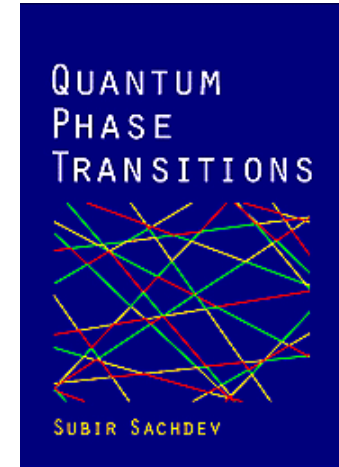


Magnetic phases and critical points of insulators and superconductors

Colloquium article:
Reviews of Modern Physics, **75**, 913 (2003).



Quantum Phase Transitions
Cambridge University Press



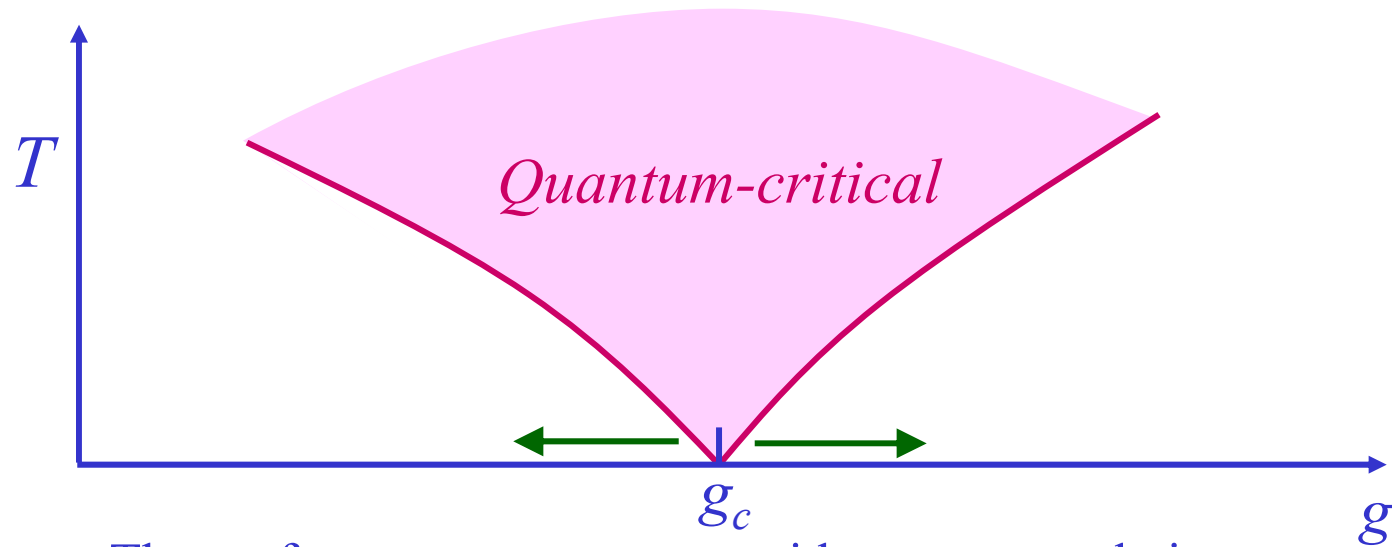
Talks online:
Google™ Sachdev



What is a quantum phase transition ?

Non-analyticity in ground state properties as a function of some control parameter g

Why study quantum phase transitions ?



- Theory for a quantum system with strong correlations: describe phases on either side of g_c by expanding in deviation from the quantum critical point.
- Critical point is a novel state of matter without quasiparticle excitations
- Critical excitations control dynamics in the wide *quantum-critical* region at non-zero temperatures.

(A) Insulators

Coupled dimer antiferromagnet

Coupled Dimer Antiferromagnet

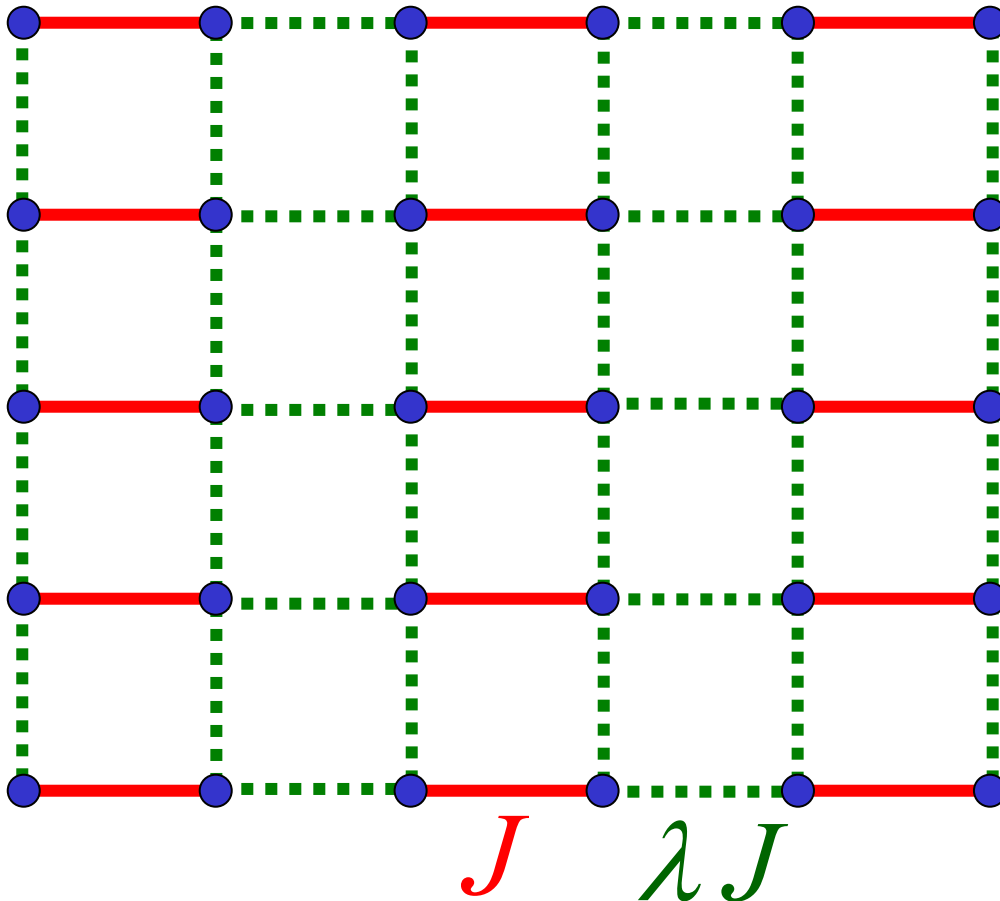
M. P. Gelfand, R. R. P. Singh, and D. A. Huse, *Phys. Rev. B* **40**, 10801-10809 (1989).

N. Katoh and M. Imada, *J. Phys. Soc. Jpn.* **63**, 4529 (1994).

J. Tworzydło, O. Y. Osman, C. N. A. van Duin, J. Zaanen, *Phys. Rev. B* **59**, 115 (1999).

M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev. B* **65**, 014407 (2002).

$S=1/2$ spins on coupled dimers



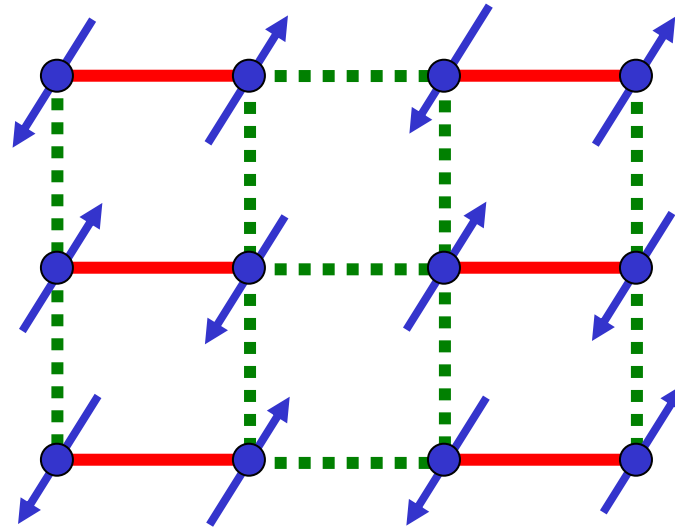
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$0 \leq \lambda \leq 1$$

λ close to 1

Square lattice antiferromagnet

Experimental realization: La_2CuO_4



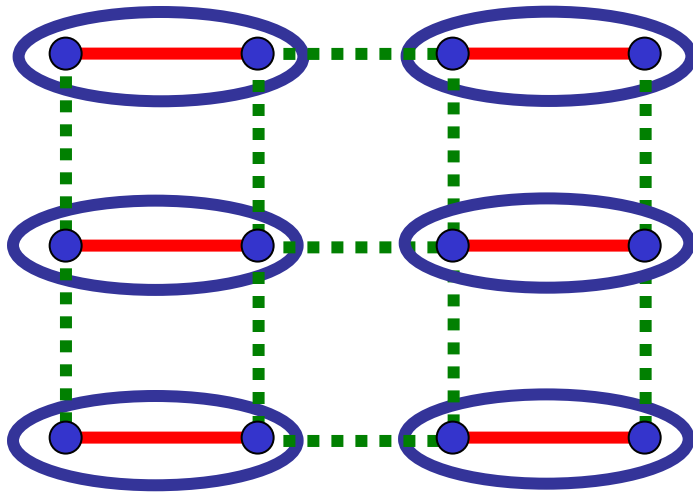
Ground state has long-range magnetic (Neel or spin density wave) order

$$\langle \vec{S}_i \rangle = (-1)^{i_x + i_y} N_0 \neq 0$$

Excitations: 2 spin waves (*magnons*) $\varepsilon_p = \sqrt{c_x^2 p_x^2 + c_y^2 p_y^2}$

