Radiation-Induced Magnetoresistance Oscillations in a 2D Electron Gas

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Outline

I. Introduction

II. Experiments – Mani *et al*. and Zudov *et al*.

III. Physical Picture

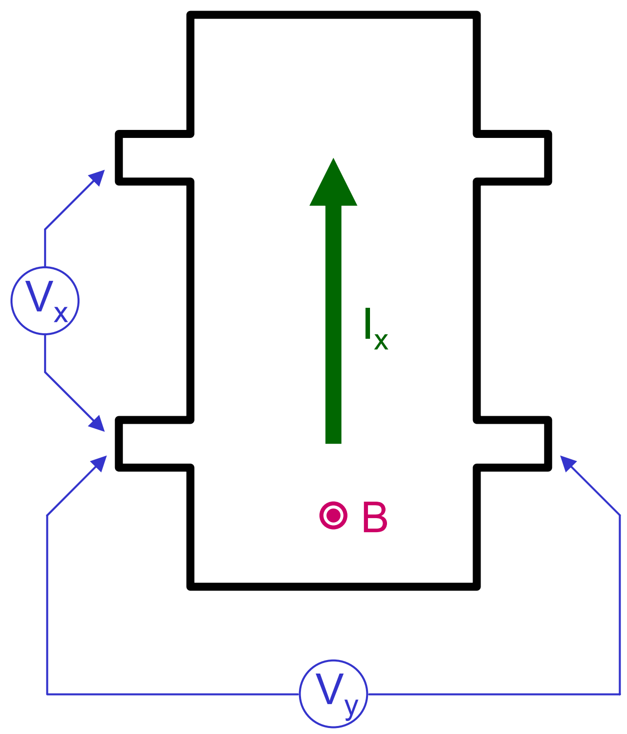
IV. Diagrammatic Conductivity Calculation

