Quantum criticality in condensed matter: field theory vs. gauge-gravity duality

Northeastern University, May 1, 2013

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Talk online at sachdev.physics.harvard.edu
Sommerfeld-Pauli-Bloch theory of metals, insulators, and superconductors: many-electron quantum states are adiabatically connected to independent electron states.
Boltzmann-Landau theory of dynamics of metals:

Long-lived *quasiparticles* (and *quasiholes*) have weak interactions which can be described by a Boltzmann equation.
Modern phases of quantum matter
Not adiabatically connected
to independent electron states:
many-particle
quantum entanglement,
and no quasiparticles
Outline

1. Superfluid-insulator transition of ultracold atoms in optical lattices:
   Quantum criticality and conformal field theories

2. Gauge-gravity duality
   Black-hole horizons and quasi-normal modes

3. Strange metals:
   What lies beyond the horizon?
1. Superfluid-insulator transition of ultracold atoms in optical lattices: *Quantum criticality and conformal field theories*

2. Gauge-gravity duality *Black-hole horizons and quasi-normal modes*

3. Strange metals: *What lies beyond the horizon ?*
Superfluid-insulator transition


Ultracold $^{87}$Rb atoms - bosons
$\Psi \rightarrow$ a complex field representing the Bose-Einstein condensate of the superfluid

$\langle \Psi \rangle \neq 0$

Superfluid

$\lambda_c$

$\langle \Psi \rangle = 0$

Insulator
\[ S = \int d^2r dt \left[ |\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right] \]

\[ V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u (|\Psi|^2)^2 \]

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Particles and holes correspond to the 2 normal modes in the oscillation of \( \Psi \) about \( \Psi = 0 \).

\[ \langle \Psi \rangle \neq 0 \quad \text{Superfluid} \]

\[ \langle \Psi \rangle = 0 \quad \text{Insulator} \]
Insulator (the vacuum)
at large repulsion between bosons
Excitations of the insulator:

Particles $\sim \Psi^\dagger$
Excitations of the insulator:

Holes $\sim \Psi$
\[ S = \int d^2r dt \left[ |\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right] \]

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Nambu-Goldstone mode is the oscillation in the phase \( \Psi \) at a constant non-zero \( |\Psi| \).

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A conformal field theory in 2+1 spacetime dimensions: a CFT3

\[ \langle \Psi \rangle \neq 0 \quad \text{Superfluid} \]

\[ \langle \Psi \rangle = 0 \quad \text{Insulator} \]
Quantum state with complex, many-body, "long-range" quantum entanglement

\[ S = \int d^2r dt \left[ |\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right] \]

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No well-defined normal modes, or particle-like excitations

\[ \langle \Psi \rangle \neq 0 \quad \text{Superfluid} \]

\[ \langle \Psi \rangle = 0 \quad \text{Insulator} \]

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The Higgs mode can be excited with a periodic modulation of the classical energy density. The Higgs mode is the oscillation in the amplitude $|\Psi|$. This decays rapidly by emitting pairs of Nambu-Goldstone modes. 

\[ S = \int d^2 r dt \left[ |\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right] \]

\[ V(\Psi) = (\lambda - \lambda_c) |\Psi|^2 + u (|\Psi|^2)^2 \]
Despite rapid decay, there is a well-defined Higgs “quasi-normal mode”. This is associated with a pole in the lower-half of the complex frequency plane.

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The Higgs quasi-normal mode is at the frequency

\[ \frac{\omega_{\text{pole}}}{\Delta} = -i \frac{4}{\pi} + \frac{1}{N} \left( \frac{16 (4 + \sqrt{2} \log (3 - 2\sqrt{2}))}{\pi^2} + 2.46531203396 i \right) + O \left( \frac{1}{N^2} \right) \]

where \( \Delta \) is the particle gap at the complementary point in the “paramagnetic” state with the same value of \(|\lambda - \lambda_c|\), and \( N = 2 \) is the number of vector components of \( \Psi \).

