

Bekenstein-Hawking entropy from strange metals

Perimeter Institute
June 16, 2015

Subir Sachdev



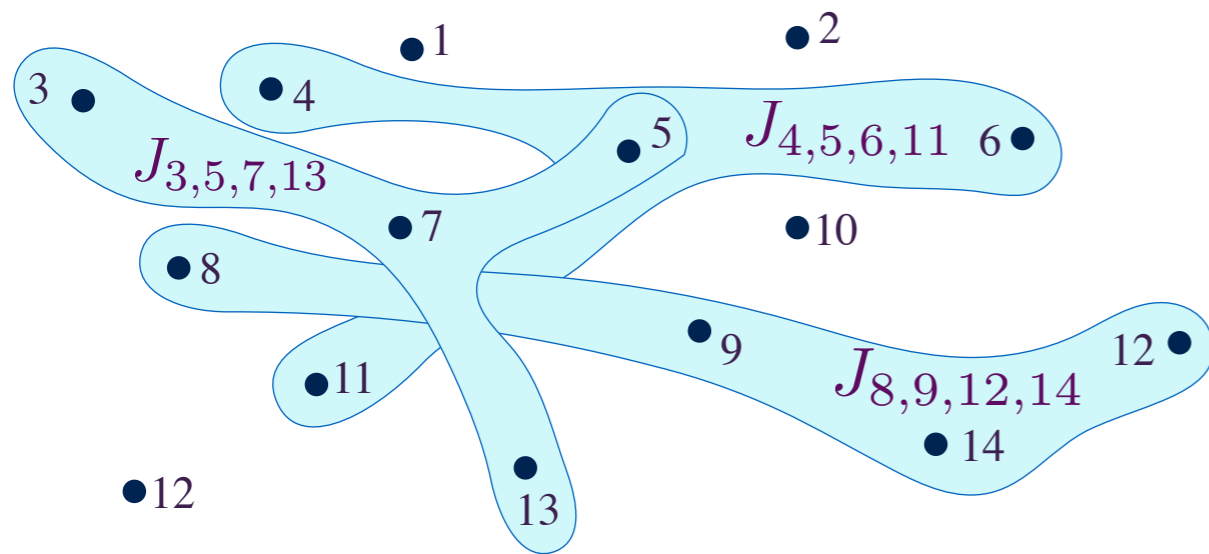
PERIMETER INSTITUTE
FOR THEORETICAL PHYSICS

PHYSICS



HARVARD

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^N J_{ij;kl} c_i^\dagger c_j^\dagger c_k c_\ell$$