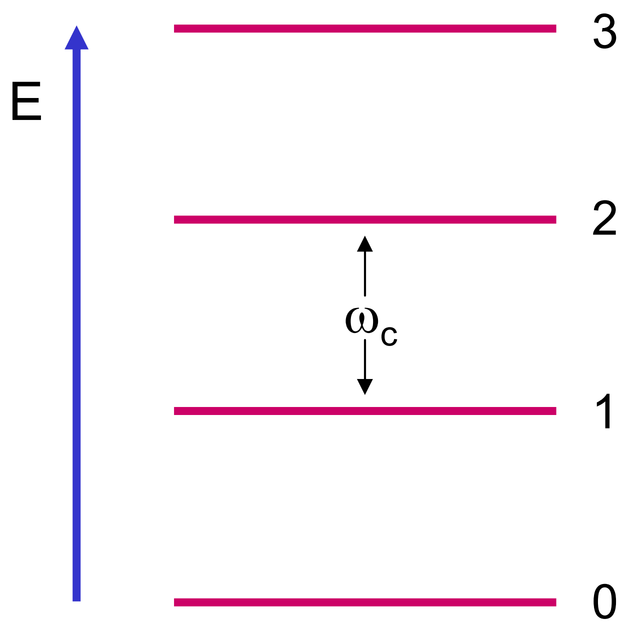
V. Calculated Resistivity Oscillations

VI. Zero-Resistivity States – Andreev, Aleiner, and Millis

VII. Conclusions

2D Electrons in Magnetic Field

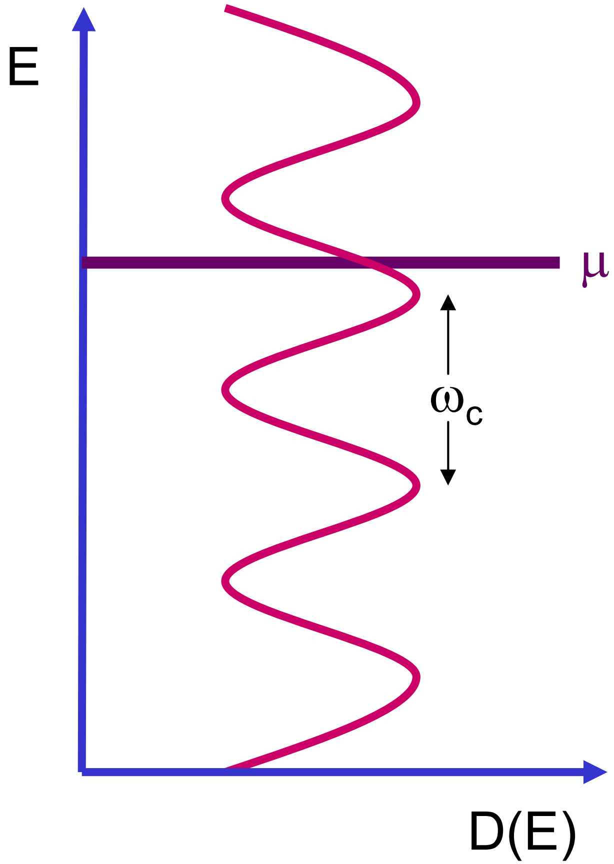


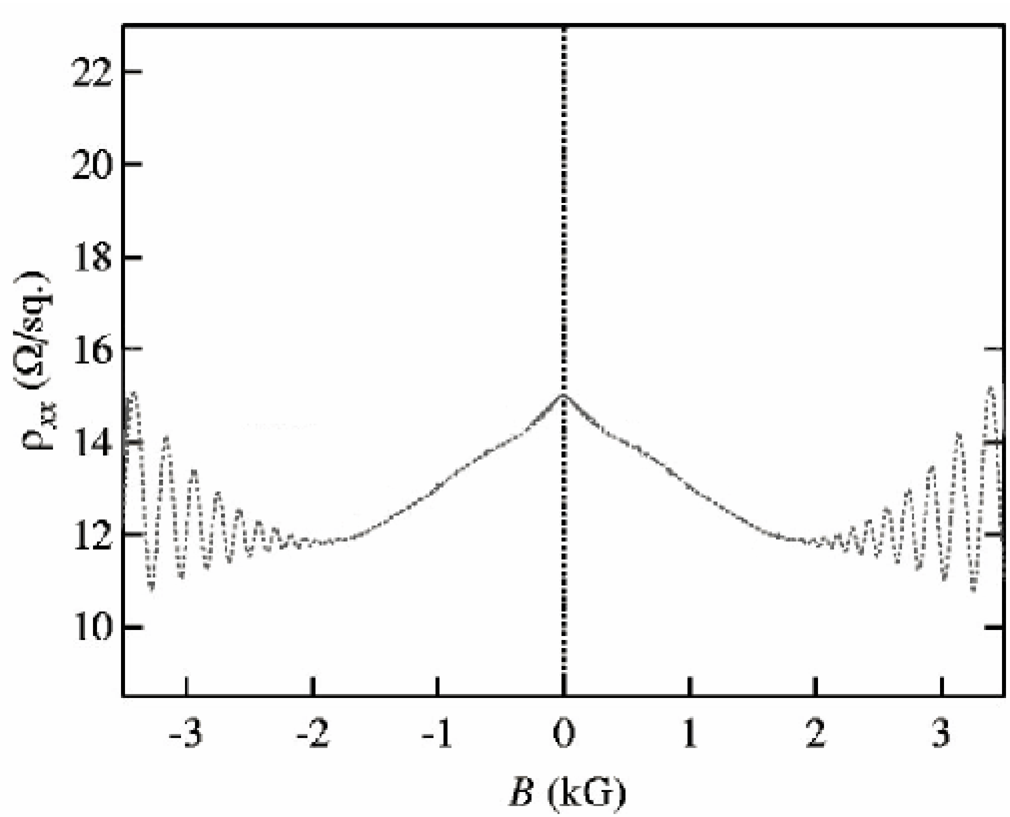


$$\varepsilon_n = n\omega_c \qquad \omega_c = \frac{eB}{m^* c}$$

$$\deg = \frac{1}{2\pi\ell^2} \qquad \ell = \sqrt{\frac{c}{eB}}$$