The universal answer is a consequence of the strong interactions in the CFT3
Observation of Higgs quasi-normal mode across the superfluid-insulator transition of ultracold atoms in a 2-dimensional optical lattice:

Response to modulation of lattice depth scales as expected from the LHP pole

Observation of Higgs quasi-normal mode in quantum Monte Carlo

Scaling of spectral response functions predicted in
D. Podolsky and S. Sachdev,

Kun Chen, Longxiang Liu,
Youjin Deng, Lode Pollet,
and Nikolay Prokof’ev,
arXiv:1301.3139

Snir Gazit, Daniel Podolsky,
and Assa Auerbach,
arXiv:1212.3759
\[
S = \int d^2r dt \left[ |\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]
\]

\[
V(\Psi) = (\lambda - \lambda_c) |\Psi|^2 + u (|\Psi|^2)^2
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A conformal field theory in 2+1 spacetime dimensions: a CFT3
The diagram illustrates a phase transition in a quantum system, with labels and axes indicating critical temperatures and parameters. The region labeled "Quantum critical" lies between the "Superfluid" and "Insulator" states, with specific lines denoted as $T_{KT}$ and $\lambda_c$.
"Boltzmann" theory of Nambu-Goldstone and vortices

Boltzmann theory of particles/holes

Superfluid

Insulator

Quantum critical

$T$

$\lambda$

$\lambda_c$

0
CFT3 at $T>0$

Quantum critical

Superfluid

Insulator

$T_{KT}$

$\lambda_c$
CFT3 at $T>0$

Boltzmann theory of particles/holes/vortices does not apply

Superfluid

Insulator

$T_K$
CFT3 at $T>0$

Needed:
Accurate theory of quantum critical dynamics
Electrical transport in a free CFT3 for $T > 0$

$\sigma$

$\sim T \delta(\omega)$

$\omega/T$
Electrical transport for a (weakly) interacting CFT

\[ \sigma(\omega, T) = \frac{e^2}{\hbar} \sum \left( \frac{\hbar \omega}{k_B T} \right) ; \quad \Sigma \to \text{a universal function} \]

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\[ \mathcal{O}(\langle u^* \rangle^2) , \]

where \( u^* \) is the fixed point interaction

Electrical transport for a (weakly) interacting CFT3

$$\sigma(\omega, T) = \frac{e^2}{h} \sum \left( \frac{\hbar \omega}{k_B T} \right) ; \quad \Sigma \rightarrow \text{a universal function}$$

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\[ \sigma(\omega, T) = \frac{e^2}{\hbar} \sum \left( \frac{\hbar \omega}{k_B T} \right) ; \quad \Sigma \to \text{a universal function} \]

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\[ \mathcal{O}\left(1/(u^*)^2\right) \]


**Needed:**

a method for computing the d.c. conductivity of interacting CFT3s
Quantum critical dynamics

Quantum “nearly perfect fluid”
with shortest possible local equilibration time, $\tau_{eq}$

$$\tau_{eq} = C \frac{\hbar}{k_B T}$$

where $C$ is a universal constant.

Response functions are characterized by poles in LHP
with $\omega \sim k_B T/\hbar$.
These poles (quasi-normal modes) appear naturally in
the holographic theory.
(Analogs of Higgs quasi-normal mode.)

Quantum critical dynamics

Transport co-efficients not determined by collision rate of quasiparticles, but by fundamental constants of nature

Conductivity

\[ \sigma = \frac{Q^2}{h} \times [\text{Universal constant } O(1)] \]

(Q is the “charge” of one boson)

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Renormalization group: $\Rightarrow$ Follow coupling constants of quantum many body theory as a function of length scale $r$
Renormalization group: \( \Rightarrow \) Follow coupling constants of quantum many body theory as a function of length scale \( r \)

Key idea: \( \Rightarrow \) Implement \( r \) as an extra dimension, and map to a local theory in \( d + 2 \) spacetime dimensions.

J. McGreevy, arXiv0909.0518
For a relativistic CFT in $d$ spatial dimensions, the metric in the holographic space is fixed by demanding the scale transformation $(i = 1 \ldots d)$

$$x_i \rightarrow \zeta x_i \ , \quad t \rightarrow \zeta t \ , \quad ds \rightarrow ds$$
This gives the unique metric

\[ ds^2 = \frac{1}{r^2} \left( -dt^2 + dr^2 + dx_i^2 \right) \]

This is the metric of anti-de Sitter space $\text{AdS}_{d+2}$. 
AdS/CFT correspondence

$AdS_4$

$\mathbb{R}^{2,1}$

Minkowski

CFT3

$r$

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This emergent spacetime is a solution of Einstein gravity with a negative cosmological constant:

\[
S_E = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) \right]
\]
For every primary operator $O(x)$ in the CFT, there is a corresponding field $\phi(x, r)$ in the bulk (gravitational) theory. For a scalar operator $O(x)$ of dimension $\Delta$, the correlators of the boundary and bulk theories are related by

$$\langle O(x_1) \ldots O(x_n) \rangle_{\text{CFT}} = \frac{Z^n}{r_1^{-\Delta} \ldots r_n^{-\Delta}} \lim_{r \to 0} \langle \phi(x_1, r_1) \ldots \phi(x_n, r_n) \rangle_{\text{bulk}}$$

where the “wave function renormalization” factor $Z = (2\Delta - D)$. 

