

# Putting competing orders in their place near the Mott transition

cond-mat/0408329 and cond-mat/0409470

Leon Balents (UCSB)

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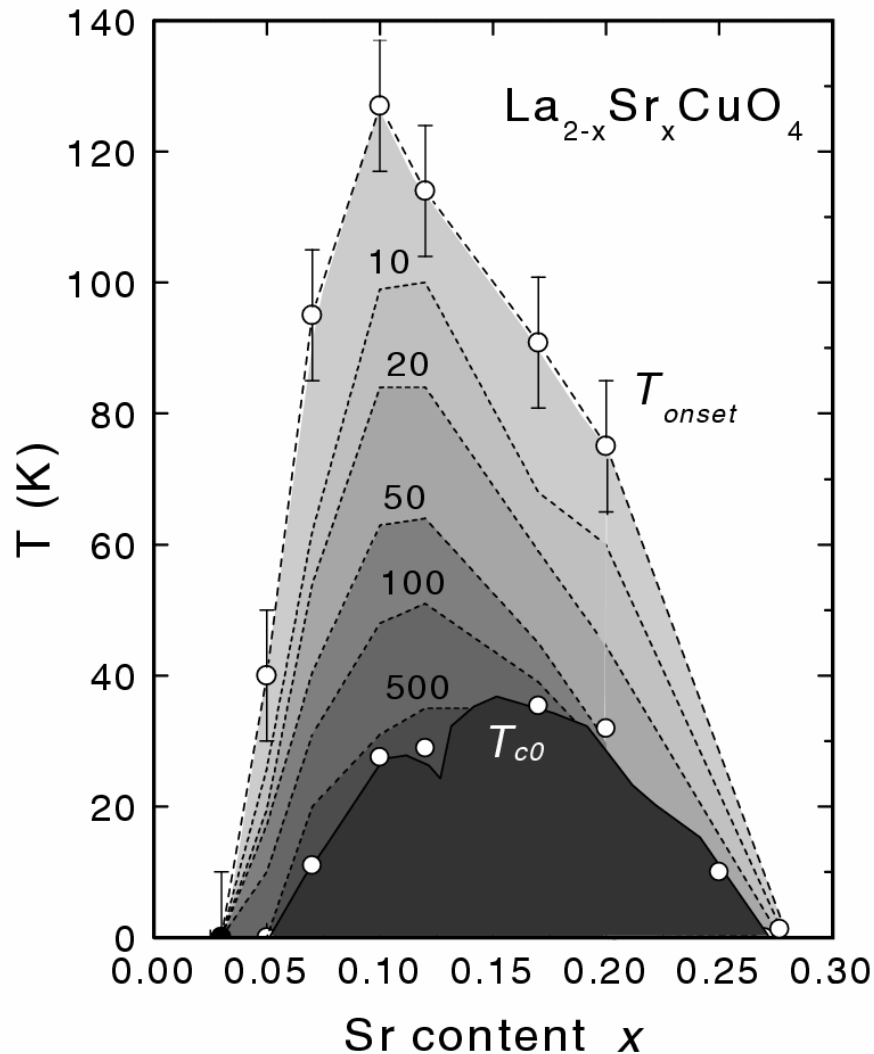
Krishnendu Sengupta (Toronto)



Talk online: Google Sachdev

# Distinct experimental characteristics of underdoped cuprates at $T > T_c$

Measurements of Nernst effect are well explained by a model of a liquid of vortices and anti-vortices

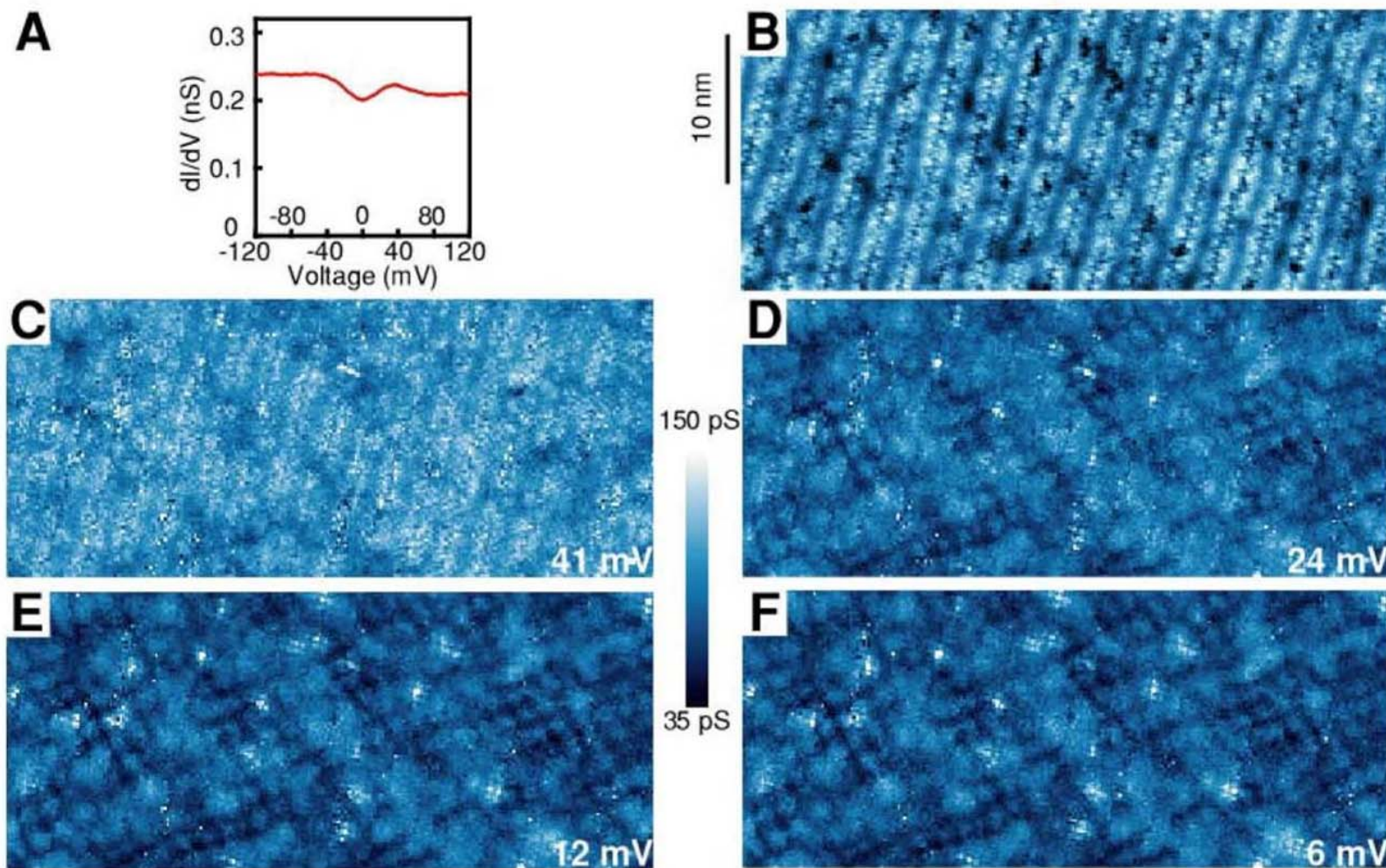


N. P. Ong, Y. Wang, S. Ono, Y. Ando, and S. Uchida, *Annalen der Physik* **13**, 9 (2004).

Y. Wang, S. Ono, Y. Onose, G. Gu, Y. Ando, Y. Tokura, S. Uchida, and N. P. Ong, *Science* **299**, 86 (2003).

# Distinct experimental characteristics of underdoped cuprates at $T > T_c$

STM measurements observe “density” modulations with a period of  $\approx 4$  lattice spacings



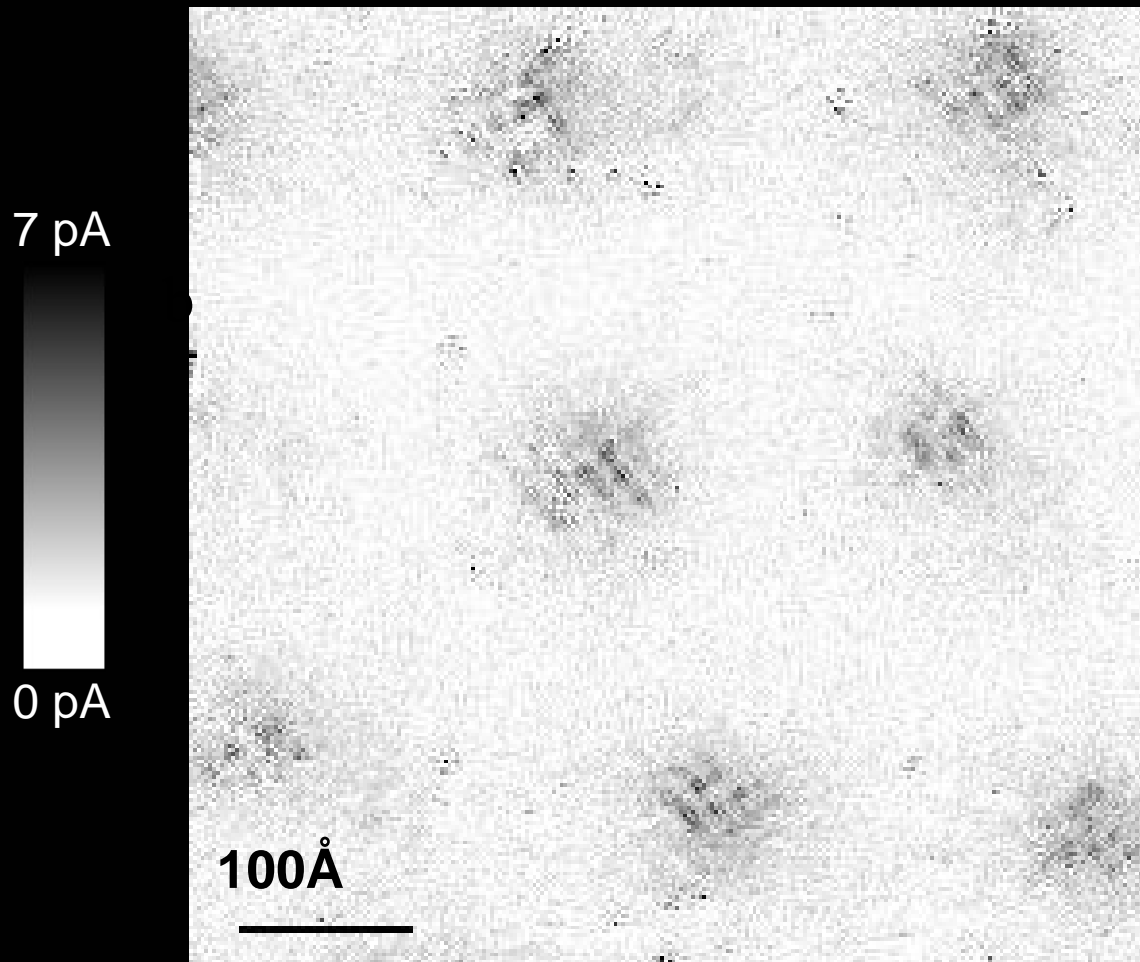
LDOS of  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$  at 100 K.

M. Vershinin, S. Misra, S. Ono, Y. Abe, Y. Ando, and A. Yazdani, *Science*, **303**, 1995 (2004).

Is there a connection between vorticity and “density” wave modulations?

“Density” wave order---modulations in pairing amplitude, exchange energy, or hole density. Equivalent to valence-bond-solid (VBS) order (except at the special period of 2 lattice spacings)

# Vortex-induced LDOS of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ integrated from 1meV to 12meV at 4K



Vortices have halos with LDOS modulations at a period  $\approx 4$  lattice spacings

J. Hoffman E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* 295, 466 (2002).

Prediction of VBS order near vortices: K. Park and S. Sachdev, *Phys. Rev. B* 64, 184510 (2001).

