Deconfined quantum criticality

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Talk online at http://sachdev.physics.harvard.edu
Outline

I. Magnetic quantum phase transitions in “dimerized” Mott insulators:  
   *Landau-Ginzburg-Wilson (LGW) theory*

II. Magnetic quantum phase transitions of Mott insulators on the square lattice  
   A. *Breakdown of LGW theory*  
   B. *Berry phases*  
   C. *Spinor formulation and deconfined criticality*
I. Magnetic quantum phase transitions in “dimerized” Mott insulators:

Landau-Ginzburg-Wilson (LGW) theory:
Second-order phase transitions described by fluctuations of an order parameter associated with a broken symmetry
TlCuCl$_3$

M. Matsumoto, B. Normand, T.M. Rice, and M. Sigrist, cond-mat/0309440.
Coupled Dimer Antiferromagnet


*S*=1/2 spins on coupled dimers

\[
H = \sum_{\langle ij \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j
\]

\[
0 \leq \lambda \leq 1
\]
$\lambda$ close to 0  Weakly coupled dimers
$\lambda$ close to 0

Weakly coupled dimers

$\left( \uparrow \downarrow \right) - \left| \downarrow \uparrow \right\rangle$

Paramagnetic ground state

$\left\langle \vec{S}_i \right\rangle = 0$
$\lambda$ close to 0

Weakly coupled dimers

$\begin{align*}
\downarrow\uparrow - \uparrow\downarrow &= 2 \\
\end{align*}$

Excitation:

$S=1$ quasiparticle

$\frac{1}{\sqrt{2}} \left( \left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$
λ close to 0

Weakly coupled dimers

\[ \frac{1}{\sqrt{2}} (|↑↓⟩ - |↓↑⟩) \]

Excitation:
\( S=1 \) quasiparticle
λ close to 0

Weakly coupled dimers

\[ \begin{pmatrix} \uparrow \downarrow - \downarrow \uparrow \end{pmatrix} = \frac{1}{\sqrt{2}} \left( \begin{pmatrix} \uparrow \downarrow \end{pmatrix} - \begin{pmatrix} \downarrow \uparrow \end{pmatrix} \right) \]

Excitation:

S=1 quasiparticle
\( \lambda \) close to 0

Weakly coupled dimers

\[ \downarrow \uparrow - \uparrow \downarrow = 2 \]

Excitation:
\[ S=1 \text{ quasiparticle} \]

\[ \rho = \frac{1}{\sqrt{2}} (|\uparrow \downarrow\rangle - |\downarrow \uparrow\rangle) \]
\[ \lambda \text{ close to 0} \]

Weakly coupled dimers

\[ \downarrow \uparrow - \uparrow \downarrow = 2 \]

Excitation:

\[ S=1 \text{ quasiparticle} \]

\[ = \frac{1}{\sqrt{2}} \left( |\uparrow \downarrow \rangle - |\downarrow \uparrow \rangle \right) \]
\[ \lambda \text{ close to 0} \]

Weakly coupled dimers

Energy dispersion away from antiferromagnetic wavevector

\[ \varepsilon_p = \Delta + \frac{c_x^2 p_x^2 + c_y^2 p_y^2}{2\Delta} \]

\[ \Delta \rightarrow \text{spin gap} \]

Excitation: \( S=1 \) quasiparticle

\[ = \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) \]
TlCuCl$_3$


FIG. 1. Measured neutron profiles in the $a^*c^*$ plane of TlCuCl$_3$ for $i=(1.35,0,0)$, $ii=(0,0,3.15)$ [r.l.u]. The spectrum at $T=1.5$ K.
Coupled Dimer Antiferromagnet
$\lambda$ close to 1  
Weakly dimerized square lattice
Weakly dimerized square lattice

Excitations:
2 spin waves (magnons)

$$\epsilon_p = \sqrt{c_x^2 p_x^2 + c_y^2 p_y^2}$$

Ground state has long-range spin density wave (Néel) order at wavevector $K = (\pi, \pi)$

spin density wave order parameter: $\bar{\phi} = \eta_i \frac{\vec{S}_i}{S}$; $\eta_i = \pm 1$ on two sublattices
TlCuCl$_3$