$$Q = \frac{1}{N} \sum_i \langle c_i^\dagger c_i \rangle.$$

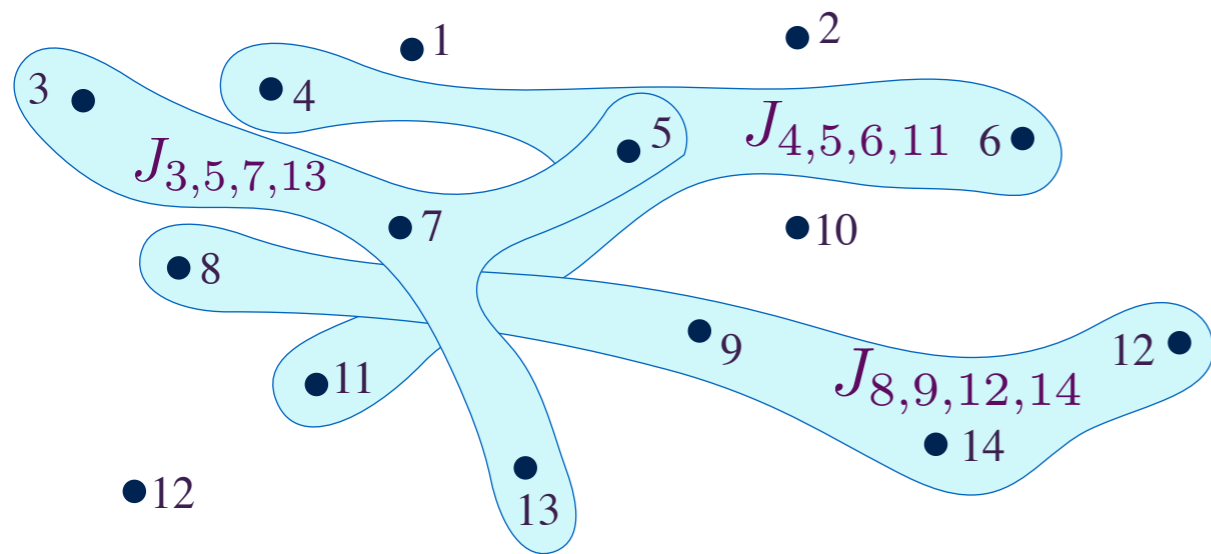
A mean-field model of a strange metal

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

A. Kitaev, unpublished

S. Sachdev, arXiv:1506.05111

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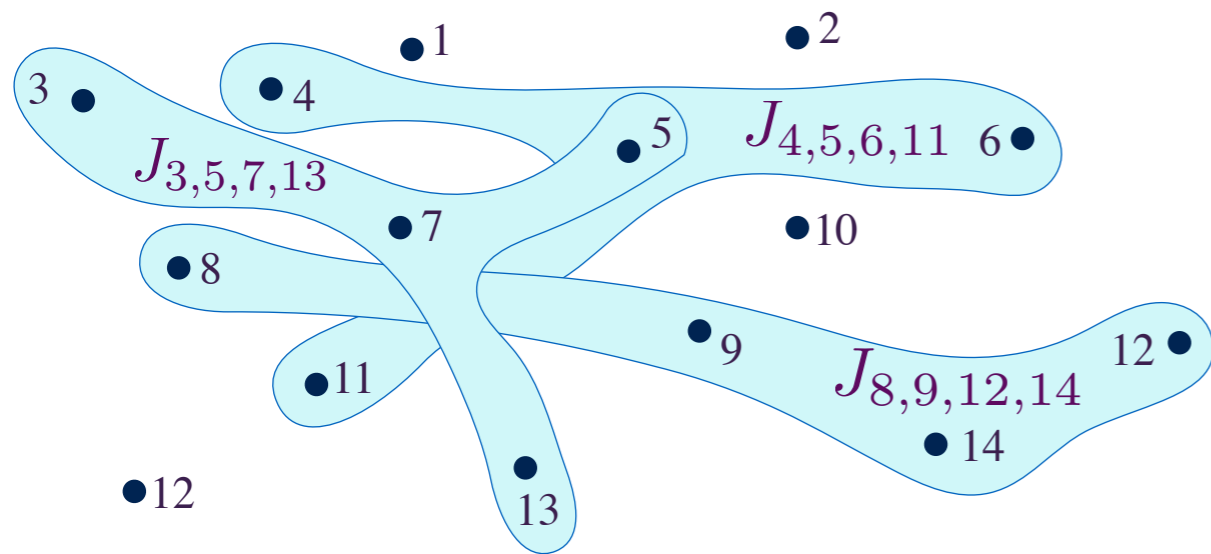
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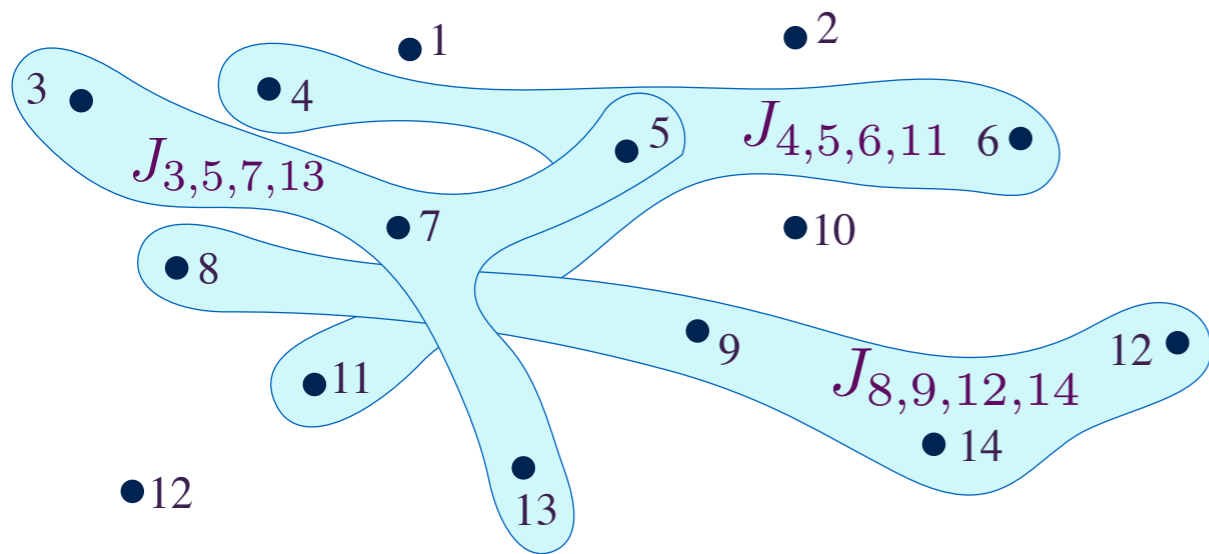
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Known ‘equation of state’
determines \mathcal{E} as a function of Q