λ close to 0

Weakly coupled dimers



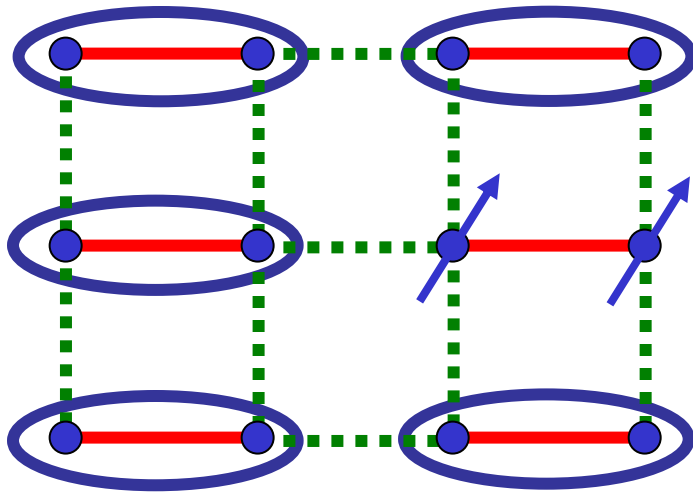
$$\text{dimer} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

Paramagnetic ground state

$$\langle \vec{S}_i \rangle = 0$$

λ close to 0

Weakly coupled dimers



$$\text{dimer} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

Excitation: $S=1$ *triplon* (*exciton*, spin collective mode)

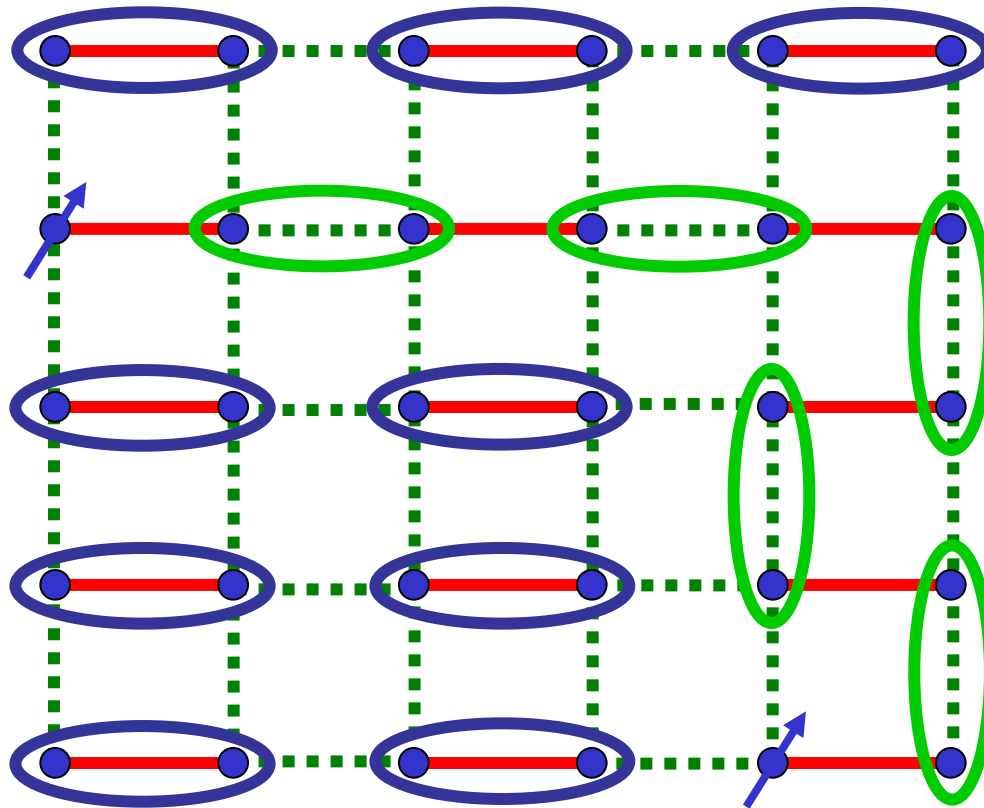
Energy dispersion away from
antiferromagnetic wavevector $\varepsilon_p = \Delta + \frac{c_x^2 p_x^2 + c_y^2 p_y^2}{2\Delta}$

