The two unusual metals in the cuprates

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Talk online: sachdev.physics.harvard.edu
What are the effective theories of charge and heat transport in systems without quasiparticles?

- A clean system has multiple different conduction mechanisms.
- Each has a characteristic timescale and imprints on the frequency-dependent conductivities in a specific way.
- Can calculate the $T$-dependence of the conductivities under various assumptions.
- As the system becomes dirtier, there is a qualitative change in the conduction mechanisms and what timescales control them.
Polarons in BEC

Related to Frolich polarons in solid state systems, electrons in magnetic systems

Used non-perturbative approaches (RG, variational wavefunctions) to study equilibrium and non-equilibrium properties of Bose polarons

RF spectroscopy

Effective mass

Universal (dimension dependent) low frequency spectrum: signature of many-body orthogonality catastrophe

Universal (dimension dependent) high frequency tail: related to two particle physics

Fabian Grusdt, et al.,
Scientific Reports 5:12124 (2015)
What are the effective theories of charge and heat transport in systems without quasiparticles?

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  - RD, Goutéraux, Hartnoll 1507.07137
- Each has a characteristic timescale and imprints on the frequency-dependent conductivities in a specific way.
  - RD, Goutéraux 1505.05092
- Can calculate the T-dependence of the conductivities under various assumptions.
  - RD, Schalm, Zaanen 1311.2451
  - RD, Goutéraux, Hartnoll 1507.07137
- As the system becomes dirtier, there is a qualitative change in the conduction mechanisms and what timescales control them.
  - RD, Goutéraux 1411.1062
Figure: K. Fujita and J. C. Seamus Davis

$YBa_2Cu_3O_{6+x}$
Conventional metal
Area enclosed by Fermi surface = $1 + \rho$

1. **Pseudogap metal at low \( p \)**
2. Strange metal

No quasiparticle excitations
1. Quasiparticle transport in ordinary metals
   *Bloch vs. Peierls*

2. Transport without quasiparticles in strange metals
   *Application to (less) strange metal in graphene*

3. The pseudogap metal
   *Fermi liquid co-existing with topological order*
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Quasiparticle transport in metals:

- Compute the scattering rate of charged quasiparticles off phonons: this leads to Bloch’s law (1930): a resistivity $\rho(T) \sim T^5$. 
Quasiparticle transport in metals:

- Compute the scattering rate of charged quasiparticles off phonons: this leads to the resistivity $\rho(T) \sim T^5$.

However, this ignores “phonon drag”

PHONON DRAG

Peierls\(^{28}\) pointed out a way in which the low temperature resistivity might decline more rapidly than $T^5$. This behavior has yet to be observed.

Rates of Momentum Flow

Electrons

Phonons

SLOW
Rates of Momentum Flow

Electrons

Phonons

Defects

SLOW

FAST
Rates of Momentum Flow

Electrons → SLOW → Process controlling resistivity (Bloch) → FAST → Defects

Phonons
Rates of Momentum Flow

Electrons

Boson $\phi$ coupled strongly to electrons
Rates of Momentum Flow

Electrons

Boson $\phi$ coupled strongly to electrons

Defects

FAST

SLOW
Rates of Momentum Flow

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Boson $\phi$ coupled strongly to electrons

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Process controlling resistivity (Peierls)
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Thermoelectric transport coefficients

Transport has two components: a “momentum drag” term, and a “quantum critical” term.

\[
\sigma = \frac{Q^2}{\mathcal{M}} \pi \delta(\omega) + \sigma_Q(\omega)
\]

\[
\alpha = \frac{S Q}{\mathcal{M}} \pi \delta(\omega) + \alpha_Q(\omega)
\]

\[
\bar{\kappa} = \frac{T S^2}{\mathcal{M}} \pi \delta(\omega) + \bar{\kappa}_Q(\omega)
\]

with entropy density \( S \), \( Q \equiv \chi_{J_x,P_x} \), and \( \mathcal{M} \equiv \chi_{P_x,P_x} \).

Obtained in hydrodynamics, holography, and by memory functions

Thermoelectric transport coefficients

Transport has two components: a “momentum drag” term, and a “quantum critical” term.

\[
\sigma = \frac{Q^2}{M} \pi \delta(\omega) + \sigma_Q(\omega), \\
\alpha = \frac{SQ}{M} \pi \delta(\omega) + \alpha_Q(\omega), \\
\bar{\kappa} = \frac{TS^2}{M} \pi \delta(\omega) + \bar{\kappa}_Q(\omega)
\]

with entropy density \(S\), \(Q \equiv \chi_{J_x, P_x}\), and \(M \equiv \chi_{P_x, P_x}\).

In theories which are relativistic at high energies (including graphene), \(T\alpha_Q(\omega) = -\mu \sigma_Q(\omega)\), \(T\bar{\kappa}_Q(\omega) = \mu^2 \sigma_Q(\omega)\), \(M = TS + \mu Q = \mathcal{H}\) the enthalpy density, and \(Q = n\) the electron density.

