Quantum matter without quasiparticles

Frontiers in Many Body Physics: Memorial for Lev Petrovich Gor’kov
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Ubiquitous “Strange”, “Bad”, or “Incoherent”, metal has a resistivity, \( \rho \), which obeys

\[
\rho \sim T,
\]

and

in some cases \( \rho \gg \hbar/e^2 \)

(in two dimensions), where \( \hbar/e^2 \) is the quantum unit of resistance.
Strange metals just got stranger…

B-linear magnetoresistance!?


P. Giraldo-Gallo et. al., arXiv:1705.05806
Strange metals just got stranger…
Scaling between $B$ and $T$ !?

\[ \rho(H, T) - \rho(0, 0) \propto \sqrt{(\alpha k_B T)^2 + (\gamma \mu_B \mu_0 H)^2} \equiv \Gamma \]

Theories of metallic states without quasiparticles without disorder

• Breakdown of quasiparticles arises from long-wavelength coupling of electrons to some bosonic collective mode. In all cases this can be written in terms of a continuum theory with a conserved momentum. The critical theory has zero resistance, even though the electron quasiparticles do not exist.

• Need to add irrelevant (umklapp) effects to obtain a non-zero resistivity, but this not yield a large linear-in-$T$ resistivity.
Theories of metallic states without quasiparticles in the presence of disorder

- Well-known perturbative theory of disordered metals has 2 classes of known fixed points, the insulator at strong disorder, and the metal at weak disorder. The latter state has long-lived, extended quasiparticle excitations (which are not plane waves).

- **Needed: a metallic fixed point at intermediate disorder and strong interactions without quasiparticle excitations.** Although disorder is present, it largely self-averages at long scales.
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- **SYK models**
The Sachdev-Ye-Kitaev (SYK) model

Pick a set of random positions
The SYK model

Place electrons randomly on some sites
The SYK model

Entangle electrons pairwise randomly
The SYK model

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Entangle electrons pairwise randomly
This describes both a strange metal and a black hole!
The SYK model

(See also: the “2-Body Random Ensemble” in nuclear physics; did not obtain the large $N$ limit; T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. 53, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij; k\ell} c_i^{\dagger} c_j^{\dagger} c_k c_\ell - \mu \sum_i c_i^{\dagger} c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^{\dagger} + c_j^{\dagger} c_i = \delta_{ij}$$

$$Q = \frac{1}{N} \sum_i c_i^{\dagger} c_i$$

$U_{ij; k\ell}$ are independent random variables with $\overline{U_{ij; k\ell}} = 0$ and $|U_{ij; k\ell}|^2 = U^2$

$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL 70, 3339 (1993)
The SYK model

Feynman graph expansion in $J_{ij..}$, and graph-by-graph average, yields exact equations in the large $N$ limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)}, \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$

$$G(\tau = 0^-) = Q.$$
The SYK model

- $T = 0$ fermion Green’s function is singular:

\[ G(\tau) \sim \frac{1}{\sqrt{\tau}} \quad \text{at large } \tau. \]

(Fermi liquids with quasiparticles have $G(\tau) \sim 1/\tau$)

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- $T > 0$ Green’s function has conformal invariance, and is ‘dephased’ at the characteristic scale $\sim k_B T/\hbar$, which is independent of $U$.

$$G \sim e^{-2\pi \mathcal{E} T \tau} \left( \frac{T}{\sin(\pi k_B T \tau / \hbar)} \right)^{1/2}$$

$\mathcal{E}$ measures particle-hole asymmetry.

A. Georges and O. Parcollet PRB 59, 5341 (1999)
S. Sachdev, PRX, 5, 041025 (2015)
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- The last property indicates $\tau_{\text{eq}} \sim \hbar/(k_B T)$, and this has been found in a recent numerical study.

A. Eberlein, V. Kasper, S. Sachdev, and J. Steinberg, arXiv:1706.07803
So the Green’s functions display thermal ‘damping’ at a scale set by $T$ alone, which is independent of $U$. 

\[ G^R(\omega)G^A(\omega) \]

\[ -\text{Re}G^R(\omega) \]

\[ -\text{Im}G^R(\omega) \]

Green’s functions away from half-filling
The basic features can be determined by a simple power-counting. Considering for simplicity scale $T$ cluster interaction of strength modes and nearly temperature with Majorana fermion modes with random all-to-all four-fermion.

