Strange metals and black holes

Michigan State University, East Lansing
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Subir Sachdev

Talk online: sachdev.physics.harvard.edu
Quantum phase transitions

Black holes

Metals, ordinary and strange

Quantum criticality in the cuprates
Quantum phase transitions

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Metals, ordinary and strange

Quantum criticality in the cuprates

The holographic connection between strange metals and black holes
Black Holes

Objects so dense that light is gravitationally bound to them.

In Einstein’s theory, the region inside the black hole horizon is disconnected from the rest of the universe.

Horizon radius $R = \frac{2GM}{c^2}$

$G$ Newton’s constant, $c$ velocity of light, $M$ mass of black hole
In a landmark study, scientists at Delft University of Technology in the Netherlands reported that they had conducted an experiment that they say proved one of the most fundamental claims of quantum theory — that objects separated by great distance can instantaneously affect each other’s behavior.

Part of the laboratory setup for an experiment at Delft University of Technology, in which two diamonds were set 1.3 kilometers apart, entangled and then shared information.
Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:

\[ \begin{array}{c}
\bullet \\
\uparrow
\end{array} \]

Hydrogen molecule:

\[ \begin{array}{c}
\bullet \\
\uparrow
\end{array} = \begin{array}{c}
\bullet \\
\uparrow
\end{array} - \begin{array}{c}
\bullet \\
\downarrow
\end{array} = \frac{1}{\sqrt{2}} \left( \begin{array}{c}
\uparrow \downarrow \\
\uparrow \downarrow
\end{array} - \begin{array}{c}
\downarrow \uparrow \\
\downarrow \uparrow
\end{array} \right) \]
Quantum Entanglement: quantum superposition with more than one particle
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Quantum Entanglement: quantum superposition with more than one particle
Einstein-Podolsky-Rosen “paradox” (1935): Measurement of one particle instantaneously determines the state of the other particle arbitrarily far away.
Quantum Entanglement across a black hole horizon
Quantum Entanglement across a black hole horizon
Quantum Entanglement across a black hole horizon
Quantum Entanglement across a black hole horizon

There is quantum entanglement between the inside and outside of a black hole.
Hawking used this to show that black hole horizons have an entropy and a temperature (because to an outside observer, the state of the electron inside the black hole is an unknown).
Black holes have an entropy and a temperature, $T_H$

The entropy is proportional to their surface area.

J. D. Bekenstein, PRD 7, 2333 (1973)
On September 14, 2015, LIGO detected the merger of two black holes, each weighing about 30 solar masses, with radii of about 100 km, 1.3 billion light years away.
The ring-down time $\frac{8\pi GM}{c^3} \sim 8$ milliseconds. Curiously, for essentially all types of black holes, the ring-down time equals $\frac{\hbar}{k_B T_H}$, where $\hbar$ is Planck’s constant, and $k_B$ is Boltzmann’s constant.
Black holes have an entropy and a temperature, $T_H$.

The entropy is proportional to their surface area.

They relax to thermal equilibrium in a Planckian time $\sim \hbar/(k_B T_H)$. 
Ordinary metals are shiny, and they conduct heat and electricity efficiently. Each atom donates electrons which are delocalized throughout the entire crystal.
Almost all many-electron systems are described by the quasiparticle concept: a quasiparticle is an “excited lump” in the many-electron state which responds just like an ordinary particle.
• The resistivity, \( \rho \), of a metal from the flow of quasiparticles is

\[
\rho = \frac{m^*}{n e^2 \tau}
\]

where \( m^* \) is the effective mass of a quasiparticle, \( n \) is the density of electrons, \( e \) is the charge of an electron, and \( \tau \) is a quasiparticle scattering time.

