Entanglement, holography, and strange metals

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Lecture at the 100th anniversary Solvay conference, Theory of the Quantum World, chair D.J. Gross.
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Modern phases of quantum matter
Not adiabatically connected
to independent electron states:

*many-particle*

*quantum entanglement*
“Complex entangled” states of quantum matter in $d$ spatial dimensions

Useful classification is provided by nature of excitations with vanishing energy:

1. Gapped systems without zero energy excitations

2. “Relativistic” systems with zero energy excitations at isolated points in momentum space

3. “Compressible” systems with zero energy excitations on $d-1$ dimensional surfaces in momentum space.
“Complex entangled” states of quantum matter in $d$ spatial dimensions

Gapped quantum matter

*Spin liquids, quantum Hall states*

Conformal quantum matter

*Graphene, ultracold atoms, antiferromagnets*

Compressible quantum matter

*Graphene, strange metals in high temperature superconductors, spin liquids*
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   Graphene, strange metals in high temperature superconductors, spin liquids
“Complex entangled” states of quantum matter in $d$ spatial dimensions

Gapped quantum matter
- Spin liquids, quantum Hall states
- Topological field theory

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- [topological field theory]
- [conformal field theory]
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An even number of electrons per unit cell
Mott insulator

Emergent excitations

An odd number of electrons per unit cell but electrons are localized by Coulomb repulsion; state has long-range entanglement
Mott insulator: Kagome antiferromagnet

\[ H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \]
Mott insulator: Kagome antiferromagnet

\[ H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \]

\[ \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]

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Quantum “disordered” state with exponentially decaying spin correlations.

non-collinear Néel state

Mott insulator: kagome antiferromagnet

Quantum “disordered” state with exponentially decaying spin correlations.

Spin liquid with topological features described by a $\mathbb{Z}_2$ gauge theory, or (equivalently) a doubled Chern-Simons field theory.

Entanglement in the $\mathbb{Z}_2$ spin liquid ground state

$|\Psi\rangle \Rightarrow$ Ground state of entire system,

$\rho = |\Psi\rangle \langle \Psi |$

$\rho_A = \text{Tr}_B \rho = \text{density matrix of region } A$

Entanglement entropy $S_{EE} = -\text{Tr} (\rho_A \ln \rho_A)$
Entanglement entropy of a band insulator:

\[ S_E = aP - b \exp(-cP) \]

where \( P \) is the surface area (perimeter) of the boundary between A and B.
Entanglement entropy of a $\mathbb{Z}_2$ spin liquid ground state:

$$S_E = aP - \ln(2)$$

where $P$ is the surface area (perimeter) of the boundary between A and B.

Entanglement entropy of a $\mathbb{Z}_2$ spin liquid:

$$S_E = aP - \ln(4)$$

where $P$ is the surface area (perimeter) of the boundary between A and B.

Entanglement entropy of a $\mathbb{Z}_2$ spin liquid ground state

\begin{equation}
S_E = aP - \ln(2)
\end{equation}

where $P$ is the surface area (perimeter) of the boundary between A and B.

Kagome antiferromagnet

Hong-Chen Jiang, Z. Wang, and L. Balents, arXiv:1205.4289

Kagome antiferromagnet: evidence for spinons

Young Lee,
APS meeting, March 2012

ZnCu$_3$(OH)$_6$Cl$_2$ (also called Herbertsmithite)
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Spinning electrons localized on a square lattice

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Examine ground state as a function of \( \lambda \)
\[ \lambda = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]
Quantum critical point described by a CFT3 (O(3) Wilson-Fisher)

Quantum critical point in a frustrated square lattice antiferromagnet

Valence bond solid (VBS) state with a nearly gapless, emergent “photon”

Critical theory for photons and deconfined spinons:

\[ S_z = \int d^2r d\tau \left[ |(\partial_\mu - iA_\mu)z_\alpha|^2 + s|z_\alpha|^2 + u(|z_\alpha|^2)^2 + \frac{1}{2e_0^2} (\epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda)^2 \right] \]


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Entanglement at the quantum critical point

- Entanglement entropy obeys $S_E = aL - \gamma$, where $\gamma$ is a shape-dependent universal number associated with the CFT3.

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• When $A$ is a circle, $e^{-\gamma} = $ partition function of CFT3 on $S^3$.