$\Delta \rightarrow$ spin gap

λ close to 0

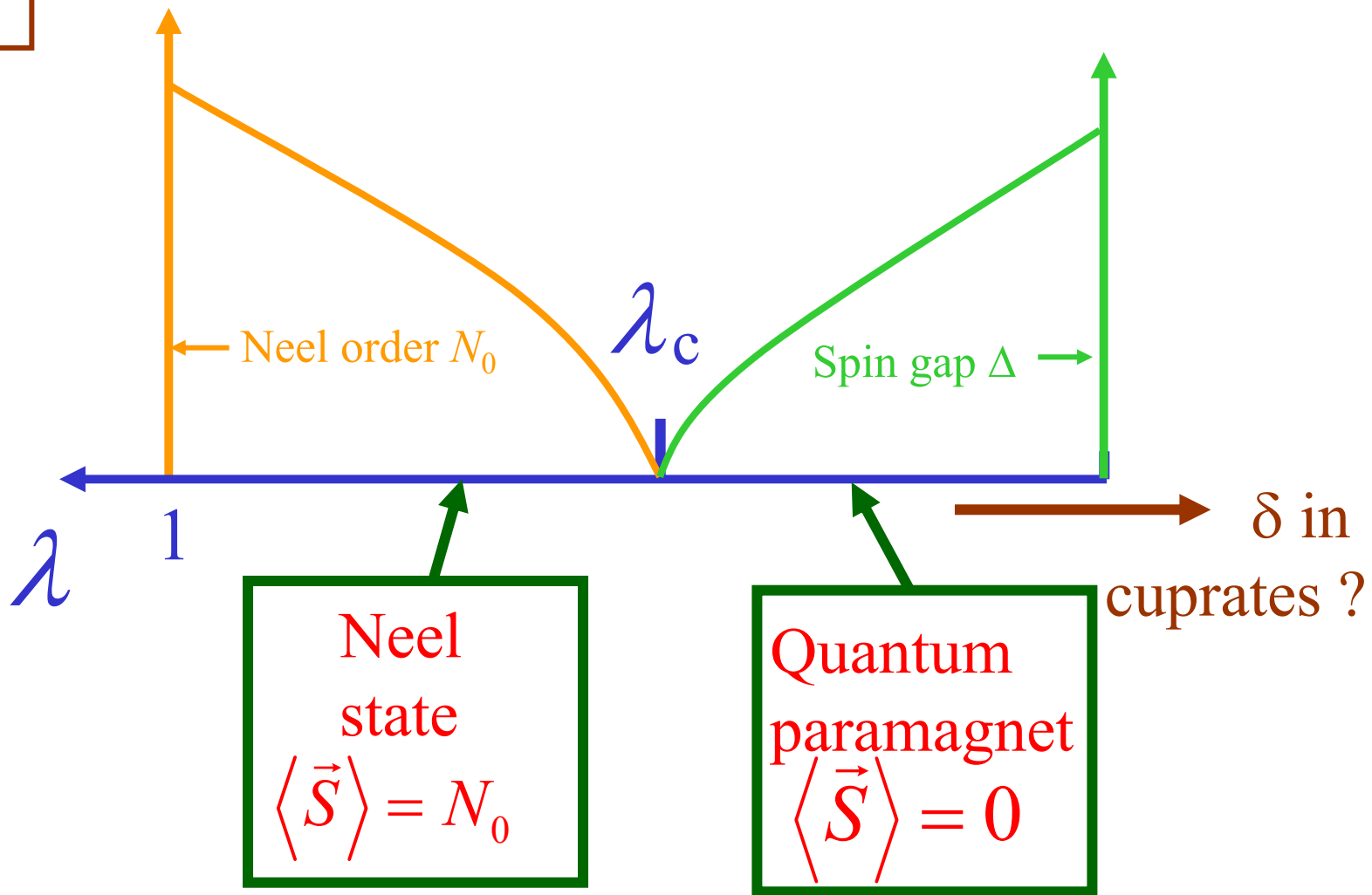
Weakly coupled dimers

$$\text{dimer} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$



$S=1/2$ spinons are confined by a linear potential into a $S=1$ triplon

$T=0$

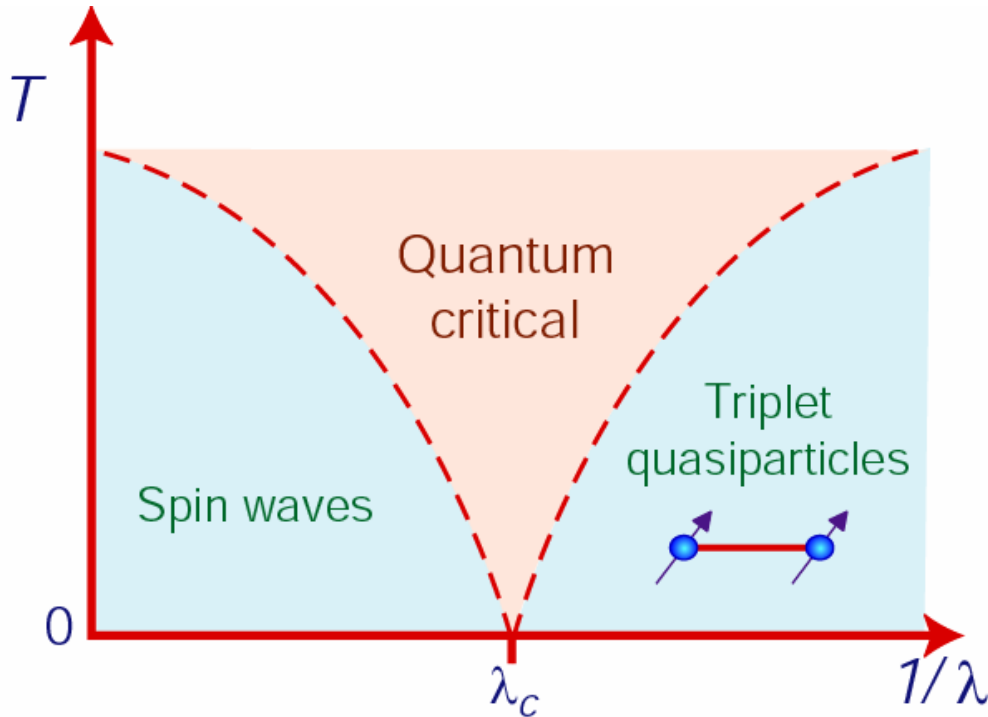


Field theory for quantum criticality

λ close to λ_c : use “soft spin” field

$$\mathcal{S}_b = \int d^2x d\tau \left[\frac{1}{2} \left((\nabla_x \phi_\alpha)^2 + c^2 (\partial_\tau \phi_\alpha)^2 + (\lambda_c - \lambda) \phi_\alpha^2 \right) + \frac{u}{4!} (\phi_\alpha^2)^2 \right]$$

$\phi_\alpha \longrightarrow$ 3-component antiferromagnetic order parameter



Quantum criticality described by strongly-coupled critical theory with universal dynamic response functions dependent on $\hbar\omega/k_B T$

$$\chi(\omega, T) = T^n g(\hbar\omega/k_B T)$$

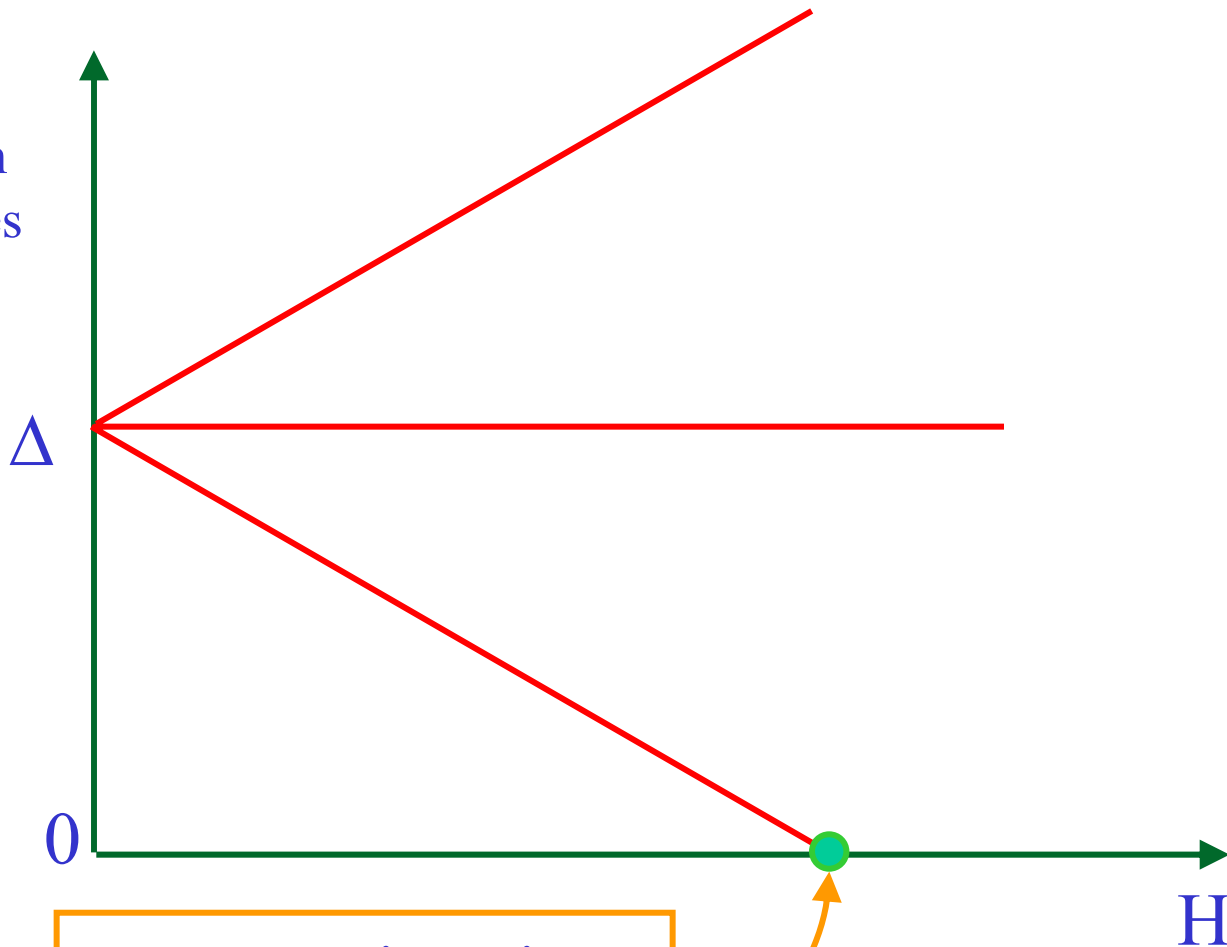
Triplon scattering amplitude is determined by $k_B T$ alone, and not by the value of microscopic coupling u

(A) Insulators

Coupled dimer antiferromagnet:
effect of a magnetic field.

Effect of a field on paramagnet

Energy of
zero
momentum
triplon states

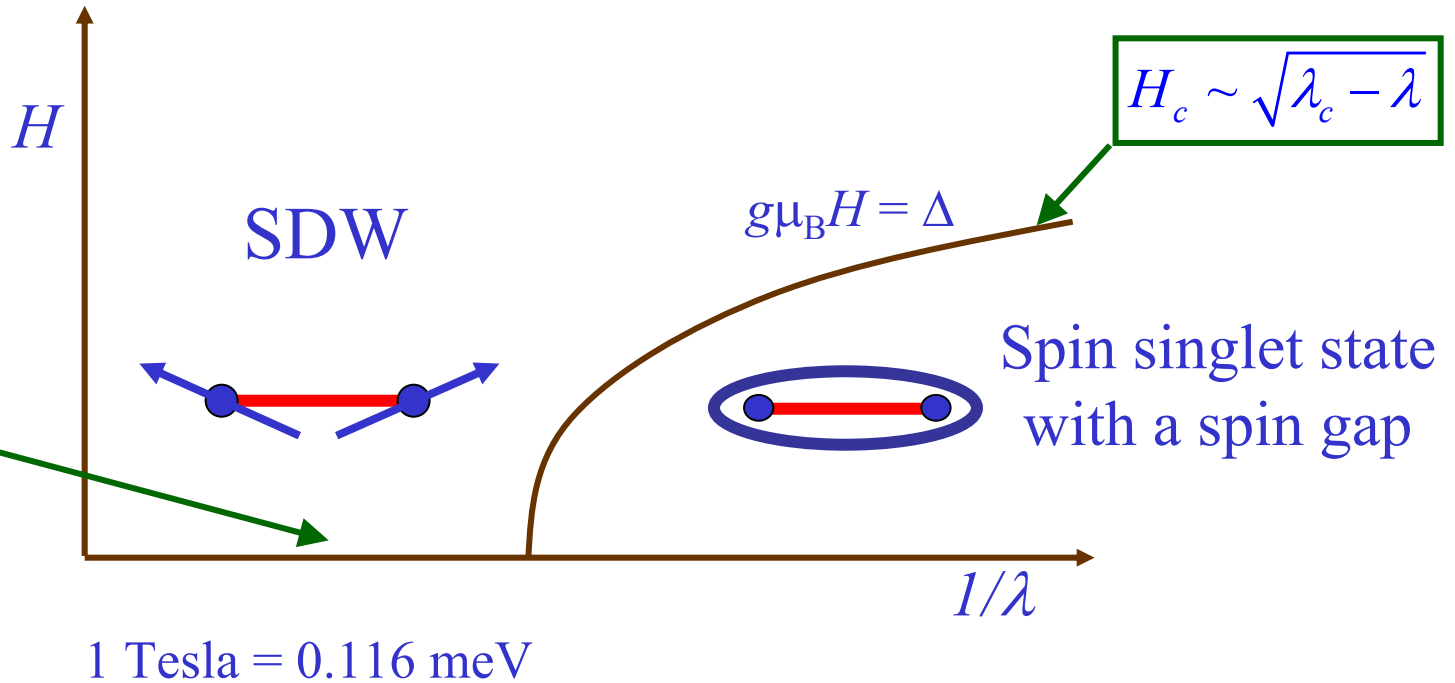


Bose-Einstein
condensation of
 $S_z=1$ triplon

Phase diagram in a magnetic field.