AdS/CFT correspondence
For a U(1) conserved current $J_\mu$ of the CFT, the corresponding bulk operator is a U(1) gauge field $A_\mu$. With a Maxwell action for the gauge field

$$S_M = \frac{1}{4g_M^2} \int d^{D+1}x \sqrt{g} F_{ab} F^{ab}$$

we have the bulk-boundary correspondence

$$\langle J_\mu(x_1) \ldots J_\nu(x_n) \rangle_{\text{CFT}} = (Z g_M^{-2})^n \lim_{r \to 0} r_1^{2-D} \ldots r_n^{2-D} \langle A_\mu(x_1, r_1) \ldots A_\nu(x_n, r_n) \rangle_{\text{bulk}}$$

with $Z = D - 2$. 
A similar analysis can be applied to the stress-energy tensor of the CFT, $T_{\mu\nu}$. Its conjugate field must be a spin-2 field which is invariant under gauge transformations: it is natural to identify this with the change in metric of the bulk theory. We write $\delta g_{\mu\nu} = (L^2/r^2)\chi_{\mu\nu}$, and then the bulk-boundary correspondence is now given by

$$\langle T_{\mu\nu}(x_1) \ldots T_{\rho\sigma}(x_n) \rangle_{\text{CFT}} =$$

$$\left( \frac{Z L^2}{\kappa^2} \right)^n \lim_{r \to 0} r_1^{-D} \ldots r_n^{-D} \langle \chi_{\mu\nu}(x_1, r_1) \ldots \chi_{\rho\sigma}(x_n, r_n) \rangle_{\text{bulk}},$$

with $Z = D$. 

**AdS/CFT correspondence**
AdS/CFT correspondence

So the minimal bulk theory for a CFT with a conserved $U(1)$ current is the *Einstein-Maxwell* theory with a cosmological constant

$$S = \frac{1}{4g_M^2} \int d^4x \sqrt{g} F_{ab} F^{ab}$$

$$+ \int d^4x \sqrt{g} \left[ -\frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) \right].$$

This action is characterized by two dimensionless parameters: $g_M$ and $L^2/\kappa^2$, which are related to the conductivity $\sigma(\omega) = \mathcal{K}$ and the central charge of the CFT.
AdS/CFT correspondence

This minimal action also fixes multi-point correlators of the CFT: however these do not have the most general form allowed for a CFT. To fix these, we have to allow for higher-gradient terms in the bulk action. For the conductivity, it turns out that only a single 4 gradient term contributes

$$S_{\text{bulk}} = \frac{1}{g_2^2 M} \int d^4 x \sqrt{g} \left[ \frac{1}{4} F_{ab} F^{ab} + \gamma L^2 C_{abcd} F^{ab} F^{cd} \right]$$

$$+ \int d^4 x \sqrt{g} \left[ -\frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) \right],$$

where $C_{abcd}$ is the Weyl tensor. The parameter $\gamma$ can be related to 3-point correlators of $J_\mu$ and $T_{\mu\nu}$. Both boundary and bulk methods show that $|\gamma| \leq 1/12$, and the bound is saturated by free fields.

D. Chowdhury, S. Raju, S. Sachdev, A. Singh, and P. Strack, arXiv:1210.5247
Gauge-gravity duality at non-zero temperatures

There is a family of solutions of Einstein gravity which describe non-zero temperatures

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\[ ds^2 = \left( \frac{L}{r} \right)^2 \left[ \frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2 \right] \]

with \( f(r) = 1 - \left( \frac{r}{R} \right)^3 \)
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A "horizon", similar to the surface of a black hole at \( r = R \)!
A 2+1 dimensional system at its quantum critical point:

\[ k_B T = \frac{3\hbar}{4\pi R}. \]

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The temperature and entropy of the horizon equal those of the quantum critical point:

\[ k_B T = \frac{3\hbar}{4\pi R} \]
Gauge-gravity duality at non-zero temperatures

A 2+1 dimensional system at its quantum critical point:
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The temperature and entropy of the horizon equal those of the quantum critical point.