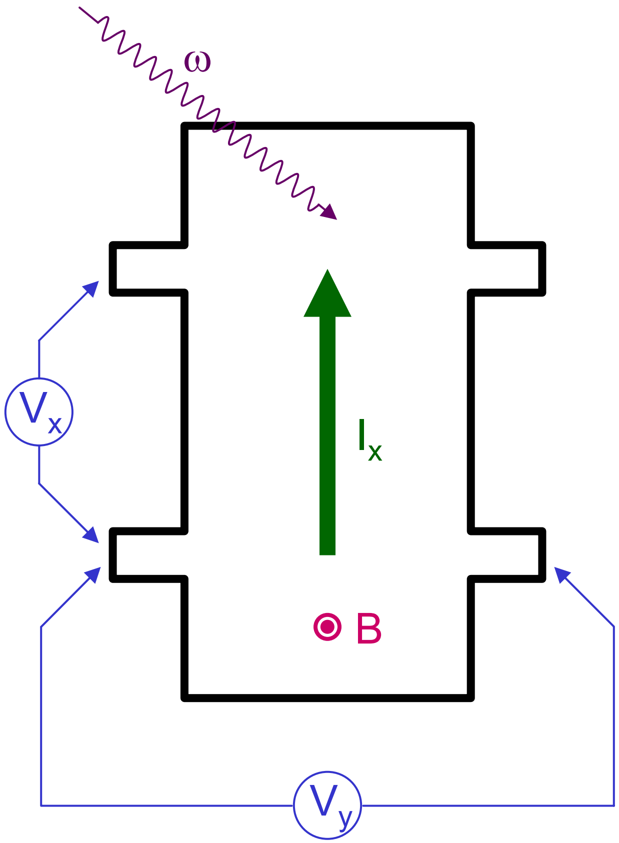
Shubnikov-de Haas Oscillations

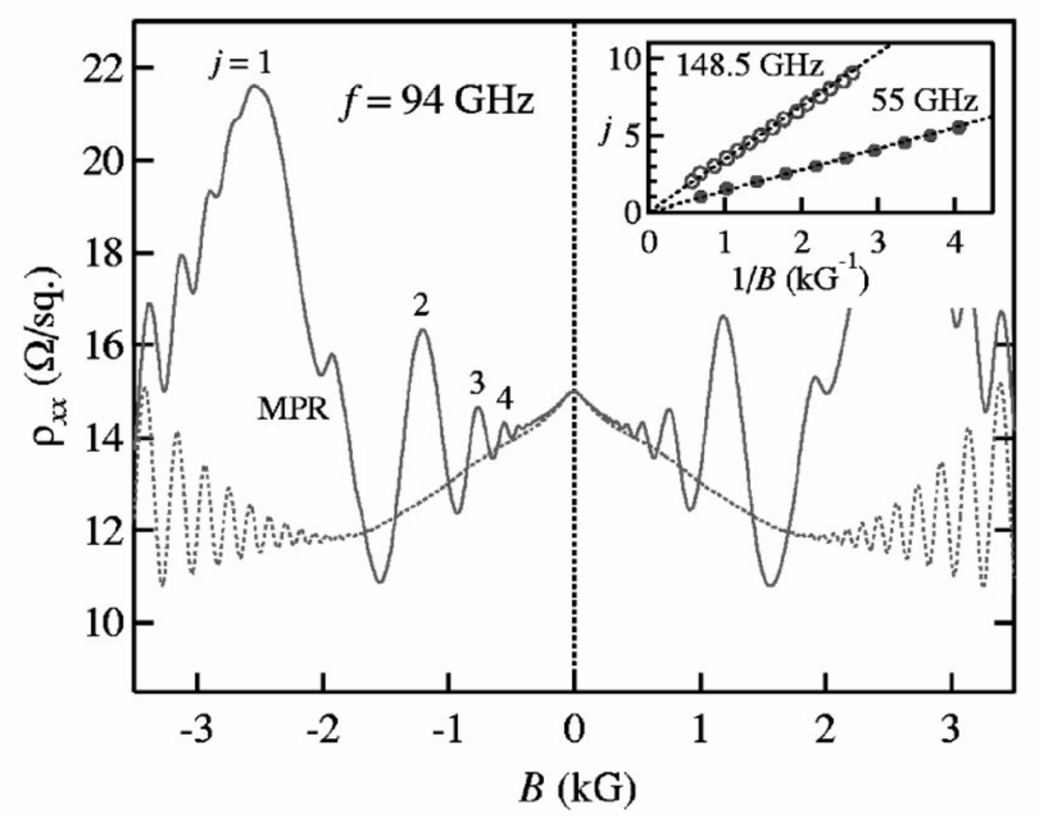




Oscillations controlled by μ/ω_c

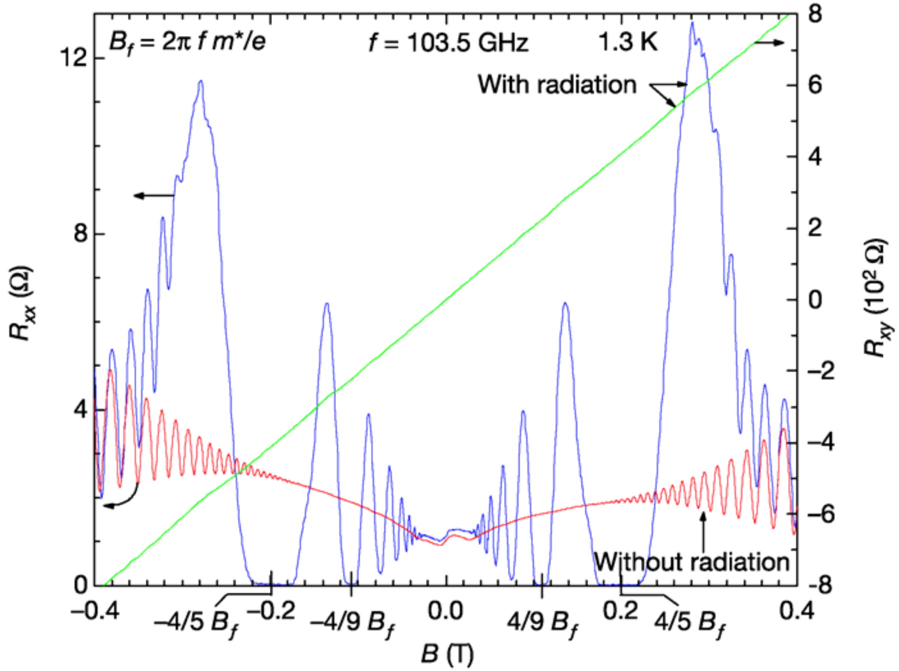
Turn on the Microwaves – Initial Experiments





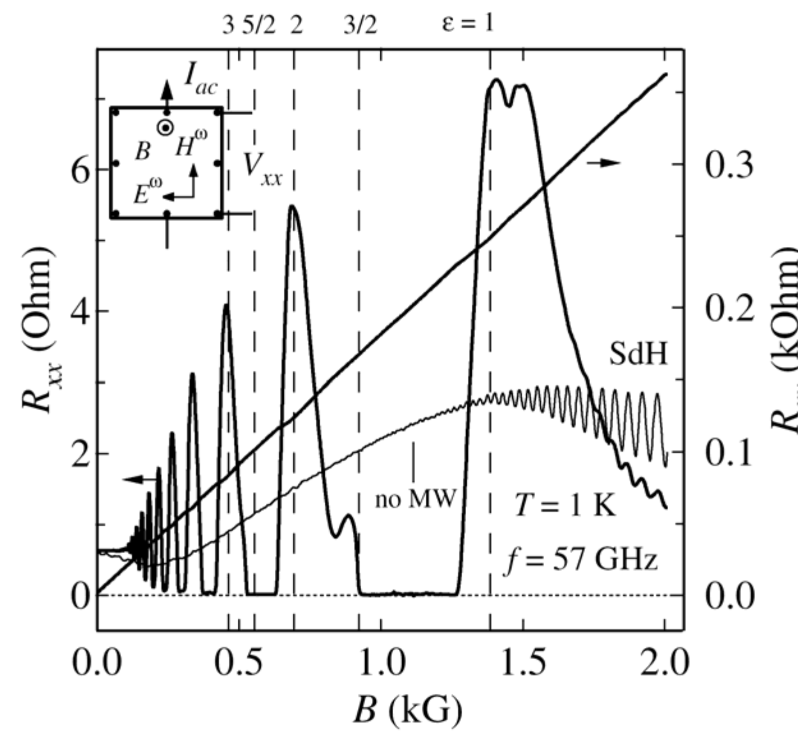
M. A. Zudov, R. R. Du, J. A. Simmons, and J. L. Reno, *Phys. Rev. B* **64**, 201311 (2001)

