Entangled states of quantum matter

Perimeter Institute
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Scientific American 308, 44 (January 2013)
Sommerfeld-Pauli-Bloch theory of metals, insulators, and superconductors: many-electron quantum states are adiabatically connected to independent electron states.
Boltzmann-Landau theory of dynamics of metals:
Long-lived \textit{quasiparticles} (and \textit{quasiholes}) have weak interactions which can be described by a Boltzmann equation.
Modern phases of quantum matter

Not adiabatically connected to independent electron states:

1. Many-particle quantum entanglement

2. (a) Quasiparticles with quantum numbers different from those of the electron

(b) No quasiparticles
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Entanglement with quasiparticles

ZnCu(OH)$_6$Cl$_2$
herbertsmithite single crystals

Tian-Heng Han, Young Lee et al, Nature 492, 406 (2012)
Entanglement with quasiparticles

Neutron scattering: excitations over a broad range of momenta and at each energy

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Kagome antiferromagnet

\[ H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \]
Mott insulator: Kagome antiferromagnet

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Thursday, May 9, 13
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Tian-Heng Han et. al., Nature 492, 406 (2012)
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Contribution of 2 spinons in a $\mathbb{Z}_2$ spin liquid;

In progress: contribution of spinon-induced vison pair-production

M. Punk, D. Chowdhury, S. Gopalakrishnan, and S. Sachdev, to appear

Also: T. Dodds, S. Bhattacharjee, and Yong Baek Kim, arXiv: 1303.1154
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1. Superfluid-insulator transition of ultracold atoms in optical lattices: 
   Conformal field theories and gauge-gravity duality

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   The pnictides and the cuprates
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Superfluid-insulator transition

Ultracold $^{87}$Rb atoms - bosons

corresponding to particle and hole excitations in our case (red and blue arrow in Figure 1).

Classical energy density of magnitude compared with earlier experiments. In the experimental sequence, this is realized by a modulation of the optical lattice potential (see main text for details).

The Higgs mode can be excited with a periodic modulation of the phase and amplitude modulations (blue and red arrows in panel 1). As the Goldstone mode to a pair of phase modes with opposite momenta. As a result, examining the low-frequency part of the response, which is expected and high-frequency parts of the response separately. We begin by about the finite spectral width of the modes, which stems from the interaction between amplitude and phase excitations. We will consider the response of a strongly interacting, two-dimensional superfluid is menisure filling in Fig. 2a. Modes at different frequencies from the low-frequency edge of the experimental data can be interpreted as the onset of spectral response. We fitted each spectrum with an error function, which is shown as vertical dashed lines in Fig. 2a. Approaching the critical point, the spectral onset becomes sharper, and the width normalized to the centre frequency becomes even more apparent for the fitted gap visible in the raw data (Fig. 2b) and becomes even more apparent for the fitted gap.

The observed softening of the onset of spectral response in the superfluid regime has led to an identification of the experimental eigenspectrum of the system in a Gutzwiller approach. To gain further insight into the full in-trap response, we calculated the and Supplementary Information). The result is a series of discrete modes in quantitative agreement with a prediction for the Higgs gap for the onset of spectral response, we fitted each spectrum with an error function, which is shown as vertical dashed lines in Fig. 2a. Approaching the critical point, the spectral onset becomes sharper, and the width normalized to the centre frequency.

The sharpness of the spectral onset can be quantified by the width of the fitted error function, which is shown as vertical dashed lines in Fig. 2a. Approaching the critical point, the spectral onset becomes sharper, and the width normalized to the centre frequency.

From quantum Monte Carlo simulations of the response, the fitted gap

\[ \Delta \text{fit} = \frac{1}{2} \left( \Delta_{\text{fit}} \right) \]

is consistent with the Mott gap

\[ \Delta_{\text{Mott}} = \frac{1}{2} \left( \Delta_{\text{Mott}} \right) \]

for the onset of spectral response, which is already visible in the raw data (Fig. 2b) and becomes even more apparent for the fitted gap.