Quasi-normal modes of quantum criticality = waves falling into black hole.
Gauge-gravity duality at non-zero temperatures

The temperature and entropy of the horizon equal those of the quantum critical point:

\[ k_B T = \frac{3\hbar}{4\pi R}. \]

Characteristic damping time of quasi-normal modes:

\[ (k_B/\hbar) \times \text{Hawking temperature} \]
AdS4 theory of electrical transport in a strongly interacting CFT3 for $T > 0$

Conductivity is independent of $\gamma = 0$. 

$\sigma$

$\frac{1}{g_M^2}$

$\omega/T$

Conductivity is independent of $\omega/T$ for $\gamma = 0$. 

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AdS4 theory of electrical transport in a strongly interacting CFT3 for $T > 0$

Consequence of self-duality of Maxwell theory in 3+1 dimensions

Conductivity is independent of $\omega/T$ for $\gamma = 0$.

C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son, 
Electrical transport in a free CFT3 for $T > 0$

$\sigma$

$\sim T \delta(\omega)$

Complementary $\omega$-dependent conductivity in the free theory
AdS4 theory of electrical transport in a strongly interacting CFT3 for $T > 0$

Conductivity is independent of $\frac{\omega}{T}$ for $\gamma = 0$. 

$$\sigma$$

$$\frac{1}{g_M^2}$$

$$\frac{\omega}{T}$$

Conductivity is independent of $\omega/T$ for $\gamma = 0$. 

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AdS\textsubscript{4} theory of “nearly perfect fluids”

The $\gamma > 0$ result has similarities to the quantum-Boltzmann result for transport of particle-like excitations.

AdS$_4$ theory of “nearly perfect fluids”

- The $\gamma < 0$ result can be interpreted as the transport of vortex-like excitations

AdS$_4$ theory of “nearly perfect fluids”

The $\gamma = 0$ case is the exact result for the large $N$ limit of SU($N$) gauge theory with $\mathcal{N} = 8$ supersymmetry (the ABJM model). The $\omega$-independence is a consequence of self-duality under particle-vortex duality ($S$-duality).

AdS$_4$ theory of “nearly perfect fluids”

Stability constraints on the effective theory ($|\gamma| < 1/12$) allow only a limited $\omega$-dependence in the conductivity

AdS$_4$ theory of quantum criticality

Poles in LHP of conductivity at $\omega \sim k_B T/\hbar$; analog of Higgs quasinormal mode–quasinormal modes of black brane

AdS$_4$ theory of quantum criticality

Poles in LHP of resistivity — quasinormal modes of S-dual theory

The holographic solutions for the conductivity satisfy two sum rules, valid for all CFT3s. (W. Witzack-Krempa and S. Sachdev, Phys. Rev. B 86, 235115 (2012))

$$\int_0^\infty d\omega \text{Re} \left[ \sigma(\omega) - \sigma(\infty) \right] = 0$$

$$\int_0^\infty d\omega \text{Re} \left[ \frac{1}{\sigma(\omega)} - \frac{1}{\sigma(\infty)} \right] = 0$$

The second rule follows from the existence of an EM-dual CFT3.

Boltzmann theory chooses a “particle” basis: this satisfies only one sum rule but not the other.

**Holographic theory satisfies both sum rules.**
Universal Scaling of the Conductivity at the Superfluid-Insulator Phase Transition

Jurij Šmakov and Erik Sørensen

Department of Physics and Astronomy, McMaster University, Hamilton, Ontario L8S 4M1, Canada

(Received 30 May 2005; published 27 October 2005)

The scaling of the conductivity at the superfluid-insulator quantum phase transition in two dimensions is studied by numerical simulations of the Bose-Hubbard model. In contrast to previous studies, we focus on properties of this model in the experimentally relevant thermodynamic limit at finite temperature $T$. We find clear evidence for deviations from $\omega_k$ scaling of the conductivity towards $\omega_k/T$ scaling at low Matsubara frequencies $\omega_k$. By careful analytic continuation using Padé approximants we show that this behavior carries over to the real frequency axis where the conductivity scales with $\omega/T$ at small frequencies and low temperatures. We estimate the universal dc conductivity to be $\sigma^* = 0.45(5) Q^2/h$, distinct from previous estimates in the $T = 0$, $\omega/T \gg 1$ limit.
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QMC yields $\sigma(0)/\sigma_\infty \approx 1.36$

Holography yields $\sigma(0)/\sigma_\infty = 1 + 4\gamma$ with $|\gamma| \leq 1/12$.