Experiments with High Mobility Samples

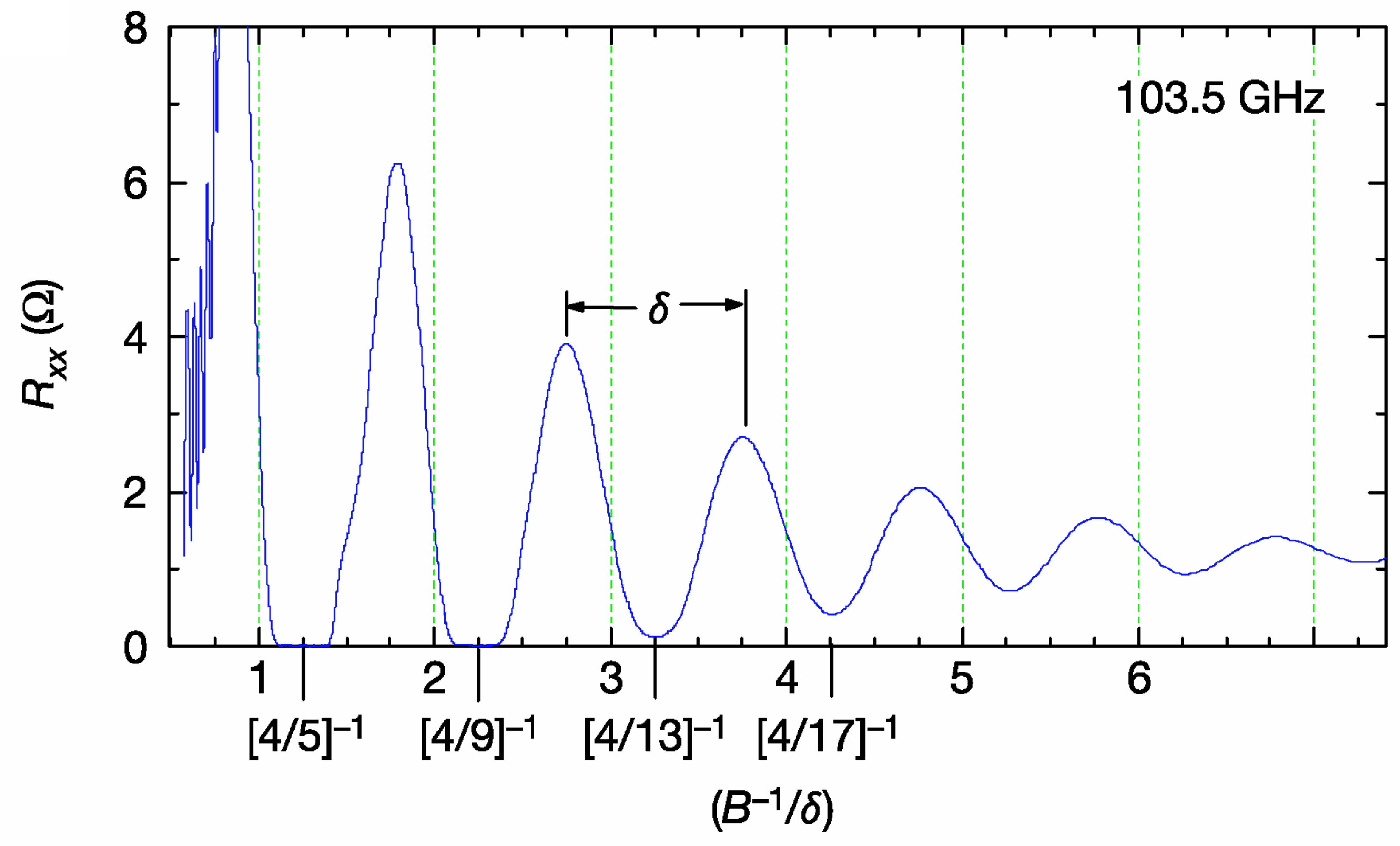


R. G. Mani, J. H. Smet, K. von Klitzing, V. Narayanamurti, W. B. Johnson, and V. Umansky, *Nature* **420**, 646 (2002)

M. A. Zudov, R. R. Du, L. N. Pfeiffer, and K. W. West, *Phys. Rev. Lett.* **90**, 46807 (2003)



Resistance Oscillations Controlled by ω/ω_c

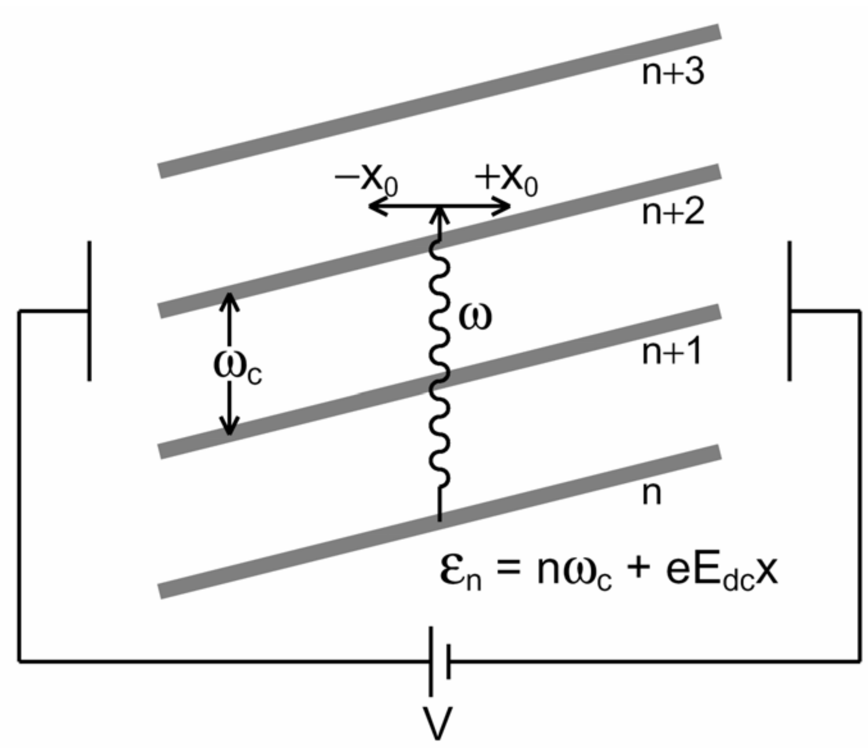


Minima near ω/ω_c = integer + 1/4

Mani *et al.*, *Nature* **420**, 646 (2002)