The fitted gaps are consistent with the Mott gap

\[ \Delta_{\text{Mott}} \]

that closes at the critical point in the same way as the gap frequency.

Here we have rescaled the fitted gap

\[ \Delta_{\text{fit}} \]

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We observe similar gapped responses in the Mott insulating regime for the onset of spectral response, we fitted each spectrum with an error function, which is shown as vertical dashed lines in Fig. 2a. Approaching the critical point, the spectral onset becomes sharper, and the width normalized to the centre frequency.

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We observe similar gapped responses in the Mott insulating regime.
The blue shading highlights the superfluid lines denote the widths of the fitted error function and characterize the experimental uncertainty of the lattice depths and the fit error for the.

(circles) show a characteristic softening close to the critical point in quantitative lattice modulation, such a divergence could be avoided and the

field simulation (grey line and shaded area) and a heuristic model (dashed line; for details see text and Methods). The simulation was done for the system.

until a rounding off takes place close to the critical point due to the finite size of the system.

The gap frequency of the lowest-lying mode corresponds to a single eigenmode, with the intensity of the colour being proportional to the line strength. The gap frequency of the lowest-lying mode like eigenmodes. Shown on the vertical axis is the strength of the response to a

invariant (for example through coupling to

absorption per particle

in the gap values to lower frequencies is clearly visible (from panel 1 to panel 3).

Figure 4 prediction (1

interaction. Each data point results from an average of the temperatures over

sions, calculated in the large-

region.

loss of this approach is a continuum of phase modes, which is

including all relevant decay and coupling processes. Lacking such a

nal is consistent with this scaling at the 'base' of the absorption feature

\[ \Psi \rightarrow \text{a complex field representing the Bose-Einstein condensate of the superfluid} \]

\[ \langle \Psi \rangle \neq 0 \]

Superfluid

\[ \langle \Psi \rangle = 0 \]

Insulator
\[ S = \int d^2r dt \left[ |\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right] \]

\[ V(\Psi) = (\lambda - \lambda_c) |\Psi|^2 + u (|\Psi|^2)^2 \]

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Particles and holes correspond to the 2 normal modes in the oscillation of \( \Psi \) about \( \Psi = 0 \).

\[ \langle \Psi \rangle \neq 0 \]

Superfluid

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Insulator

\( \lambda_c \)

\( \lambda \)
Insulator (the vacuum)
at large repulsion between bosons
Excitations of the insulator:

Particles $\sim \Psi^\dagger$
Excitations of the insulator:

$$\text{Holes} \sim \Psi$$
\[ S = \int d^2r dt \left[ |\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right] \]

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Nambu-Goldstone mode is the oscillation in the phase \( \Psi \) at a constant non-zero \( |\Psi| \).

\[ \langle \Psi \rangle \neq 0 \quad \text{Superfluid} \]

\[ \langle \Psi \rangle = 0 \quad \text{Insulator} \]
A conformal field theory in 2+1 spacetime dimensions: a CFT3

\[ S = \int d^2r dt \left[ |\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right] \]

\[ V(\Psi) = (\lambda - \lambda_c) |\Psi|^2 + u (|\Psi|^2)^2 \]

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**CFT3:** The simplest class of theories with many-body entanglement and no quasiparticles

\[ \langle \Psi \rangle \neq 0 \quad \text{Superfluid} \]

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\[ S = \int d^2r dt \left[ |\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right] \]

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Higgs mode is the oscillation in the amplitude \(|\Psi|\). This decays rapidly by emitting pairs of Nambu-Goldstone modes.

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\[ \langle \Psi \rangle = 0 \quad \text{Insulator} \]
\[ S = \int d^2r dt \left[ |\partial_t \Psi|^2 - c^2 |\nabla \Psi|^2 - V(\Psi) \right] \]

\[ V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u (|\Psi|^2)^2 \]

Despite rapid decay, there is a well-defined Higgs “quasi-normal mode”. This is associated with a pole in the lower-half of the complex frequency plane.

