” describing a “bonding” orbital between two sites

Realizes a metal with a Fermi surface of area $p$ co-existing with “topological order”

Fractionalized Fermi liquid (FL*)

\[ |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \]

Realizes a metal with a Fermi surface of area $p$ co-existing with "topological order"

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Realizes a metal with a Fermi surface of area $p$ co-existing with “topological order”

Fractionalized Fermi liquid (FL*)

\[
\begin{align*}
\left| \uparrow \downarrow \right> - \left| \downarrow \uparrow \right>
\end{align*}
\]

Realizes a metal with a Fermi surface of area $p$ co-existing with “topological order”

Fractionalized Fermi liquid (FL*)

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Fractionalized Fermi liquid (FL*)

\[ = |↑↓⟩ - |↓↑⟩ \]

Realizes a metal with a Fermi surface of area \( p \) co-existing with "topological order"

Topological order

Place pseudogap metal on a torus;
Topological order

Place pseudogap metal on a torus;

obtain “topological” states nearly degenerate with the ground state:

cchange sign of every dimer across red line
Topological order

\[ = |↑↓⟩ - |↓↑⟩ \]

Place pseudogap metal on a torus; obtain “topological” states nearly degenerate with the ground state: change sign of every dimer across red line.
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Place pseudogap metal on a torus; to change overall sign, a pair of “spinons” have to be moved globally around a circumference of the torus.
Topological order

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Place pseudogap metal on a torus; to change overall sign, a pair of “spinons” have to be moved globally around a circumference of the torus.
1. The pseudogap metal
   Fermi liquid co-existing with topological order

2. The strange metal
   Metal without quasiparticles
   Infinite-range model: dual to extremal charged black holes and yields Bekenstein-Hawking entropy
Many experimental indications of a quantum state which has:

- a continuously variable density at zero temperature,
- bulk excitations of arbitrarily low energy,
- and no long-lived quasiparticles.
\[ H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;kl} c_i^\dagger c_j^\dagger c_k c_\ell \]

\[ Q = \frac{1}{N} \sum_i \langle c_i^\dagger c_i \rangle. \]

**An infinite-range model of a strange metal**


A. Kitaev, unpublished

S. Sachdev, arXiv:1506.05111
\[ H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;\ell} c_i^\dagger c_j^\dagger c_k c_\ell \]

\[ Q = \frac{1}{N} \sum_i \langle c_i^\dagger c_i \rangle. \]

Local fermion density of states

\[ \rho(\omega) \sim \begin{cases} 
\omega^{-1/2}, & \omega > 0 \\
e^{-2\pi\epsilon |\omega|^{-1/2}}, & \omega < 0 
\end{cases} \]

\[ H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;kl} c_i^\dagger c_j c_k c_\ell \]

\[ c_i c_j + c_j c_i = 0 \]
\[ c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij} \]
\[ J_{ij;kl} \text{ independent random numbers} \]

\[ Q = \frac{1}{N} \sum_i \langle c_i^\dagger c_i \rangle. \]

Local fermion density of states
\[ \rho(\omega) \sim \begin{cases} \omega^{-1/2}, & \omega > 0 \\ e^{-2\pi \mathcal{E}} |\omega|^{-1/2}, & \omega < 0. \end{cases} \]

Known ‘equation of state’ determines \( \mathcal{E} \) as a function of \( Q \)
\[ Q = \frac{1}{4} \left( 3 - \tanh(2\pi \mathcal{E}) \right) - \frac{1}{\pi} \tan^{-1} \left( e^{2\pi \mathcal{E}} \right) \]

A. Georges, O. Parcollet, and S. Sachdev
\[ H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;\ell} c_i^\dagger c_j^\dagger c_k c_\ell \]

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Known ‘equation of state’ determines \( \mathcal{E} \) as a function of \( \mathcal{Q} \)

\[ \frac{\partial \mathcal{S}}{\partial \mathcal{Q}} = 2\pi \mathcal{E} \]