Elastic scattering intensity

$$I[H] = I[0] + a \left(\frac{H}{J} \right)^2$$

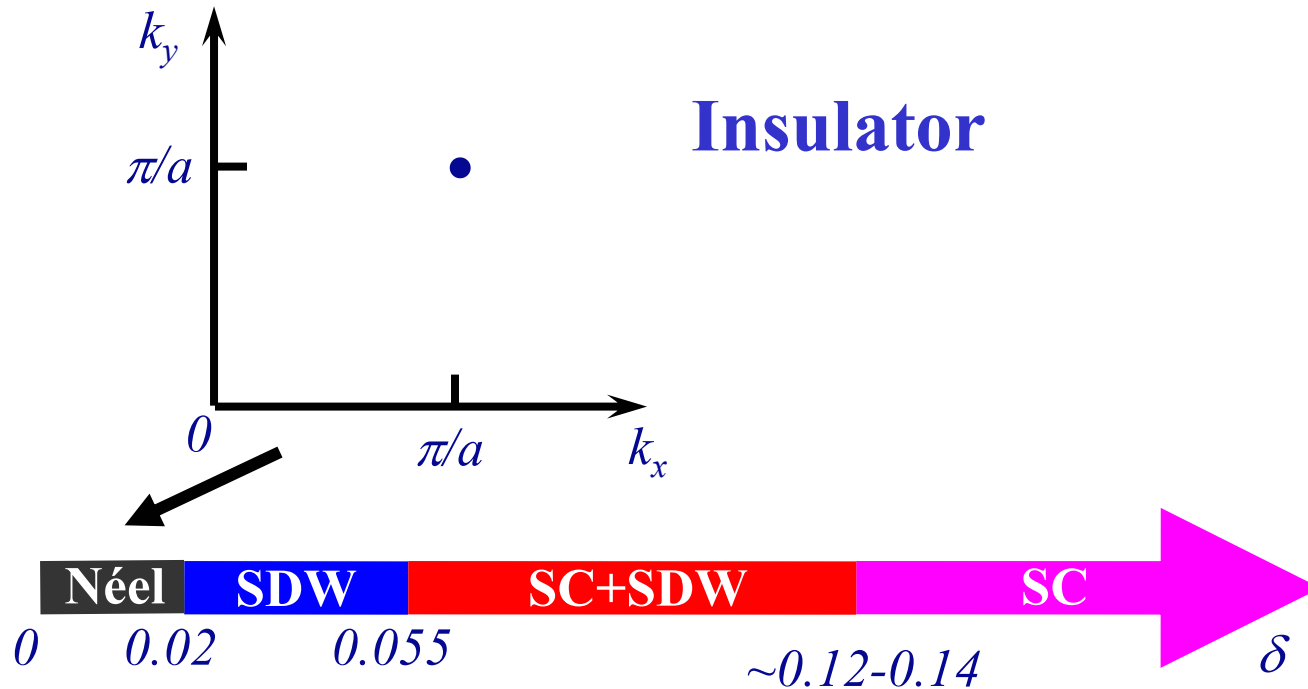


(B) Superconductors

Magnetic transitions in a superconductor:
effect of a magnetic field.

Interplay of SDW and SC order in the cuprates

T=0 phases of LSCO



(additional commensurability effects near $\delta=0.125$)

J. M. Tranquada *et al.*, *Phys. Rev. B* **54**, 7489 (1996).

G. Aeppli, T.E. Mason, S.M. Hayden, H.A. Mook, J. Kulda, *Science* **278**, 1432 (1997).

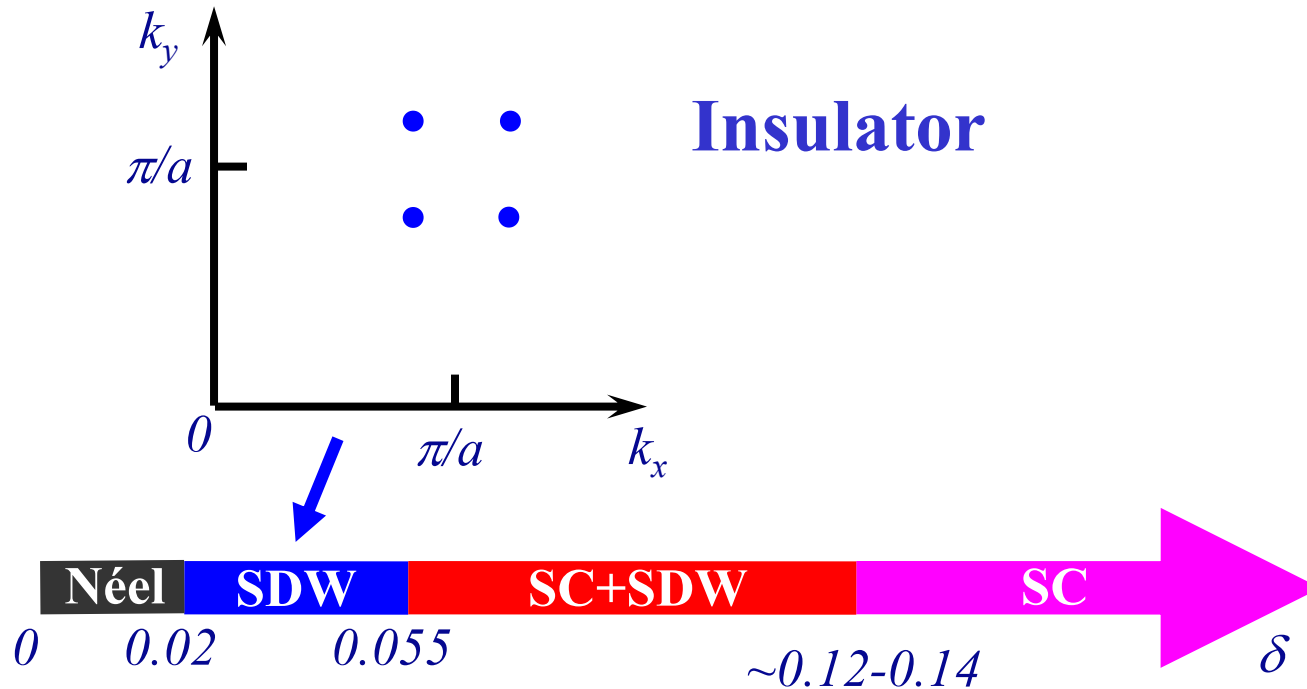
S. Wakimoto, G. Shirane *et al.*, *Phys. Rev. B* **60**, R769 (1999).

Y.S. Lee, R. J. Birgeneau, M. A. Kastner *et al.*, *Phys. Rev. B* **60**, 3643 (1999)

S. Wakimoto, R.J. Birgeneau, Y.S. Lee, and G. Shirane, *Phys. Rev. B* **63**, 172501 (2001).

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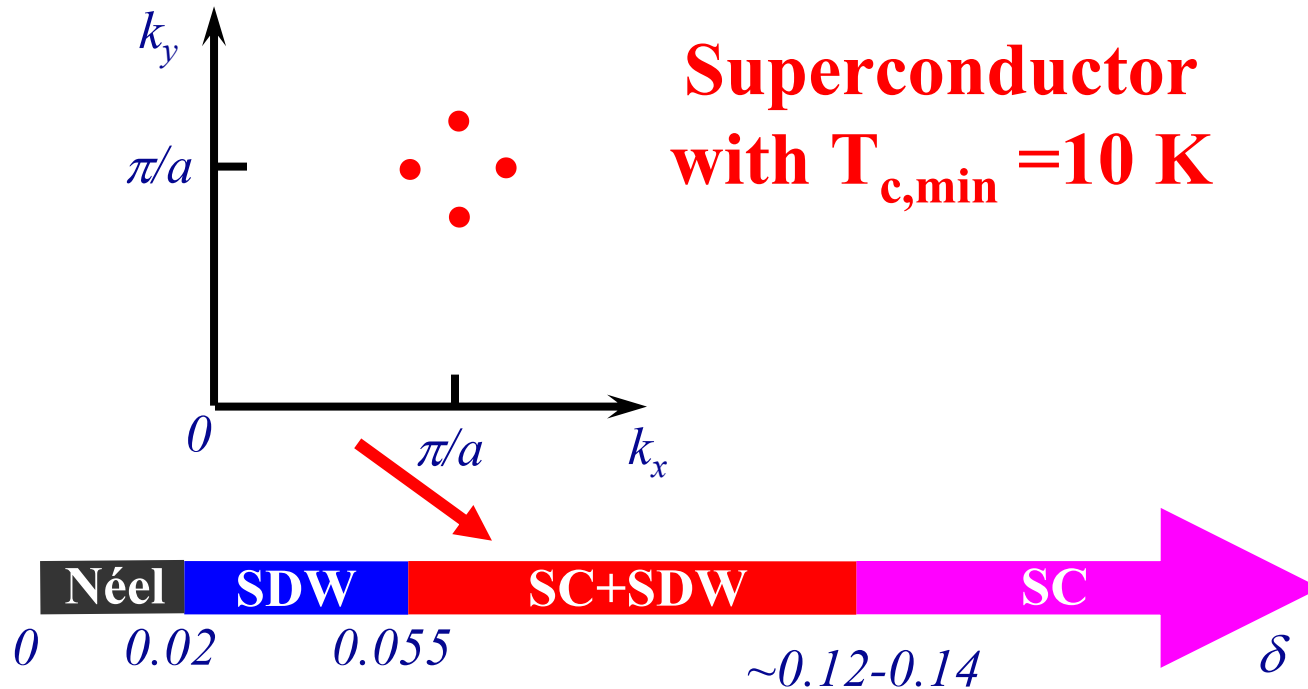
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S. Wakimoto, G. Shirane *et al.*, *Phys. Rev. B* **60**, R769 (1999).