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\[ S = \int d^2r dt \left[ |\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right] \]

\[ V(\Psi) = (\lambda - \lambda_c) |\Psi|^2 + u (|\Psi|^2)^2 \]


\[ \frac{\omega_{\text{pole}}}{\Delta} = -i \frac{4}{\pi} + \frac{1}{N} \left( \frac{16 \left( 4 + \sqrt{2} \log \left( 3 - 2\sqrt{2} \right) \right)}{\pi^2} + 2.46531203396 i \right) + O \left( \frac{1}{N^2} \right) \]

where \( \Delta \) is the particle gap at the complementary point in the insulator state, and \( N = 2 \) is the number of vector components of \( \Psi \).
The fitted gap values (1) denoted in the Gutzwiller approximation. a.u., arbitrary units.

Figure 2. The fitted gap values (1) denoted in the Gutzwiller approximation. a.u., arbitrary units.

Figure 3. (a) A diagonalization of the trapped Hamiltonian (grey line and shaded area) and a heuristic model (dashed line; red) as a function of the modulation frequency. The black line is a fit of the form $k_B T/U = \Delta$ (green), which shows a characteristic softening close to the critical point in quantitative lattice modulation, such as a divergence could be avoided and the leading correction to the gap as calculated in the Gutzwiller approximation. 

Figure 4. Vertical dashed lines mark the frequency region of the response in the superfluid regime showing a scaling compatible with the scaling of the homogeneous system based on an O($U$) field theory in two dimensions.

The inset shows the same line for a response function describing the localized systems from the Gutzwiller approach (Fig. 3a) with the line shape for a critical point at $T/T_c = 1$.

A fit of the form $k_B T/U = \Delta$, $j/j_c = 2.9(5)$. The inset shows the same line shape for a critical point at $T/T_c = 1$.


Figure 4 | Scaling of the low-frequency response. The low-frequency response in the superfluid regime shows a scaling compatible with the prediction \((1 - j/j_c)^{-2} v^3\) (Methods). Shown is the temperature response rescaled with \((1 - j/j_c)^2\) for \(V_0 = 10E_r\) (grey), \(9.5E_r\) (black), \(9E_r\) (green), \(8.5E_r\) (blue) and \(8E_r\) (red) as a function of the modulation frequency. The black line is a fit of the form \(a v^b\) with a fitted exponent \(b = 2.9(5)\). The inset shows the same data points without rescaling, for comparison. Error bars, s.e.m.

Observation of Higgs quasi-normal mode in quantum Monte Carlo


Snir Gazit, Daniel Podolsky, and Assa Auerbach, arXiv:1212.3759
\[ S = \int d^2 r dt \left[ |\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right] \]

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A conformal field theory in 2+1 spacetime dimensions: a CFT3

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\( \lambda_c \)
The diagram illustrates the phase behavior of a system as a function of temperature ($T$) and a parameter ($\lambda$). The region labeled "Quantum critical" separates the superfluid phase from the insulator phase. The line $T_{KT}$ marks a transition line within the superfluid phase. The critical point $\lambda_c$ demarcates the boundary between the superfluid and insulator phases.
“Boltzmann” theory of Nambu-Goldstone and vortices