## Landau-Ginzburg-Wilson theory of multiple order parameters:

- “Vortex/phase fluctuations” (“preformed pairs”)

Complex superconducting order parameter:  $\Psi_{sc}$

$\Psi_{sc} \rightarrow \Psi_{sc} e^{i\theta}$  symmetry encodes number conservation

- “Charge/valence-bond/pair-density/stripe” order

Order parameters:

$$\rho(\mathbf{r}) = \sum_{\mathbf{Q}} \rho_{\mathbf{Q}} e^{i\mathbf{Q} \cdot \mathbf{r}}$$

$\rho_{\mathbf{Q}} \rightarrow \rho_{\mathbf{Q}} e^{i\theta}$  encodes space group symmetry

## Landau-Ginzburg-Wilson theory of multiple order parameters:

LGW free energy:

$$F = F_{sc} [\Psi_{sc}] + F_{\text{charge}} [\rho_Q] + F_{\text{int}}$$

$$F_{sc} [\Psi_{sc}] = r_1 |\Psi_{sc}|^2 + u_1 |\Psi_{sc}|^4 + \dots$$

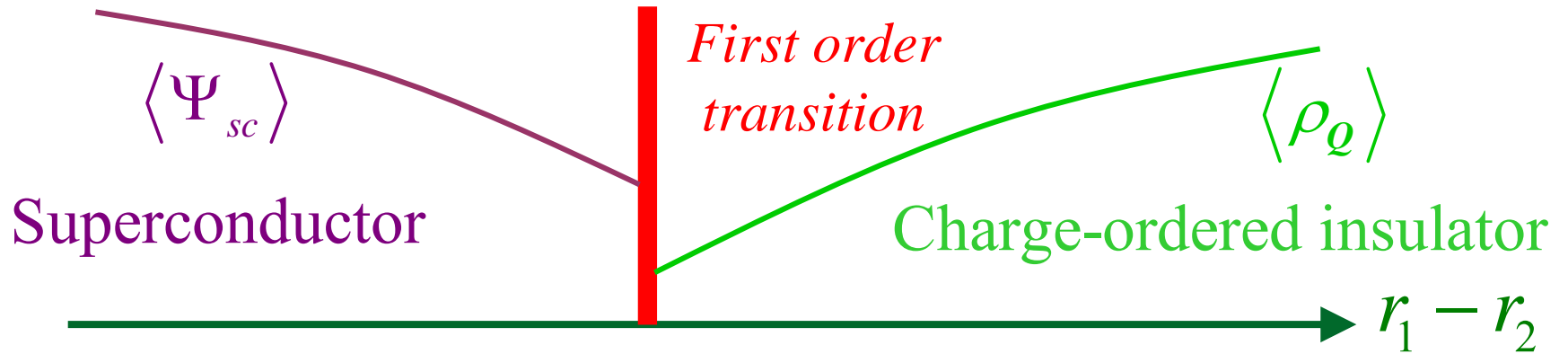
$$F_{\text{charge}} [\rho_Q] = r_2 |\rho_Q|^2 + u_2 |\rho_Q|^4 + \dots$$

$$F_{\text{int}} = v |\Psi_{sc}|^2 |\rho_Q|^2 + \dots$$

Distinct symmetries of order parameters permit couplings only between their energy densities (there are no symmetries which “rotate” two order parameters into each other)

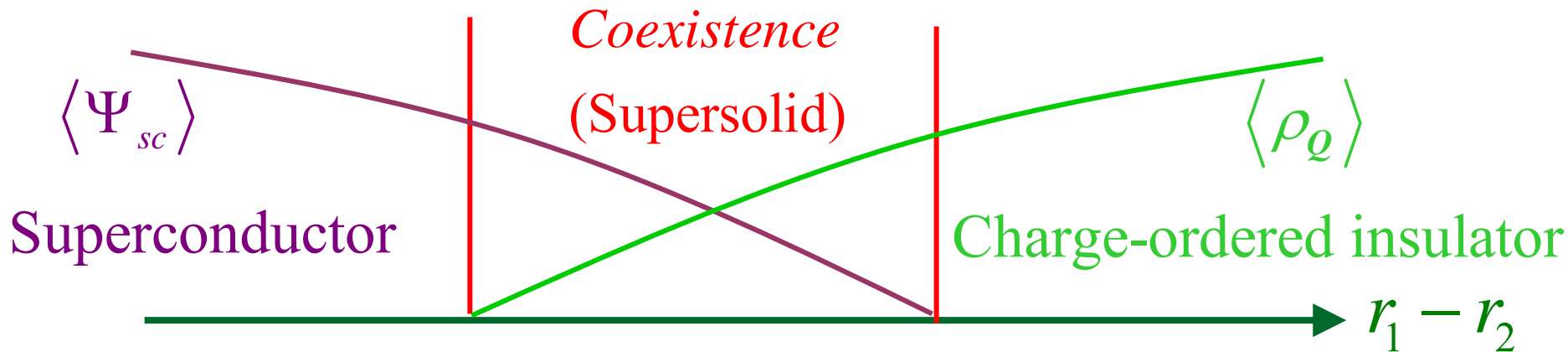
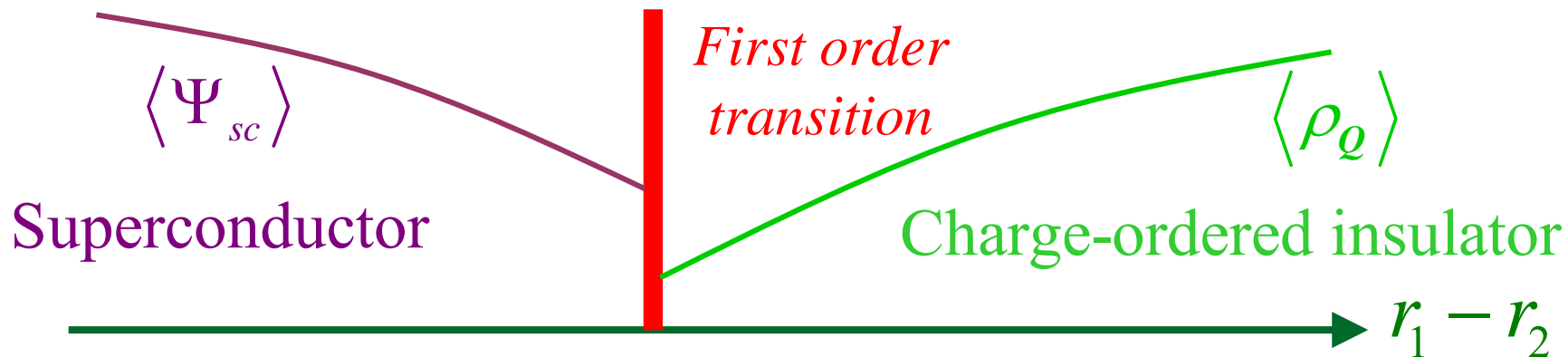
For large positive  $v$ , there is a correlation between vortices and density wave order

## Predictions of LGW theory

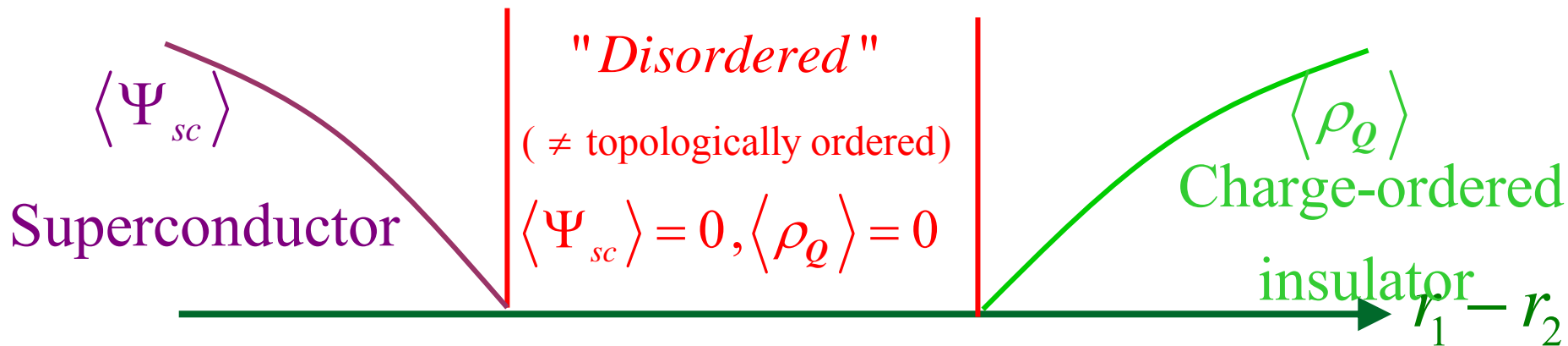
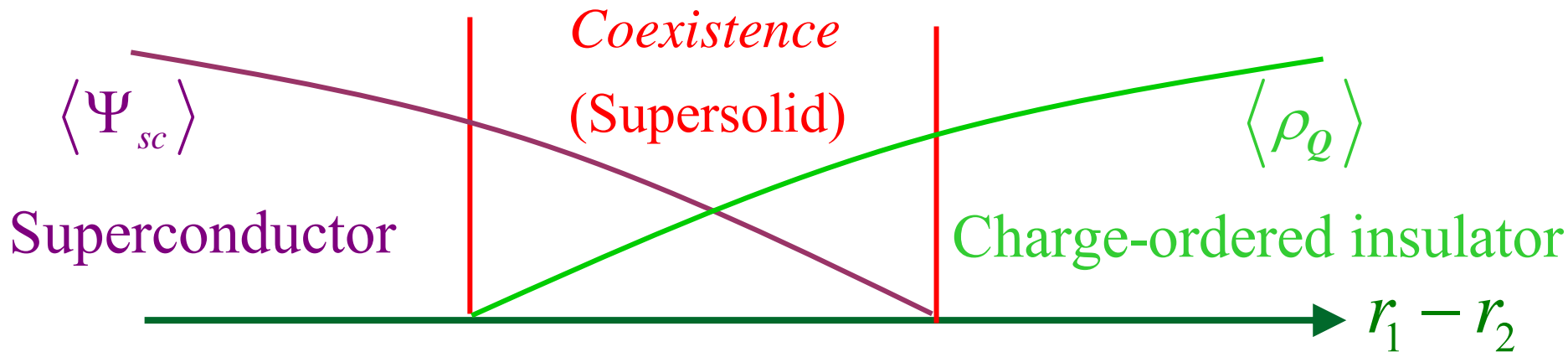
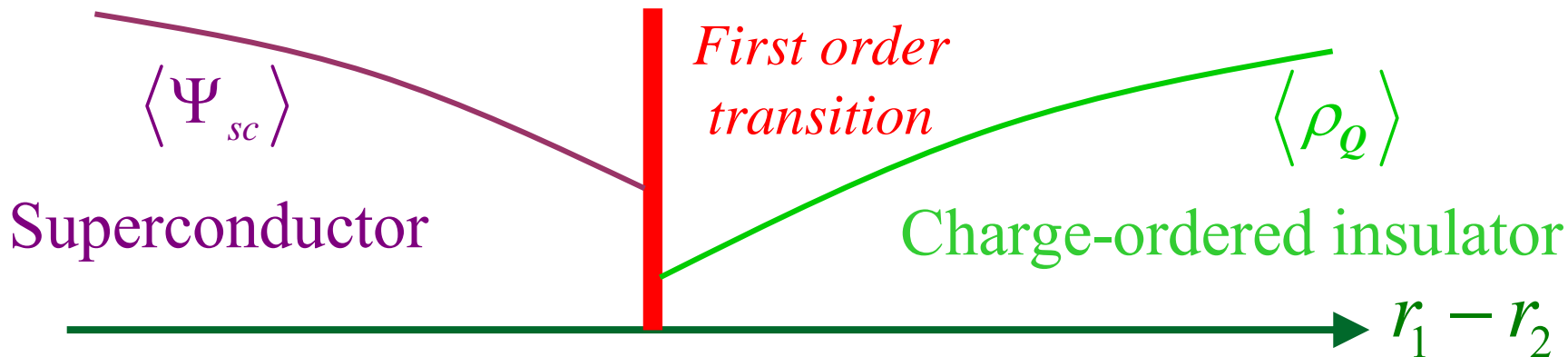