The theory of ordinary metals implies that as the temperature \( T \to 0 \)

\[
\tau \sim \frac{1}{T^2} \gg \frac{\hbar}{k_B T}
\]
High temperature superconductors

\[ \text{YBa}_2\text{Cu}_3\text{O}_{6+x} \]
Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University
Insulating antiferromagnet
Antiferromagnet doped with hole density $p$
Strange Metal

Temperature (K)
AF insulator

Superconductor

Hole doping, ρ

Inset: Structure with labels for Cu$^{2+}$, Cu$^{3+}$, O$^{2-}$, Y$^{3+}$, Ba$^{2+}$, and lattice parameters.
Remarkable recent observation of ‘Planckian’ strange metal transport in cuprates, pnictides, magic-angle graphene, and ultracold atoms: the resistivity, $\rho$, is

$$\rho = \frac{m^*}{n e^2} \frac{1}{\tau}$$

with a universal scattering rate

$$\frac{1}{\tau} \approx \frac{k_B T}{\hbar} ,$$

independent of the strength of interactions!
<table>
<thead>
<tr>
<th>Material</th>
<th>$n$ (10^{27} \text{m}^{-3})$</th>
<th>$m^*$ ($m_0$)</th>
<th>$A_1 / d$ (\Omega / K)</th>
<th>$h / (2e^2 T_F)$ (\Omega / K)</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bi2212</td>
<td>$p = 0.23$</td>
<td>6.8</td>
<td>8.4 ± 1.6</td>
<td>8.0 ± 0.9</td>
<td>7.4 ± 1.4</td>
</tr>
<tr>
<td>Bi2201</td>
<td>$p \sim 0.4$</td>
<td>3.5</td>
<td>7 ± 1.5</td>
<td>8 ± 2</td>
<td>8 ± 2</td>
</tr>
<tr>
<td>LSCO</td>
<td>$p = 0.26$</td>
<td>7.8</td>
<td>9.8 ± 1.7</td>
<td>8.2 ± 1.0</td>
<td>8.9 ± 1.8</td>
</tr>
<tr>
<td>Nd-LSCO</td>
<td>$p = 0.24$</td>
<td>7.9</td>
<td>12 ± 4</td>
<td>7.4 ± 0.8</td>
<td>10.6 ± 3.7</td>
</tr>
<tr>
<td>PCCO</td>
<td>$x = 0.17$</td>
<td>8.8</td>
<td>2.4 ± 0.1</td>
<td>1.7 ± 0.3</td>
<td>2.1 ± 0.1</td>
</tr>
<tr>
<td>LCCO</td>
<td>$x = 0.15$</td>
<td>9.0</td>
<td>3.0 ± 0.3</td>
<td>3.0 ± 0.45</td>
<td>2.6 ± 0.3</td>
</tr>
<tr>
<td>TMTSF</td>
<td>$P = 11$ kbar</td>
<td>1.4</td>
<td>1.15 ± 0.2</td>
<td>2.8 ± 0.3</td>
<td>2.8 ± 0.4</td>
</tr>
</tbody>
</table>

Slope of $T$-linear resistivity vs Planckian limit in seven materials.

\[ \frac{1}{\tau} = \alpha \frac{k_B T}{\hbar} \]

Black holes

Metals, ordinary and strange

Quantum criticality in the cuprates
Strange Metal

Temperature (K)

Hole doping, $\rho$

AF insulator

Superconductor

$T_N$

$T^*$

$T_c$
The long-range entanglement in the “quantum critical” state near $p = p_c$ leads to the non-quasiparticle current flow with relaxation time $\sim \frac{\hbar}{k_B T}$ in the strange metal. Is there a quantum phase transition at a critical $p = p_c$?
Real-space view at small $p$

$p$ mobile holes in a background of fluctuating spins
Real-space view at small $p$

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$p$ mobile holes in a background of fluctuating spins
Real-space view at small $p$

$p$ mobile holes in a background of fluctuating spins
Momentum-space view at large $p$

$1+p$ mobile holes in a filled band
Momentum-space view at large $p$.

The low-energy electronic structure of very overdoped Tl$_2$201 is dominated by a single CuO band. In this sense, the Tl-Cu nonstoichiometry and the presence of an overdoped Tl$_2$2201 sample could not be followed via the shift of the lattice constant. With parameters determined by the total FS volume, the TlO band is emptied of its electrons and the LDA FS reduces to the potential in the calculations in a rigid-band-like fashion to match the doping level of our Tl$_2$2201-OD30 sample (as found by the ARPES intensity at $T=10$ K, week ending 077001).