Boltzmann theory of particles/holes

Superfluid

Insulator

Quantum critical

$T$, $\lambda_c$, $\lambda$
CFT3 at $T>0$
CFT3 at $T > 0$

Boltzmann theory of particles/holes/vortices does not apply

Superfluid

Insulator

$T_K$

$T$

$\lambda$

$\lambda_c$

$0$
CFT3 at $T>0$

Needed:
Accurate theory of quantum critical dynamics
Electrical transport in a free CFT3 for $T > 0$
Electrical transport for a CFT3, assuming quasiparticles with weak interactions

\[ \sigma(\omega, T) = \frac{e^2}{\hbar} \sum \left( \frac{\hbar \omega}{k_B T} \right) ; \quad \Sigma \to \text{a universal function} \]

\[ \mathcal{O}(1/(u^*)^2) \]

\[ \mathcal{O}((u^*)^2), \]

where \( u^* \) is the fixed point interaction

Quantum critical dynamics

Quantum “nearly perfect fluid” with shortest possible local equilibration time, $\tau_{eq}$

$$\tau_{eq} = C \frac{\hbar}{k_B T}$$

where $C$ is a universal constant.

Response functions are characterized by poles in LHP with $\omega \sim k_B T / \hbar$. These poles (quasi-normal modes) appear naturally in the holographic theory. (Analogs of Higgs quasi-normal mode.)

Quantum critical dynamics

Transport co-efficients not determined by collision rate of quasiparticles, but by fundamental constants of nature

Conductivity

\[ \sigma = \frac{Q^2}{\hbar} \times \text{[Universal constant } O(1) \text{]} \]

(Q is the “charge” of one boson)


Gauge-gravity duality at non-zero temperatures

A 2+1 dimensional system at $T > 0$ with couplings at its quantum critical point

$$S_E = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) \right]$$
A “horizon”, similar to the surface of a black hole!

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$$S_E = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) \right]$$
The temperature and entropy of the horizon equal those of the quantum critical point.

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Gauge-gravity duality at non-zero temperatures.
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A 2+1 dimensional system at $T > 0$ with couplings at its quantum critical point

Quasi-normal modes of quantum criticality = waves falling into black hole

The temperature and entropy of the horizon equal those of the quantum critical point
Characteristic damping time of quasi-normal modes:

\[ \frac{k_B}{\hbar} \times \text{Hawking temperature} \]
**AdS$_4$ theory of quantum criticality**

Most general effective holographic theory for linear charge transport with 4 spatial derivatives:

\[
S_{\text{bulk}} = \frac{1}{g_M^2} \int d^4x \sqrt{g} \left[ \frac{1}{4} F_{ab} F^{ab} + \gamma L^2 C_{abcd} F^{ab} F^{cd} \right] \\
+ \int d^4x \sqrt{g} \left[ -\frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) \right],
\]

This action is characterized by 3 dimensionless parameters, which can be linked to data of the CFT (OPE coefficients): 2-point correlators of the conserved current $J_\mu$ and the stress energy tensor $T_{\mu\nu}$, and a 3-point $T, J, J$ correlator.

AdS$_4$ theory of quantum criticality

AdS$_4$ theory of quantum criticality

- Stability constraints on the effective theory ($|\gamma| < 1/12$) allow only a limited $\omega$-dependence in the conductivity. This contrasts with the Boltzmann theory in which $\sigma(\omega)/\sigma(\infty)$ becomes very large in the regime of its validity.

Universal Scaling of the Conductivity at the Superfluid-Insulator Phase Transition

Jurij Šmakov and Erik Sørensen

Department of Physics and Astronomy, McMaster University, Hamilton, Ontario L8S 4M1, Canada
(Received 30 May 2005; published 27 October 2005)

The scaling of the conductivity at the superfluid-insulator quantum phase transition in two dimensions is studied by numerical simulations of the Bose-Hubbard model. In contrast to previous studies, we focus on properties of this model in the experimentally relevant thermodynamic limit at finite temperature $T$. We find clear evidence for deviations from $\omega_k$ scaling of the conductivity towards $\omega_k/T$ scaling at low Matsubara frequencies $\omega_k$. By careful analytic continuation using Padé approximants we show that this behavior carries over to the real frequency axis where the conductivity scales with $\omega/T$ at small frequencies and low temperatures. We estimate the universal dc conductivity to be $\sigma^* = 0.45(5)Q^2/h$, distinct from previous estimates in the $T = 0$, $\omega/T \gg 1$ limit.