Maximum possible holographic value $\sigma(0)/\sigma_\infty = 1.33$

Traditional CMT

- Identify quasiparticles and their dispersions
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- Compute scattering matrix elements of quasiparticles (or of collective modes)
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Holography and black-branes

- Start with strongly interacting CFT without particle- or wave-like excitations
**Traditional CMT**

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**Holography and black-branes**

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- Compute OPE co-efficients of operators of the CFT
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Holography and black-branes

- Start with strongly interacting CFT without particle- or wave-like excitations
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- Relate OPE co-efficients to couplings of an effective gravitational theory on AdS
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<td>- Solve Einstein-Maxwell-... equations, allowing for a horizon at non-zero temperatures.</td>
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Iron pnictides:

ea new class of high temperature superconductors
Resistivity $\sim \rho_0 + AT^{\alpha}$

BaFe$_2$(As$_{1-x}$P$_x$)$_2$


Short-range entanglement in state with Neel (AF) order

Resistivity \( \sim \rho_0 + AT^\alpha \)


Superconductivity

Bose condensate of pairs of electrons

Short-range entanglement

Resistivity

\( \sim \rho_0 + AT^\alpha \)
Superconductivity

BaFe$_2$(As$_{1-x}$P$_x$)$_2$

Resistivity $\sim \rho_0 + AT^\alpha$


Superconductivity

\[ \text{BaFe}_2(\text{As}_{1-x}\text{P}_x)_2 \]

Resistivity \( \sim \rho_0 + AT^\alpha \)


Strange Metal

\[ \text{Resistivity } \sim \rho_0 + AT^\alpha \]


Strange Metal

no quasiparticles, Landau-Boltzmann theory does not apply

\[ \text{Resistivity} \sim \rho_0 + AT^\alpha \]


Electrons (fermions) occupy states inside a Fermi “surface” (circle) of radius $k_F$ which is determined by the density of electrons, $Q$. 
Can bosons form a metal?

Each quark is charged under an emergent gauge force, which encapsulates the entanglement in the ground state. The quarks have “hidden” Fermi surfaces of radius $k_F$. Can bosons form a metal?
Can bosons form a metal?

Yes, if each boson, \( b \), \textit{fractionalizes} into 2 fermions (‘quarks’)

\[ b = f_1 f_2 \]
Can bosons form a metal?

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Can bosons form a metal?

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- Each quark is charged under an emergent gauge force, which encapsulates the entanglement in the ground state.
- The quarks have “hidden” Fermi surfaces of radius \( k_F \).
The density of particles $Q$ creates an electric flux $\mathcal{E}_r$ which modifies the metric of the emergent spacetime.
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Holographic theory of a strange metal

Hidden Fermi surfaces of “quarks”?  

The general metric transforms under rescaling as

$$x_i \rightarrow \zeta x_i, \quad t \rightarrow \zeta^z t, \quad ds \rightarrow \zeta^{\theta/d} ds.$$  

Recall: conformal matter has $\theta = 0$, $z = 1$, and the metric is anti-de Sitter
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The value \( \theta = d - 1 \) reproduces all the essential characteristics of the entropy and entanglement entropy of a strange metal.
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\[ x_i \rightarrow \zeta x_i, \quad t \rightarrow \zeta^{\frac{z}{d}} t, \quad ds \rightarrow \zeta^{\theta/d} ds. \]

The null-energy condition of gravity yields \( z \geq 1 + \theta/d \). In \( d = 2 \), this leads to \( z \geq 3/2 \). Field theory on strange metal yields \( z = 3/2 \) to 3 loops!

Conclusions

Conformal quantum matter

New insights and solvable models for diffusion and transport of strongly interacting systems near quantum critical points
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Good prospects for experimental tests of frequency-dependent, non-linear, and non-equilibrium transport
More complex examples in metallic states are experimentally ubiquitous, but pose difficult strong-coupling problems to conventional methods of field theory.
Conclusions

String theory and gravity in emergent dimensions offer a remarkable new approach to describing states with many-particle quantum entanglement.

Much recent progress offers hope of a holographic description of “strange metals”