Neutron Diffraction Study of the Pressure-Induced Magnetic Ordering in the Spin Gap System TlCuCl$_3$

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Fig. 3. Temperature dependence of the magnetic Bragg peak intensity for $Q = (1,0,-3)$ reflection measured at $P = 1.48$ GPa in TlCuCl$_3$. 

\[ \lambda_c = 0.52337(3) \]
M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, 

T=0

\[ \langle \tilde{\phi} \rangle \neq 0 \]
Néel state

\[ \langle \tilde{\phi} \rangle = 0 \]
Quantum paramagnet

Pressure in TlCuCl\(_3\)

**LGW theory for quantum criticality**

Landau-Ginzburg-Wilson theory: write down an effective action for the antiferromagnetic order parameter $\bar{\phi}$ by expanding in powers of $\bar{\phi}$ and its spatial and temporal derivatives, while preserving all symmetries of the microscopic Hamiltonian.

$$S_{\phi} = \int d^2 x d\tau \left[ \frac{1}{2} \left( (\nabla_x \bar{\phi})^2 + c^2 (\partial_\tau \bar{\phi})^2 + (\lambda_c - \lambda) \bar{\phi}^2 \right) + \frac{u}{4!} (\bar{\phi}^2)^2 \right]$$

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For $\lambda < \lambda_c$, oscillations of $\bar{\phi}$ about $\bar{\phi} = 0$ constitute the triplon excitation.

I. Magnetic quantum phase transitions in “dimerized” Mott insulators: 
   \textit{Landau-Ginzburg-Wilson (LGW) theory}

II. Magnetic quantum phase transitions of Mott insulators on the square lattice
   \begin{enumerate}
   \item \textit{Breakdown of LGW theory}
   \item \textit{Berry phases}
   \item \textit{Spinor formulation and deconfined criticality}
   \end{enumerate}
II. Magnetic quantum phase transitions of Mott insulators on the square lattice:

A. Breakdown of LGW theory
Ground state has long-range Néel order

Order parameter \( \tilde{\phi} = \eta_i \tilde{S}_i \)

\( \eta_i = \pm 1 \) on two sublattices

\[ \langle \tilde{\phi} \rangle \neq 0 \]
Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \quad ; \quad \vec{S}_i \Rightarrow \text{spin operator with } S=1/2 \]

Destroy Neel order by perturbations which preserve full square lattice symmetry e.g. second-neighbor or ring exchange.

What is the state with \( \langle \phi \rangle = 0 \) ?
Square lattice antiferromagnet

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$$S_\varphi = \int d^2x d\tau \left[ \frac{1}{2} \left( (\nabla_x \bar{\phi})^2 + c^2 (\partial_\tau \bar{\phi})^2 + r \bar{\phi}^2 \right) + \frac{u}{4!} (\bar{\phi}^2)^2 \right]$$

The ground state for $r > 0$ has no broken symmetry and a gapped $S=1$ quasiparticle excitation (oscillations of $\bar{\phi}$ about $\bar{\phi} = 0$)
Problem: there is no state with a gapped, stable $S=1$ quasiparticle and no broken symmetries
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“Liquid” of valence bonds has fractionalized $S=1/2$ excitations.
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“Liquid” of valence bonds has fractionalized $S=1/2$ excitations
Another possible state, with $\langle \phi \rangle = 0$, is the valence bond solid (VBS)
Another possible state, with $\langle \tilde{\phi} \rangle = 0$, is the valence bond solid (VBS)

Such a state breaks the symmetry of rotations by $n\pi / 2$ about lattice sites, and has $\langle \Psi_{\text{vbs}} \rangle \neq 0$, where $\Psi_{\text{vbs}}$ is the VBS order parameter.

$$\Psi_{\text{vbs}}(i) = \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j e^{i \arctan(r_j - r_i)}$$
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$$
\Psi_{\text{vbs}} (i) = \sum_{\langle ij \rangle} \tilde{S}_i \cdot \tilde{S}_j e^{i \arctan(r_j - r_i)}
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\[
\Psi = \sum_{G} \Psi_{vbs} \phi_{G} \phi_{vbs}(j)
\]