A. Georges, O. Parcollet, and S. Sachdev
Phys. Rev. B **63**, 134406 (2001)

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Microscopic zero temperature
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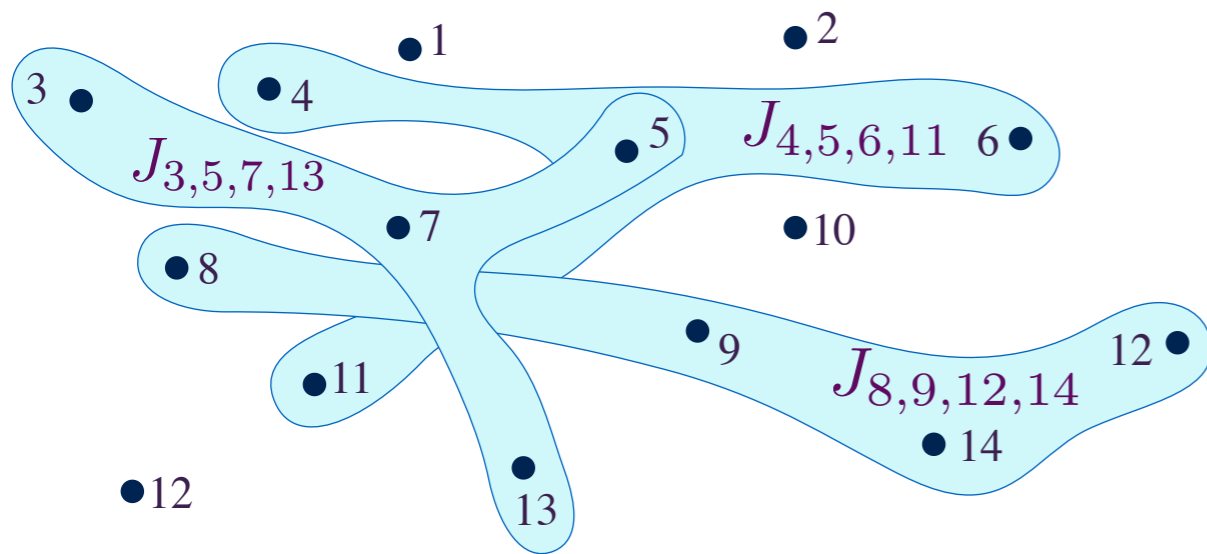
$$\frac{\partial \mathcal{S}}{\partial Q} = 2\pi\mathcal{E}$$

O. Parcollet, A. Georges, G. Kotliar, and A. Sengupta
Phys. Rev. B **58**, 3794 (1998)

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Einstein-Maxwell theory
+ cosmological constant