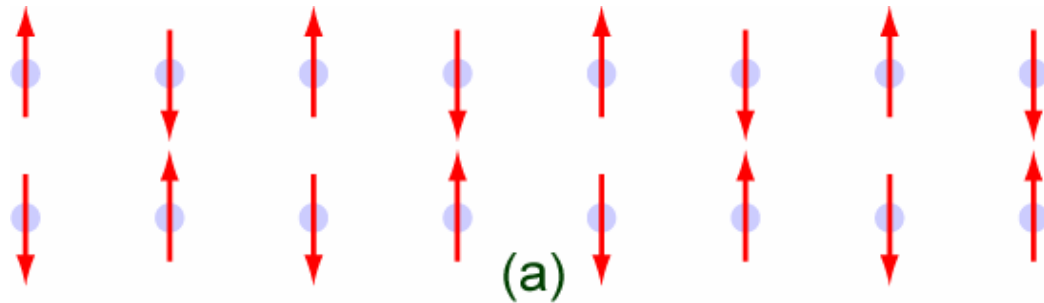
Y.S. Lee, R. J. Birgeneau, M. A. Kastner *et al.*, *Phys. Rev. B* **60**, 3643 (1999)

S. Wakimoto, R.J. Birgeneau, Y.S. Lee, and G. Shirane, *Phys. Rev. B* **63**, 172501 (2001).

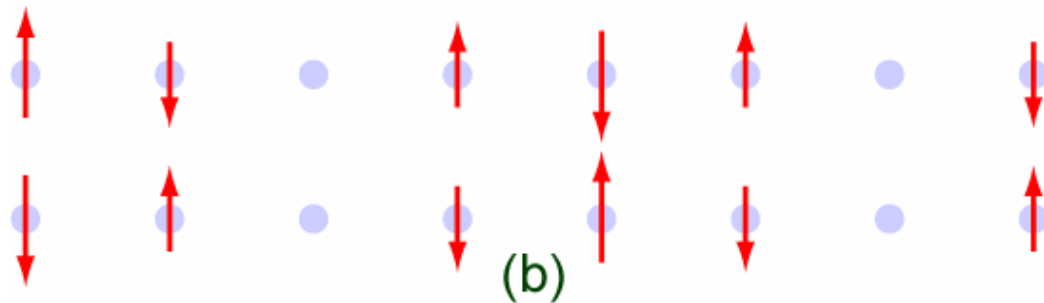
Collinear magnetic (spin density wave) order

$$\langle \mathbf{S}_j \rangle = N_1 \cos(\vec{K} \cdot \vec{r}_j) + N_2 \sin(\vec{K} \cdot \vec{r}_j)$$

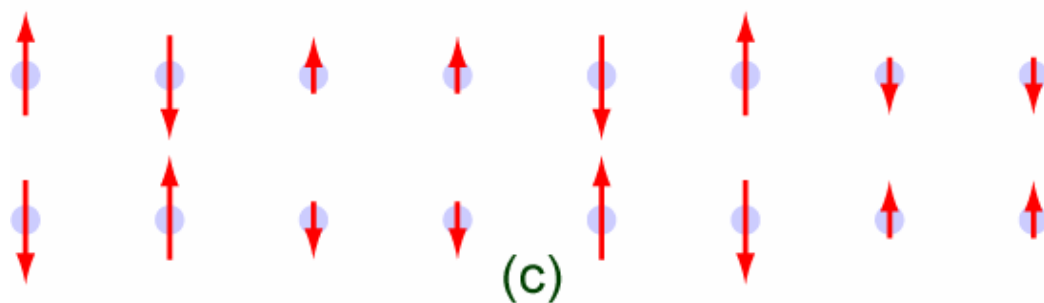
Collinear spins



$$\vec{K} = (\pi, \pi) ; N_2 = 0$$



$$\vec{K} = (3\pi/4, \pi) ; N_2 = 0$$

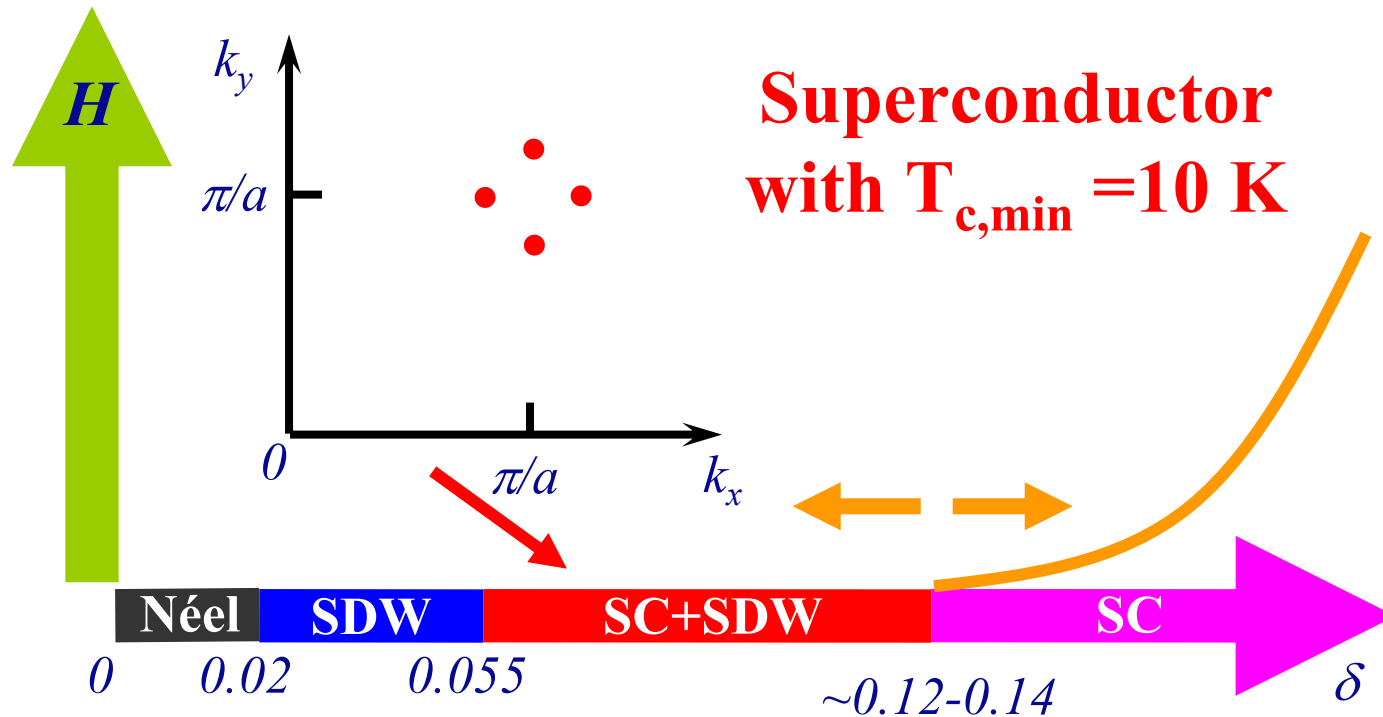


$$\vec{K} = (3\pi/4, \pi) ;$$

$$N_2 = (\sqrt{2} - 1) N_1$$

Interplay of SDW and SC order in the cuprates

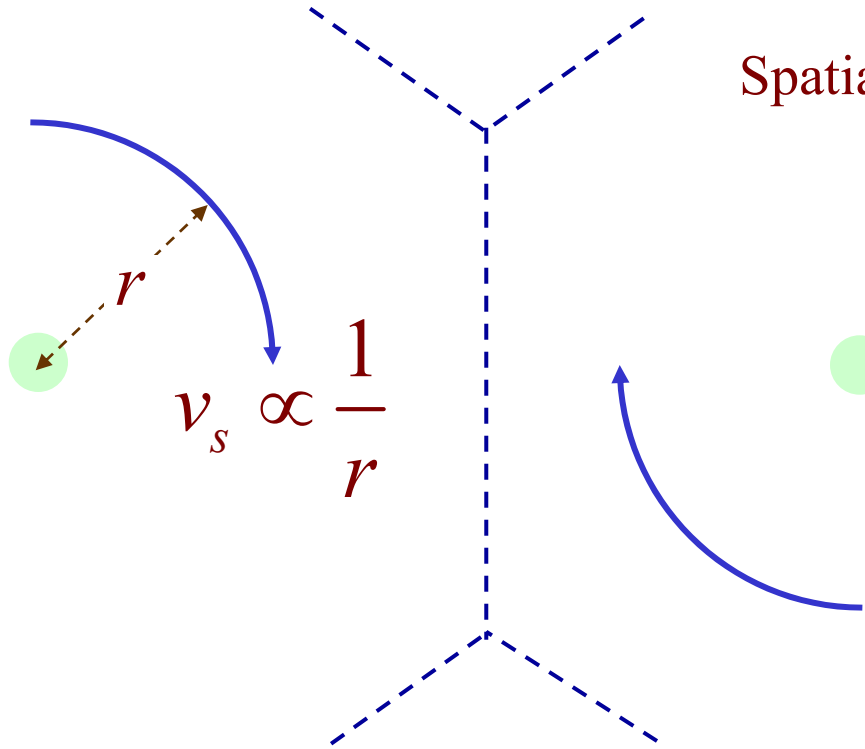
T=0 phases of LSCO



Use simplest assumption of a direct second-order quantum phase transition between SC and SC+SDW phases