Here, we take one step closer to realism by considering a strongly correlated metal. Combining the imaginary time path integral with a zero-dimensional non-Fermi liquid with emergent conformal symmetry and complete absence of quasi- the ubiquity of this phenomenology, strong correlations and quantum fluctuations make it challenging to A strongly correlated metal built from Sachdev-Ye-Kitaev models...
Title: A strongly correlated metal built from Sachdev-Ye-Kitaev models
Authors: Xue-Yang Song, Chao-Ming Jian, Leon Balents

Large $N$ equations

$$G(i\omega_n)^{-1} = i\omega_n + \mu - \Sigma_4(i\omega_n) - zt_0^2 G(i\omega_n),$$

$$\Sigma_4(\tau) = -U_0^2 G(\tau)^2 G(-\tau) ,$$
Title: A strongly correlated metal built from Sachdev-Ye-Kitaev models
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- There is a low coherence scale $E_c \sim t_0^2/U$, with SYK criticality at $T > E_c$ and heavy Fermi liquid behavior for $T < E_c$. 

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- There is a low coherence scale $E_c \sim t_0^2 / U$, with SYK criticality at $T > E_c$ and heavy Fermi liquid behavior for $T < E_c$.

- From the Kubo formula, we have the conductivity

$$\text{Re}[\sigma(\omega)] \propto t_0^2 \int d\Omega \frac{f(\omega + \Omega) - f(\Omega)}{\omega} A(\Omega) A(\omega + \Omega)$$

where $A(\omega) = \text{Im}[G^R(\omega)]$. At $T > E_c$, using $A(\omega) \sim \omega^{-1/2} F(\omega/T)$, this yields the bad metal behavior

$$\sigma \sim \frac{e^2}{h} \frac{t_0^2}{U} \frac{1}{T} ; \quad \rho \sim \left(\frac{h}{e^2}\right) \frac{T}{E_c}$$
arXiv:1712.05026
Title: Magnetotransport in a model of a disordered strange metal
Authors: Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev
Infecting a Fermi liquid and making it SYK

Mobile electrons ($c$, green) interacting with SYK quantum dots ($f$, blue) with exchange interactions. This yields the first model agreeing with magnetotransport in strange metals!

\[
H = -t \sum_{\langle rr' \rangle; \ i=1}^{M} (c_{r'i}^{\dagger}c_{ri} + \text{h.c.}) - \mu_c \sum_{r; \ i=1}^{M} c_{r'i}^{\dagger}c_{ri} - \mu \sum_{r; \ i=1}^{N} f_{ri}^{\dagger}f_{ri}
\]

\[
+ \frac{1}{NM^{1/2}} \sum_{r; \ i,j=1}^{N} \sum_{k,l=1}^{M} g_{ijkl}^{rr'} f_{ri}^{\dagger}f_{rj}^{\dagger}c_{rk}^{\dagger}c_{rl} + \frac{1}{N^{3/2}} \sum_{r; \ i,j,k,l=1}^{N} J_{ijkl}^{rr'} f_{ri}^{\dagger}f_{rj}^{\dagger}f_{rk}f_{rl}.
\]


Similar results in non-random models by Y. Werman, D. Chowdhury, T. Senthil, and E. Berg, to appear
Infected a Fermi liquid and making it SYK

\[
\Sigma(\tau - \tau') = -J^2 G^2(\tau - \tau')G(\tau' - \tau) - \frac{M}{N} g^2 G(\tau - \tau')G^c(\tau - \tau')G^c(\tau' - \tau),
\]

\[
G(i\omega_n) = \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)}, \quad (f \text{ electrons})
\]

\[
\Sigma^c(\tau - \tau') = -g^2 G^c(\tau - \tau')G(\tau - \tau')G(\tau' - \tau),
\]

\[
G^c(i\omega_n) = \sum_k \frac{1}{i\omega_n - \epsilon_k + \mu_c - \Sigma^c(i\omega_n)}, \quad (c \text{ electrons})
\]

Exactly solvable in the large $N,M$ limits!

- Low-$T$ phase: $c$ electrons form a Marginal Fermi-liquid (MFL), $f$ electrons are local SYK models

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Infecteding a Fermi liquid and making it SYK

\[ G_c(\tau) = -\frac{C_c}{\sqrt{1 + e^{-4\pi\mathcal{E}c}}} \left( \frac{T}{\sin(\pi T \tau)} \right)^{1/2} e^{-2\pi\mathcal{E}c T \tau} \]

\[ G(\tau) = -\frac{C}{\sqrt{1 + e^{-4\pi\mathcal{E}}}} \left( \frac{T}{\sin(\pi T \tau)} \right)^{1/2} e^{-2\pi\mathcal{E} T \tau}, \quad 0 \leq \tau < \beta \]

- High-\( T \) phase: \( c \) electrons form an “incoherent metal” (IM), with local Green’s function, and no notion of momentum; \( f \) electrons remain local SYK models.