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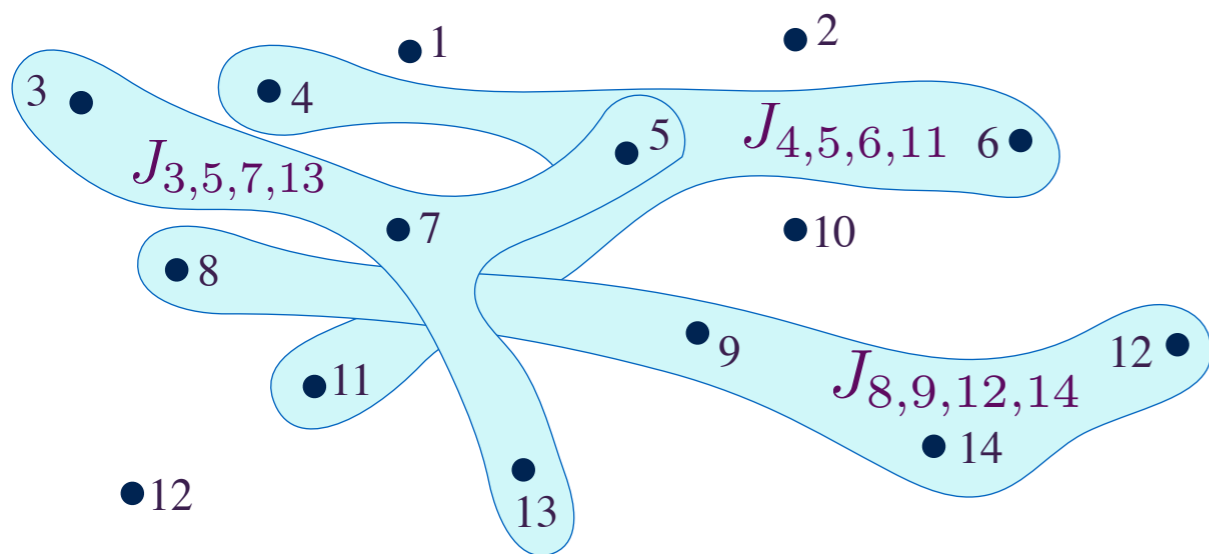
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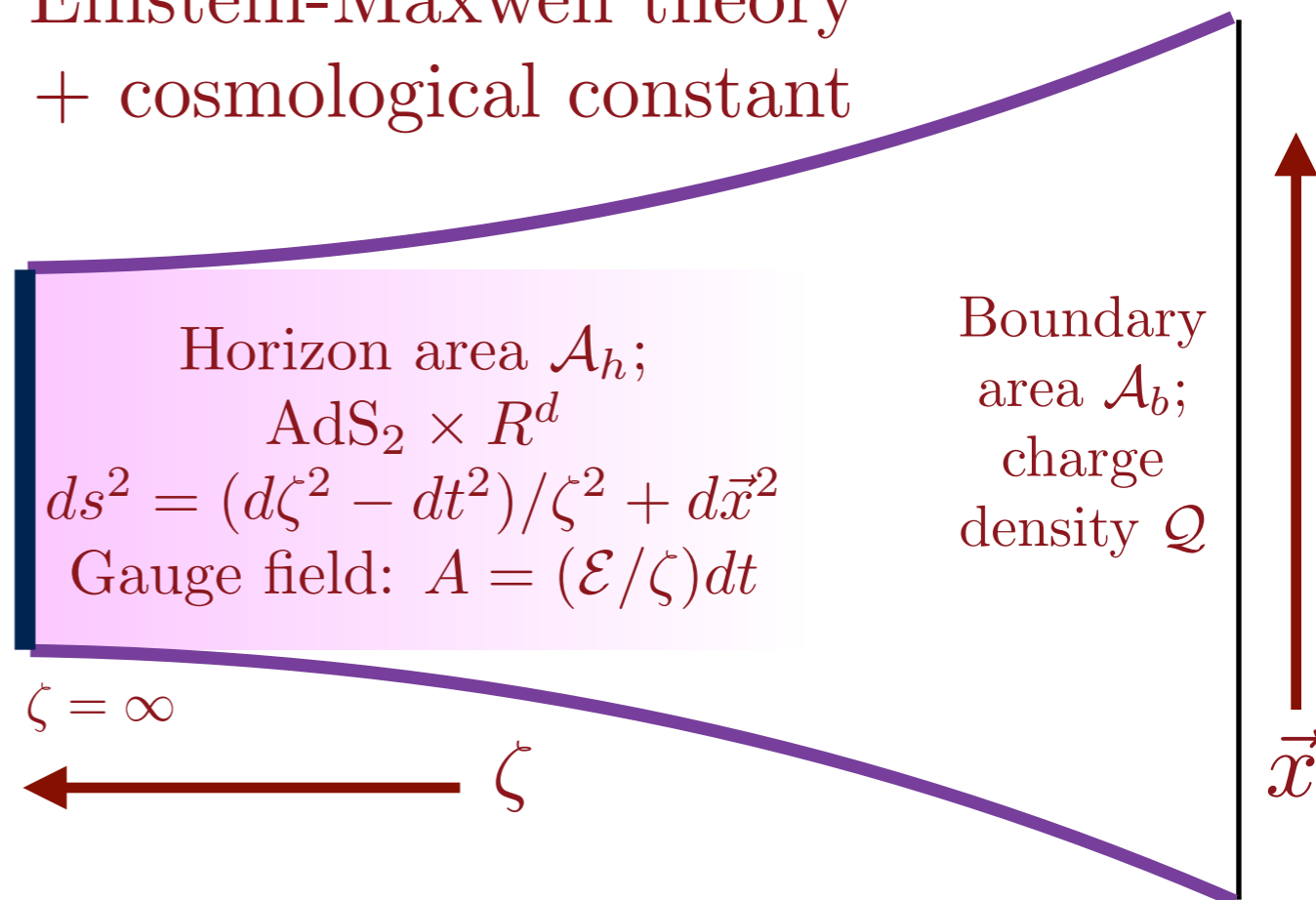