O. Parcollet, A. Georges, G. Kotliar, and A. Sengupta

A. Georges, O. Parcollet, and S. Sachdev
\[ H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij; k\ell} c_i^\dagger c_j^\dagger c_k c_{\ell} \]

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Known ‘equation of state’ determines \( \mathcal{E} \) as a function of \( Q \)

Microscopic zero temperature entropy density, \( S \), obeys

\[ \frac{\partial S}{\partial Q} = 2\pi \mathcal{E} \]
$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{i;j,k;\ell} c_i^\dagger c_j^\dagger c_k c_\ell$
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\[ \rho(\omega) \sim \begin{cases} \omega^{-1/2}, & \omega > 0 \\ e^{-2\pi\varepsilon}|\omega|^{-1/2}, & \omega < 0. \end{cases} \]

Known ‘equation of state’ determines \( \mathcal{E} \) as a function of \( Q \)

Microscopic zero temperature entropy density, \( S \), obeys

\[ \frac{\partial S}{\partial Q} = 2\pi\varepsilon \]

Einstein-Maxwell theory + cosmological constant

Horizon area \( A_h \):

\[ \text{AdS}_2 \times R^d \]

\[ ds^2 = (d\xi^2 - dt^2)/\xi^2 + d\vec{x}^2 \]

Gauge field: \( A = (\mathcal{E}/\zeta)dt \)

\[ \mathcal{L} = \bar{\psi} \Gamma^\alpha D_\alpha \psi + m\bar{\psi}\psi \]

Local fermion density of states

\[ \rho(\omega) \sim \begin{cases} \omega^{-1/2}, & \omega > 0 \\ e^{-2\pi\varepsilon}|\omega|^{-1/2}, & \omega < 0. \end{cases} \]

T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh

$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;kl} c_i^{\dagger} c_j^{\dagger} c_k c_{\ell}$

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ds^2 = (d\zeta^2 - dt^2)/\zeta^2 + d\vec{x}^2
\]

Gauge field: $A = (\mathcal{E}/\zeta)dt$

\[
\zeta = \infty
\]

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‘Equation of state’ relating $\mathcal{E}$ and $Q$ depends upon the geometry of spacetime far from the AdS$_2$

Eliminate $r_0$ between
\[
Q = \frac{r_0^{d-1}\sqrt{2d [(d-1)R^2 + (d+1)r_0^2]}}{\kappa^2 g_F}
\]
\[
\mathcal{E} = \frac{g_F r_0 \sqrt{2d [(d-1)R^2 + (d+1)r_0^2]}}{2 [(d-1)^2R^2 + d(d+1)r_0^2]}
\]

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Microscopic zero temperature entropy density, \( S \), obeys
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‘Equation of state’ relating \( \mathcal{E} \) and \( Q \) depends upon the geometry of spacetime far from the AdS\(_2\)

Black hole thermodynamics (classical GR) yields
\[ \frac{1}{\mathcal{A}_b} \frac{\partial \mathcal{A}_h}{\partial Q} = 8\pi G_N \mathcal{E} \]

\[ H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij; k\ell} c^\dagger_i c^\dagger_j c_k c_\ell \]

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**Einstein-Maxwell theory + cosmological constant**

**Horizon area** \( A_h \);

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‘Equation of state’ relating \( \mathcal{E} \) and \( Q \) depends upon the geometry of spacetime far from the AdS_2

**Black hole thermodynamics (classical GR) yields**

\[ S = \frac{A_h}{4G_N A_b} \]

\[ \frac{1}{A_b} \frac{\partial A_h}{\partial Q} = 8\pi G_N \mathcal{E} \]

S. Sachdev, arXiv:1506.05111
Figure: K. Fujita and J. C. Seamus Davis

- Pseudogap
- Strange metal

Diagram showing phase transitions in a material under varying temperature ($T$) and density ($\rho$) conditions.
1. The pseudogap metal

   Fermi liquid co-existing with topological order

2. The strange metal

   Metal without quasiparticles
   
   Infinite-range model: dual to extremal charged black holes and yields
   
   Bekenstein-Hawking entropy