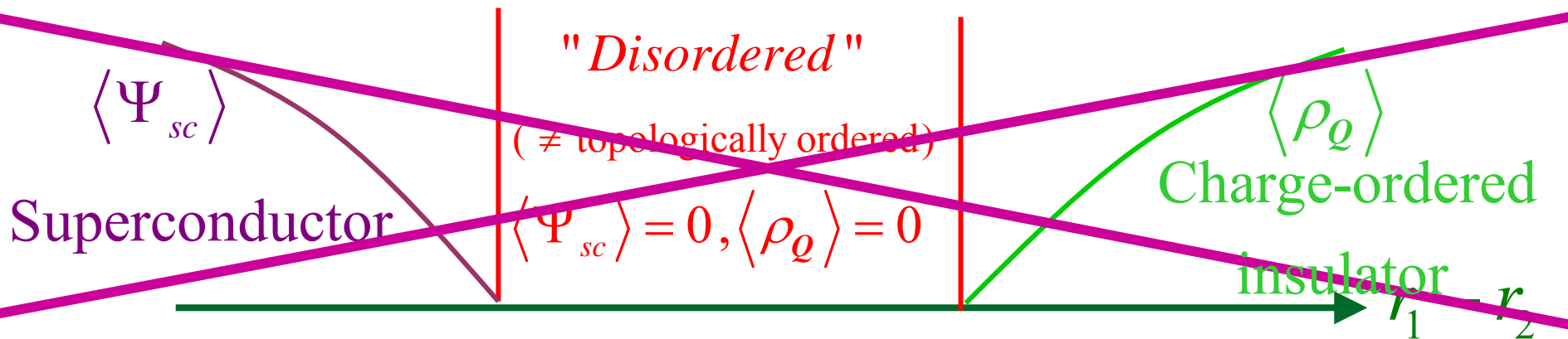
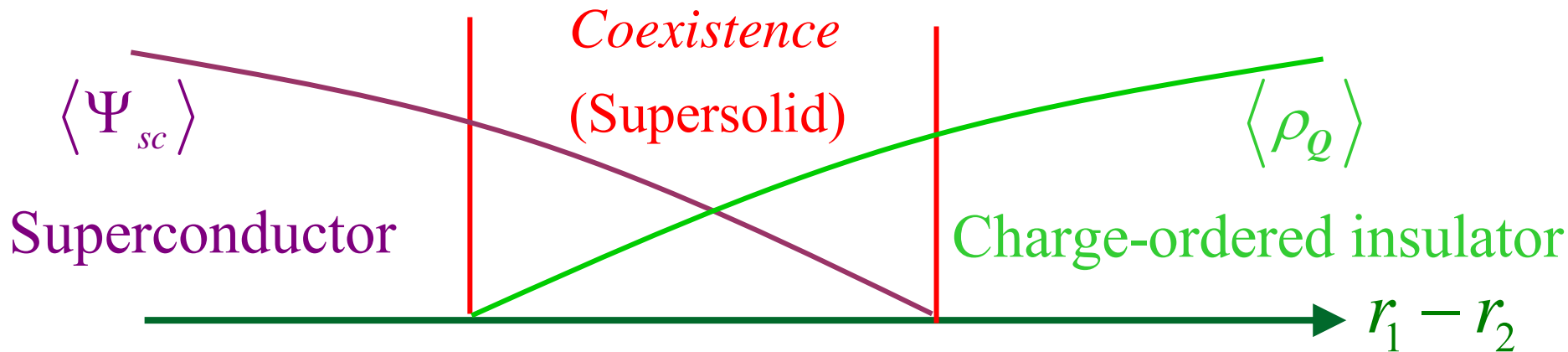
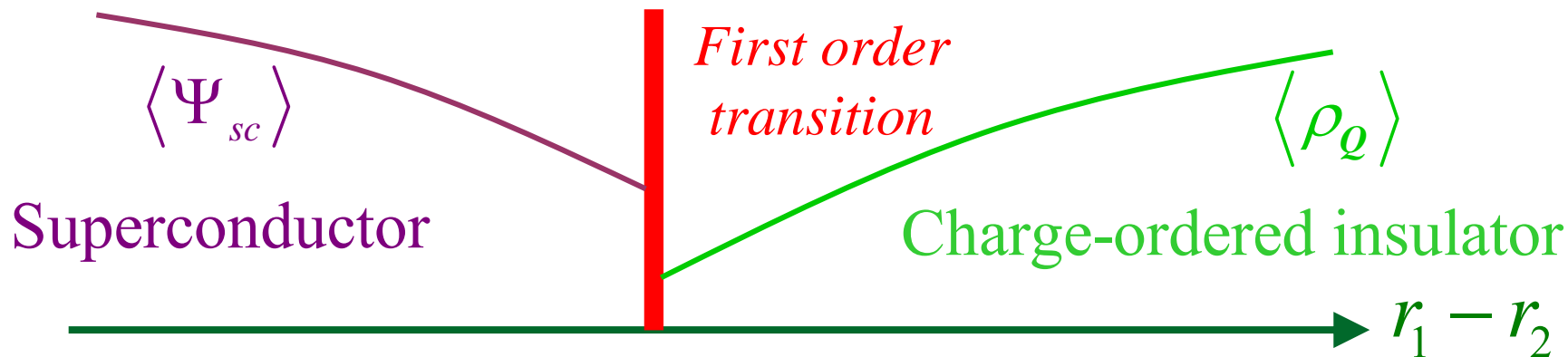
# Predictions of LGW theory



# Predictions of LGW theory



# Predictions of LGW theory



Non-superconducting quantum phase must have some other “order”:

- Charge order in an insulator
- Fermi surface in a metal
- “Topological order” in a spin liquid
- .....

This requirement is not captured by LGW theory.

*Needed:* a theory of precursor  
fluctuations of the density  
wave order of the insulator  
within the superconductor.

*i.e.* a connection between  
vortices and density wave  
order

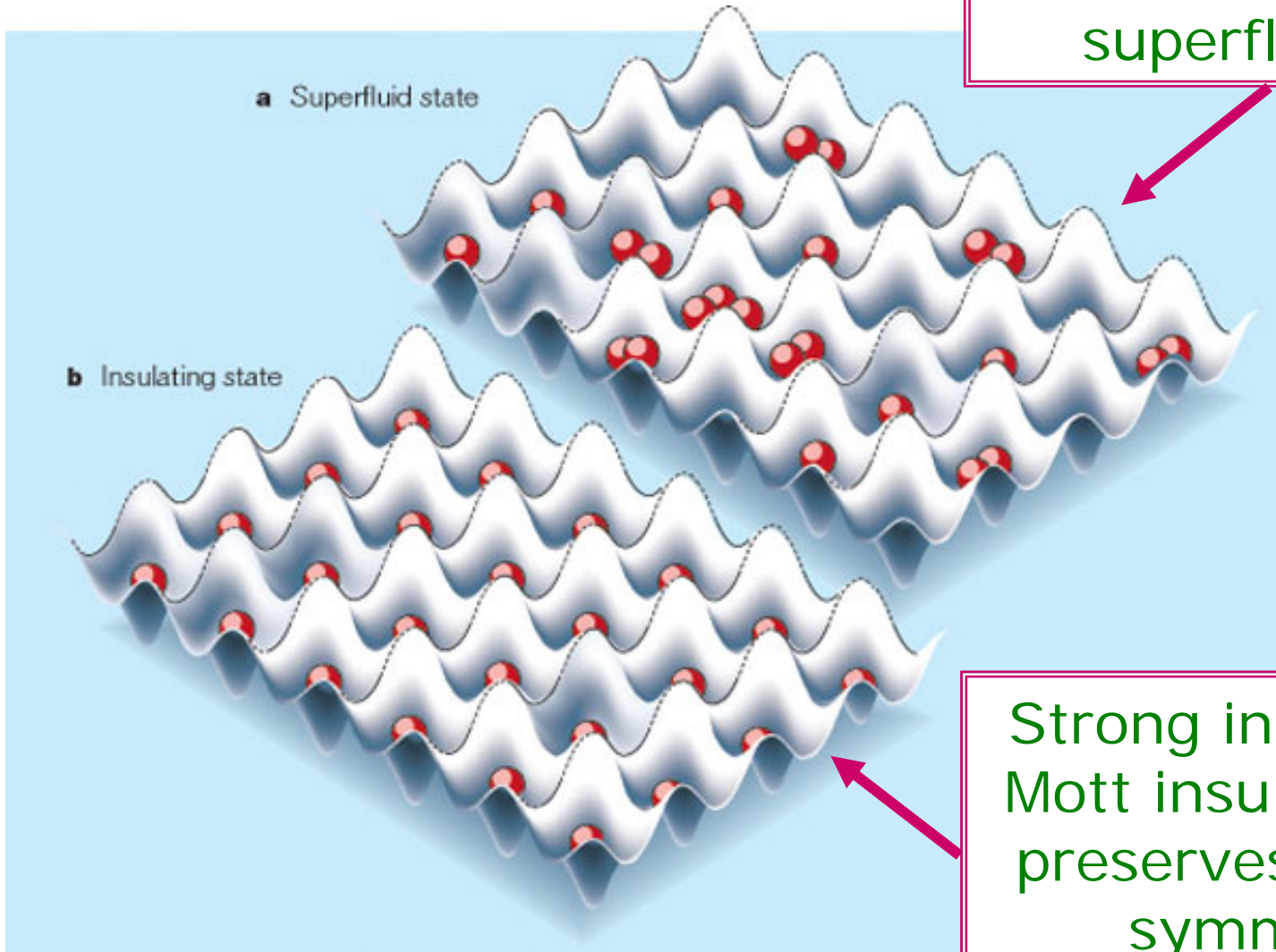
# Outline

- A. Superfluid-insulator transitions of bosons on the square lattice at fractional filling  
*Quantum mechanics of vortices in a superfluid proximate to a commensurate Mott insulator*
  
- B. Application to a short-range pairing model for the cuprate superconductors  
*Competition between VBS order and  $d$ -wave superconductivity*

A. Superfluid-insulator transitions of bosons  
on the square lattice at fractional filling

*Quantum mechanics of vortices in a  
superfluid proximate to a commensurate  
Mott insulator*