Physical Picture – Radiation-Induced Disorder-Assisted Transitions

- Landau levels tilted by dc bias
- Radiation excites electrons
- Disorder scatters electrons
- Electrons scattered to the left enhance dc current
- Electrons scattered to the right diminish dc current
- If rate to scatter right exceeds rate to scatter left, then photo-excited electrons flow uphill



Toy Calculation

- Electron at center of *n*th Landau level absorbs photon of energy ω and scatters a distance *x* in the *x*-direction
- Energy with respect to local energy levels

$$\varepsilon = n\omega_c + \omega - eE_{dc}x$$

- Scattering rate via Fermi's golden rule

$$R(x) = 2\pi |M(x)|^2 \mathcal{N}(n\omega_c + \omega - eE_{dc}x)$$

- Photo-excited electrons contribute -*exR*(*x*) to dc current
- Left-right symmetry → $M(-x) = M(x)$
- Periodicity of Landau levels → $\mathcal{N}(\epsilon + n\omega_c) = \mathcal{N}(\epsilon)$
- Radiation-induced change in current density

$$\Delta J_x = \frac{2\pi e}{L} \int_0^{\frac{L}{2}} dx\, x |M(x)|^2 \left[\mathcal{N}(\omega + eE_{dc}x) - \mathcal{N}(\omega - eE_{dc}x)\right]$$

- Linear response limit ($E_{dc} \to 0$) yields derivative

$$\Delta\sigma_{xx} \sim \left.\frac{\partial \mathcal{N}}{\partial \epsilon}\right|_{\epsilon=\omega}$$

- Inverting conductivity tensor and noting $\rho_{xx} \ll \rho_{xy} \approx \rho^0_{xy} = B/nec$

$$\rho_{xx} = \frac{\sigma_{xx}}{\sigma^2_{xx} + \sigma^2_{xy}} \approx \rho^2_{xy}\sigma_{xx}$$

- Landau level periodicity of density of states yields resistivity oscillations

$$\mathcal{N}(\epsilon) = \mathcal{N}_0 + \mathcal{N}_1 \cos(2\pi\epsilon/\omega_c)$$

$$\Delta\rho_{xx} \sim \left.\frac{\partial \mathcal{N}}{\partial \epsilon}\right|_{\epsilon=\omega} \sim -\sin(2\pi\omega/\omega_c)$$

← Correct period and phase

Magnetic Field + Radiation + Disorder → Resistivity Oscillations

Diagrammatic Conductivity Calculation

$$H = \sum_{k,n} n\omega_c \, c^\dagger_{nk} c_{nk} + V_{\text{imp}} + \frac{eE\ell}{\sqrt{2}} \sum_{k,n} \sqrt{n} \left(c^\dagger_{nk} c_{n-1,k} e^{-i\omega t} + c^\dagger_{n-1,k} c_{nk} e^{i\omega t} \right)$$

- Magnetic field

  Landau levels

- Disorder

  Self-consistent Born approximation (SCBA)

  δ-correlated disorder → $\overline{V_{\text{imp}}(\mathbf{r}) V_{\text{imp}}(\mathbf{r}')} = \frac{2\gamma}{m^*} \delta(\mathbf{r} - \mathbf{r}')$ $\quad \gamma = \frac{1}{2\tau}$

- Radiation

  Rotating-wave approximation

  Non-equilibrium → Kadanoff-Baym-Keldysh formalism

Green's Functions

Three Green's functions: $G^R_{nm}(t_1,t_2)$ $G^A_{nm}(t_1,t_2)$ $G^<_{nm}(t_1,t_2)$

Define $\bar{G}^{R,A,<}_{nm}(t,\mathcal{T}) = e^{in\omega t_1} G^{R,A,<}_{nm}(t_1,t_2) e^{-im\omega t_2}$ $t \equiv t_1 - t_2$ $\mathcal{T} \equiv (t_1+t_2)/2$

$$\sum_p \left[\left(z - n(\omega_c - \omega) - \Sigma^{R,A}(z+n\omega) \pm i\delta\right)\delta_{np} - \frac{eE\ell}{\sqrt{2}}\left(\sqrt{n}\,\delta_{n,p+1} + \sqrt{p}\,\delta_{n+1,p}\right)\right] \bar{G}^{R,A}_{pm}(z) = \delta_{nm}$$

$$\bar{G}^<_{nm}(z) = \sum_p \bar{G}^R_{np}(z)\left[n_F(z+p\omega)2i\delta + \Sigma^<(z+p\omega)\right]\bar{G}^A_{pm}(z)$$

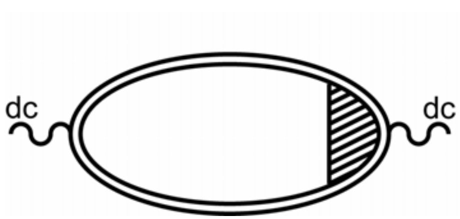
$$\Sigma^{R,A,<}(z) = \frac{\gamma\omega_c}{\pi}\sum_n \bar{G}^{R,A,<}_{nn}(z-n\omega)$$

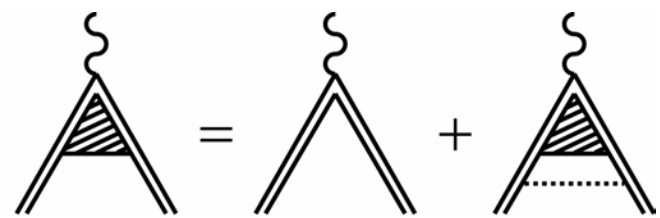
Electrical Conductivity

Kubo formula

$$\sigma_{ij} = -\lim_{\Omega \to 0} \frac{\mathrm{Im}\, \Pi^R_{ij}(\Omega)}{\Omega} \qquad i, j = \{x, y\}$$