Infecting a Fermi liquid and making it SYK

• Low-\(T\) phase: \(c\) electrons form a Marginal Fermi-liquid (MFL), \(f\) electrons are local SYK models

\[
\Sigma^c(i\omega_n) = \frac{ig^2\nu(0)T}{2J \cosh^{1/2}(2\pi\mathcal{E})\pi^{3/2}} \left( \frac{\omega_n}{T} \ln \left( \frac{2\pi Te^{\gamma_E-1}}{J} \right) + \frac{\omega_n}{T} \psi \left( \frac{\omega_n}{2\pi T} \right) + \pi \right),
\]

\[
\Sigma^c(i\omega_n) \to \frac{ig^2\nu(0)}{2J \cosh^{1/2}(2\pi\mathcal{E})\pi^{3/2}} \omega_n \ln \left( \frac{|\omega_n|e^{\gamma_E-1}}{J} \right), \quad |\omega_n| \gg T \quad (\nu(0) \sim 1/t)
\]

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Linear-in-$T$ resistivity

*Both* the MFL and the IM are not translationally-invariant and have linear-in-$T$ resistivities!

\[
\sigma_0^{\text{MFL}} = 0.120251 \times MT^{-1} J \times \left( \frac{v_F^2}{g^2} \right) \cosh^{1/2}(2\pi \mathcal{E}). \quad (v_F \sim t)
\]

\[
\sigma_0^{\text{IM}} = \left( \frac{\pi^{1/2}}{8} \right) \times MT^{-1} J \times \left( \frac{\Lambda}{\nu(0)g^2} \right) \frac{\cosh^{1/2}(2\pi \mathcal{E})}{\cosh(2\pi \mathcal{E}_c)}.
\]

[Can be obtained straightforwardly from Kubo formula in the large-$N,M$ limits]

The IM is also a “Bad metal” with \( \sigma_0^{\text{IM}} \ll 1 \)

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Magnetotransport: Marginal-Fermi liquid

- Thanks to large $N,M$, we can also exactly derive the linear-response Boltzmann equation for non-quantizing magnetic fields...

\[
(1 - \partial_\omega \text{Re}[\Sigma_R^c(\omega)]) \partial_t \delta n(t, k, \omega) + v_F \hat{k} \cdot \mathbf{E}(t) = \nu_f(\omega) + v_F (\hat{k} \times \mathbf{B}) \cdot \nabla_k \delta n(t, k, \omega) = 2\delta n(t, k, \omega) \text{Im}[\Sigma_R^c(\omega)],
\]

\[
(\mathcal{B} = eBa^2/\hbar) \text{ (i.e. flux per unit cell)}
\]

\[
\sigma_{L}^{\text{MFL}} = M \frac{v_F^2 \nu(0)}{16T} \int_{-\infty}^{\infty} \frac{dE_1}{2\pi} \text{sech}^2 \left( \frac{E_1}{2T} \right) \frac{-\text{Im}[\Sigma_R^c(E_1)]}{\text{Im}[\Sigma_R^c(E_1)]^2 + (v_F/(2k_F))^2 \mathcal{B}^2},
\]

\[
\sigma_{H}^{\text{MFL}} = -M \frac{v_F^2 \nu(0)}{16T} \int_{-\infty}^{\infty} \frac{dE_1}{2\pi} \text{sech}^2 \left( \frac{E_1}{2T} \right) \frac{(v_F/(2k_F))\mathcal{B}}{\text{Im}[\Sigma_R^c(E_1)]^2 + (v_F/(2k_F))^2 \mathcal{B}^2}.
\]

\[
\sigma_{L}^{\text{MFL}} \sim T^{-1} s_L((v_F/k_F)(\mathcal{B}/T)), \quad \sigma_{H}^{\text{MFL}} \sim -\mathcal{B}T^{-2} s_H((v_F/k_F)(\mathcal{B}/T)).
\]

\[
s_{L,H}(x \to \infty) \propto 1/x^2, \quad s_{L,H}(x \to 0) \propto x^0.
\]

Scaling between magnetic field and temperature in orbital magnetotransport!
Magnetotransport with mesoscopic homogeneity

- No macroscopic momentum, so equations describing charge transport are just
  \[ \nabla \cdot \mathbf{I}(\mathbf{x}) = 0, \quad \mathbf{I}(\mathbf{x}) = \sigma(\mathbf{x}) \cdot \mathbf{E}(\mathbf{x}), \quad \mathbf{E}(\mathbf{x}) = -\nabla \Phi(\mathbf{x}). \]

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• Current path length increases linearly with \( B \) at large \( B \) due to local Hall effect, which causes the global resistance to increase linearly with \( B \) at large \( B \).