# Bosons at density $f = 1$



Weak interactions:  
superfluidity

Strong interactions:  
Mott insulator which  
preserves all lattice  
symmetries

LGW theory: continuous quantum transitions between these states

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

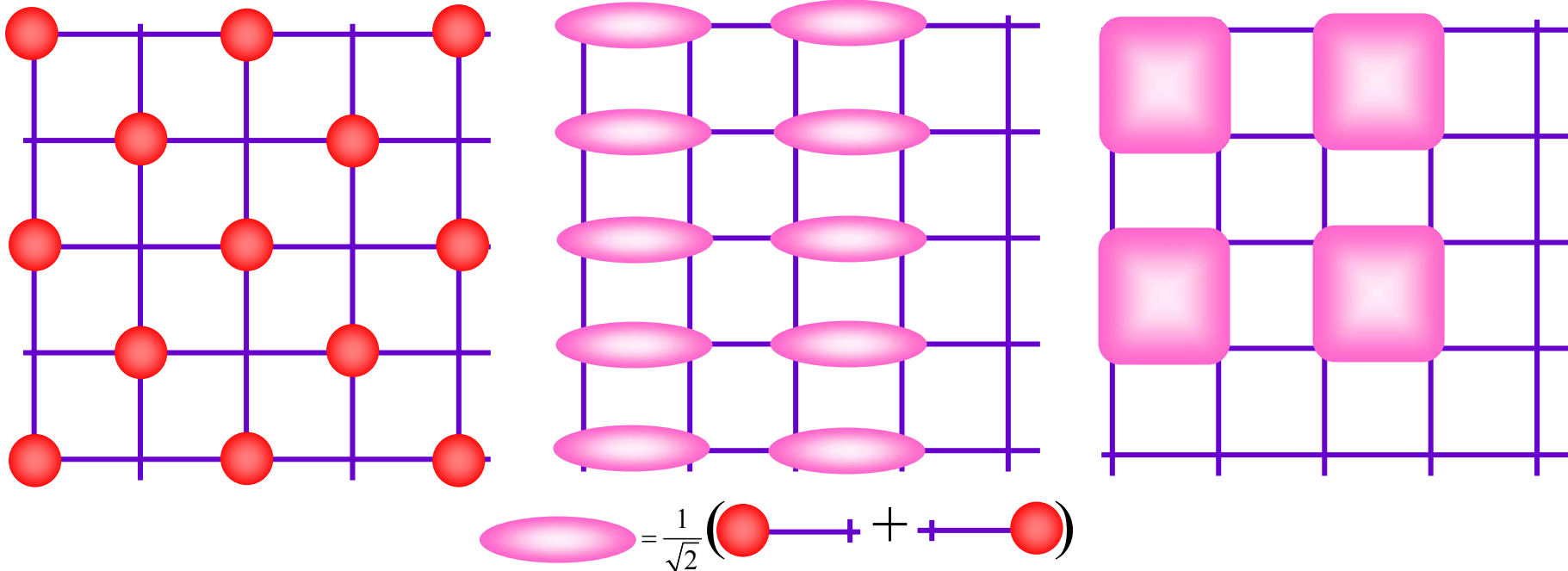


# Bosons at density $f = 1/2$ (equivalent to $S=1/2$ AFMs)

Weak interactions: superfluidity

$$\langle \Psi_{sc} \rangle \neq 0$$

Strong interactions: Candidate insulating states

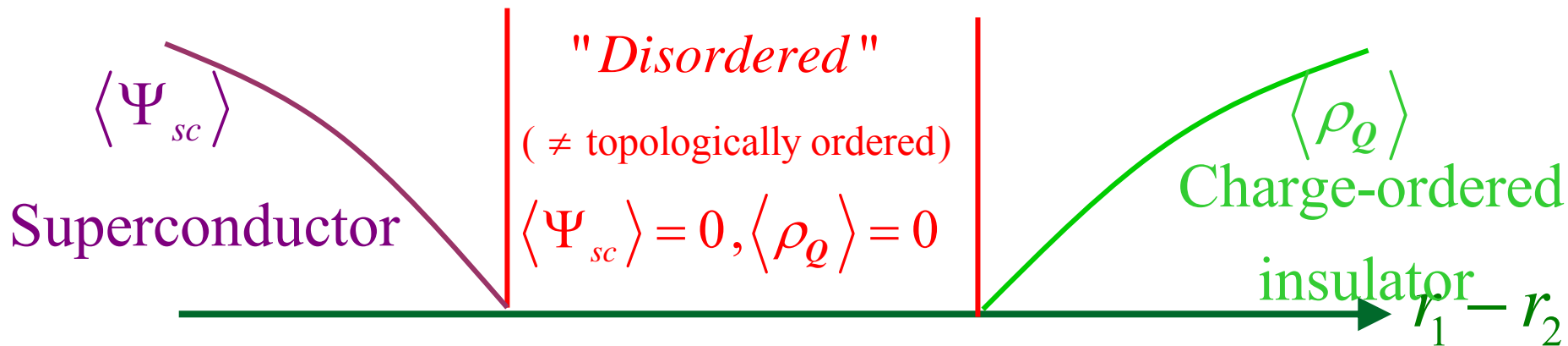
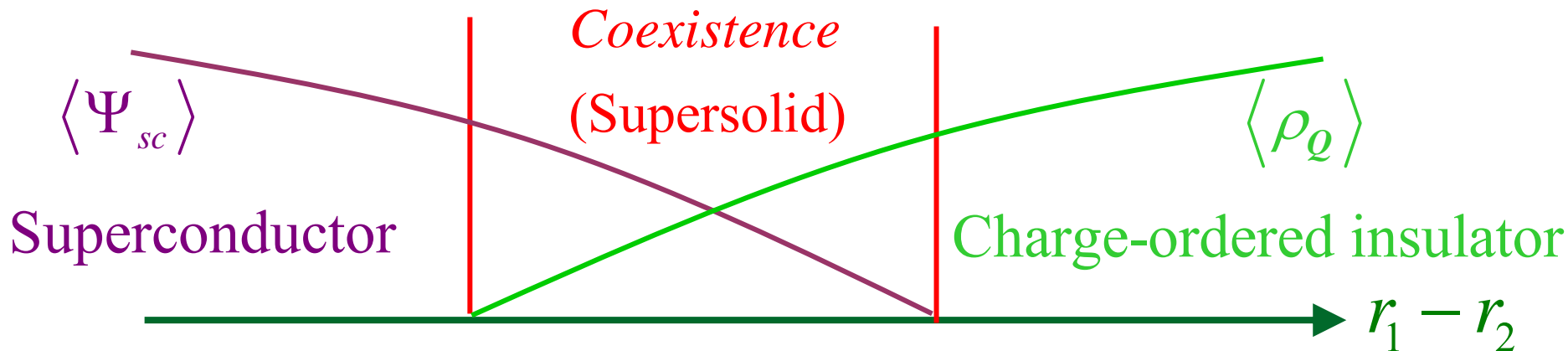
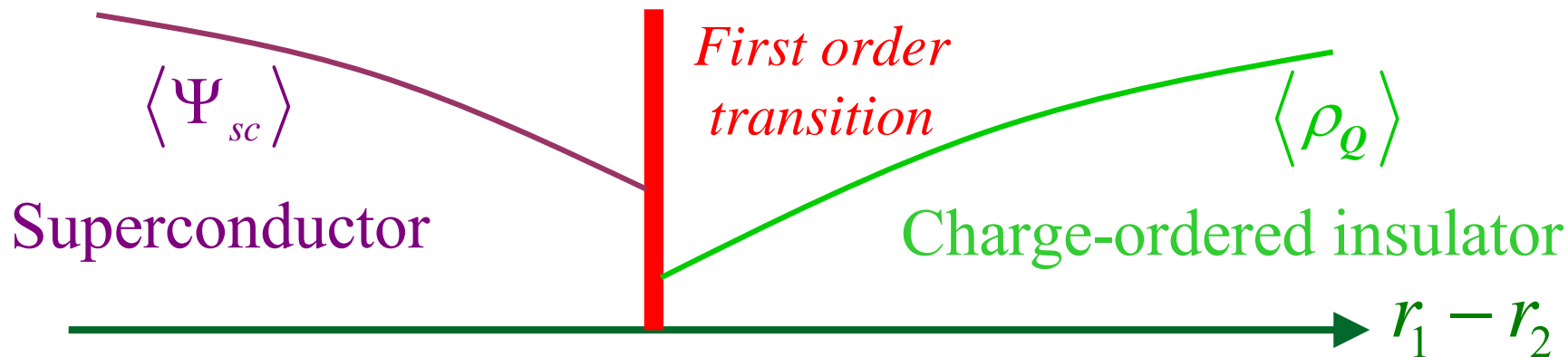


All insulating phases have density-wave order  $\rho(\mathbf{r}) = \sum_{\mathbf{q}} \rho_{\mathbf{q}} e^{i\mathbf{q} \cdot \mathbf{r}}$  with  $\langle \rho_{\mathbf{q}} \rangle \neq 0$

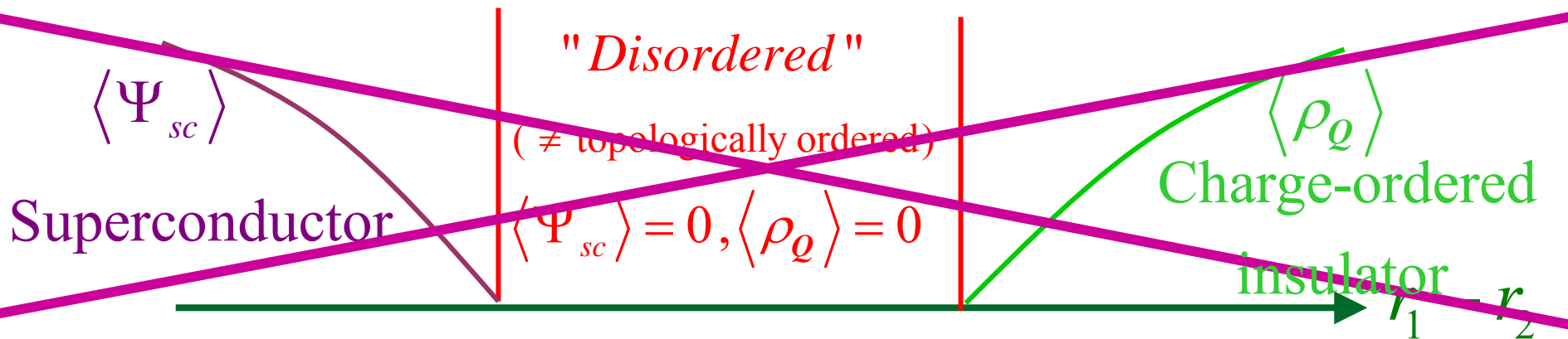
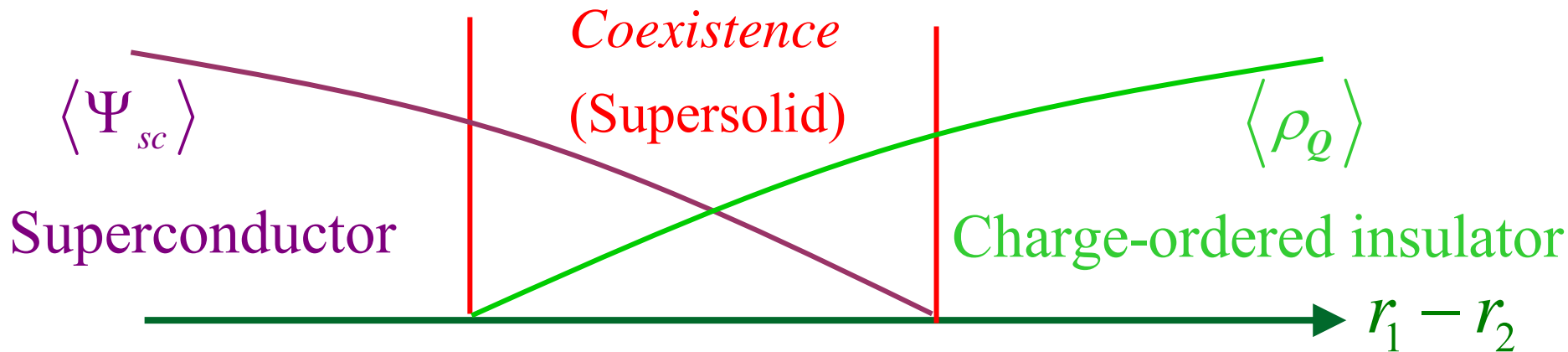
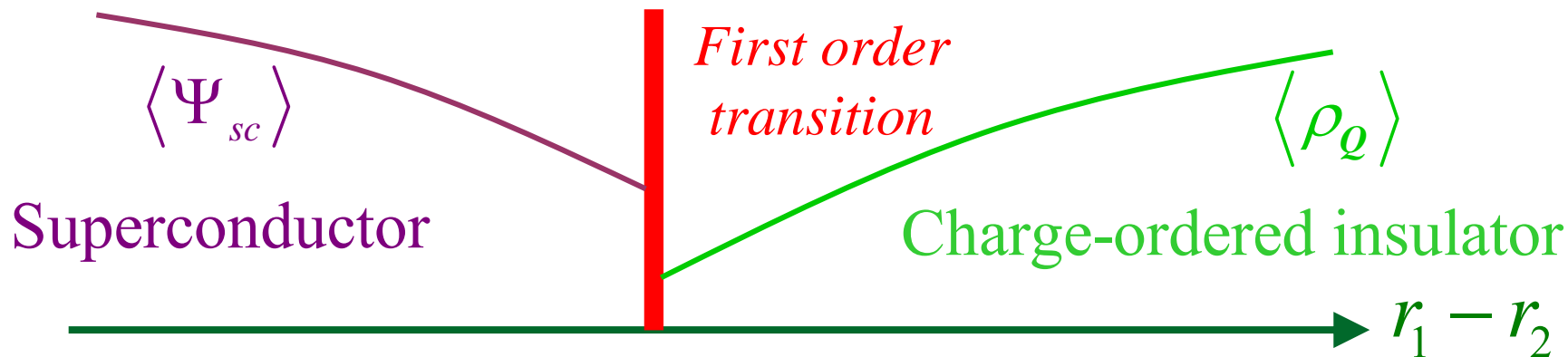
C. Lannert, M.P.A. Fisher, and T. Senthil, *Phys. Rev. B* **63**, 134510 (2001)