Key idea: \( \Rightarrow \) Implement \( r \) as an extra dimension, and map to a local theory in \( d + 2 \) spacetime dimensions.
For a relativistic CFT in $d$ spatial dimensions, the metric in the holographic space is uniquely fixed by demanding the following scale transformation ($i = 1 \ldots d$)

\[ x_i \rightarrow \zeta x_i \ , \quad t \rightarrow \zeta t \ , \quad ds \rightarrow ds \]
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$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad ds \to ds$$

This gives the unique metric

$$ds^2 = \frac{1}{r^2} (-dt^2 + dr^2 + dx_i^2)$$

Reparametrization invariance in $r$ has been used to the prefactor of $dx_i^2$ equal to $1/r^2$. This fixes $r \to \zeta r$ under the scale transformation. This is the metric of the space $\text{AdS}_{d+2}$. 
AdS/CFT correspondence

$\text{AdS}_4$

$R^{2,1}$

Minkowski

$CFT_3$

$r$
AdS/CFT correspondence

AdS$_4$ \quad $R^{2,1}$ \quad Minkowski

$\ldots$ \quad A

$r$ \quad CFT3
AdS/CFT correspondence

Minimal surface area measures entanglement entropy

- Minimal surface area yields $S_E = aL - \gamma$, where $\gamma$ is a shape-dependent universal number.

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Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge $Q$ (the “electron density”) in spatial dimension $d > 1$.

Describe zero temperature phases where $d\langle Q\rangle/d\mu \neq 0$, where $\mu$ (the “chemical potential”) which changes the Hamiltonian, $H$, to $H - \mu Q$. 

Compressible quantum matter

Compressible systems must be gapless.

Conformal quantum matter is compressible in $d = 1$, but not for $d > 1$. 

Compressible quantum matter
Conformal quantum matter
Compressible quantum matter
The Fermi liquid

\[ \mathcal{L} = f_\sigma^\dagger \left( \partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f_\sigma + \text{short-range 4-Fermi terms} \]
The Fermi liquid

\[ \mathcal{L} = f^\dagger_\sigma \left( \partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f_\sigma \]

+ short-range 4-Fermi terms

- Area enclosed by the Fermi surface \( A = Q \), the fermion density
The Fermi liquid

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- Area enclosed by the Fermi surface \( \mathcal{A} = \mathcal{Q} \), the fermion density
- Particle and hole of excitations near the Fermi surface with energy \( \omega \sim |q| \).
The Fermi liquid

\[ \mathcal{L} = f_\sigma^\dagger \left( \partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f_\sigma \]

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- Fermion Green’s function \( G_f^{-1} = \omega - v_F q + i\mathcal{O}(\omega^2, q^2) \).
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- Fermion Green's function \( G_f^{-1} = \omega - v_F q + i \mathcal{O}(\omega^2, q^2) \).

- The phase space density of fermions is effectively one-dimensional, so the entropy density \( S \sim T^{d_{\text{eff}}} \) with \( d_{\text{eff}} = 1 \).
Logarithmic violation of “area law”: \[ S_{EE} = \frac{1}{12} (k_F P) \ln(k_F P) \]

for a circular Fermi surface with Fermi momentum \( k_F \), where \( P \) is the perimeter of region A with an arbitrary smooth shape.


Non-Fermi liquids

To obtain a compressible state which is not a Fermi liquid, take a Fermi surface in $d = 2$, and couple it to any gapless scalar field, $\phi$, which has low energy excitations near $q = 0$. 

- ferromagnetic order
- breaking of point-group symmetry (Ising-nematic order)
- breaking of time-reversal symmetry
- circulating currents
- transverse component of an Abelian or non-Abelian gauge field.
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- ...
Non-Fermi liquids

- $\phi$ fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm \vec{k}_0$. 
Non-Fermi liquids

- $\phi$ fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm \vec{k}_0$.

- Expand fermion kinetic energy at wavevectors about $\pm \vec{k}_0$ and boson ($\phi$) kinetic energy about $\vec{q} = 0$. 

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Non-Fermi liquids

\[ \mathcal{L}[\psi_{\pm}, \phi] = \]

\[ \psi_+^\dagger \left( \partial_\tau - i \partial_x - \partial_y^2 \right) \psi_+ + \psi_-^\dagger \left( \partial_\tau + i \partial_x - \partial_y^2 \right) \psi_- \]

\[ -g\phi \left( \psi_+^\dagger \psi_+ \pm \psi_-^\dagger \psi_- \right) + \left( \partial_y \phi \right)^2 \]

Non-Fermi liquids

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Non-Fermi liquids

- Area enclosed by the Fermi surface $A = Q$, the fermion density

- Critical continuum of excitations near the Fermi surface with energy $\omega \sim |q|^z$, where $q = |k| - k_F$ is the distance from the Fermi surface and $z$ is the dynamic critical exponent.