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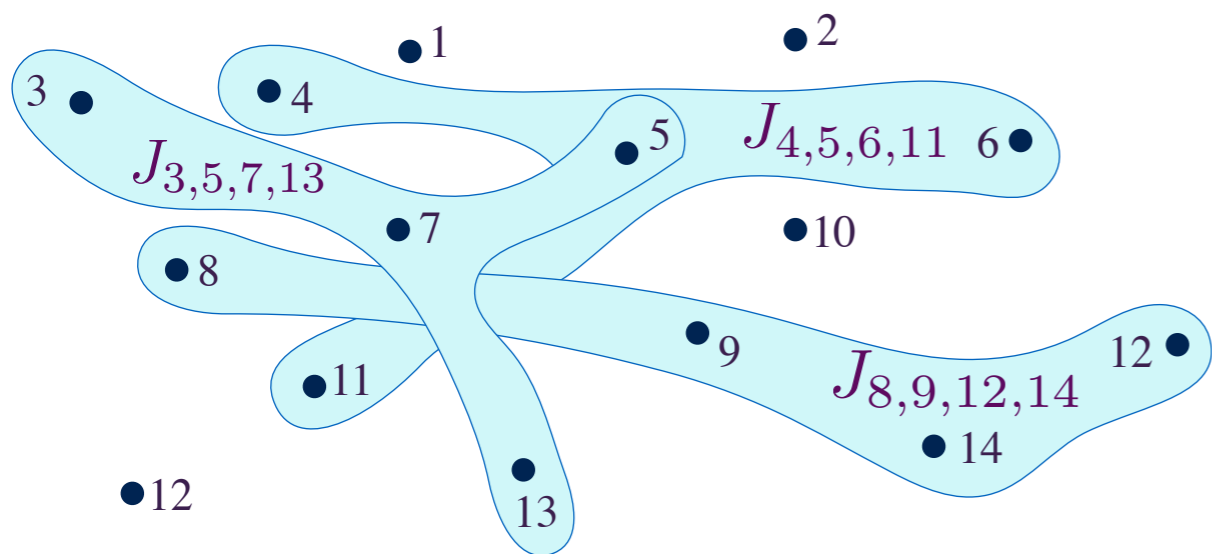
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A. Chamblin, R. Emparan, C.V. Johnson, and R.C. Myers
Phys. Rev. D **60**, 064018 (1999)