S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

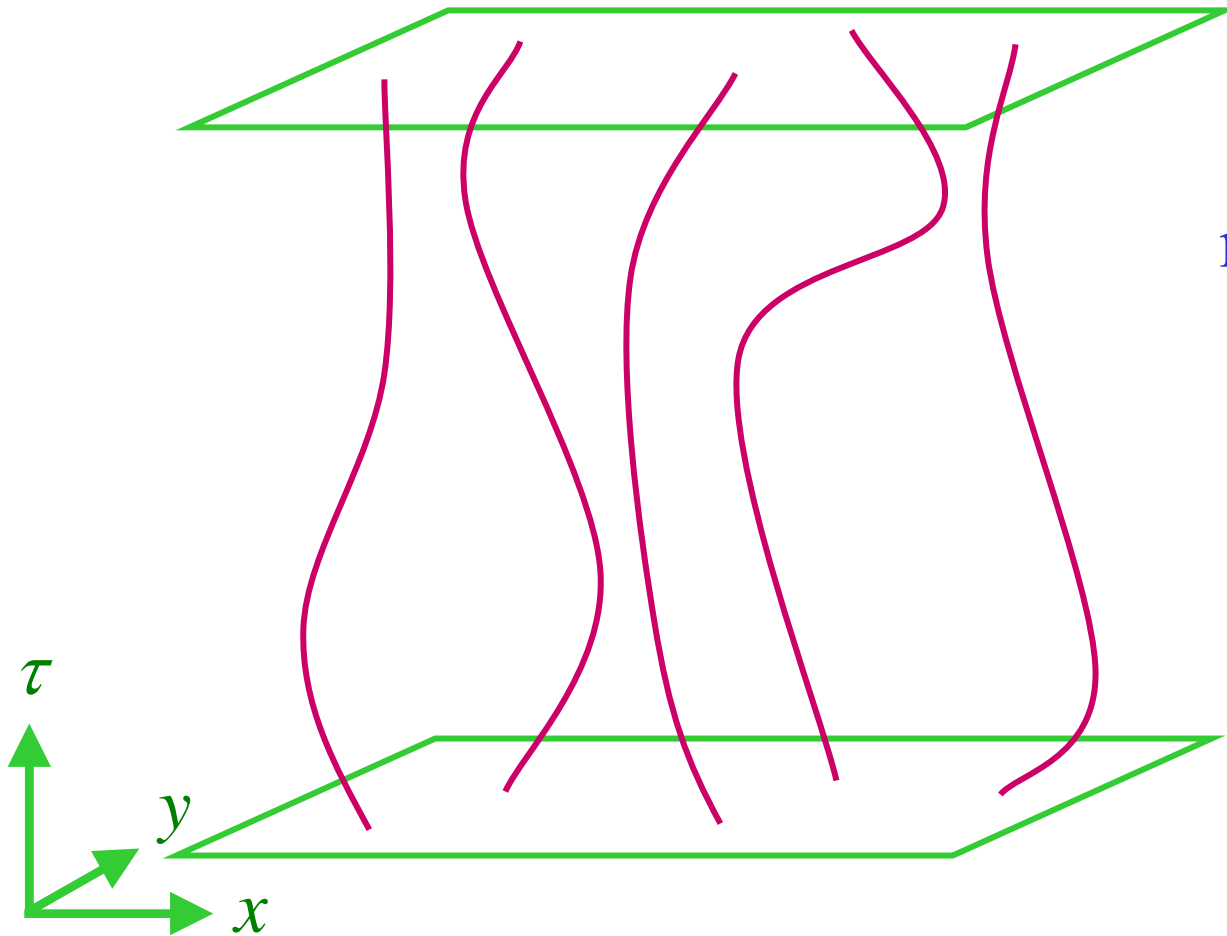
# Predictions of LGW theory



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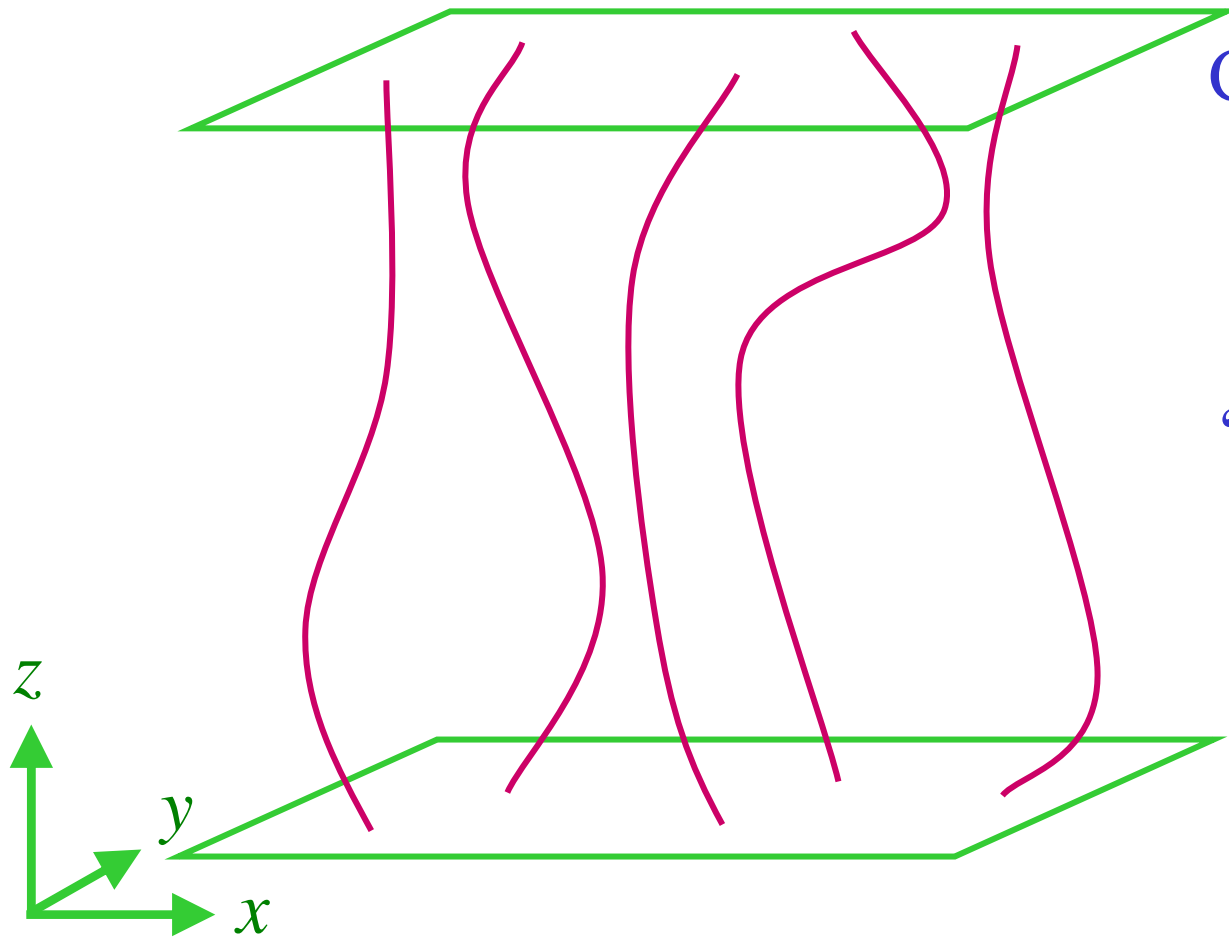


# Boson-vortex duality



Quantum  
mechanics of two-  
dimensional  
bosons: world  
lines of bosons in  
spacetime

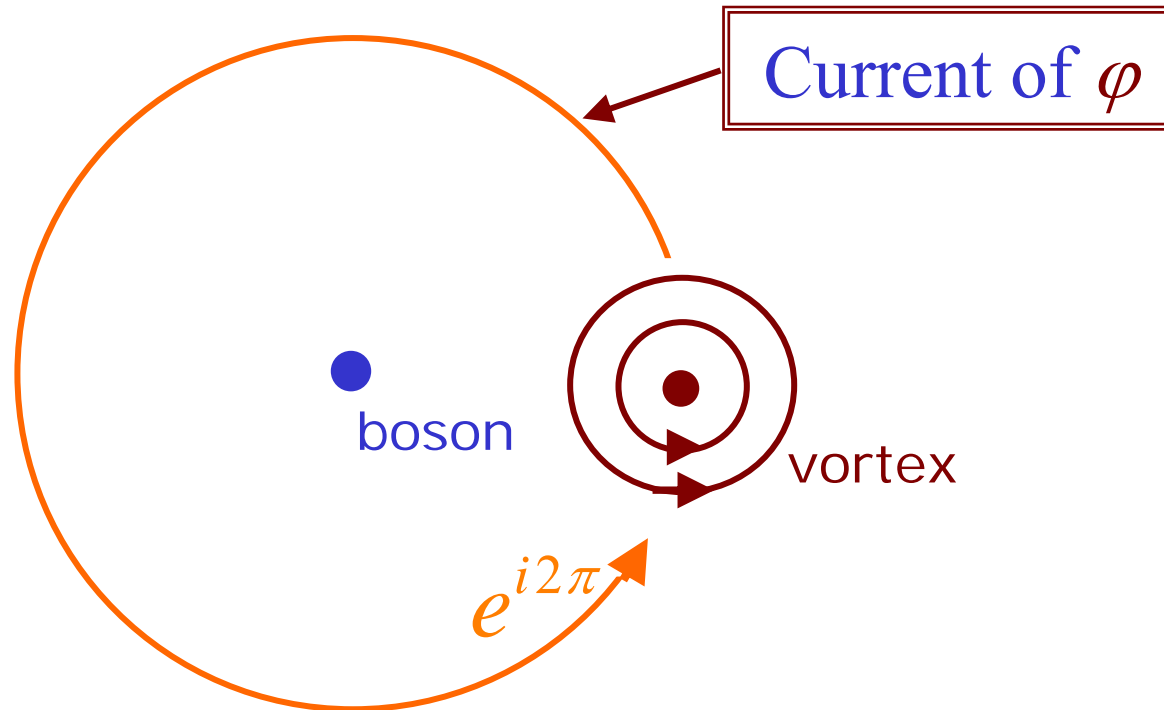
# Boson-vortex duality



Classical statistical mechanics of a “dual” three-dimensional “superconductor”, with order parameter  $\varphi$ : trajectories of vortices in a “magnetic” field

Strength of “magnetic” field on dual superconductor  $\varphi$   
= density of bosons =  $f$  flux quanta per plaquette

# Boson-vortex duality



The wavefunction of a vortex acquires a phase of  $2\pi$  each time the vortex encircles a boson

Strength of “magnetic” field on dual superconductor  $\varphi$   
= density of bosons =  $f$  flux quanta per plaquette

C. Dasgupta and B.I. Halperin, *Phys. Rev. Lett.* **47**, 1556 (1981); D.R. Nelson, *Phys. Rev. Lett.* **60**, 1973 (1988); M.P.A. Fisher and D.-H. Lee, *Phys. Rev. B* **39**, 2756 (1989);

## Boson-vortex duality

Statistical mechanics of dual “superconductor”  $\varphi$ , is invariant under the square lattice space group:

$T_x, T_y$  : Translations by a lattice spacing in the  $x, y$  directions

$R$  : Rotation by 90 degrees.

Magnetic space group:

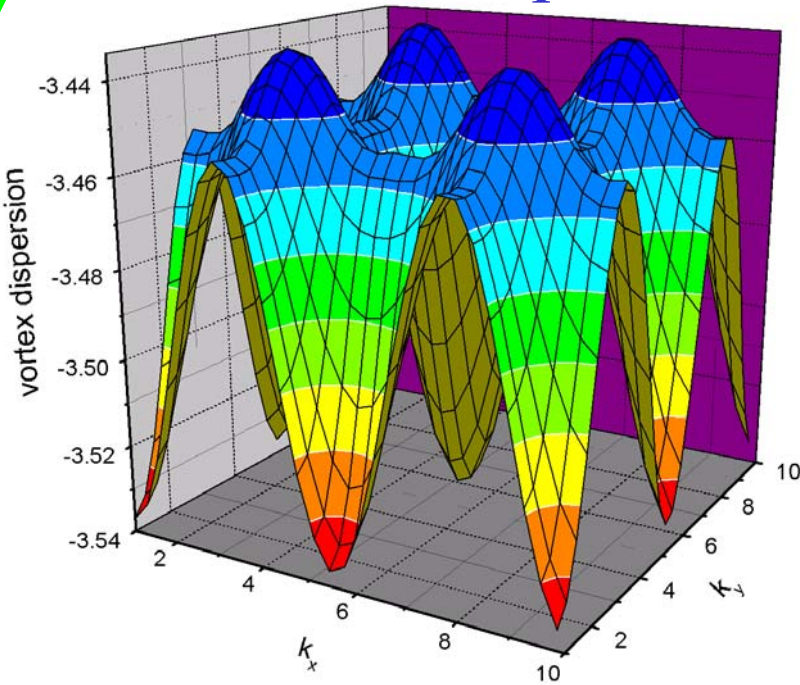
$$T_x T_y = e^{2\pi i f} T_y T_x \ ;$$

$$R^{-1} T_y R = T_x \ ; \ R^{-1} T_x R = T_y^{-1} \ ; \ R^4 = 1$$

Strength of “magnetic” field on dual superconductor  $\varphi$   
= density of bosons =  $f$  flux quanta per plaquette

# Boson-vortex duality

## Hofstadter spectrum of dual “superconducting” order $\varphi$



At density  $f = p / q$  ( $p, q$  relatively prime integers) there are  $q$  species of vortices,  $\varphi_\ell$  (with  $\ell = 1 \dots q$ ), associated with  $q$  gauge-equivalent regions of the Brillouin zone

Magnetic space group:

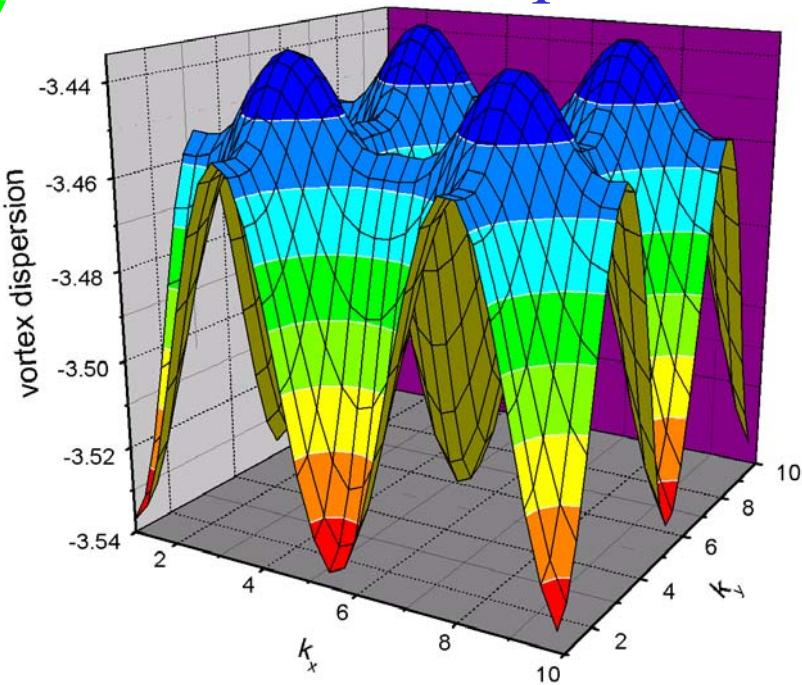
$$T_x T_y = e^{2\pi i f} T_y T_x ;$$

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# Boson-vortex duality

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The  $q$  vortices form a *projective* representation of the space group

$$T_x : \varphi_\ell \rightarrow \varphi_{\ell+1} \quad ; \quad T_y : \varphi_\ell \rightarrow e^{2\pi i \ell f} \varphi_\ell$$

$$R : \varphi_\ell \rightarrow \frac{1}{\sqrt{q}} \sum_{m=1}^q \varphi_m e^{2\pi i \ell m f}$$

## Boson-vortex duality

The  $q \varphi_\ell$  vortices characterize *both* superconducting and density wave orders

Superconductor/insulator :  $\langle \varphi_\ell \rangle = 0 / \langle \varphi_\ell \rangle \neq 0$

# Boson-vortex duality

The  $q$   $\varphi_\ell$  vortices characterize *both* superconducting and density wave orders

Density wave order:

Status of space group symmetry determined by

density operators  $\rho_{\mathbf{Q}}$  at wavevectors  $\mathbf{Q}_{mn} = \frac{2\pi p}{q}(m, n)$