For equivalent doping, all indicated that the low-energy electronic structure of very overdoped Tl$_2$2201 is dominated by a single CuO band. In this sense, the Tl-Cu nonstoichiometry and the presence of an overdoped Tl$_2$2201 sample could not be followed via the shift of the lattice constant. With parameters determined by the total FS volume, the TlO band is emptied of its electrons and the LDA FS reduces to the potential in the calculations in a rigid-band-like fashion to match the doping level of our Tl$_2$2201-OD30 sample (as found by the ARPES intensity at $T=10$ K, week ending 077001).

As discussed, with a behavior which is ubiquitous among the cuprates, the presence of a peak at $E_F$; near $\pi/2$ for the lattice constant. With parameters determined by the total FS volume, the TlO band is emptied of its electrons and the LDA FS reduces to the potential in the calculations in a rigid-band-like fashion to match the doping level of our Tl$_2$2201-OD30 sample (as found by the ARPES intensity at $T=10$ K, week ending 077001).

This, however, is no surprise even within the independent particle picture. In fact, adjusting the chemical potential in the calculations in a rigid-band-like fashion to match the doping level of our Tl$_2$2201-OD30 sample (as found by the ARPES intensity at $T=10$ K, week ending 077001).
The remarkable underlying ground states of cuprate superconductors

Cyril Proust and Louis Taillefer, arXiv:1807.0507

Figure 6

Across the quantum critical point. (a) Normal-state electronic specific heat in the $T=0$ limit as a function of doping, plotted as $C_{el}/T$ vs $p$ (red symbols) in Eu-LSCO (squares), Nd-LSCO (circles) and LSCO (diamonds). From ref. (75). We also show $C_{el}/T$ in YBCO (blue dots (18)) and in Tl2201 (green dot (76)). The vertical grey lines mark the limits of the CDW phase in Nd-LSCO, between $p=0.08$ and $p'>0.19$.

(b) Normal-state Hall number $n_H$ in the $T=0$ limit from doping, in YBCO (blue circles (21), $p'=0.19$) and Nd-LSCO (red squares (4), $p'=0.23$). We also show $n_H$ in LSCO (grey squares (67)) and YBCO (grey circles (68)) at low doping, and $n_H$ in Tl2201 (white diamond (29)) at high doping.

5. PSEUDOGAP PHASE

DOS: Density of states ($N_F$)

$E$: Condensation energy

$H_c$: Lower critical field

$C(T)$: Residual linear term in the specific heat, $C(T)$ at $T=0$

The two traditional signatures of the pseudogap phase are: 1) a loss of density of states (DOS) below $p'$; 2) the opening of a partial spectral gap below $T'$, see by ARPES (Figs. 1c, 1d) and optical conductivity, for example. Here we summarize recent high-field measurements of the specific heat in the LSCO family (75) showing that there is a large mass enhancement at $p'$. The new data show that the pseudogap does not simply cause a loss of DOS below $p'$; instead, there is huge peak in the DOS at $p'$ (Fig. 6a) – much larger than expected from a van Hove singularity (75, 80). We then show how high-field measurements of the Hall coefficient reveal a new signature of the pseudogap phase – a rapid drop in the carrier density, at $p'$ (Fig. 6c). These new properties alter profoundly our view of the pseudogap phase, and of the strange metal just above it (sec. 6).