Polarization Bubble



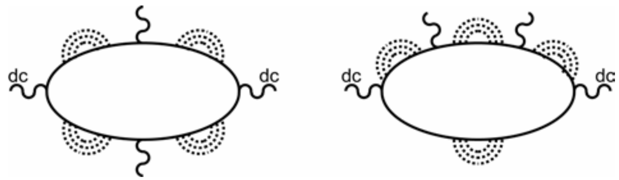


Vertex dressed with ladders of disorder lines

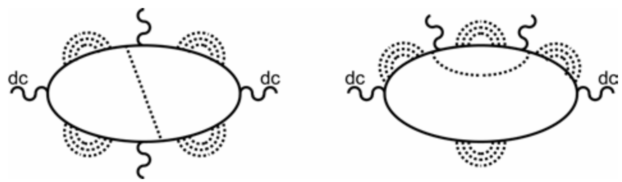
Conserving Approximation

- Neglect vertex corrections
- Neglect diagrams where disorder lines cross photon insertions

Included:



Neglected:

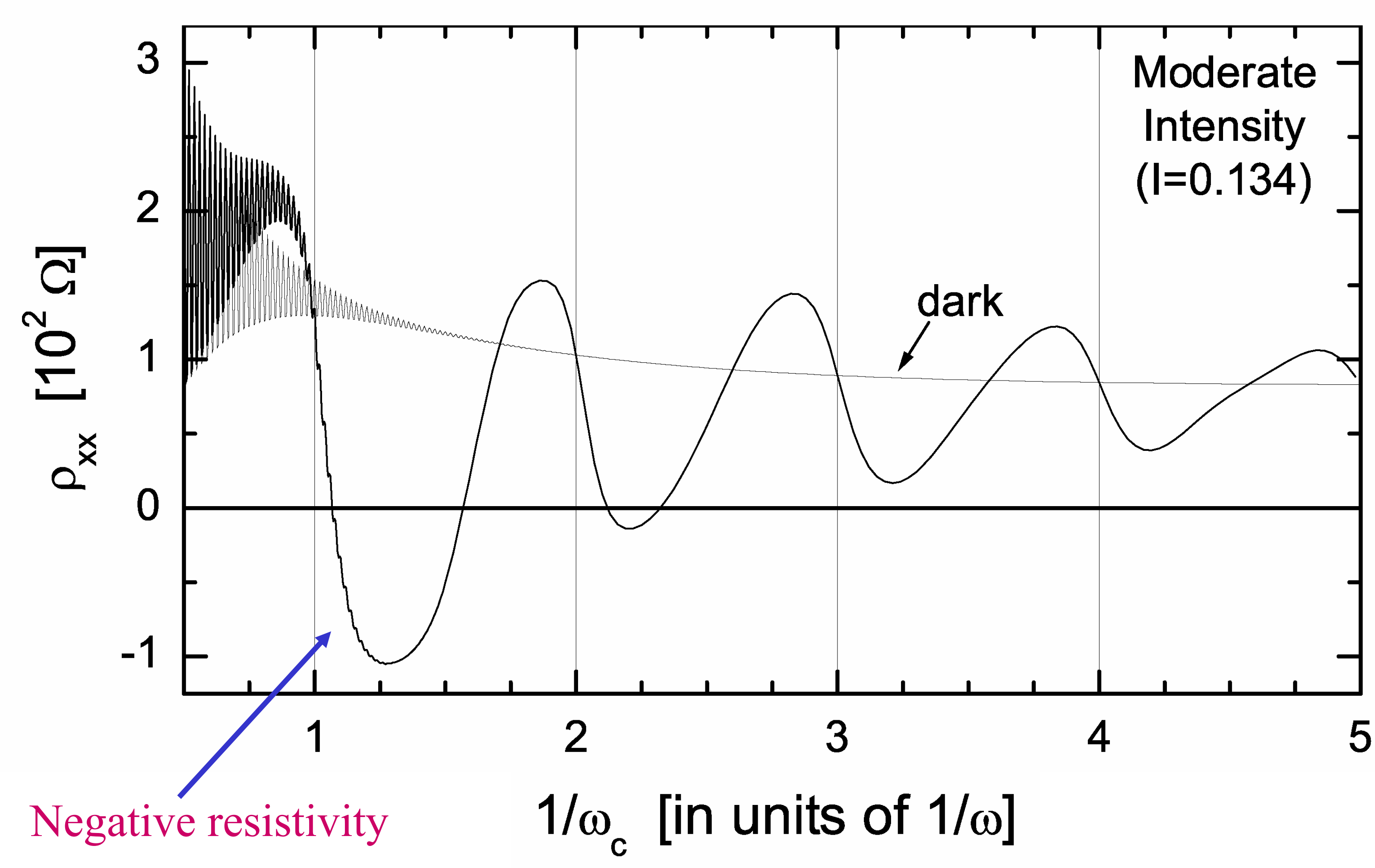


- Equivalent to replacing fully self-consistent Σ's with Σ's calculated in absence of radiation but still self-consistent in disorder
- For sufficient disorder ($2\pi\gamma > \omega_c$)

$$\Sigma(z) \approx -i\gamma\left[1 + 2e^{-\frac{2\pi\gamma}{\omega_c}} \exp\left(i\frac{2\pi z}{\omega_c}\right)\right]$$