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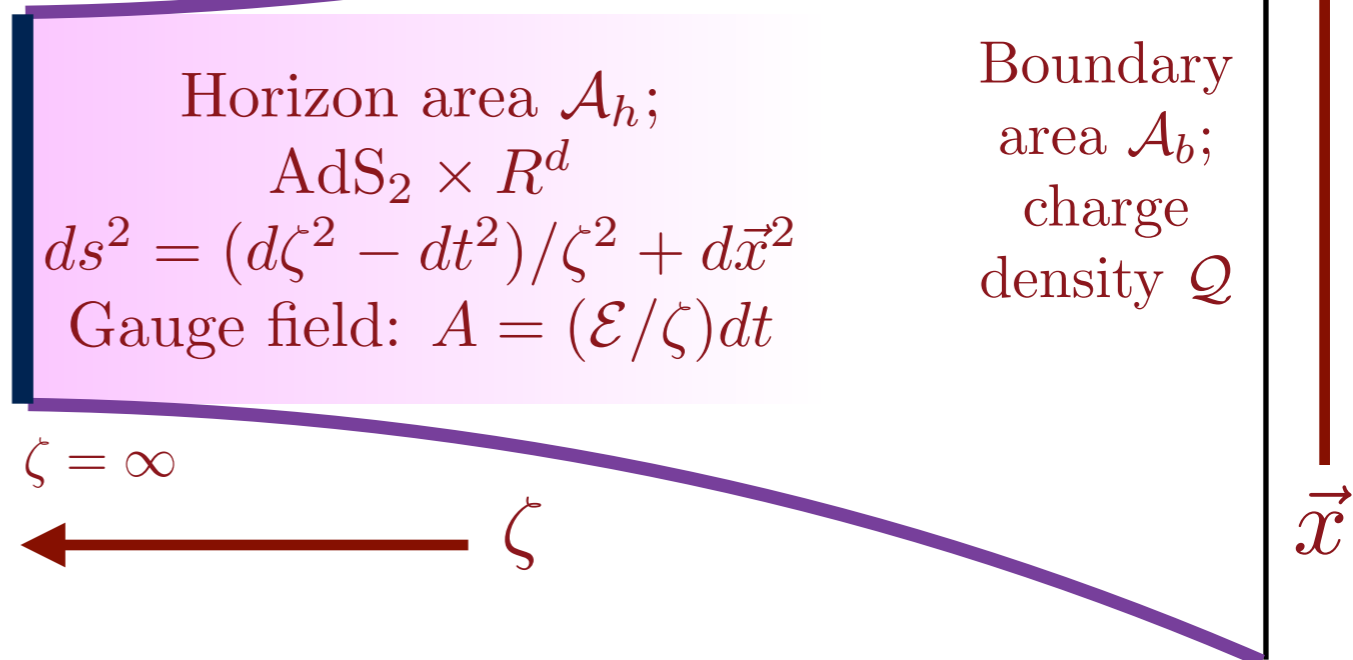
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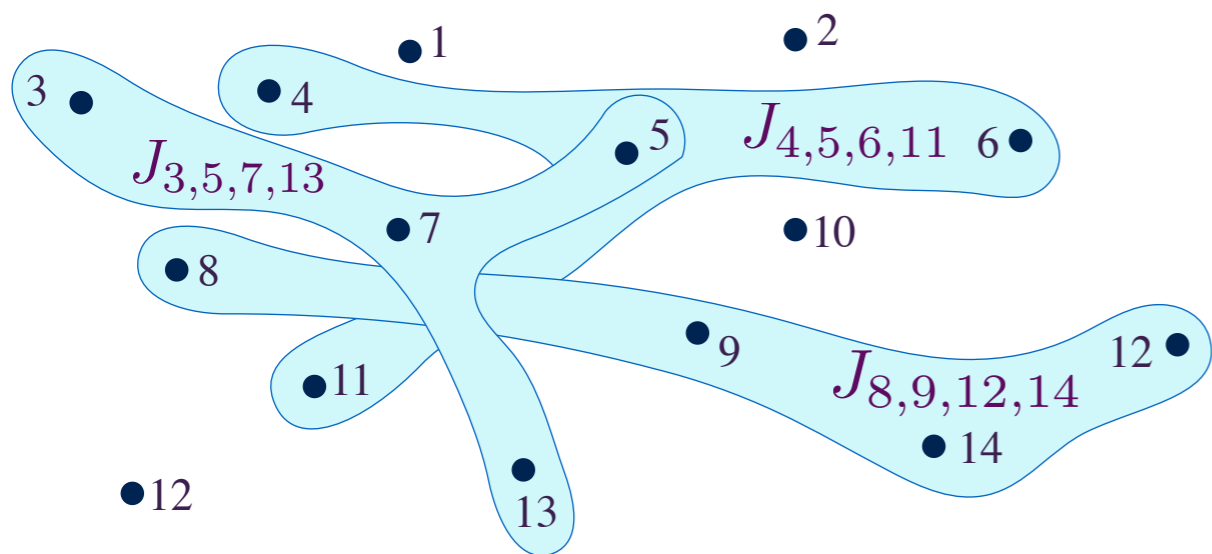