5.1. Density of states

5.1.1. Condensation energy.

One way to access the DOS, $N_F$, is via the superconducting condensation energy $E$, since $E = N_F (20/4)$, where $0$ is the $d$-wave gap maximum. Experimentally, and in the framework of BCS theory, $E$ can be measured using the upper and lower critical fields, $H_{c2}$ and $H_{c1}$, to get the thermodynamic field $H_c$ via $H_c^2 = H_{c1} H_{c2} / (\ln(\pi) + 0.5)$, given that $E = H_{c2}^2 / 2 \mu_0$. In Fig. 2b, we plot $E/T_{c2}$ vs $p$ thus obtained for YBCO (17). We see that $E/T_{c2}/N_F$ drops by a factor 8-9 between $p=0.18$ and $p'=0.1$, in agreement with the drop reported earlier from an analysis of specific heat data measured in low fields up to $T>T_c$ in YBCO (71) and Bi2212 (72). Note
Hole doped cuprates


Hidden magnetism at the pseudogap critical point of a high temperature superconductor

Mehdi Frachet\textsuperscript{1,}\textsuperscript{†}, Igor Vinograd\textsuperscript{1,}\textsuperscript{†}, Rui Zhou\textsuperscript{1,2}, Siham Benhabib\textsuperscript{1}, Shangfei Wu\textsuperscript{1}, Hadrien Mayaffre\textsuperscript{1}, Steffen Krämer\textsuperscript{1}, Sanath K. Ramakrishna\textsuperscript{3}, Arneil P. Reyes\textsuperscript{3}, Jérôme Debray\textsuperscript{4}, Tohru Kurosawa\textsuperscript{5}, Naoki Momono\textsuperscript{6}, Migaku Oda\textsuperscript{5}, Seiki Komiy\textsuperscript{7}, Shimpei Ono\textsuperscript{7}, Masafumi Horio\textsuperscript{8}, Johan Chang\textsuperscript{8}, Cyril Proust\textsuperscript{1}, David LeBoeuf\textsuperscript{1,}\textsuperscript{*}, Marc-Henri Julien\textsuperscript{1,}\textsuperscript{*}

\texttt{arXiv:1909.10258}

Quasi-static magnetism in the pseudogap state of La\textsubscript{2-x}Sr\textsubscript{x}CuO\textsubscript{4}. Temperature – doping phase diagram representing $T_{\text{min}}$, the temperature of the minimum in the sound velocity, at different fields. Since superconductivity precludes the observation of $T_{\text{min}}$ in zero-field, the dashed line (brown area) represents the extrapolated $T_{\text{min}}(B=0)$. While not exactly equal to the freezing temperature $T_{\text{f}}$ (see Fig. 2), $T_{\text{min}}$ is closely tied to $T_{\text{f}}$ and so is expected to have the same doping dependence, including a peak around $p = 0.12$ in zero/low fields (ref. 2). Onset temperatures of charge order are from ref. 33 (squares) and 35 (hexagons).
Square lattice of Cu sites at $p=p_c$

Remove fraction $p$ electrons

\[ = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \]
Square lattice of Cu sites at $p=p_c$

Electrons entangle in ("Cooper") pairs into chemical bonds

$\begin{array}{c}
\text{ Electrons} \\
\text{entangle in} \\
\text{("Cooper")} \\
\text{pairs into} \\
\text{chemical} \\
\text{bonds} \\
\end{array}$
Electrons entangle “en masse” by exchanging partners, and there is long-range quantum entanglement

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$$\begin{array}{c}
\text{= } |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle
\end{array}$$
Electrons entangle “en masse” by exchanging partners, and there is long-range quantum entanglement.
Electrons entangle “en masse” by exchanging partners, and there is long-range quantum entanglement.
We consider the hole-doped case, with no double occupancy.

\[ \alpha = \uparrow, \downarrow, \quad \{ c_{i\alpha}, c_{j\beta}^{\dagger} \} = \delta_{ij} \delta_{\alpha\beta}, \quad \{ c_{i\alpha}, c_{j\beta} \} = 0 \]

\[ \vec{S}_i = \frac{1}{2} c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \sum_{\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} \leq 1, \quad \frac{1}{N} \sum_{i\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} = 1 - p \]
The Sachdev-Ye-Kitaev (SYK) model


Variation described in
We consider the hole-doped case, with no double occupancy.