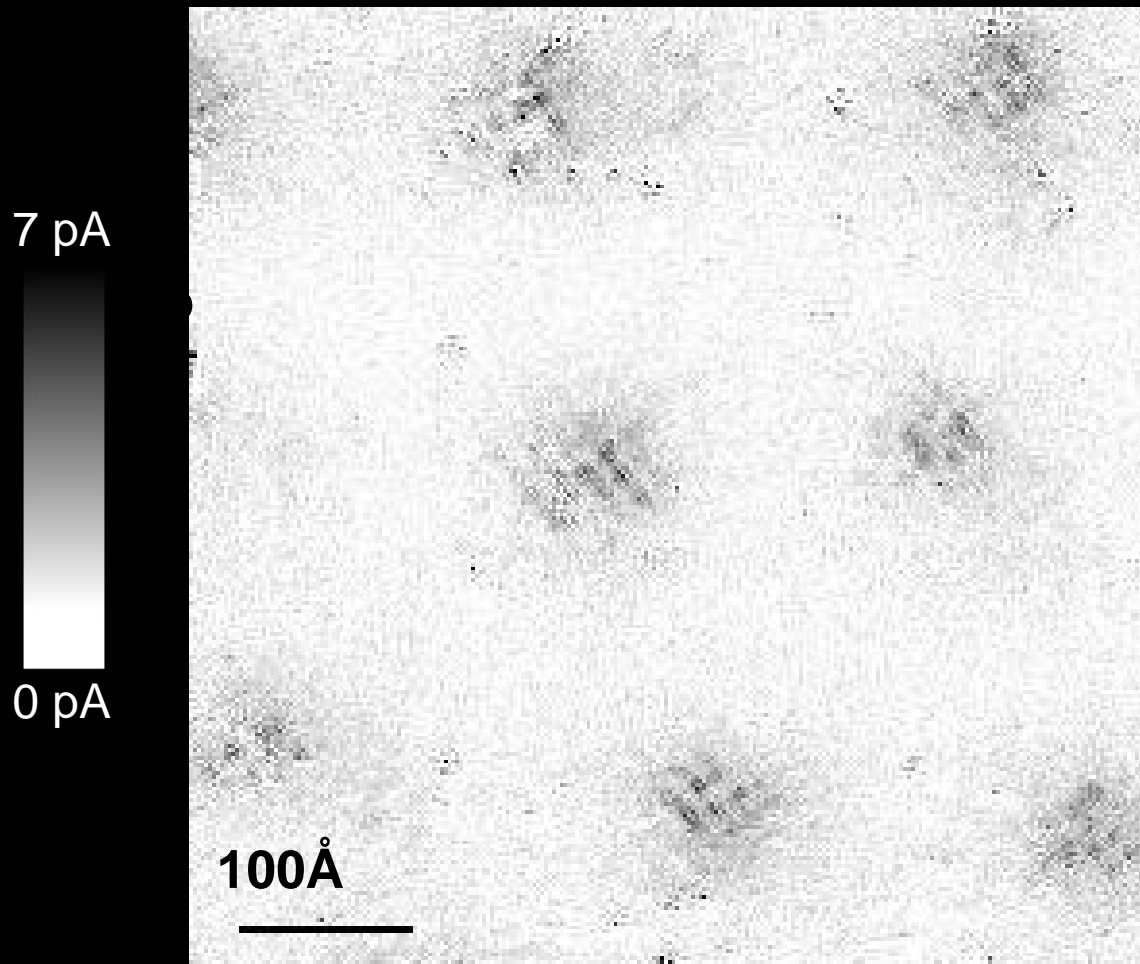
$$\rho_{mn} = e^{i\pi mnf} \sum_{\ell=1}^q \varphi_\ell^* \varphi_{\ell+n} e^{2\pi i \ell m f}$$

$$T_x : \rho_{\mathbf{Q}} \rightarrow \rho_{\mathbf{Q}} e^{i\mathbf{Q} \cdot \hat{x}} \quad ; \quad T_y : \rho_{\mathbf{Q}} \rightarrow \rho_{\mathbf{Q}} e^{i\mathbf{Q} \cdot \hat{y}}$$

$$R : \rho(\mathbf{Q}) \rightarrow \rho(R\mathbf{Q})$$

Each pinned vortex in the superfluid has a halo of density wave order over a length scale  $\approx$  the zero-point quantum motion of the vortex. This scale diverges upon approaching the Mott insulator

# Vortex-induced LDOS of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ integrated from 1meV to 12meV at 4K

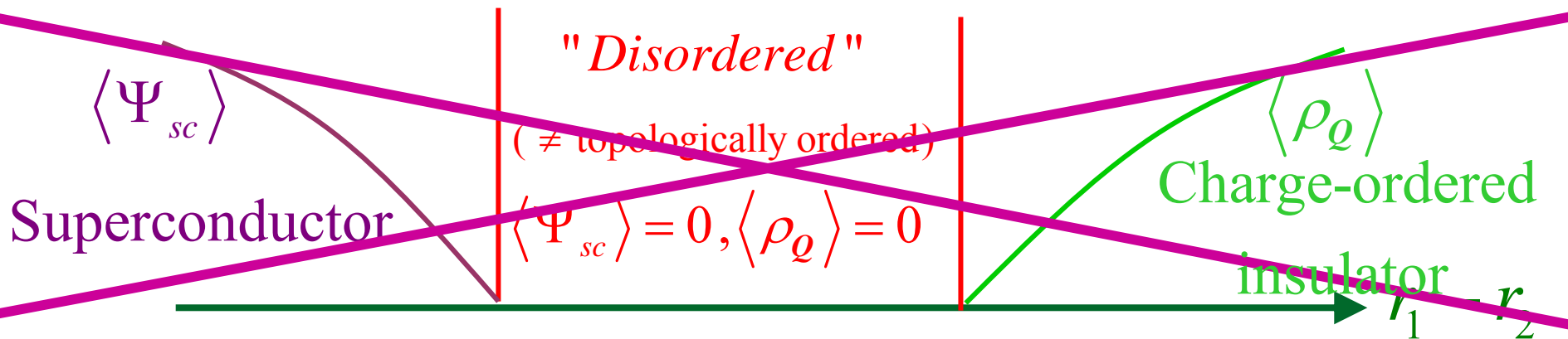
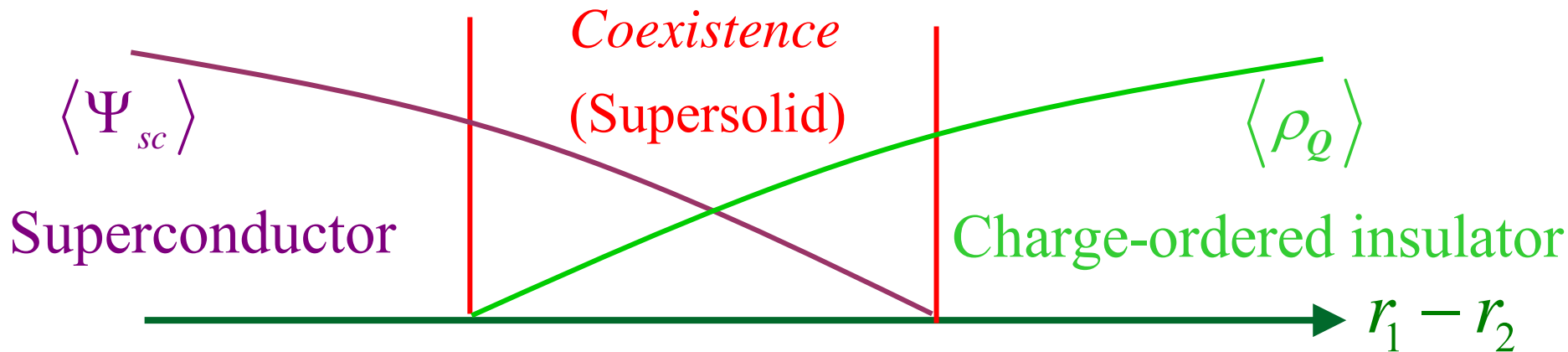
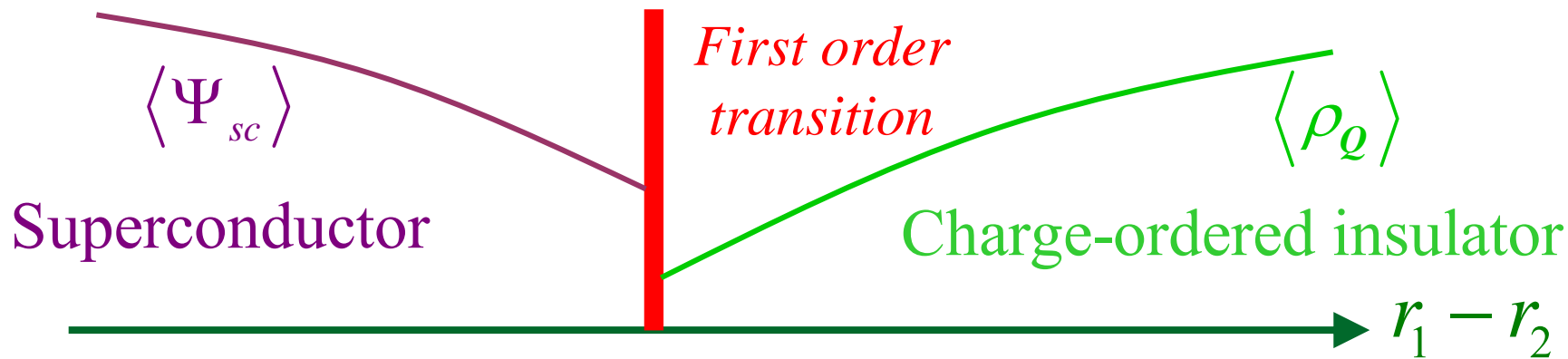


Vortices have halos with LDOS modulations at a period  $\approx 4$  lattice spacings

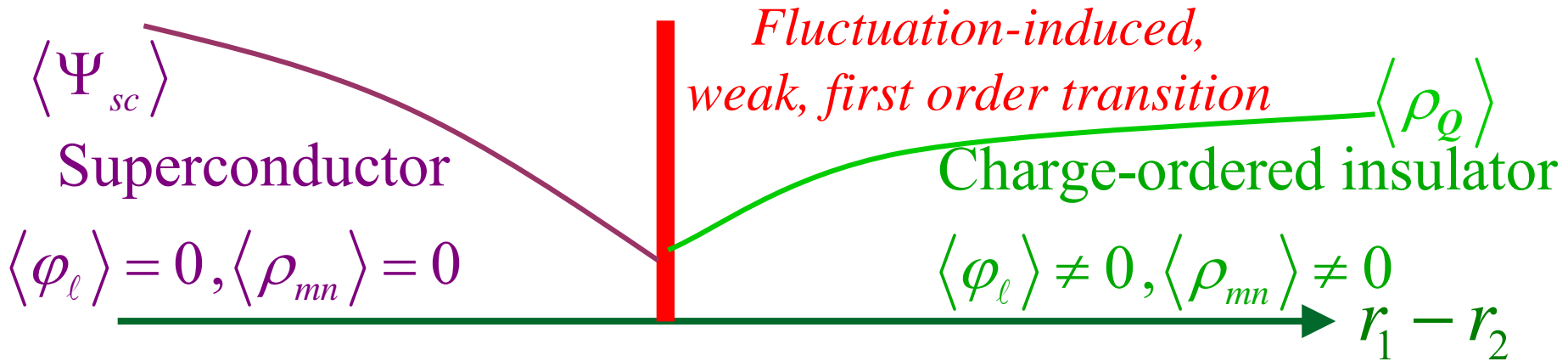
J. Hoffman E. W. Hudson, K. M. Lang,  
V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida,  
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Prediction of VBS order near vortices: K. Park and S. Sachdev, *Phys. Rev. B* 64, 184510 (2001).