Non-Fermi liquids

• Gauge-dependent Green’s function $G_f^{-1} = q^{1-\eta} F(\omega/q^z)$. Three-loop computation shows $\eta \neq 0$ and $z = 3/2$.

Non-Fermi liquids

\[ \mathcal{A} \rightarrow |q| \leftarrow \]

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\begin{align*}
\text{S.-S. Lee, Phys. Rev. B} & \text{ 80, 165102 (2009)} \\
\text{M. A. Metlitski and S. Sachdev, Phys. Rev. B} & \text{ 82, 075127 (2010)} \\
\end{align*}
Non-Fermi liquids

Simple scaling argument for $z = 3/2$.

\[
\mathcal{L} = \psi_+^\dagger \left( \partial_\tau - i \partial_x - \partial_y^2 \right) \psi_+ + \psi_-^\dagger \left( \partial_\tau + i \partial_x - \partial_y^2 \right) \psi_-
- g\phi \left( \psi_+^\dagger \psi_+ \pm \psi_-^\dagger \psi_- \right) + (\partial_y \phi)^2
\]
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$$- g\phi \left( \psi_+^\dagger \psi_+ \pm \psi_-^\dagger \psi_- \right) + (\partial_y \phi)^2$$

Under the rescaling $x \to x/s$, $y \to y/s^{1/2}$, and $\tau \to \tau/s^z$, we find invariance provided

$$\phi \to \phi s^{(2z+1)/4}$$

$$\psi \to \psi s^{(2z+1)/4}$$

$$g \to g s^{(3-2z)/4}$$

So the action is invariant provided $z = 3/2$. 
Logarithmic violation of “area law”: \( S_{EE} = \frac{1}{12} (k_F P) \ln(k_F P) \)

for a circular Fermi surface with Fermi momentum \( k_F \), where \( P \) is the perimeter of region A with an arbitrary smooth shape.

Non-Fermi liquids have, at most, the “1/12” prefactor modified.

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Computations in the $1/N$ expansion

All planar graphs of $\psi_+$ alone are as important as the leading term

Graph mixing $\psi_+$ and $\psi_-$ is $O(N^{3/2})$ (instead of $O(N)$), violating genus expansion


Consider the metric which transforms under rescaling as

\[ x_i \rightarrow \zeta x_i \]
\[ t \rightarrow \zeta^z t \]
\[ ds \rightarrow \zeta^{\theta/d} ds. \]

This identifies \( z \) as the dynamic critical exponent (\( z = 1 \) for “relativistic” quantum critical points).

\( \theta \) is the violation of hyperscaling exponent.
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The most general choice of such a metric is

\[
    ds^2 = \frac{1}{r^2} \left( -\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)
\]

We have used reparametrization invariance in \( r \) to choose so that it scales as \( r \rightarrow \zeta^{(d-\theta)/d} r \).
At $T > 0$, there is a “black-brane” at $r = r_h$.

The Beckenstein-Hawking entropy of the black-brane is the thermal entropy of the quantum system $r = 0$.

The entropy density, $S$, is proportional to the “area” of the horizon, and so $S \sim r_h^{-d}$.
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Under rescaling $r \rightarrow \zeta^{(d-\theta)/d} r$, and the temperature $T \sim t^{-1}$, and so

$$S \sim T^{(d-\theta)/z} = T^{d_{\text{eff}}/z}$$

where $\theta = d - d_{\text{eff}}$ measures “dimension deficit” in the phase space of low energy degrees of a freedom.
The thermal entropy density scales as

\[ S \sim T^{(d-\theta)/z}. \]

The third law of thermodynamics requires \( \theta < d \).
Area of minimal surface equals entanglement entropy.
The thermal entropy density scales as

$$S \sim T^{(d-\theta)/z}.$$  

The third law of thermodynamics requires $\theta < d$.

The entanglement entropy, $S_E$, of an entangling region with boundary surface ‘area’ $P$ scales as

$$S_E \sim \begin{cases} P, & \text{for } \theta < d - 1 \\ P \ln P, & \text{for } \theta = d - 1 \\ P^{\theta/(d-1)}, & \text{for } \theta > d - 1 \end{cases}$$

All local quantum field theories obey the “area law” (upto log violations) and so $\theta \leq d - 1$.  

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  All local quantum field theories obey the “area law” (upto log violations) and so \( \theta \leq d - 1 \).