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$$\Psi_{vbs} (i) = \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j e^{i \arctan(r_j - r_i)}$$
The VBS state does have a stable $S=1$ quasiparticle excitation

$$\langle \Psi_{vbs} \rangle \neq 0, \quad \langle \phi \rangle = 0$$
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$$\langle \Psi_{vbs} \rangle \neq 0, \quad \langle \phi \rangle = 0$$
LGW theory of multiple order parameters

\[ F = F_{\text{vbs}} \left[ \Psi_{\text{vbs}} \right] + F_{\phi} \left[ \phi \right] + F_{\text{int}} \]

\[ F_{\text{vbs}} \left[ \Psi_{\text{vbs}} \right] = r_1 \left| \Psi_{\text{vbs}} \right|^2 + u_1 \left| \Psi_{\text{vbs}} \right|^4 + \cdots \]

\[ F_{\phi} \left[ \phi \right] = r_2 \left| \phi \right|^2 + u_2 \left| \phi \right|^4 + \cdots \]

\[ F_{\text{int}} = \nu \left| \Psi_{\text{vbs}} \right|^2 \left| \phi \right|^2 + \cdots \]

Distinct symmetries of order parameters permit couplings only between their energy densities
LGW theory of multiple order parameters

First order transition

Neel order \[ \langle \tilde{\phi} \rangle \]

VBS order \[ \langle \Psi_{vbs} \rangle \]

Coexistence

"disordered"

Neel order \[ \langle \tilde{\phi} \rangle \]

VBS order \[ \langle \Psi_{vbs} \rangle \]
LGW theory of multiple order parameters

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Quantum theory for destruction of Neel order

Ingredient missing from LGW theory:
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$e^{iSA}$
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Discretize imaginary time: path integral is over fields on the sites of a cubic lattice of points $a$
Quantum theory for destruction of Neel order

Discretize imaginary time: path integral is over fields on the sites of a cubic lattice of points \( a \)

Recall \( \vec{\phi}_a = 2\eta_a \vec{S}_a \rightarrow \vec{\phi} = (0,0,1) \) in classical Neel state;

\( \eta_a \rightarrow \pm 1 \) on two square sublattices;

\[
(\mu = x, y, \tau)
\]
Quantum theory for destruction of Neel order

Discretize imaginary time: path integral is over fields on the sites of a cubic lattice of points \( a \)

Recall \( \phi_a = 2 \eta_a \vec{S}_a \rightarrow \phi_a = (0,0,1) \) in classical Neel state;

\( \eta_a \rightarrow \pm 1 \) on two square sublattices ;

\( A_{a\mu} \rightarrow \text{half} \) oriented area of spherical triangle

formed by \( \phi_a, \phi_{a+\mu}, \) and an arbitrary reference point \( \phi_0 \)
Quantum theory for destruction of Neel order

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$A_{a\mu} \rightarrow \text{half oriented area of spherical triangle}$ formed by $\vec{\phi}_a$, $\vec{\phi}_{a+\mu}$, and an arbitrary reference point $\vec{\phi}_0$

\[2A_{a\mu} \rightarrow 2A_{a\mu} - \gamma_{a+\mu} + \gamma_a\]

Change in choice of $\vec{\phi}_0$ is like a “gauge transformation”
Quantum theory for destruction of Neel order

Discretize imaginary time: path integral is over fields on the sites of a cubic lattice of points \( a \)

Recall \( \vec{\phi}_a = 2\eta_a \vec{S}_a \rightarrow \vec{\phi}_a = (0,0,1) \) in classical Neel state;
\( \eta_a \rightarrow \pm 1 \) on two square sublattices;
\( A_{a\mu} \rightarrow \text{half} \) oriented area of spherical triangle formed by \( \vec{\phi}_a, \vec{\phi}_{a+\mu}, \) and an arbitrary reference point \( \vec{\phi}_0 \)

\[
2A_{a\mu} \rightarrow 2A_{a\mu} - \gamma_{a+\mu} + \gamma_a
\]