Follow intensity of elastic Bragg spots in a magnetic field

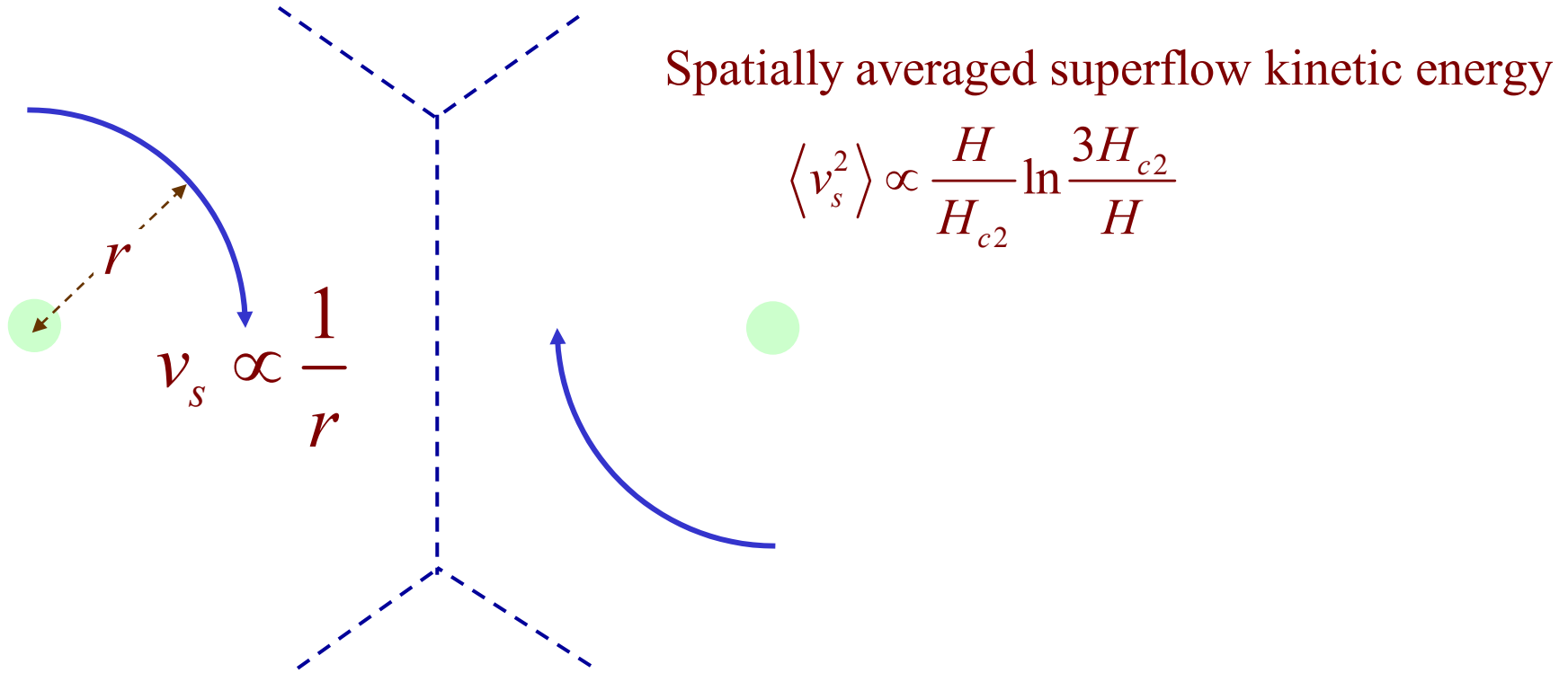
A magnetic field applied to a superconductor induces a lattice of vortices in superflow



Spatially averaged superflow kinetic energy

$$\langle v_s^2 \rangle \propto \frac{H}{H_{c2}} \ln \frac{3H_{c2}}{H}$$

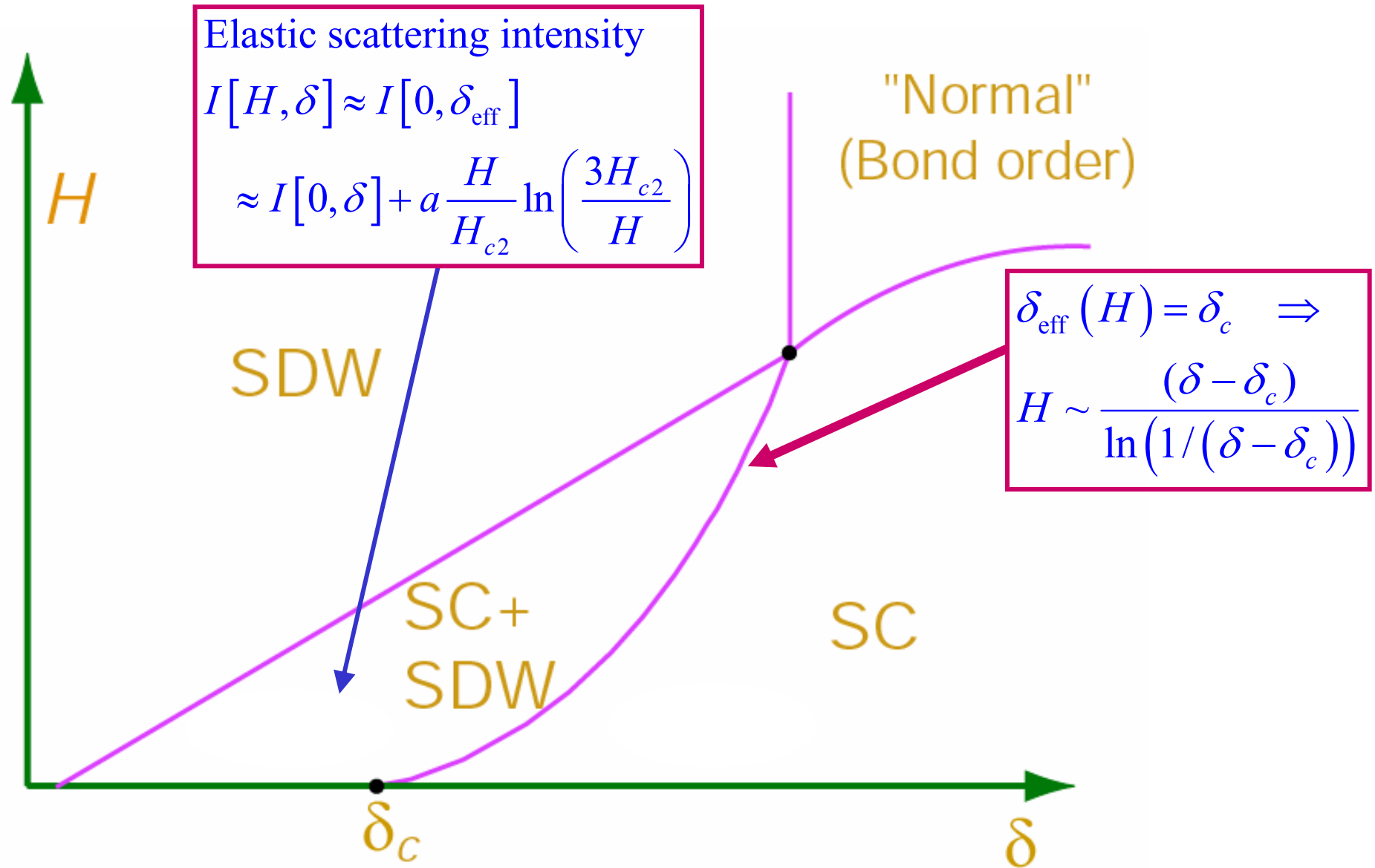
Dominant effect with coexisting superconductivity: **uniform** softening of triplon spin excitations by superflow kinetic energy



The suppression of SC order appears to the SDW order as a **uniform** effective "doping" δ :