\[ H = \sum_{i,j=1}^{N} t_{ij} c_{i\alpha}^{\dagger} c_{j\alpha} + \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

\[ \alpha = \uparrow, \downarrow, \{ c_{i\alpha}, c_{j\beta}^{\dagger} \} = \delta_{ij} \delta_{\alpha\beta}, \{ c_{i\alpha}, c_{j\beta} \} = 0 \]

\[ \vec{S}_i = \frac{1}{2} c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \sum_{\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} \leq 1, \frac{1}{N} \sum_{i\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} = 1 - p \]
\[ H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

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\[ J_{ij} \text{ random, } \overline{J_{ij}} = 0, \quad \overline{J_{ij}^2} = J^2 \]
\[ t_{ij} \text{ random, } \overline{t_{ij}} = 0, \quad \overline{t_{ij}^2} = t^2 \]
Allow electron motion and bond exchange between ANY pair of sites, all with a random amplitude.
Allow electron motion and bond exchange between ANY pair of sites, all with a random amplitude.

\[ |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \]
Allow electron motion and bond exchange between ANY pair of sites, all with a random amplitude.
Allow electron motion and bond exchange between ANY pair of sites, all with a random amplitude.

\[ = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \]

**Diagram:**
- Blue circles represent sites.
- Green arrows indicate the direction of electron motion.
- Dotted lines represent bond exchange between sites.
- The equation \[ = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \] is shown below the diagram.
Allow electron motion and bond exchange between ANY pair of sites, all with a random amplitude.

\[
\begin{bmatrix}
|1\uparrow\downarrow\rangle \\
|1\downarrow\uparrow\rangle \\
\end{bmatrix}
\]
Allow electron motion and bond exchange between ANY pair of sites, all with a random amplitude.
Allow electron motion and bond exchange between ANY pair of sites, all with a random amplitude.
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Allow electron motion and bond exchange between ANY pair of sites, all with a random amplitude.

\[ |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle = 0 \]
Allow electron motion and bond exchange between ANY pair of sites, all with a random amplitude
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Allow electron motion and bond exchange between ANY pair of sites, all with a random amplitude.

\[ \begin{align*}
  \text{OVal} &= |\uparrow\downarrow\downarrow\rangle - |\downarrow\downarrow\uparrow\rangle
\end{align*} \]
Allow electron motion and bond exchange between ANY pair of sites, all with a random amplitude.
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Allow electron motion and bond exchange between ANY pair of sites, all with a random amplitude
**$t$-$\bar{j}$ model**

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij} \, c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy. Each site has 3 states which we map to the ‘superspin’ space of a boson $b$ (the holon) and a fermion $f_\alpha$ (the spinon):

$$|0\rangle \Rightarrow b^\dagger |v\rangle \quad , \quad c_{\alpha}^\dagger |0\rangle \Rightarrow f_\alpha^\dagger |v\rangle$$

$$c_\alpha = f_\alpha b^\dagger$$

$$\vec{S} = \frac{1}{2} f_\alpha^\dagger \sigma_{\alpha\beta} f_\beta$$

$$f_\alpha^\dagger f_\alpha + b^\dagger b = 1$$

U(1) gauge invariance, \hspace{1cm} b \rightarrow be^{i\phi}, \quad f_\alpha \rightarrow f_\alpha e^{i\phi}$$

The physical electron ($c_\alpha$) and spin ($\vec{S}$) operators are rotations in this SU(1|2) superspin space.
We consider the hole-doped case, with no double occupancy. Each site has 3 states which we map to the ‘superspin’ space of a boson $b$ (the holon) and a fermion $f_\alpha$ (the spinon):

$$
|0\rangle \Rightarrow f^\dagger |v\rangle, \quad c^\dagger_\alpha |0\rangle \Rightarrow b^\dagger_\alpha |v\rangle
$$

$$
c_\alpha = b_\alpha f^\dagger
$$

$$
\vec{S} = \frac{1}{2} b^\dagger_\alpha \sigma_{\alpha\beta} b_\beta
$$

$$
b^\dagger_\alpha b_\alpha + f^\dagger f = 1
$$

U(1) gauge invariance, \quad f \rightarrow fe^{i\phi}, \quad b_\alpha \rightarrow b_\alpha e^{i\phi}