Obtained in hydrodynamics, holography, and by memory functions

Thermoelectric transport coefficients

Transport has two components: a “momentum drag” term, and a “quantum critical” term.

\[
\sigma = \frac{Q^2}{\mathcal{M}} \frac{1}{(-i\omega + 1/\tau)} + \sigma_Q(\omega)
\]

\[
\alpha = \frac{SQ}{\mathcal{M}} \frac{1}{(-i\omega + 1/\tau)} + \alpha_Q(\omega)
\]

\[
\bar{k} = \frac{TS^2}{\mathcal{M}} \frac{1}{(-i\omega + 1/\tau)} + \bar{k}_Q(\omega)
\]

Momentum relaxation by an external source \( h \) coupling to the operator \( \mathcal{O} \)

\[
H = H_0 - \int d^d x \ h(x) \ \mathcal{O}(x).
\]

\[
\frac{\mathcal{M}}{\tau} = \lim_{\omega \to 0} \int d^d q \ |h(q)|^2 \frac{q_x^2 \ \text{Im} \left( G^R_{\mathcal{O}\mathcal{O}}(q, \omega) \right)}{\omega} H_0 + \text{higher orders in} \ h
\]

Observation of the Dirac fluid
and the breakdown of the Wiedemann-Franz law in graphene


Thermal conductivity $\kappa = \bar{\kappa} - T\alpha^2/\sigma$

Lorenz ratio $L = \kappa/(T\sigma) = \frac{\mathcal{H}\tau}{T^2\sigma_Q} \frac{1}{\left(1 + n^2\tau/(\mathcal{H}\sigma_Q)\right)^2}$
Strange metal

No quasiparticle excitations
Transport has two components: a “momentum drag” term, and a “quantum critical” term.

\[
\sigma_{xx} = \frac{(\tau^{-1} - i\omega)M\sigma_Q + Q^2 + B^2\sigma_Q^2}{Q^2B^2 + ((\tau^{-1} - i\omega)M + B^2\sigma_Q)^2}M\left(\frac{1}{\tau} - i\omega\right),
\]

\[
\sigma_{xy} = \frac{2(\tau^{-1} - i\omega)M\sigma_Q + Q^2 + B^2\sigma_Q^2}{Q^2B^2 + ((\tau^{-1} - i\omega)M + B^2\sigma_Q)^2}BQ.
\]

Electrical and thermal magnetotransport with no additional parameters
(assuming \(\sigma_Q\) is field-independent)
Thermoelectric transport coefficients

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\[
\sigma_{xx} = \frac{(\tau^{-1} - i\omega) M \sigma_Q + Q^2 + B^2 \sigma_Q^2}{Q^2 B^2 + ((\tau^{-1} - i\omega) M + B^2 \sigma_Q)^2} M \left( \frac{1}{\tau} - i\omega \right),
\]

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\]

Electrical and thermal magnetotransport with no additional parameters (assuming \(\sigma_Q\) is field-independent)

Blake and Donos: With \(\sigma_Q \sim 1/T\) and \(\tau \sim 1/T^2\), we obtain \(\sigma_{xx} \sim 1/T\) and \(\tan(\theta_H) = \sigma_{xy}/\sigma_{xx} \sim 1/T^2\), in agreement with data on cuprates (Ong, PRL 1991); such data cannot be explained in a quasiparticle model.

M. Blake and A. Donos, PRL 114, 021601 (2015)


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Pseudogap metal at low $p$
Evolution of the Hall Coefficient and the Peculiar Electronic Structure of the Cuprate Superconductors

Yoichi Ando,* Y. Kurita,† Seiki Komiya, S. Ono, and Kouji Segawa

PRL 92, 197001 (2004)

T-independent Hall effect in a magnetic field of fermions of charge +e and density \( \rho \)
Electrical and optical evidence for Fermi surface of long-lived quasiparticles of density $p$

Spectroscopic evidence for Fermi liquid-like energy and temperature dependence of the relaxation rate in the pseudogap phase of the cuprates

Seyed Iman Mirzaei$^a$, Damien Stricker$^a$, Jason N. Hancock$^{a,b}$, Christophe Berthod$^a$, Antoine Georges$^{a,c,d}$, Erik van Heumen$^{a,e}$, Mun K. Chan$^f$, Xudong Zhao$^{f,g}$, Yuan Li$^h$, Martin Greven$^f$, Neven Barisic$^{f,i,j}$, and Dirk van der Marel$^{a,1}$

PNAS 110, 5774 (2013)

$$\sigma_{xx} \sim \frac{1}{(-i\omega + 1/\tau)}$$

with $$\frac{1}{\tau} \sim \omega^2 + T^2$$

Fig. 6. Collapse of the frequency and temperature dependence of the relaxation rate of underdoped cuprate materials. Normal state $M_2(\omega, T)$ as a function of $\xi^2 \equiv (\hbar \omega)^2 + (p\pi k_B T)^2$. 

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In summary, we have shown from optical spectroscopy measurements that the ungapped near-nodal excitations of underdoped cuprates.

**Inset**: $\xi$ dependence of the spin gap for $T < T_c$. We find the spin gap below 140 meV is clearly observable for temperatures below 100 K.

$\xi_0(0123456789\ldots)$.