Change in choice of \( \vec{\phi}_0 \) is like a “gauge transformation”

The area of the triangle is uncertain modulo \( 4\pi \), and the action has to be invariant under \( A_{a\mu} \rightarrow A_{a\mu} + 2\pi \)
Quantum theory for destruction of Neel order

Ingredient missing from LGW theory: Spin Berry Phases

\[ \exp \left( i \sum_a \eta_a A_{a \tau} \right) \]

Sum of Berry phases of all spins on the square lattice.
Quantum theory for destruction of Neel order

Partition function on cubic lattice

\[ Z = \prod_a \int d\vec{\phi}_a \delta (\vec{\phi}_a^2 - 1) \exp \left( \frac{1}{g} \sum_{a,\mu} \vec{\phi}_a \cdot \vec{\phi}_{a+\mu} \right) \]

LGW theory: weights in partition function are those of a classical ferromagnet at a “temperature” \( g \)

Small \( g \) \( \Rightarrow \) ground state has Neel order with \( \langle \vec{\phi} \rangle \neq 0 \)

Large \( g \) \( \Rightarrow \) paramagnetic ground state with \( \langle \vec{\phi} \rangle = 0 \)
Quantum theory for destruction of Neel order

Partition function on cubic lattice

\[ Z = \prod_a \int d\tilde{\phi}_a \delta(\tilde{\phi}^2_a - 1) \exp \left( \frac{1}{g} \sum_{a,\mu} \tilde{\phi}_a \cdot \tilde{\phi}_{a+\mu} + i \sum_a \eta_a A_{a\tau} \right) \]

Modulus of weights in partition function: those of a classical ferromagnet at a “temperature” \( g \)

Small \( g \Rightarrow \) ground state has Neel order with \( \langle \tilde{\phi} \rangle \neq 0 \)

Large \( g \Rightarrow \) paramagnetic ground state with \( \langle \tilde{\phi} \rangle = 0 \)

Berry phases lead to large cancellations between different time histories \( \Rightarrow \) need an effective action for \( A_{a\mu} \) at large \( g \)

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C. Spinor formulation and deconfined criticality
Quantum theory for destruction of Neel order

Partition function on cubic lattice

\[ Z = \prod_a \int d\Phi_a \delta(\Phi_a^2 - 1) \exp \left( \frac{1}{g} \sum_{a,\mu} \Phi_a \cdot \Phi_{a+\mu} + i \sum_a \eta_a A_{a\tau} \right) \]

Rewrite partition function in terms of spinors \( z_{a\alpha} \),
with \( \alpha = \uparrow, \downarrow \) and

\[ \Phi_a = z^*_{a\alpha} \sigma_{\alpha\beta} z_{a\beta} \]

Quantum theory for destruction of Neel order

Partition function on cubic lattice

\[ Z = \prod_a \int d\tilde{\phi}_a \delta(\tilde{\phi}_a^2 - 1) \exp \left( \frac{1}{g} \sum_{a,\mu} \tilde{\phi}_a \cdot \tilde{\phi}_{a+\mu} + i \sum_a \eta_a A_{a\tau} \right) \]

Rewrite partition function in terms of spinors \( z_{a\alpha} \), with \( \alpha = \uparrow, \downarrow \) and

\[ \tilde{\phi}_a = z_{a\alpha}^* \sigma_{\alpha\beta} z_{a\beta} \]

Remarkable identity from spherical trigonometry

\[ \text{Arg} \left[ z_{a\alpha}^* z_{a+\mu,\alpha} \right] = A_{a\mu} \]

Quantum theory for destruction of Neel order

Partition function on cubic lattice

\[ Z = \prod_a \int d\phi_a \, \delta(\phi_a^2 - 1) \exp \left( \frac{1}{g} \sum_{a,\mu} \phi_a \cdot \phi_{a+\mu} + i \sum_a \eta_a A_{a\tau} \right) \]