The physical electron ($c_\alpha$) and spin ($\vec{S}$) operators are rotations in this SU(2|1) superspin space.
We consider the hole-doped case, with no double occupancy. Each site has 3 states which we map to the ‘superspin’ space of a boson \( b \) (the holon) and a fermion \( f_\alpha \) (the spinon):

\[
|0\rangle \Rightarrow f^\dagger |v\rangle, \quad c^\dagger_\alpha |0\rangle \Rightarrow b^\dagger_\alpha |v\rangle
\]

\[
c_\alpha = b_\alpha f^\dagger, \quad \vec{S} = \frac{1}{2} b^\dagger_\alpha \sigma_{\alpha\beta} b_\beta
\]

\[
b^\dagger_\alpha b_\alpha + f^\dagger f = 1
\]

\[
U(1) \text{ gauge invariance, } \quad f \rightarrow fe^{i\phi}, \quad b_\alpha \rightarrow b_\alpha e^{i\phi}
\]

The physical electron (\( c_\alpha \)) and spin (\( \vec{S} \)) operators are rotations in this SU(2|1) superspin space.
t-$j$ model phase diagram

Deconfined quantum critical point

\[ \langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{|\tau|} \]

D. Joshi, Chanyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822
Deconfined quantum critical point

\[ \langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{|\tau|} \]

Zeroth order, \( p_c = 1/3 \)

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822
\( t/J \) model phase diagram

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Deconfined quantum critical point

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Zeroth order, \( p_c = 1/3 \)

Disordered Fermi liquid.
Condense holon \( b \),
\( f_\alpha \) carrier density \( 1 + p \)

\[ f_\uparrow \langle |v\rangle \quad f_\downarrow \langle |v\rangle \]

\[ b^\dagger \langle |v\rangle \]

\[ \langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{\tau^2} \]

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822
Deconfined quantum critical point

\[ \langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \rangle \sim \frac{1}{|\tau|} \]

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D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822
**t-J model phase diagram**

- **Metallic spin glass.**
  - Condense spinon $b_\alpha$, $f$ carrier density $p$
  - **SU(2|1) theory**

- **Strange Metal**

- **Disordered Fermi liquid.**
  - Condense holon $b$, $f_\alpha$ carrier density $1 + p$
  - **SU(1|2) theory**

**Parameters:**
- $T$ (Temperature)
- $p$ (Carrier density)
\[ \rho \sim \frac{\hbar T}{e^2 E_c} \]

with \( E_c \sim \left( t\langle b \rangle^2 \right)^2 / J \)

\( A. \) Georges and O. Parcollet PRB \textbf{59}, 5341 (1999)

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**t-J model phase diagram**

<table>
<thead>
<tr>
<th>Region</th>
<th>Description</th>
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<tbody>
<tr>
<td>Metallic spin glass</td>
<td>Condense spinon ( b_\alpha ), ( f ) carrier density ( p )</td>
</tr>
<tr>
<td>Disordered Fermi liquid</td>
<td>Condense holon ( b ), ( f_\alpha ) carrier density ( 1+p )</td>
</tr>
</tbody>
</table>

---

**SU(2|1) theory**

**SU(1|2) theory**

---

\( p_c \)
\[
\rho = \rho_0 + aT
\]

in a numerical large-\(M\) study of similar EDMFT equations of a non-random model.