**Figure 1**: As seen in the results, Fermi liquid behavior is very much about the energy dependence of the dc resistivity, as seen from transport measurements. Below 140 meV is clearly observable for temperatures below 100 K.

*For a Fermi liquid, $1/T$ is expected to collapse to $1/T_0$ at $T_0 = 67 K$.

**Figure 3** displays the temperature dependences of samples of the same composition and quality. The dc transport data, owing to the higher precision, allow for a comparison of different samples of the same composition and quality. The theoretical accuracy of the temperature dependence of the dc resistivity, as seen from transport measurements, the width of this peak corresponds to the spin gap for $T < T_c$.

**Inset**: $\chi(\omega)$ (frequency-independent) relaxation rate with a frequency-dependent behavior. Below 140 meV is clearly observable for temperatures below 100 K.
Electrical and optical evidence for Fermi surface of long-lived quasiparticles of density $p$

In-Plane Magnetoresistance Obeys Kohler’s Rule in the Pseudogap Phase of Cuprate Superconductors

M. K. Chan,1,* M. J. Veit,1 C. J. Dorow,1,† Y. Ge,1 Y. Li,1 W. Tabis,1,2 Y. Tang,1 X. Zhao,1,3
N. Barišić,1,4,5,‡ and M. Greven1,§

PRL 113, 177005 (2014)

We report in-plane resistivity ($\rho$) and transverse magnetoresistance (MR) measurements for underdoped HgBa$_2$CuO$_{4+\delta}$ (Hg1201). Contrary to the long-standing view that Kohler’s rule is strongly violated in underdoped cuprates, we find that it is in fact satisfied in the pseudogap phase of Hg1201. The transverse MR shows a quadratic field dependence, $\delta\rho/\rho_0 = aH^2$, with $a(T) \propto T^{-4}$. In combination with the observed $\rho \propto T^2$ dependence, this is consistent with a single Fermi-liquid quasiparticle scattering rate. We show that this behavior is typically masked in cuprates with lower structural symmetry or strong disorder effects.

$$\rho_{xx} \sim \frac{1}{\tau} (1 + aH^2\tau^2 + \ldots)$$

with $\frac{1}{\tau} \sim T^2$
Can we have a metal with no broken translational symmetry, and with long-lived electron-like quasiparticles on a Fermi surface of size $p$?
Can we have a metal with no broken translational symmetry, and with long-lived electron-like quasiparticles on a Fermi surface of size $p$?

**Answer:** Yes.

There can be a Fermi surface of size $p$, but it must be accompanied by topological order, in a “fractionalized Fermi liquid”. At $T=0$, such a metal must be separated from a Fermi liquid (with a Fermi surface of size $1+p$) by a quantum phase transition.
Fractionalized Fermi liquid (FL*)

\[ | \uparrow\downarrow \rangle - | \downarrow\uparrow \rangle \]

Realizes a metal with a Fermi surface of area $p$ co-existing with “topological order”

Fractionalized Fermi liquid (FL*)

\[ | \uparrow \downarrow \rangle - | \downarrow \uparrow \rangle \]

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Fractionalized Fermi liquid (FL*)

\[
\begin{pmatrix}
| \uparrow \downarrow \rangle \\
- | \downarrow \uparrow \rangle
\end{pmatrix}
\]

Realizes a metal with a Fermi surface of area $p$ co-existing with “topological order”

A fermionic “dimer” describing a “bonding” orbital between two sites

Realizes a metal with a Fermi surface of area $p$ co-existing with “topological order”

Density of fermionic dimers = $p$; density of holes relative to filled band = $1 + p$

Fractionalized Fermi liquid (FL*)

\[ \frac{1}{4} | \uparrow \downarrow \rangle - | \downarrow \uparrow \rangle \]

Realizes a metal with a Fermi surface of area \( p \) co-existing with "topological order"

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\[ = | \uparrow \downarrow \rangle - | \downarrow \uparrow \rangle \]

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Fractionalized Fermi liquid (FL*)

\[ | \uparrow \downarrow \rangle - | \downarrow \uparrow \rangle \]

Realizes a metal with a Fermi surface of area \( p \) co-existing with "topological order"

Dispersion and quasiparticle residue of a single fermionic dimer for \( J = V = 1 \), and hopping parameters obtained from the \( t-J \) model for the cuprates, 
\( t_1 = -1.05, \ t_2 = 1.95 \text{ and } t_3 = -0.6 \), on a \( 8 \times 8 \) lattice.

M. Punk, A. Allais, and S. Sachdev, arXiv:1501.00978
“Back side” of Fermi surface is suppressed for observables which change electron number in the square lattice


Pseudogap at low $p$
A fractionalized Fermi liquid (FL*) — with electron-like quasiparticles on a Fermi surface of size $p$, coexisting with topological order.

Pseudogap

Density wave (DW) order at low $T$ and $p$

Identified as a predicted “d-form factor density wave”

\[ Q = (\pi/2, 0) \]

Pseudogap
\[ Q = (\pi/2, 0) \]
\( Q = \left( \frac{\pi}{2}, 0 \right) \)
The high $T$ FL* can help explain the “d-form factor density wave” observed at low $T$.

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