**QMC yields** $\sigma(0)/\sigma_\infty \approx 1.36$

**Holography yields** $\sigma(0)/\sigma_\infty = 1 + 4\gamma$ with $|\gamma| \leq 1/12$.

**Maximum possible holographic value** $\sigma(0)/\sigma_\infty = 1.33$

Excellent agreement of $\omega$ dependence between QMC and holography for $\gamma \approx 1/12$.

Traditional CMT

- Identify quasiparticles and their dispersions
- Compute scattering matrix elements of quasiparticles (or of collective modes)
- These parameters are input into a quantum Boltzmann equation
- Deduce dissipative and dynamic properties at non-zero temperatures
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Holography and black-branes

- Start with strongly interacting CFT without particle- or wave-like excitations
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Entanglement but no quasiparticles

1. Superfluid-insulator transition of ultracold atoms in optical lattices:
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2. Metals with antiferromagnetism, and high temperature superconductivity
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Iron pnictides:
a new class of high temperature superconductors
Resistivity \( \sim \rho_0 + AT^\alpha \)

Superconductivity

\[ \text{Resistivity} \sim \rho_0 + AT^\alpha \]

Short-range entanglement in state with Neel (AF) order

\[ \text{BaFe}_2(\text{As}_{1-x}\text{P}_x)_2 \]


Superconductor

Bose condensate of pairs of electrons

Short-range entanglement

Resistivity

\[ \rho \sim \rho_0 + AT^\alpha \]
Superconductivity

$\text{BaFe}_2(\text{As}_{1-x}\text{P}_x)_2$

\[
\text{Resistivity } \sim \rho_0 + AT^{\alpha}
\]


Ordinary metal (Fermi liquid)
Strange Metal

no quasiparticles, Landau-Boltzmann theory does not apply

Resistivity \( \sim \rho_0 + AT^\alpha \)

\[ \text{BaFe}_2(\text{As}_{1-x}\text{P}_x)_2 \]


High temperature superconductors

\[ \text{YBa}_2\text{Cu}_3\text{O}_{6+x} \]
Strange Metal

Temperature [K]

AF insulator

Superconductor

$T_c$

$T^*$

$T_N$

Hole doping, $\rho$

Smaller hole Fermi-pockets

Large hole Fermi surface

K.M. Shen et al., Science 2005

M. Plaât et al., PRL 2005
The electron spin polarization obeys

$$\langle \vec{S}(\mathbf{r}, \tau) \rangle = \varphi(\mathbf{r}, \tau)e^{i\mathbf{K} \cdot \mathbf{r}}$$

where $\mathbf{K}$ is the ordering wavevector.
Fermi surface + antiferromagnetism

Metal with “large” Fermi surface
Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$. 
Fermi surface + antiferromagnetism

“Hot” spots
Fermi surface + antiferromagnetism

Electron and hole pockets in antiferromagnetic phase with antiferromagnetic order parameter $\langle \varphi \rangle \neq 0$
Fermi surface + antiferromagnetism

Metal with electron and hole pockets

Metal with “large” Fermi surface

\[ \langle \varphi \rangle \neq 0 \]

\[ \langle \varphi \rangle = 0 \]
d-wave superconductor: particle-particle pairing at and near hot spots, with sign-changing pairing amplitude

\[ \left\langle c_{k\alpha}^{\dagger} c_{-k\beta}^{\dagger} \right\rangle = \varepsilon_{\alpha\beta} \Delta_S (\cos k_x - \cos k_y) \]

Sign-problem-free Quantum Monte Carlo for antiferromagnetism in metals

$s/d$ pairing amplitudes $P_+/P_-$ as a function of the tuning parameter $r$

Fermi surface+antiferromagnetism

- Fluctuating Fermi pockets
- Quantum Critical
- Large Fermi surface
- Increase SDW order $T^*$
- Underlying SDW ordering quantum critical point in metal at $x = x_m$

Spin density wave (SDW)
QCP for the onset of SDW order is actually within a superconductor
Strange Metal

no quasiparticles, Landau-Boltzmann theory does not apply

Resistivity \( \sim \rho_0 + AT^\alpha \)


What about the pseudogap?
There is an approximate pseudospin symmetry in metals with antiferromagnetic spin correlations.