- The null energy condition implies \( z \geq 1 + \frac{\theta}{d} \).
\[ ds^2 = \frac{1}{r^2} \left( -\frac{dt^2}{r^2 d(z-1)/(d-\theta)} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right) \]

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- The null energy condition implies \( z \geq 1 + \frac{\theta}{d} \).
The value of $\theta$ is fixed by requiring that the thermal entropy density $S \sim T^{1/z}$ for general $d$. Conjecture: this metric then describes a compressible state with a hidden Fermi surface of quarks coupled to gauge fields.

\[ ds^2 = \frac{1}{r^2} \left( -\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right) \]

$\theta = d - 1$
The value of $\theta$ is fixed by requiring that the thermal entropy density $S \sim T^{1/z}$ for general $d$. Conjecture: this metric then describes a compressible state with a hidden Fermi surface of quarks coupled to gauge fields.

The null energy condition yields the inequality $z \geq 1 + \theta/d$. For $d = 2$ and $\theta = 1$ this yields $z \geq 3/2$. The field theory analysis gave $z = 3/2$ to three loops!

\[ ds^2 = \frac{1}{r^2} \left( -\frac{dt^2}{r^{2d/(d-1)}} + \frac{r^{2\theta/(d-\theta)}}{d-\theta} dr^2 + dx_i^2 \right) \]

$\theta = d - 1$

---

The entanglement entropy exhibits logarithmic violation of the area law only for this value of $\theta$!!

The logarithmic violation is of the form $P \ln P$, where $P$ is the perimeter of the entangling region. This form is independent of the shape of the entangling region, just as is expected for a (hidden) Fermi surface!!

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Holographic theory of a non-Fermi liquid (NFL)

Add a relevant “dilaton” field

\[ \mathcal{S} = \int d^{d+2}x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( R - 2(\nabla \Phi)^2 - \frac{V(\Phi)}{L^2} \right) - \frac{Z(\Phi)}{4e^2} F_{ab}F^{ab} \right] \]

with \( Z(\Phi) = Z_0 e^{\alpha \Phi} \), \( V(\Phi) = -V_0 e^{-\beta \Phi} \), as \( \Phi \to \infty \).


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\[ \mathcal{E}_r = \langle Q \rangle \]

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with \( Z(\Phi) = Z_0 e^{\alpha \Phi} \), \( V(\Phi) = -V_0 e^{-\beta \Phi} \), as \( \Phi \to \infty \).

This is a “bosonization” of the Fermi surface.
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\[ \mathcal{E}_r = \langle Q \rangle \]

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Leads to metric

\[ ds^2 = L^2 \left( -f(r)dt^2 + g(r)dr^2 + \frac{dx^2 + dy^2}{r^2} \right) \]

with \( f(r) \sim r^{-\gamma} \), \( g(r) \sim r^{\delta} \), \( \Phi(r) \sim \ln(r) \) as \( r \to \infty \).

Holography of non-Fermi liquids

\[ ds^2 = \frac{1}{r^2} \left( -\frac{dt^2}{r^2 d(z-1)/(d-\theta)} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right) \]

\[ \theta = d - 1 \]

- The entanglement entropy has log-violation of the area law

\[ S_E = \Xi Q^{(d-1)/d} P \ln P. \]

where \( P \) is surface area of the entangling region, and \( \Xi \) is a dimensionless constant which is independent of all UV details, of \( Q \), and of any property of the entangling region. Note \( Q^{(d-1)/d} \sim k_F^{d-1} \) via the Luttinger relation, and then \( S_E \) is just as expected for a Fermi surface !!!!

Gauss Law and the "attractor" mechanism ⇔ Luttinger theorem on the boundary
Holographic theory of a fractionalized-Fermi liquid (FL*)

A state with partial confinement


These are spectators, and are expected to have well-defined quasiparticle excitations.
Confining geometry leads to a state which has all the properties of a Landau Fermi liquid.

- Visible Fermi surfaces of “mesinos”

S. Sachdev, Physical Review D 84, 066009 (2011)
Conclusions

Compressible quantum matter

Field theory of a non-Fermi liquid obtained by coupling a Fermi surface to a gapless scalar field with low energy excitations near zero wavevector
Evidence for *hidden Fermi surfaces* in compressible states obtained for a class of holographic Einstein-Maxwell-dilaton theories. These theories describe a *non-Fermi liquid* (NFL) state of gauge theories at non-zero density.
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Thursday, June 7, 2012