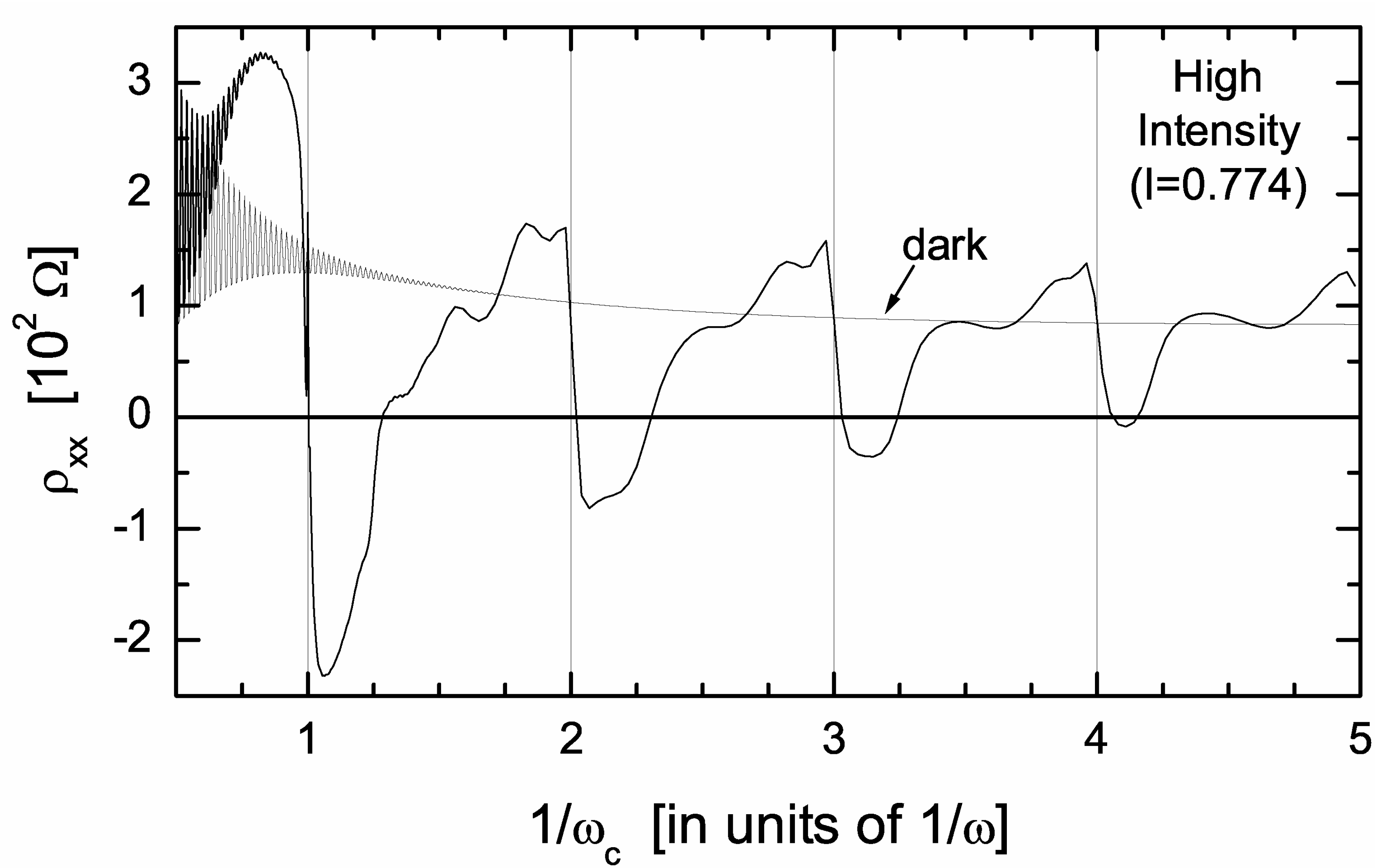
T. Ando, *J. Phys. Soc. Japan*, **37**, 1233 (1974)

Calculated Resistivity Oscillations



$\mu=50\omega$ $k_BT=\omega/4$ $\gamma=0.08\omega$ $I=0.134m^*\omega^3$

Multi-Photon Effects



$\mu=50\omega$ $k_BT=\omega/4$ $\gamma=0.08\omega$ $I=0.774m^*\omega^3$

Small-Angle Scattering

- Look at the numbers – Calculated dark ρ_{xx} is 50 times too big
- Simplifying approximation – δ-correlated disorder

$$\overline{V_{\text{imp}}(\mathbf{r})V_{\text{imp}}(\mathbf{r}')} = \frac{2\gamma}{m^*}\delta(\mathbf{r}-\mathbf{r}') \qquad \gamma = 1/2\tau \qquad \rho^0_{xx} \sim \gamma$$

- Reality – Smooth disorder potential with long-ranged correlations

$$\rho^0_{xx} \sim \gamma_{tr} \qquad \gamma_{tr} = 1/2\tau_{tr} \qquad \gamma/\gamma_{tr} \approx 50$$

Transport scattering rate discounts importance of small-angle scattering

- Consequence for Hall resistivity
  - Dark ρ_{xy} is insensitive to disorder $\qquad \rho^0_{xy} \approx B/nec$
  - $\Delta\rho_{xy}$ is comparable to $\Delta\rho_{xx}$ but dark ρ_{xy} is effectively too small
  - Radiation-induced oscillations appear in calculated ρ_{xy}
  - Effect is negligible after correcting for factor of 50
- Solution – Consider realistic disorder – Much more difficult calculation

Mechanism for Energy Relaxation

- Microwave radiation pumps energy into the electronic system
- How does it get out?
- Reality (probably)
  - Continuous radiation heats up electronic system
  - Mechanism for energy loss required for relaxation to steady state
  - Electron-phonon interactions allow energy loss to lattice
- Conserving approximation skirts the issue
  - Approximation is insufficient to produce heating effects
  - Mimics effect of energy loss mechanisms
  - Allows for steady state without electron-phonon interactions
- Energy relaxation mechanism required to go beyond this approximation