The pseudospin partner of $d$-wave superconductivity is an incommensurate $d$-wave bond order.

These orders form a pseudospin doublet, which is responsible for the “pseudogap” phase.

S. Sachdev and R. La Placa, arXiv:1303.2114
Pseudospin symmetry of the exchange interaction

\[ H_J = \sum_{i<j} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

with \( \vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \sigma_{\alpha\beta} c_{i\beta} \) is the antiferromagnetic exchange interaction.

Introduce the Nambu spinor

\[
\Psi_{i\uparrow} = \begin{pmatrix} c_{i\uparrow} \\ c_{i\downarrow} \end{pmatrix}, \quad \Psi_{i\downarrow} = \begin{pmatrix} c_{i\downarrow} \\ -c_{i\uparrow}^\dagger \end{pmatrix}
\]

Then we can write

\[
H_J = \frac{1}{8} \sum_{i<j} J_{ij} \left( \Psi_{i\alpha}^\dagger \sigma_{\alpha\beta} \Psi_{i\beta} \right) \cdot \left( \Psi_{j\gamma}^\dagger \sigma_{\gamma\delta} \Psi_{i\delta} \right)
\]

which is invariant under independent SU(2) pseudospin transformations on each site

\[ \Psi_{i\alpha} \rightarrow U_i \Psi_{i\alpha} \]

This pseudospin (gauge) symmetry is important in classifying spin liquid ground states of \( H_J \).

References:
Pseudospin symmetry of the exchange interaction

\[ H_{tJ} = - \sum_{i,j} t_{ij} c_{i\uparrow}^\dagger c_{j\alpha} + \sum_{i<j} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

with \( \vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta} \) is the antiferromagnetic exchange interaction. Introduce the Nambu spinor

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\Psi_{i\uparrow} = \begin{pmatrix} c_{i\uparrow} \\ c_{i\downarrow} \end{pmatrix}, \quad \Psi_{i\downarrow} = \begin{pmatrix} c_{i\downarrow}^\dagger \\ -c_{i\uparrow} \end{pmatrix}
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\]

which is invariant under independent SU(2) pseudospin transformations on each site

\[
\Psi_{i\alpha} \rightarrow U_i \Psi_{i\alpha}
\]

This pseudospin (gauge) symmetry is important in classifying spin liquid ground states of \( H_J \). It is fully broken by the electron hopping \( t_{ij} \) but does have remnant consequences in doped spin liquid states.

Pseudospin symmetry of the exchange interaction

\[ H_{tJ} = -\sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \sum_{i<j} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

with \( \vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \sigma_{\alpha\beta} c_{i\beta} \) is the antiferromagnetic exchange interaction. Introduce the Nambu spinor

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Then we can write

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\]

which is invariant under independent SU(2) pseudospin transformations on each site

\[ \Psi_{i\alpha} \rightarrow U_i \Psi_{i\alpha} \]

We will start with the Néel state, and find important consequences of the pseudospin symmetry in metals with antiferromagnetic correlations.