Figure 3 Visualization of currents and voltages at large magnetic field in a 10 \times 10 random network of disks with radii 1 (arbitrary units), where the potential difference \( U = -1 \) V. The black arrows represent the currents, and arrow size depicts the magnitude of the current. The major current path is perpendicular to the applied voltage for a significant proportion of the time, which implies that the magnetoresistance is provided internally by the Hall effect, which is therefore linear in \( H \).

Solvable toy model: two-component disorder

- Two types of domains \(a, b\) with different carrier densities and lifetimes randomly distributed in approximately equal fractions over sample.

- Effective medium equations can be solved exactly

\[
\left( I + \frac{\sigma^a - \sigma^e}{2\sigma^e_L} \right)^{-1} \cdot (\sigma^a - \sigma^e) + \left( I + \frac{\sigma^b - \sigma^e}{2\sigma^e_L} \right)^{-1} \cdot (\sigma^b - \sigma^e) = 0.
\]

\[
\rho^e_L \equiv \frac{\sigma^e_L}{\sigma^e_L + \sigma^e_H} = \frac{\sqrt{(B/m)^2 \left( \gamma_a \sigma^\text{MFL}_{0a} - \gamma_b \sigma^\text{MFL}_{0b} \right)^2 + \gamma_a^2 \gamma_b^2 \left( \sigma^\text{MFL}_{0a} + \sigma^\text{MFL}_{0b} \right)^2}}{\gamma_a \gamma_b (\sigma^\text{MFL}_{0a} \sigma^\text{MFL}_{0b})^{1/2} (\sigma^\text{MFL}_{0a} + \sigma^\text{MFL}_{0b})},
\]

\[
\rho^e_H \equiv -\frac{\sigma^e_H/B}{\sigma^e_L + \sigma^e_H} = \frac{\gamma_a + \gamma_b}{m \gamma_a \gamma_b (\sigma^\text{MFL}_{0a} + \sigma^\text{MFL}_{0b})} \cdot (m = k_F/v_F \sim 1/t)
\]

\(\gamma_{a,b} \sim T^*\) (i.e. effective transport scattering rates)

\[
\rho^e_L \sim \sqrt{c_1 T^2 + c_2 B^2}
\]

Scaling between \(B\) and \(T\) at microscopic orbital level has been transferred to global MR!
Engineered a model of a Fermi surface coupled to SYK quantum dots which leads to a marginal Fermi liquid with a linear-in-$T$ resistance, with a magnetoresistance which scales as $B \sim T$. Mesoscopic disorder then leads to linear-in-$B$ magnetoresistance, and a combined dependence which scales as $\sim B^2 + T^2$. Higher temperatures lead to an incoherent metal with a local Green's function and a linear-in-$T$ resistance, but negligible magnetoresistance.
• Engineered a model of a Fermi surface coupled to SYK quantum dots which leads to a marginal Fermi liquid with a linear-in-$T$ resistance, with a magnetoresistance which scales as $B \sim T$.

• Mesoscopic disorder then leads to linear-in-$B$ magnetoresistance, and a combined dependence which scales as $\sim \sqrt{B^2 + T^2}$.
Magnetotransport in strange metals

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• Mesoscopic disorder then leads to linear-in-$B$ magnetoresistance, and a combined dependence which scales as $\sim \sqrt{B^2 + T^2}$

• Higher temperatures lead to an incoherent metal with a local Green’s function and a linear-in-$T$ resistance, but negligible magnetoresistance.
This simple two-component model describes a new state of matter which is realized by electrons in the presence of strong interactions and disorder.

Can such a model be realized as a fixed-point of a generic theory of strongly-interacting electrons in the presence of disorder?

Can we start from a single-band Hubbard model with disorder, and end up with such two-band fixed point, with emergent local conservation laws?
Electrons in doped silicon appear to separate into two components: localized spin moments and itinerant electrons.

**FIG. 1.** Temperature dependence of normalized susceptibility $\chi/\chi_{Pauli}$ of three Si:P:B samples with different normalized electron densities, $n/n_c = 0.58$, 1.1, and 1.8. Solid lines through data are a guide to the eye.