$$\mathcal{L} = \bar{\psi} \Gamma^\alpha D_\alpha \psi + m \bar{\psi} \psi$$

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T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh
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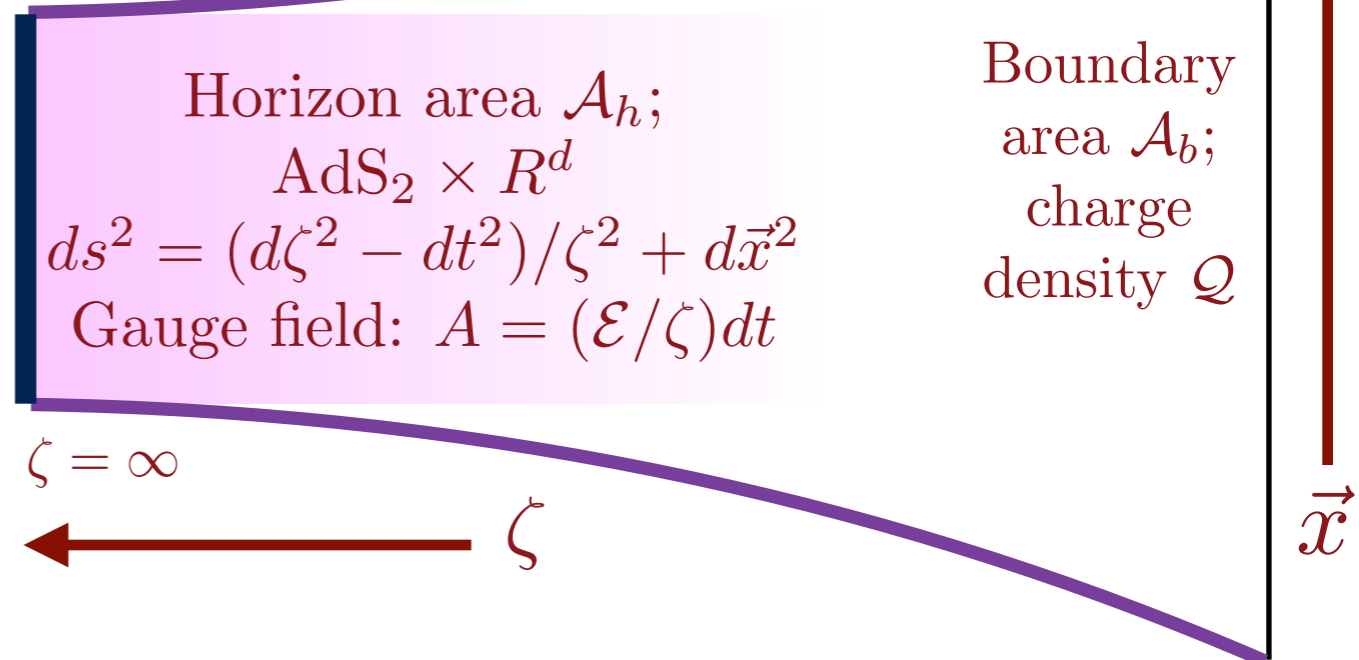
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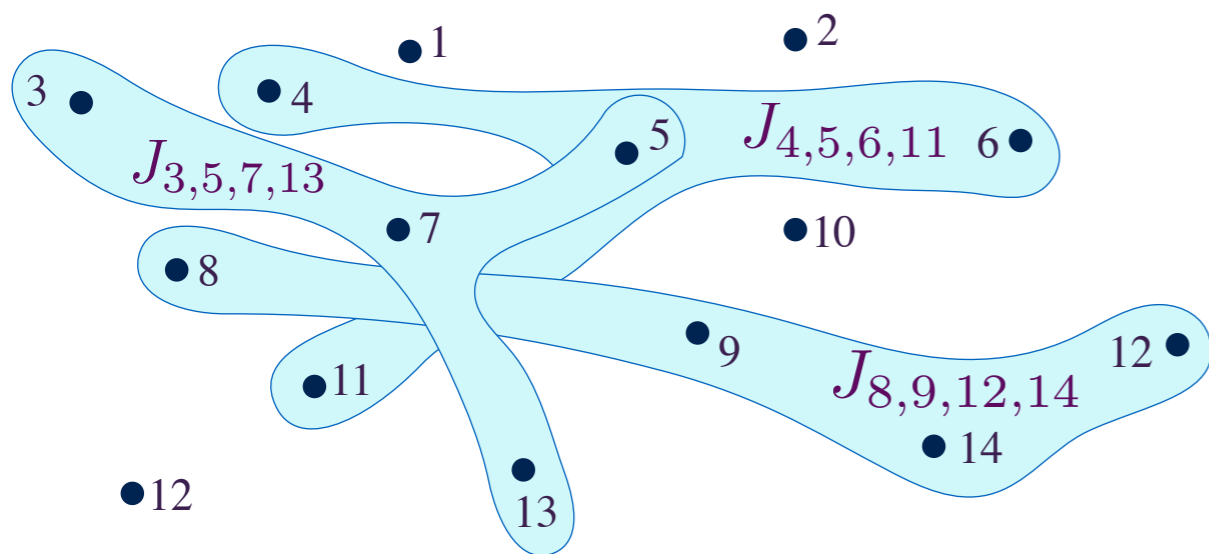
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‘Equation of state’ relating \mathcal{E} and Q depends upon the geometry of spacetime far from the AdS_2