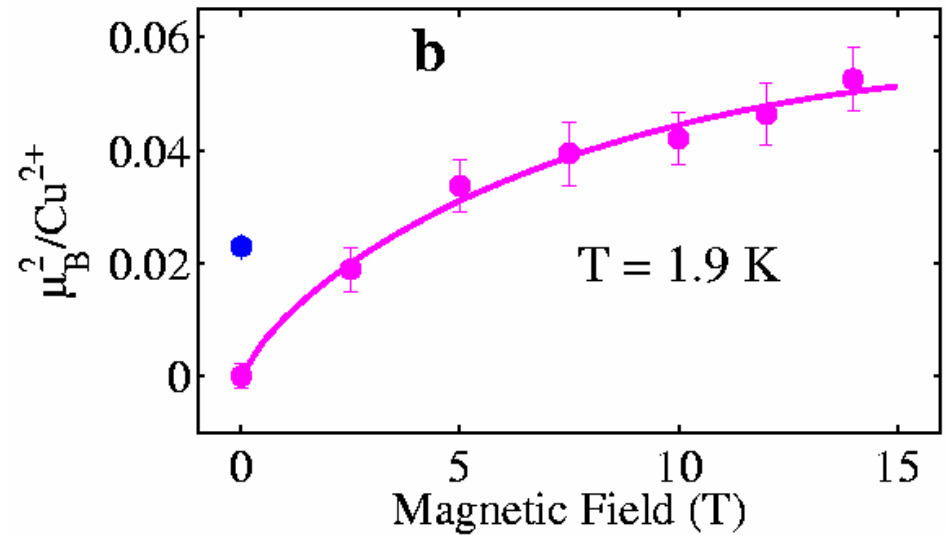
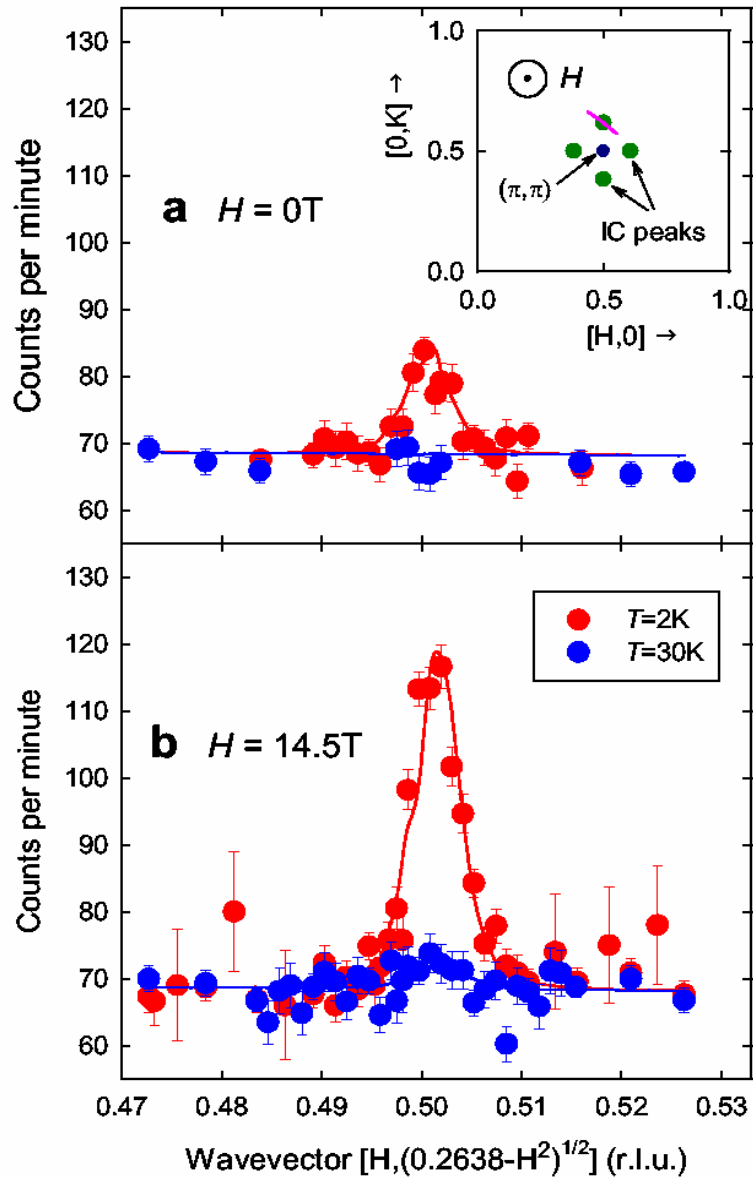
$$\delta_{\text{eff}}(H) = \delta - C \frac{H}{H_{c2}} \ln \left(\frac{3H_{c2}}{H} \right)$$

Phase diagram of SC and SDW order in a magnetic field



Neutron scattering of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ at $x=0.1$

B. Lake, H. M. Rønnow, N. B. Christensen, G. Aeppli, K. Lefmann, D. F. McMorrow, P. Vorderwisch, P. Smeibidl, N. Mangkorntong, T. Sasagawa, M. Nohara, H. Takagi, T. E. Mason, *Nature*, **415**, 299 (2002).



Solid line - fit to :
$$I(H) = a \frac{H}{H_{c2}} \ln \left(\frac{H_{c2}}{H} \right)$$

See also S. Katano, M. Sato, K. Yamada, T. Suzuki, and T. Fukase, *Phys. Rev. B* **62**, R14677 (2000).

Neutron scattering measurements of static spin correlations of the superconductor+spin-density-wave (SC+CM) in a magnetic field

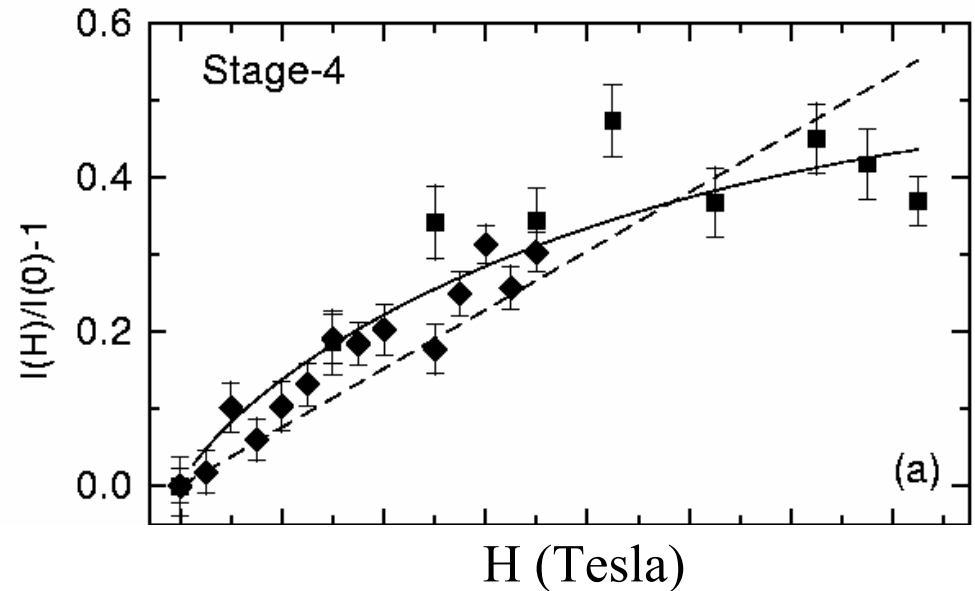
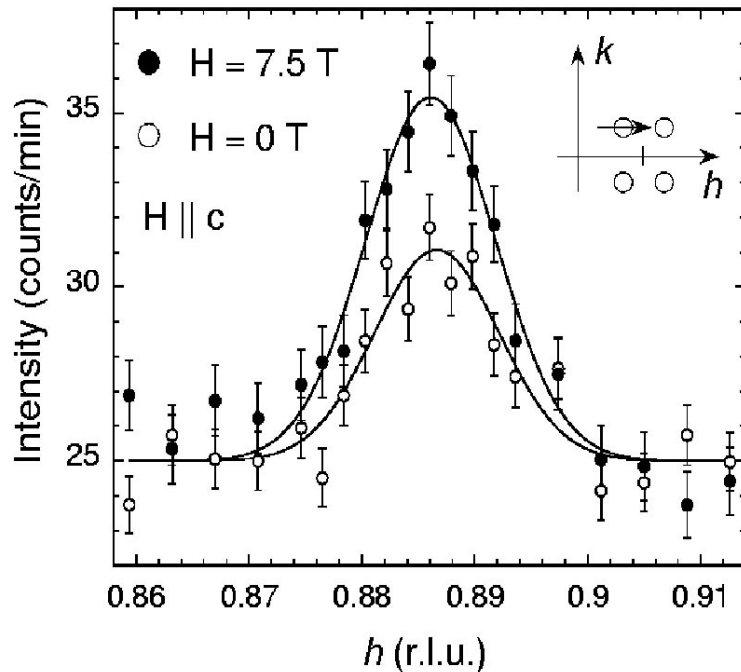
Elastic neutron scattering off $\text{La}_2\text{CuO}_{4+y}$

B. Khaykovich, Y. S. Lee, S. Wakimoto,

K. J. Thomas, M. A. Kastner,

and R.J. Birgeneau, *Phys. Rev. B* **66**,

014528 (2002).

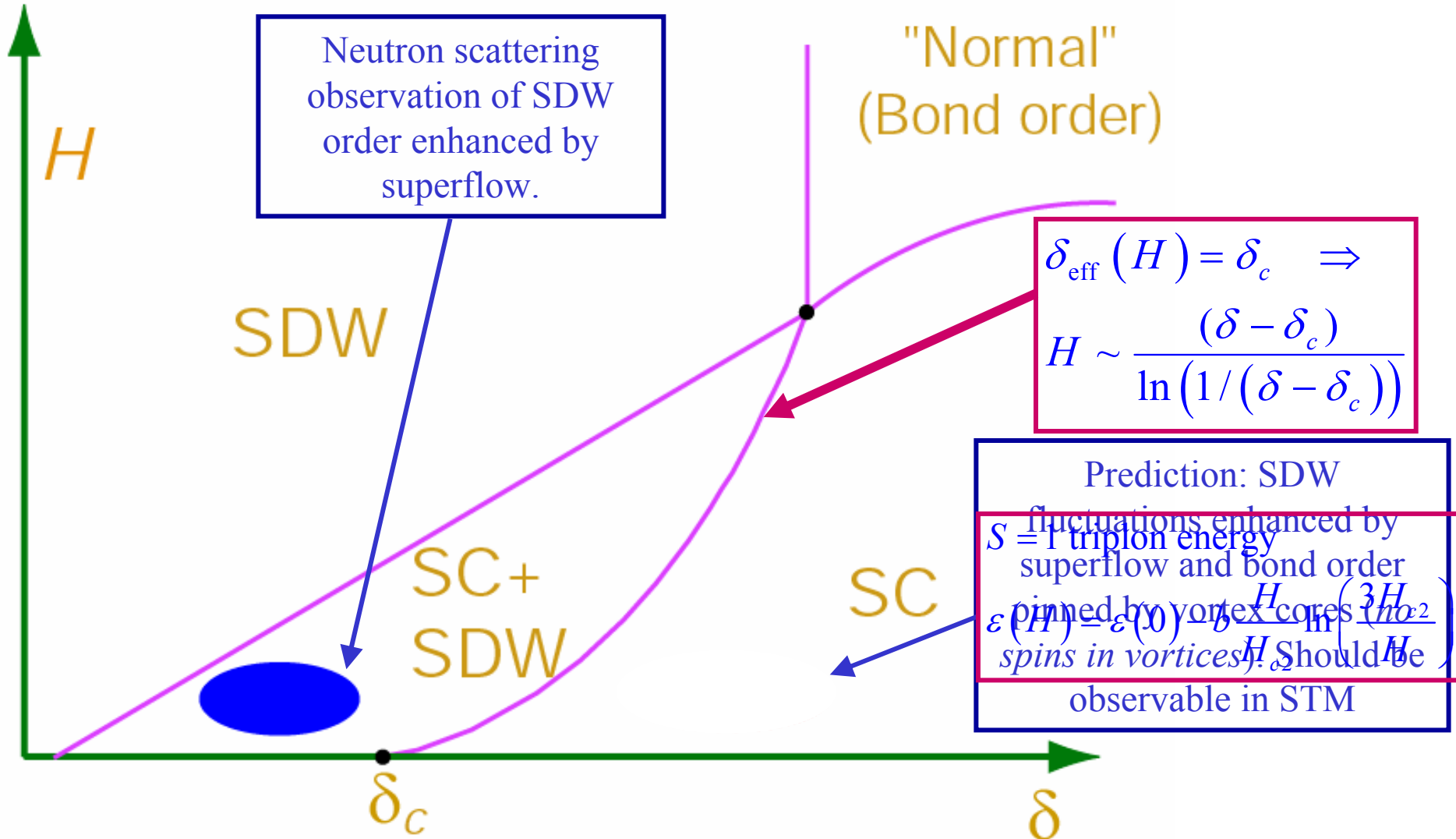


Solid line --- fit to : $\frac{I(H)}{I(0)} = 1 + a \frac{H}{H_{c2}} \ln\left(\frac{3.0H_{c2}}{H}\right)$