$t$-$j$ model entropy

$S$ vs $T$

$p = p_c$

$S_0$

$\frac{C}{T} = \frac{dS}{dT}$
t-J model entropy

\[ p = p_c \]

\[ |p - p_c| > 0 \]

\[ \frac{C}{T} = \frac{dS}{dT} \]
**$t/J$ model entropy**

![Diagram showing the entropy $S$ as a function of temperature $T$, with the condition $|p - p_c| > 0$ and $|p - p_c| \gg 0$.]

\[
\frac{C}{T} = \frac{dS}{dT}
\]
Hole doped cuprates

The remarkable underlying ground states of cuprate superconductors

Cyril Proust and Louis Taillefer, arXiv:1807.0507

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### Figure 6

**a)** Normal-state electronic specific heat in the $T=0$ limit as a function of doping, plotted as $C_{el}/T$ vs $p$ (red symbols) in Eu-LSCO (squares), Nd-LSCO (circles) and LSCO (diamonds). From ref. (75). We also show $C_{el}/T$ in YBCO (blue dots (18)) and in Tl2201 (green dot (76)). The vertical grey lines mark the limits of the CDW phase in Nd-LSCO, between $p_0.08$ and $p_0.19$.

**b)** Normal-state Hall number $n_H$ in the $T=0$ limit as a function of doping, in YBCO (blue circles (21), $p_0.19$) and Nd-LSCO (red squares (4), $p_0.23$). We also show $n_H$ in LSCO (grey squares (67)) and YBCO (grey circles (68)) at low doping, and $n_H$ in Tl2201 (white diamond (29)) at high doping.

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5. **PSEUDOGAP PHASE**

**DOS:** Density of states ($N_F$)

**$E_c$:** Condensation energy

**$H_{c1}$:** Lower critical field

**$C(T)$:** Residual linear term in the specific heat, $C(T)$ at $T=0$, purely electronic

The two traditional signatures of the pseudogap phase are: 1) a loss of density of states (DOS) below $p_0.18$; 2) the opening of a partial spectral gap below $T_0$, see by ARPES (Figs. 1c, 1d) and optical conductivity, for example. Here we summarize recent high-field measurements of the specific heat in the LSCO family (75) showing that there is a large mass enhancement at $p_0.18$. The new data show that the pseudogap does not simply cause a loss of DOS below $p_0.18$; instead, there is huge peak in the DOS at $p_0.18$ (Fig. 6a) – much larger than expected from a van Hove singularity (75, 80). We then show how high-field measurements of the Hall coefficient reveal a new signature of the pseudogap phase – a rapid drop in the carrier density, at $p_0.18$ (Fig. 6c). These new properties alter profoundly our view of the pseudogap phase, and of the strange metal just above it (sec. 6).

5.1. **Density of states**

5.1.1. **Condensation energy.** One way to access the DOS, $N_F$, is via the superconducting condensation energy $E_c$, since $E_c = N_F^{2/0}$, where $0$ is the $d$-wave gap maximum. Experimentally, and in the framework of BCS theory, $E_c$ can be measured using the upper and lower critical fields, $H_{c2}$ and $H_{c1}$, to get the thermodynamic field $H_c$ via

$$H_c^2 = H_{c1}H_{c2}/(\ln(\pi) + 0.5),$$

given that $E_c = H_{c2}/2\mu_0$. In Fig. 2b, we plot $E_c/T_0$ vs $p_0$ thus obtained for YBCO (17). We see that $E_c/T_0/N_F$ drops by a factor 8-9 between $p_0.18$ and $p_0.19$, in agreement with the drop reported earlier from an analysis of specific heat data measured in low fields up to $T>T_c$ in YBCO (71) and Bi2212 (72). Note
Black holes

Metals, ordinary and strange

Quantum criticality in the cuprates

The holographic connection between strange metals and black holes
• Black holes have an entropy and a temperature, $T_H$.

• The entropy is proportional to their surface area.

• They relax to thermal equilibrium in a Planckian time $\sim \hbar/(k_B T_H)$.
Quantum Black holes

- Black holes have an entropy and a temperature, $T_H$
- The entropy is proportional to their surface area.
- They relax to thermal equilibrium in a Planckian time $\sim \hbar / (k_B T_H)$.

**Holography:**
Quantum black holes “look like” quantum-critical many-particle systems without quasiparticle excitations, residing “on” the surface of the black hole.

Maxwell’s electromagnetism and Einstein’s general relativity allow black hole solutions with a net charge.
Maxwell’s electromagnetism and Einstein’s general relativity allow black hole solutions with a net charge.