S. Sachdev, arXiv:1506.05111

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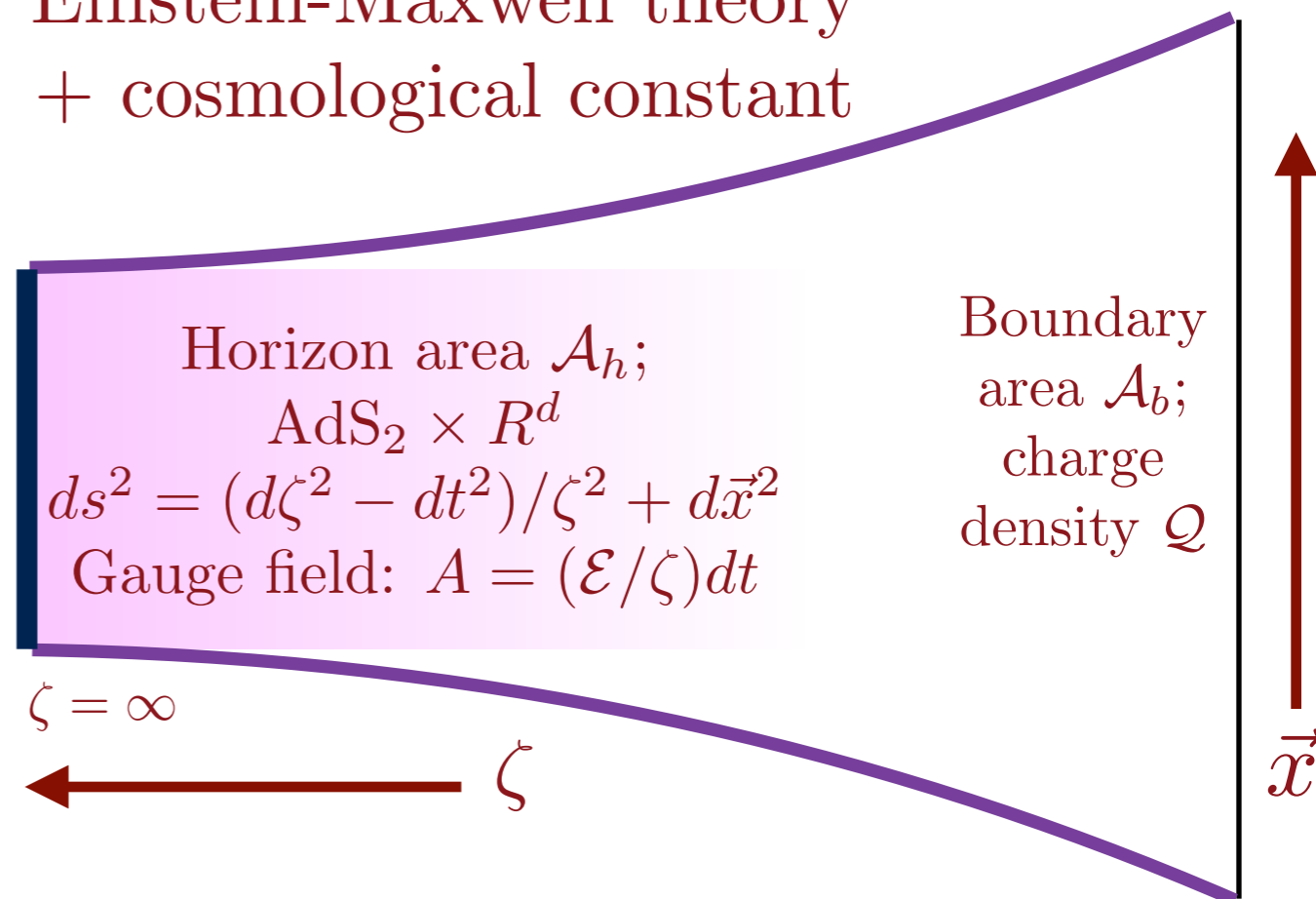
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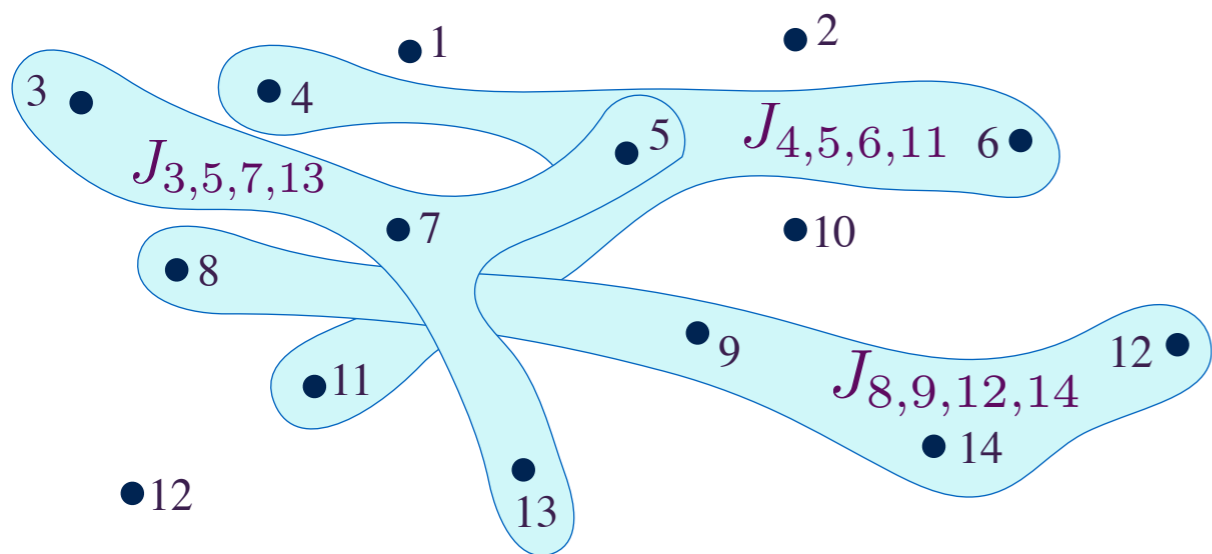
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Black hole thermodynamics (classical GR) yields

$$\frac{1}{A_b} \frac{\partial A_h}{\partial Q} = 8\pi G_N \mathcal{E}$$

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Combination:

$$\mathcal{S} = \frac{\mathcal{A}_h}{4G_N \mathcal{A}_b}$$

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Einstein-Maxwell theory + cosmological constant

Horizon area \mathcal{A}_h ;
 $\text{AdS}_2 \times R^d$
 $ds^2 = (d\zeta^2 - dt^2)/\zeta^2 + d\vec{x}^2$
 Gauge field: $A = (\mathcal{E}/\zeta)dt$

Boundary area \mathcal{A}_b ;
 charge density Q

$\zeta = \infty$

ζ

\vec{x}

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