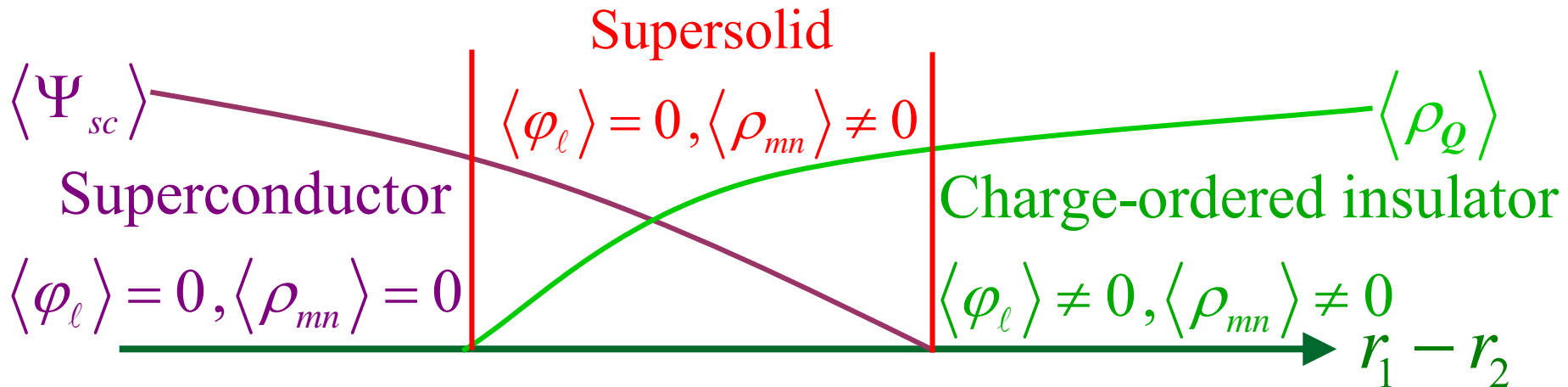
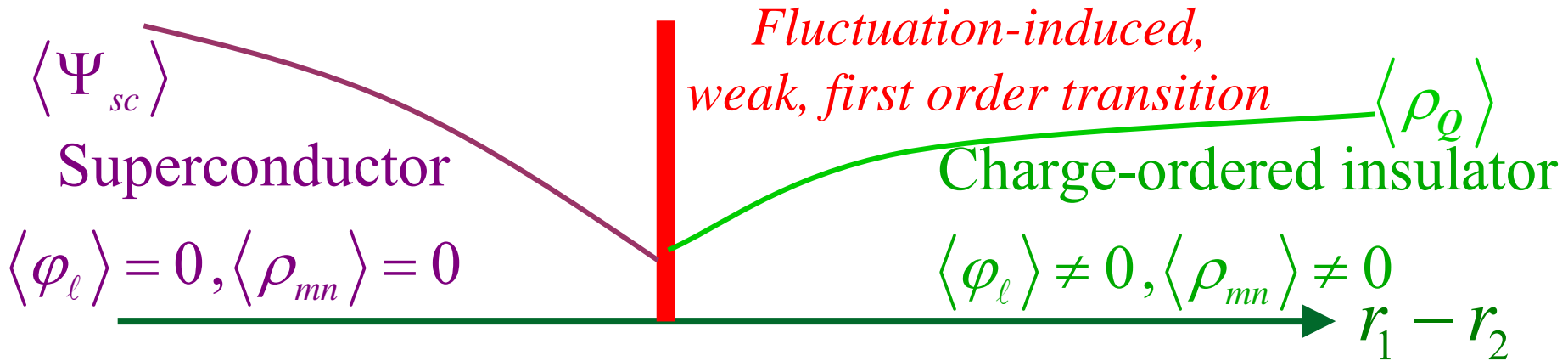
# Predictions of LGW theory



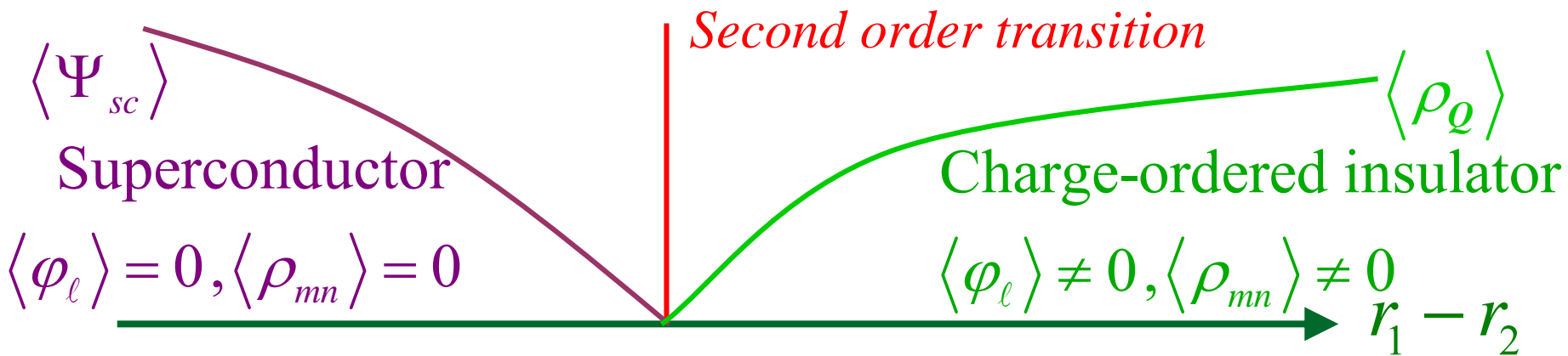
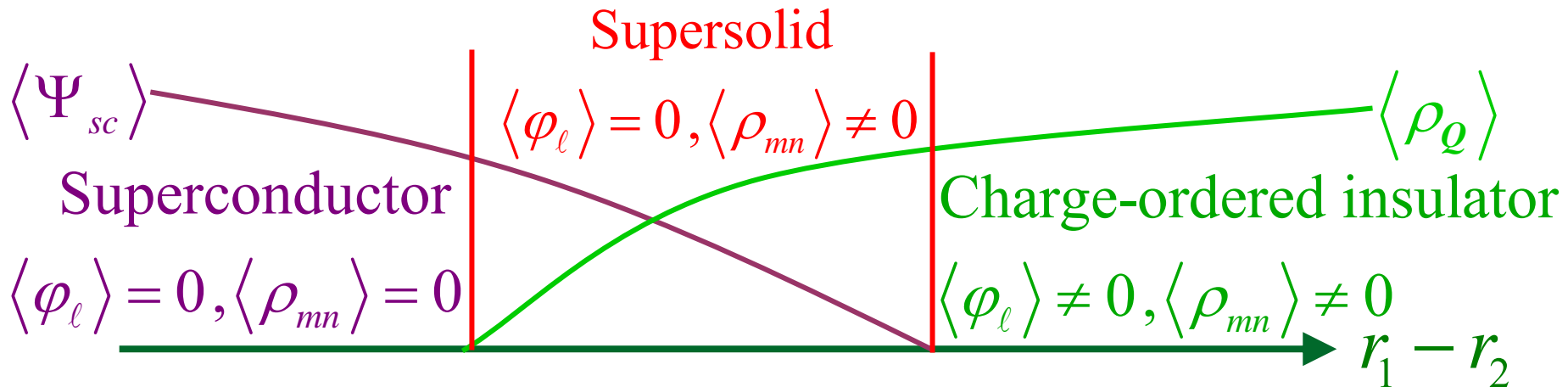
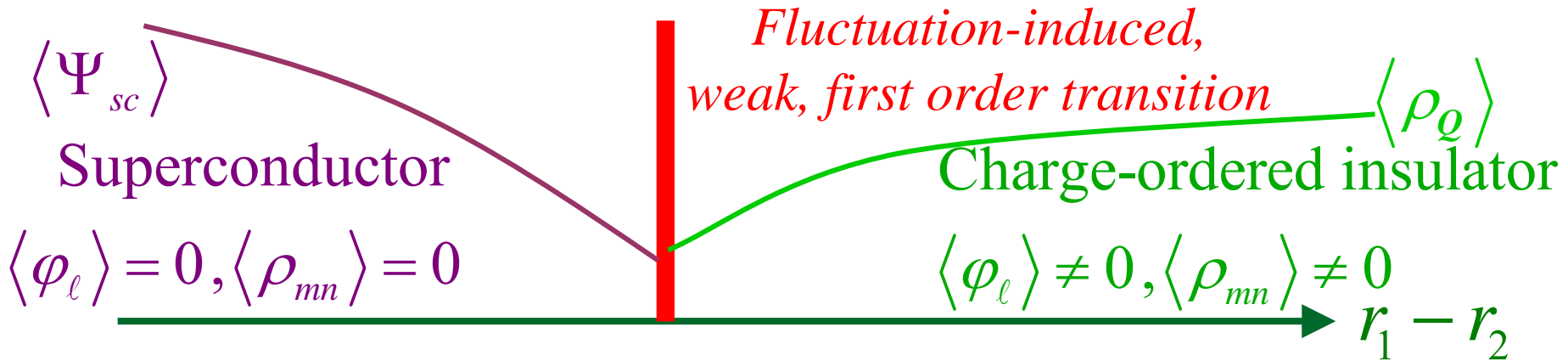
# Analysis of “extended LGW” theory of projective representation



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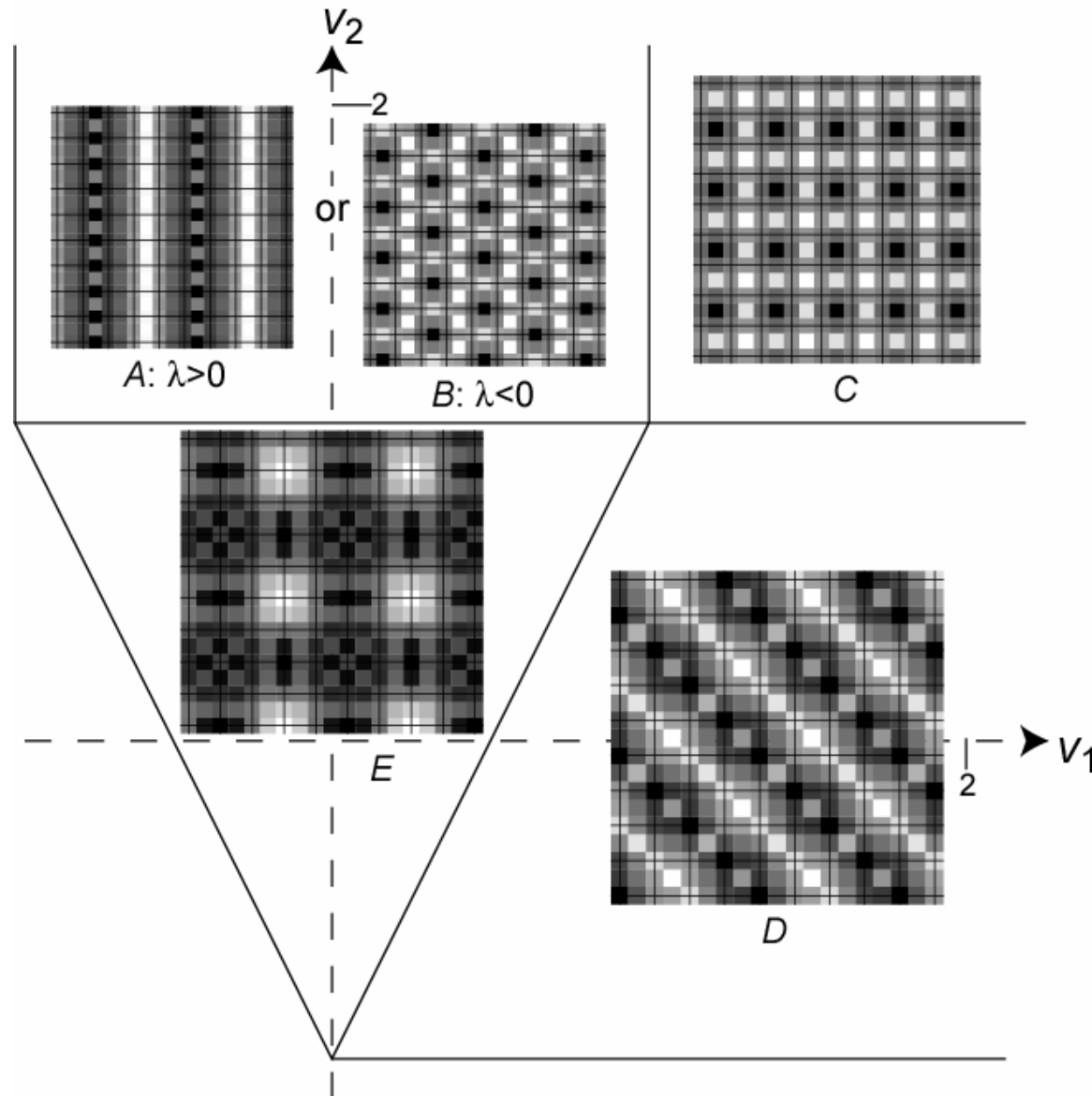
# Analysis of “extended LGW” theory of projective representation





# Analysis of “extended LGW” theory of projective representation

Spatial structure of insulators for  $q=4$  ( $f=1/4$  or  $3/4$ )



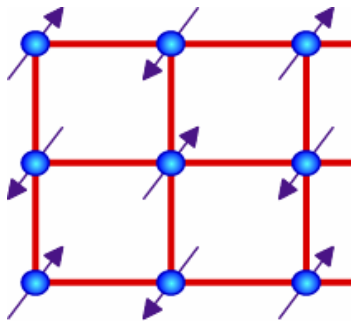
$a \times b$  unit cells;  
 $q/a, q/b, ab/q,$   
all integers

## B. Application to a short-range pairing model for the cuprate superconductors

*Competition between VBS order and d-wave  
superconductivity*

# Phase diagram of doped antiferromagnets

$g$  = parameter controlling strength of quantum fluctuations in a semiclassical theory of the destruction of Neel order