Physics on Internet Time

Magnetic Field + Radiation + Disorder → Negative-Resistivity Minima

Durst, Sachdev, Read, and Girvin, cond-mat/0301569

January 29, 2003

↓ 6 days later

Negative-Resistivity Minima → Zero-Resistivity States

Andreev, Aleiner, and Millis, cond-mat/0302063

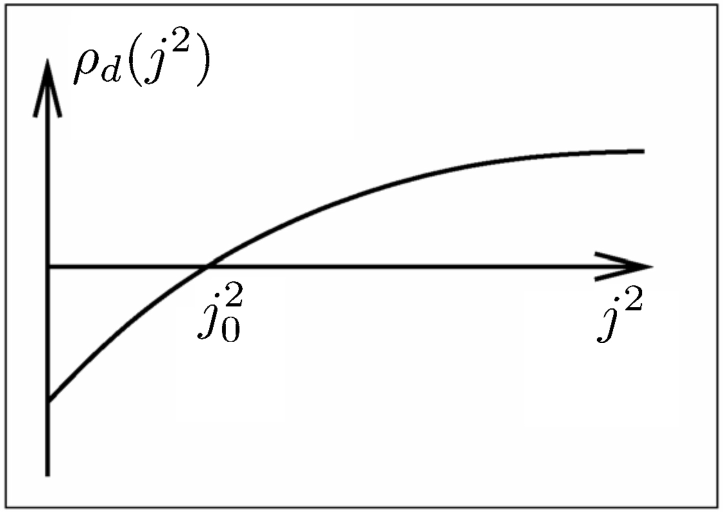
February 4, 2003

Nonlinear Response

$$\mathbf{E} = \mathbf{j}\rho_d(j^2) + [\mathbf{j} \times \mathbf{z}]\rho_H$$

$$\rho_d(j^2 = 0) < 0$$

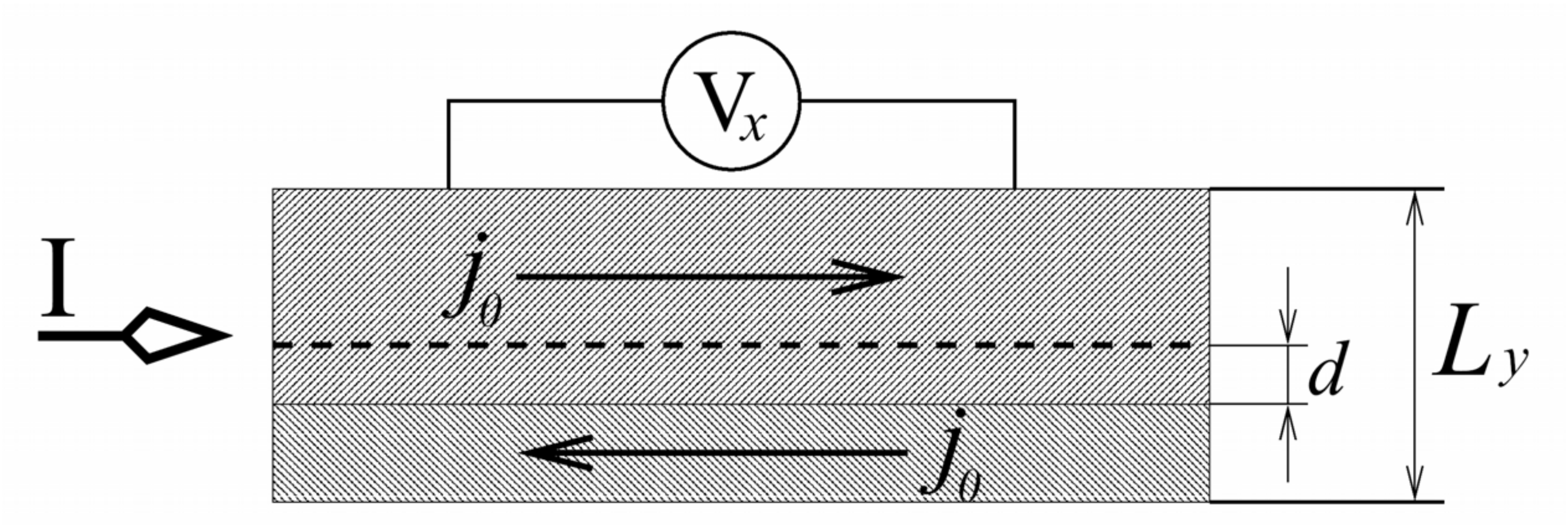
$$\rho_d(j_0^2) = 0$$



- Consider Coulomb interactions within macroscopic Poisson equation
- Homogeneous current distribution with $\rho_d < 0$ is unstable to formation of inhomogeneous current distribution with $\rho_d = 0$

A. V. Andreev, I. L. Aleiner, and A. J. Millis, cond-mat/0302063

Zero-Resistivity States via Dynamical Symmetry Breaking



$$I = 2dj_0 \qquad V_x = \rho_d(j_0^2)\, j_0 L_x = 0$$

$$V_y = -\rho_H\left[j_0\left(L_y/2 + d\right) - j_0\left(L_y/2 - d\right)\right] = -\rho_H I$$

A. V. Andreev, I. L. Aleiner, and A. J. Millis, cond-mat/0302063

Conclusions

- Basic ingredients required for radiation-induced magnetoresistance oscillations in a 2d electron gas are: magnetic field, radiation, and disorder
- Photo-excited disorder-scattered electrons yield photocurrent proportional to the derivative of the density of states at the radiation frequency
- Diagrammatic calculation yields oscillations in the longitudinal resistivity which are controlled by ω/ω_c and exhibit minima near ω/ω_c = integer + 1/4
- Oscillations increase with radiation intensity, easily exceeding the dark resistivity and resulting in negative-resistivity minima
- Andreev, Aleiner, and Millis have shown that homogeneous current distributions with negative resistivity are unstable to the formation of inhomogeneous current distributions with zero resistivity
- Our result, taken together with theirs, explains the zero-resistance states observed in recent experiments