Partition function expressed as a gauge theory of spinor degrees of freedom

\[ Z \approx \prod_a \int dz_{a\alpha} \, dA_{a\mu} \, \delta \left( |z_{a\alpha}|^2 - 1 \right) \]

\[ \times \exp \left( \frac{1}{g} \sum_{a,\mu} z^*_{a\alpha} e^{iA_{a\mu}} z_{a+\mu,\alpha} + i \sum_a \eta_a A_{a\tau} \right) \]

Large $g$ effective action for the $A_{a\mu}$ after integrating $z_{\alpha\mu}$

$$Z = \prod_{a,\mu} \int dA_{a\mu} \exp \left( \frac{1}{2e^2} \sum \cos(\Delta_{\mu} A_{av} - \Delta_{\nu} A_{a\mu}) - i \sum \eta_a A_{a\tau} \right)$$

with $e^2 \sim g^2$

This is compact QED in 3 spacetime dimensions with static charges $\pm 1$ on two sublattices.

This theory can be reliably analyzed by a duality mapping.

The gauge theory is in a **confining** phase, and there is VBS order in the ground state. (Proliferation of monopoles in the presence of Berry phases).

---

\[ Z \approx \prod_a \int d z_{a\alpha} d A_{a\mu} \delta \left( |z_{a\alpha}|^2 - 1 \right) \exp \left( \frac{1}{g} \sum_{a,\mu} z_{a\alpha}^* e^{i A_{a\mu}} z_{a+\mu,\alpha} + i \sum_a \eta_a A_{a\tau} \right) \]

Neel order
\[ \langle \tilde{\phi} \rangle \neq 0 \]

VBS order
\[ \langle \Psi_{vbs} \rangle \neq 0 \]
Not present in LGW theory of \( \tilde{\phi} \) order
Ordering by quantum fluctuations
Ordering by quantum fluctuations
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\[ Z \approx \prod_a \int d\bar{z}_{a\alpha} dA_{a\mu} \delta \left( |z_{a\alpha}|^2 - 1 \right) \exp \left\{ \frac{1}{g} \sum_{a,\mu} z^*_{a\alpha} e^{iA_{a\mu}} z_{a+\mu,\alpha} + i \sum_a \eta_a A_{a\tau} \right\} \]

**Neel order**

\[ \langle \tilde{\phi} \rangle \neq 0 \]

**VBS order**

\[ \langle \Psi_{\text{vbs}} \rangle \neq 0 \]

Not present in LGW theory of \( \tilde{\phi} \) order
**Theory of a second-order quantum phase transition between Neel and VBS phases**

At the quantum critical point:

- \( A_\mu \to A_\mu + 2\pi \) periodicity can be ignored
  (Monopoles interfere destructively and are dangerously irrelevant).
- \( S=1/2 \) spinons \( z_\alpha \), with \( \bar{\phi} \sim z_\alpha^* \bar{\sigma}_\alpha \beta z_\beta \), are globally propagating degrees of freedom.

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**Second-order critical point described by emergent fractionalized degrees of freedom \((A_\mu \text{ and } z_\alpha)\); Order parameters \((\phi \text{ and } \Psi_{\text{vbs}})\) are “composites” and of secondary importance**


Phase diagram of S=1/2 square lattice antiferromagnet

Neel order

\[ \langle \phi \rangle \sim \langle z_\alpha^* \sigma_{\alpha\beta} z_\beta \rangle \neq 0 \]

VBS order \( \langle \Psi_{vbs} \rangle \neq 0 \) (associated with condensation of monopoles in \( A_\mu \)),

\[ S = 1/2 \text{ spinons } z_\alpha \text{ confined,} \]

\[ S = 1 \text{ triplon excitations} \]

Second-order critical point described by

\[
S_{\text{critical}} = \int d^2x d\tau \left[ |(\partial_\mu - iA_\mu)z_\alpha|^2 + r |z_\alpha|^2 + \frac{u}{2} (|z_\alpha|^2)^2 + \frac{1}{4e^2} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \right]
\]

at its critical point \( r = r_c \), where \( A_\mu \) is non-compact

Conclusions

• New quantum phases induced by Berry phases: VBS order in the antiferromagnet
• Critical resonating-valence-bond states describes the quantum phase transition from the Neel to the VBS
• Emergent gauge fields are essential for a full description of the low energy physics.