Zooming into the near-horizon region of a charged black hole at low temperature, yields a quantum theory in one space ($\zeta$) and one time dimension.
Maxwell’s electromagnetism and Einstein’s general relativity allow black hole solutions with a net charge.

The quantum versions of Maxwell’s and Einstein’s equations in this two-dimensional spacetime are also the equations describing electron entanglement in the SYK model.
Maxwell’s electromagnetism and Einstein’s general relativity allow black hole solutions with a net charge. This has led to a deeper understanding of entanglement in superconductors and of Hawking’s black hole information “paradox.”
Quantum phase transitions
Black holes
Metals, ordinary and strange
Quantum criticality in the cuprates
The holographic connection between strange metals and black holes
The near-horizon region of a charged black hole has the geometry of (1+1)-dimensional anti-de Sitter spacetime. By holography, this should map to a zero-dimensional quantum system: this turns out to be the SYK model.
SYK model and charged black holes

Black hole horizon

Bekenstein-Hawking entropy of AdS\(_2\) horizon at \(T = 0\) \(\Leftrightarrow\) \(N s_0\) entropy of SYK model.

\[
\frac{d s_0}{d Q} = 2\pi \mathcal{E}
\]

can be obtained from the Einstein equations for the black hole, and the quantum theory of the SYK model, and \(\mathcal{E}\) determines identical fermion spectral functions.

Remarkably, the correspondence between charged black holes and the SYK model also holds for the leading fluctuations at higher temperatures: both are described by a ‘Schwarzian’ theory with emergent SL(2,R) and U(1) gauge symmetries. For the black hole, the Schwarzian describes the fluctuations of the boundary between AdS\(_2\) and AdS\(_4\).
Main result

A. Kitaev (2015)


J. Maldacena, D. Stanford, and Zhenbin Yang, PTEP 12C104 (2016)


J. Engelsoy, T.G. Mertens, and H. Verlinde, JHEP 1607 (2016) 139


P. Chaturvedi, Yingfei Gu, Wei Song, Boyang Yu, arXiv:1808.08062


S. Sachdev, arXiv:1902.04078
Main result

SYK model of fermions with random interactions of mean-square-value $U$, with total fermion number $Q$, at temperatures $T \ll U$
Main result

SYK model of fermions with random interactions of mean-square-value $U$, with total fermion number $Q$, at temperatures $T \ll U$

and

Charged black holes in 3+1 dimensions of radius $R_h$, with total charge $Q$, at temperatures $T \ll 1/R_h$

are described by a common low energy quantum theory in 0 + 1 dimensions
Main result

The common low $T$ path integral is $Z = \int \mathcal{D}f \mathcal{D}\phi e^{-I}$. This can be exactly evaluated, and the action is

$$I = -s_0 + \int_0^{1/T} d\tau \left\{ \frac{K}{2} \left( \frac{\partial \phi}{\partial \tau} + i(2\pi \mathcal{E} T) \frac{\partial f}{\partial \tau} \right)^2 - \frac{\gamma}{4\pi^2} \text{Sch}[\tan(\pi T f(\tau)), \tau] \right\},$$

where $f(\tau)$ is a monotonic reparameterization of the temporal circle with

$$f(\tau + 1/T) = f(\tau) + 1/T,$$

$\phi$ is a phase conjugate to the charge density with

$$\phi(\tau + 1/T) = \phi(\tau) + 2\pi n, \text{ } n \text{ integer},$$

$\text{Sch}[g[\tau], \tau]$ is the Schwarzian derivative of $g(\tau)$. The couplings are related to the entropy $S(T, Q)$ and the chemical potential $\mu$ via

$$S(T \rightarrow 0, Q) = s_0 + \gamma T, \quad K = \left( \frac{dQ}{d\mu} \right)_{T \rightarrow 0}, \quad 2\pi \mathcal{E} = \frac{ds_0}{dQ}.$$
Quantum phase transitions

Black holes

Metals, ordinary and strange

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