Fermi surface + antiferromagnetism

Metal with electron and hole pockets

\[ \langle \bar{\varphi} \rangle \neq 0 \]

Metal with “large” Fermi surface

\[ \langle \bar{\varphi} \rangle = 0 \]
Fermi surface + antiferromagnetism

Focus on this region

\[ \langle \tilde{\varphi} \rangle \neq 0 \]

Metal with electron and hole pockets

\[ \langle \tilde{\varphi} \rangle = 0 \]

Metal with "large" Fermi surface

\( \gamma \)
Fermi surface + antiferromagnetism

“Hot” spots
Fermi surface + antiferromagnetism

Low energy theory for critical point near hot spots

Thursday, May 9, 13
Fermi surface + antiferromagnetism

Low energy theory for critical point near hot spots
Theory has fermions $\psi_{1,2}$ (with Fermi velocities $v_{1,2}$) and boson order parameter $\bar{\varphi}$, interacting with coupling $\lambda$.

\[ S = \int d^2r d\tau \left[ \psi_{1\alpha}^\dagger \left( \partial_\tau - i \mathbf{v}_1 \cdot \nabla_r \right) \psi_{1\alpha} + \psi_{2\alpha}^\dagger \left( \partial_\tau - i \mathbf{v}_2 \cdot \nabla_r \right) \psi_{2\alpha} \right. \\
+ \frac{1}{2} \left( \nabla_r \phi \right)^2 + \frac{s}{2} \phi^2 + \frac{u}{4} \phi^4 - \lambda \phi \cdot \left( \psi_{1\alpha}^\dagger \bar{\sigma}_{\alpha\beta} \psi_{2\beta} + \psi_{2\alpha}^\dagger \bar{\sigma}_{\alpha\beta} \psi_{1\beta} \right) \]
This low-energy theory is invariant under independent SU(2) pseudospin rotations on each pair of hot-spots: there is a global $SU(2) \times SU(2) \times SU(2) \times SU(2)$ pseudospin symmetry.
\[ \langle c_{k\alpha}^\dagger c_{-k\beta}^\dagger \rangle = \varepsilon_{\alpha\beta} \Delta_S (\cos k_x - \cos k_y) \]

d-wave superconductor: particle-particle pairing at \textit{and near} hot spots, with sign-changing pairing amplitude

Incommensurate d-wave bond order: particle-hole pairing at \textit{and near} hot spots, with sign-changing pairing amplitude

\[
\left\langle c_{k-Q/2,\alpha}^\dagger c_{k+Q/2,\alpha} \right\rangle = \Delta_Q (\cos k_x - \cos k_y)
\]
Incommensurate $d$-wave bond order

Consider modulation in an off-site “density” like variable at sites $r_i$ and $r_j$

$$
\langle c_{i\alpha}^\dagger c_{j\alpha} \rangle \sim \left[ \sum_k \Delta_Q(k) e^{i k \cdot (r_i - r_j)} \right] e^{i Q \cdot (r_i + r_j)/2}
$$

The wavevector $Q$ is associated with a modulation in the average co-ordinate $(r_i - r_j)/2$: this determines the wavevector of the neutron/X-ray scattering peak.

The interesting part is the dependence on the relative co-ordinate $r_i - r_j$. Assuming time-reversal, the order parameter $\Delta_Q(k)$ can always be expanded as

$$
\Delta_Q(k) = c_s + c_{s'} (\cos k_x + \cos k_y) + c_d (\cos k_x - \cos k_y) + \ldots
$$

The usual charge-density-wave has only $c_s \neq 0$.

The bond-ordered state we find has

$$
|c_d| \gg c_s, c_{s'}, \ldots
$$
Incommensurate $d$-wave bond order

\[
\langle c^\dagger_{r\alpha} c_{s\alpha} \rangle = \sum_Q \sum_k e^{iQ \cdot (r+s)/2} e^{-i k \cdot (r-s)} \langle c^\dagger_{k-Q/2,\alpha} c_{k+Q/2,\alpha} \rangle
\]

where $Q$ extends over $Q = (\pm Q_0, \pm Q_0)$ with $Q_0 = 2\pi/(7.3)$ and

\[
\langle c^\dagger_{k-Q/2,\alpha} c_{k+Q/2,\alpha} \rangle = \Delta_Q (\cos k_x - \cos k_y)
\]

Note $\langle c^\dagger_{r\alpha} c_{s\alpha} \rangle$ is non-zero only when $r, s$ are nearest neighbors.
Incommensurate $d$-wave bond order