a is the only fitting parameter

Best fit value - $a = 2.4$ with $H_{c2} = 60 \text{ T}$

Phase diagram of a superconductor in a magnetic field

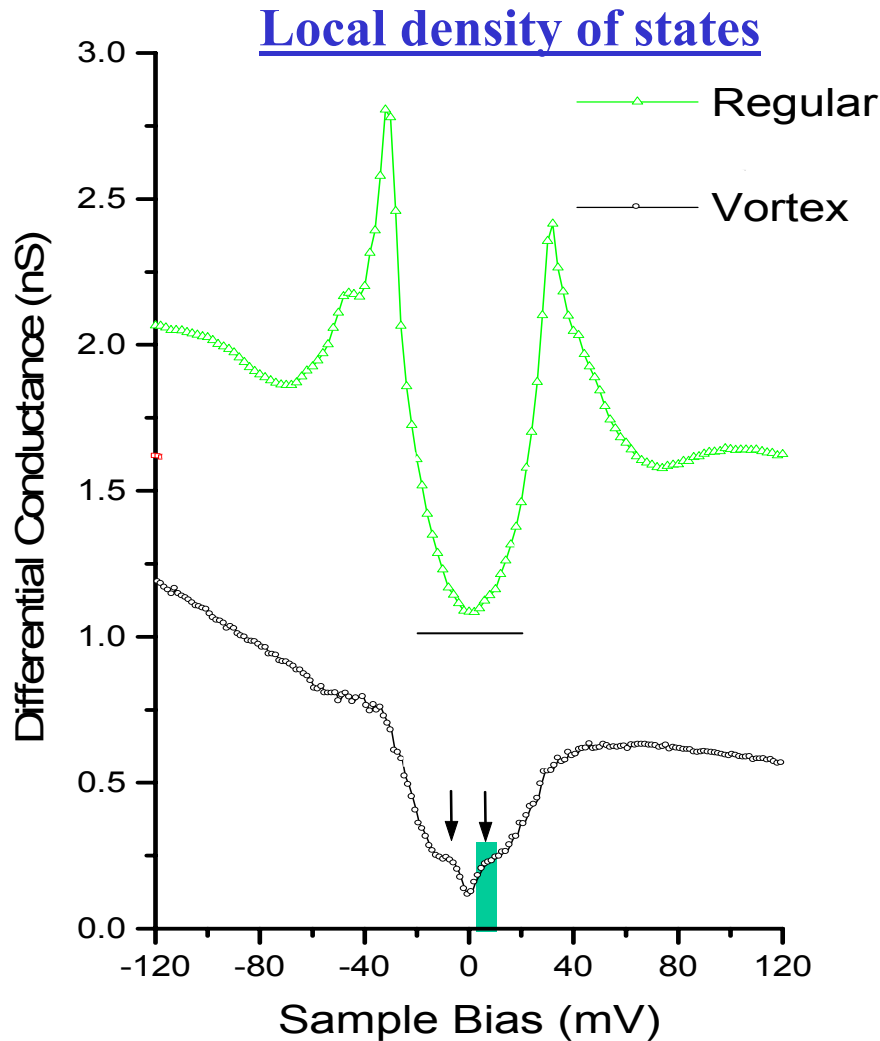


K. Park and S. Sachdev *Physical Review B* **64**, 184510 (2001);

E. Demler, S. Sachdev, and Ying Zhang, *Phys. Rev. Lett.* **87**, 067209 (2001);
 Y. Zhang, E. Demler and S. Sachdev, *Physical Review B* **66**, 094501 (2002).

STM around vortices induced by a magnetic field in the superconducting state

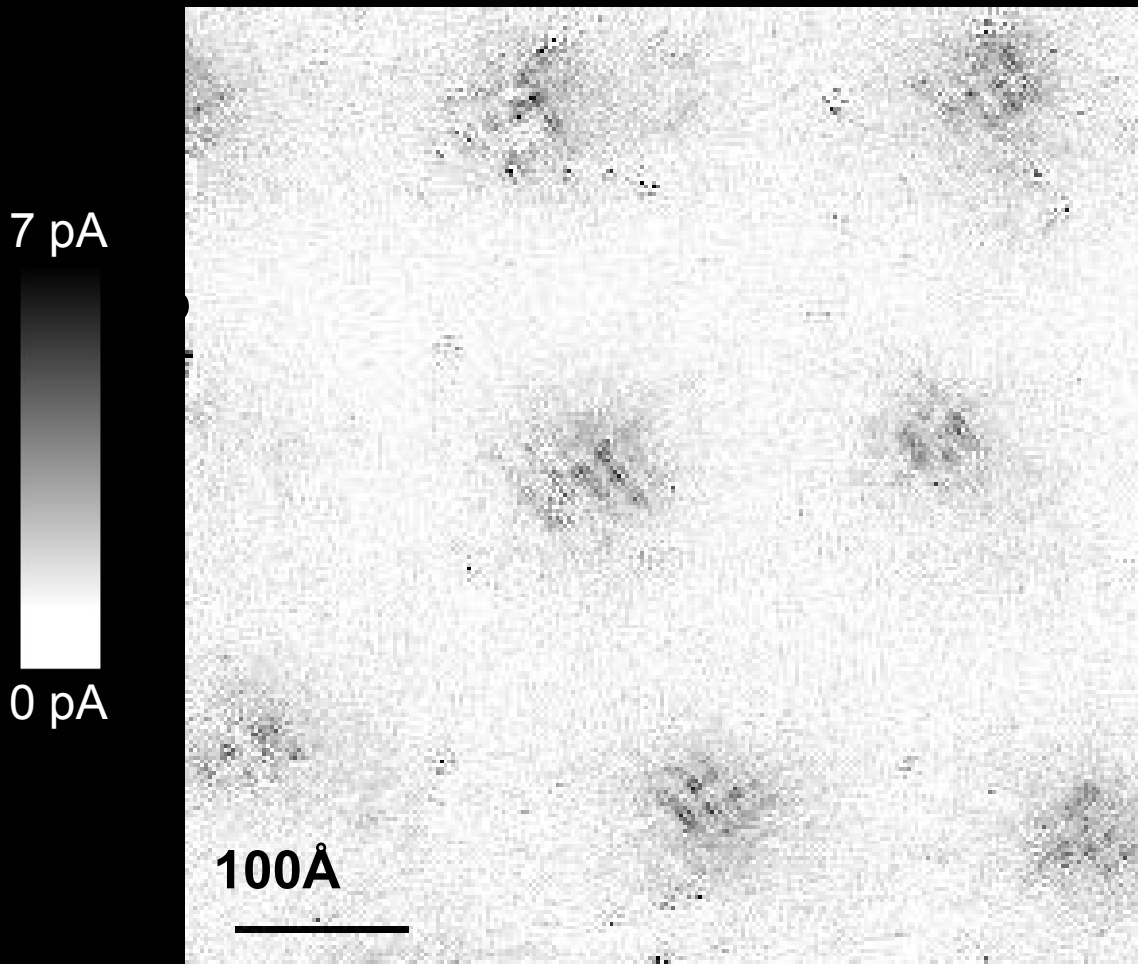
J. E. Hoffman, E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan,
H. Eisaki, S. Uchida, and J. C. Davis, *Science* **295**, 466 (2002).



1Å spatial resolution
image of integrated
LDOS of
 $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$
(1meV to 12 meV)
at B=5 Tesla.

S.H. Pan *et al.* *Phys. Rev. Lett.* **85**, 1536 (2000).

Vortex-induced LDOS of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ integrated from 1meV to 12meV



Our interpretation:
LDOS modulations are
signals of bond order of
period 4 revealed in
vortex halo

See also:

S. A. Kivelson, E. Fradkin,
V. Oganesyan, I. P. Bindloss,
J. M. Tranquada,
A. Kapitulnik, and
C. Howald,
cond-mat/0210683.

J. Hoffman E. W. Hudson, K. M. Lang,
V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida,
and J. C. Davis, *Science* 295, 466 (2002).

Conclusions

- I. Introduction to magnetic quantum criticality in coupled dimer antiferromagnet.
- II. Theory of quantum phase transitions provides semi-quantitative predictions for neutron scattering measurements of spin-density-wave order in superconductors; theory also proposes a connection to STM experiments.