M.A. Metlitski and S. Sachdev, 

\[
\left\langle c_{k-Q/2,\alpha}^{\dagger} c_{k+Q/2,\alpha} \right\rangle = \Delta_Q (\cos k_x - \cos k_y)
\]
Incommensurate $d$-wave bond order

High $T$ pseudogap: Fluctuating composite order parameter of nearly degenerate $d$-wave pairing and incommensurate $d$-wave bond order. (Approximate) SU(2) symmetry of composite order prevents long-range order $T > 0$.


$$\left\langle c^\dagger_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Delta_{\mathbf{Q}} (\cos k_x - \cos k_y)$$
Incommensurate $d$-wave bond order

Our computations show that the charge order is predominantly $d$-wave also at this $Q$. This $Q$ is preferred in computations of bond order within the superconducting phase.

S. Sachdev and R. La Placa, arXiv:1303.2114
A Four Unit Cell Periodic Pattern of Quasi-Particle States Surrounding Vortex Cores in Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$

J. E. Hoffman,¹ E. W. Hudson,¹,²* K. M. Lang,¹ V. Madhavan,¹ H. Eisaki,³† S. Uchida,³ J. C. Davis¹,²‡

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Moreover, the energy dependence of the CDW order in the planes is shown to result from a spatial modulation of energies of the Cu 2p to 3d_{x^2-y^2} transition, similar to stripe-ordered 214 cuprates.

These energy shifts are interpreted as a spatial modulation of the electronic structure and may point to a valence-bond-solid interpretation of the stripe phase.
Direct observation of competition between superconductivity and charge density wave order in YBa$_2$Cu$_3$O$_{6.67}$

J. Chang$^{1,2,*}$, E. Blackburn$^3$, A. T. Holmes$^3$, N. B. Christensen$^4$, J. Larsen$^{4,5}$, J. Mesot$^{1,2}$, Ruixing Liang$^{6,7}$, D. A. Bonn$^{6,7}$, W. N. Hardy$^{6,7}$, A. Watenphul$^8$, M. v. Zimmermann$^8$, E. M. Forgan$^3$ and S. M. Hayden$^9$

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**Figure 1:**

- **a:** Temperature ($T$) vs. doping ($p$) plot for YBCO with transitions marked: $T_c$, $T_{CDW}$, $T_H$, $T^{*}$, $T_{N}$. The shaded regions indicate different phases: AF (antiferromagnetic), SC (superconducting).

- **b:** Intensity ($I$) vs. $h$ (in hexagonal reciprocal lattice units) for $T = 2K$, 17T, and 0T.

- **c:** Intensity ($I$) vs. $h$ (in hexagonal reciprocal lattice units) for $T = 150K$, 17T, and 0T.
Magnetic-field-induced charge-stripe order in the high-temperature superconductor YBa$_2$Cu$_3$O$_y$

Tao Wu$^1$, Hadrien Mayaffre$^1$, Steffen Krämer$^1$, Mladen Horvatić$^1$, Claude Berthier$^1$, W. N. Hardy$^{2,3}$, Ruixing Liang$^{2,3}$, D. A. Bonn$^{2,3}$ & Marc-Henri Julien$^1$

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Summary

Conformal quantum matter

- New insights and solvable models for diffusion and transport of strongly interacting systems near quantum critical points using the methods of gauge-gravity duality.

- The description is far removed from, and complementary to, that of the quantum Boltzmann equation which builds on the quasiparticle/vortex picture.

- Good prospects for experimental tests of frequency-dependent, non-linear, and non-equilibrium transport
Antiferromagnetic quantum criticality leads to d-wave superconductivity (supported by sign-problem-free Monte Carlo simulations)

Metals with antiferromagnetic spin correlations have nearly degenerate instabilities: to d-wave superconductivity, and to a charge density wave with a d-wave form factor. This is a promising explanation of the pseudogap regime.