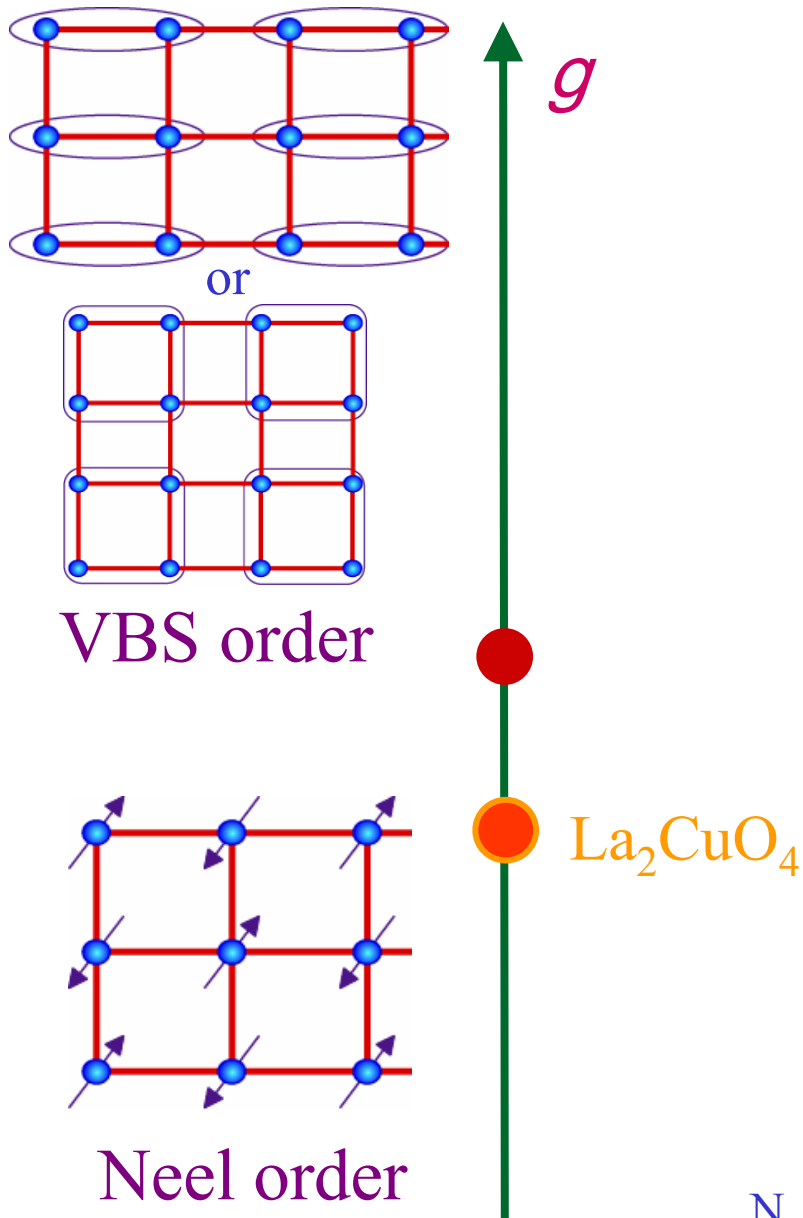


Neel order



$\text{La}_2\text{CuO}_4$

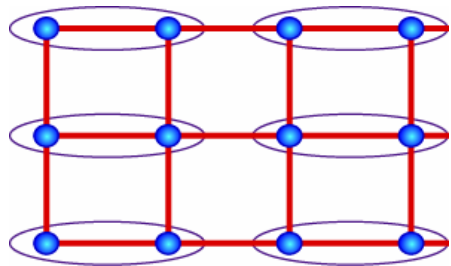
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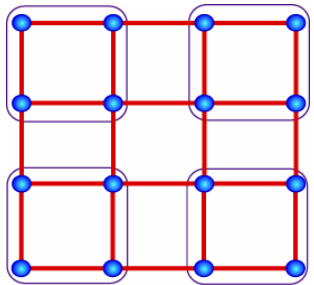
N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

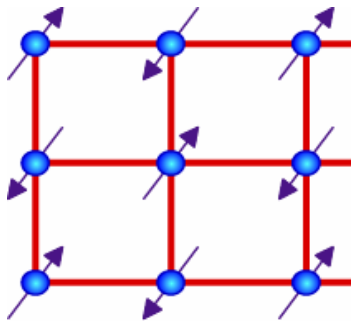
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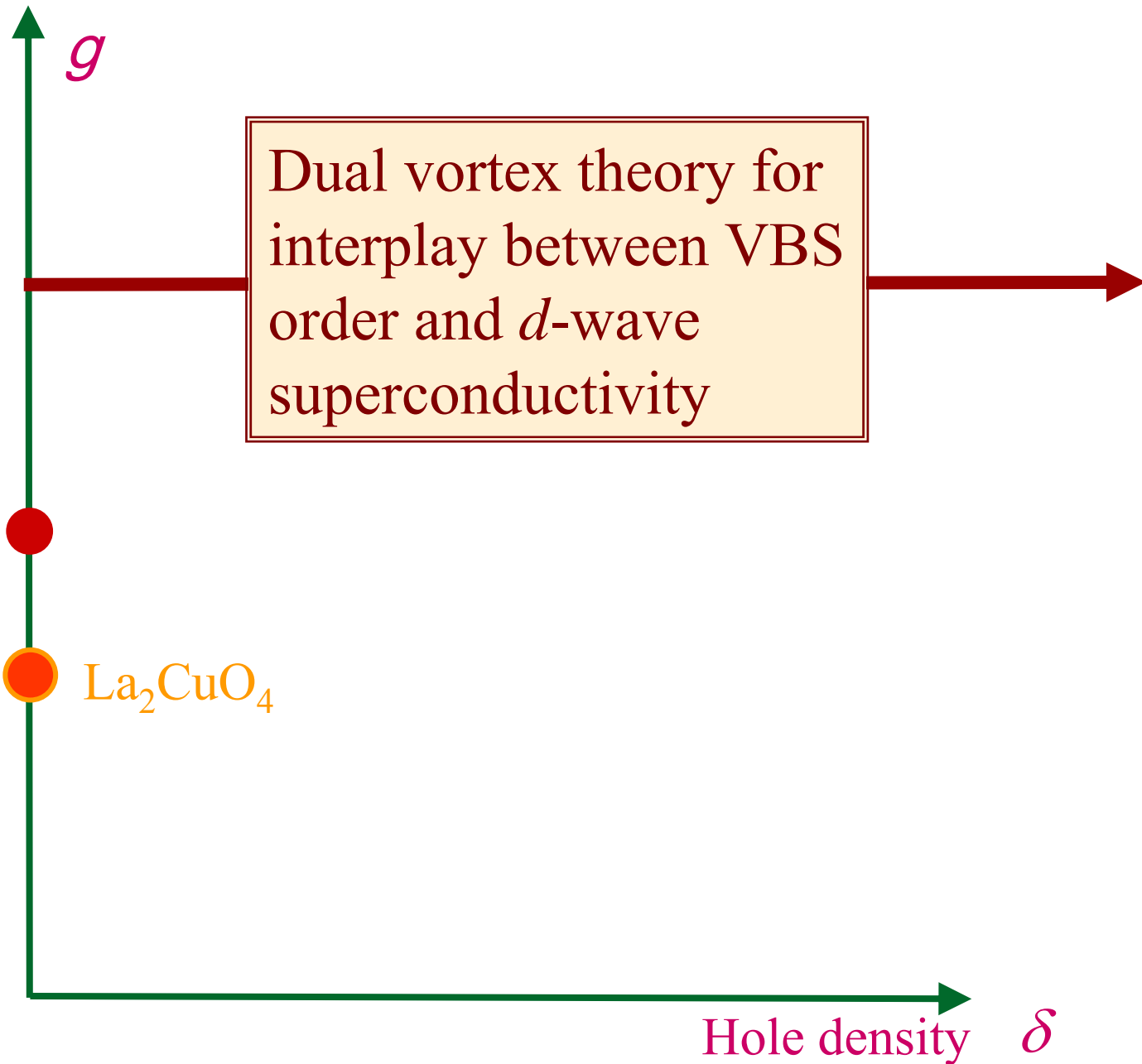
or



VBS order



Neel order



Dual vortex theory for interplay between VBS order and  $d$ -wave superconductivity

Hole density  $\delta$

A convenient derivation of the dual theory for vortices is obtained from the doped quantum dimer model

$$\begin{aligned}
 H_{dqd} = & J \sum_{\square} (| \begin{array}{cc} \bullet & \bullet \\ | & | \\ \bullet & \bullet \end{array} \rangle \langle \begin{array}{cc} \bullet & \bullet \\ \text{---} & \text{---} \\ \bullet & \bullet \end{array} | + | \begin{array}{cc} \bullet & \bullet \\ \text{---} & \text{---} \\ \bullet & \bullet \end{array} \rangle \langle \begin{array}{cc} | & | \\ \bullet & \bullet \end{array} |) \\
 - t \sum_{\triangle} (| \begin{array}{c} \circ \\ | \\ \bullet \end{array} \rangle \langle \begin{array}{cc} \bullet & \bullet \\ \text{---} & \text{---} \\ \circ & \circ \end{array} | + | \begin{array}{cc} \bullet & \bullet \\ \text{---} & \text{---} \\ \circ & \circ \end{array} \rangle \langle \begin{array}{c} \circ \\ | \\ \bullet \end{array} |) - \dots
 \end{aligned}$$

Density of holes =  $\delta$

E. Fradkin and S. A. Kivelson, *Mod. Phys. Lett. B* **4**, 225 (1990).

## Duality mapping of doped dimer model shows:

Vortices in the superconducting state obey the magnetic translation algebra

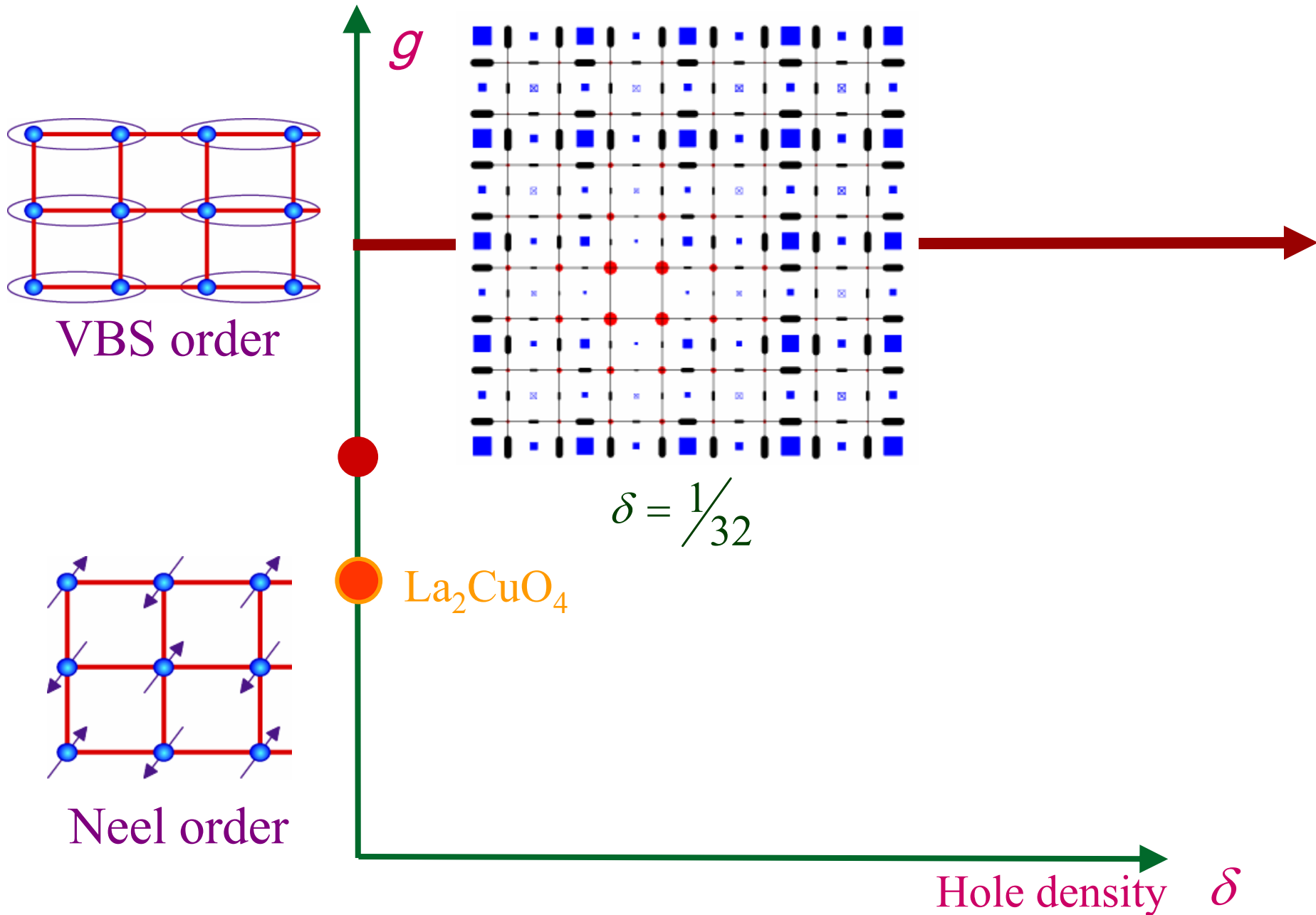
$$T_x T_y = e^{2\pi i f} T_y T_x$$

with  $f = \frac{p}{q} = \frac{1 - \delta_{MI}}{2}$

where  $\delta_{MI}$  is the density of holes in the proximate Mott insulator (for  $\delta_{MI} = 1/8$ ,  $f = 7/16 \Rightarrow q = 16$ )

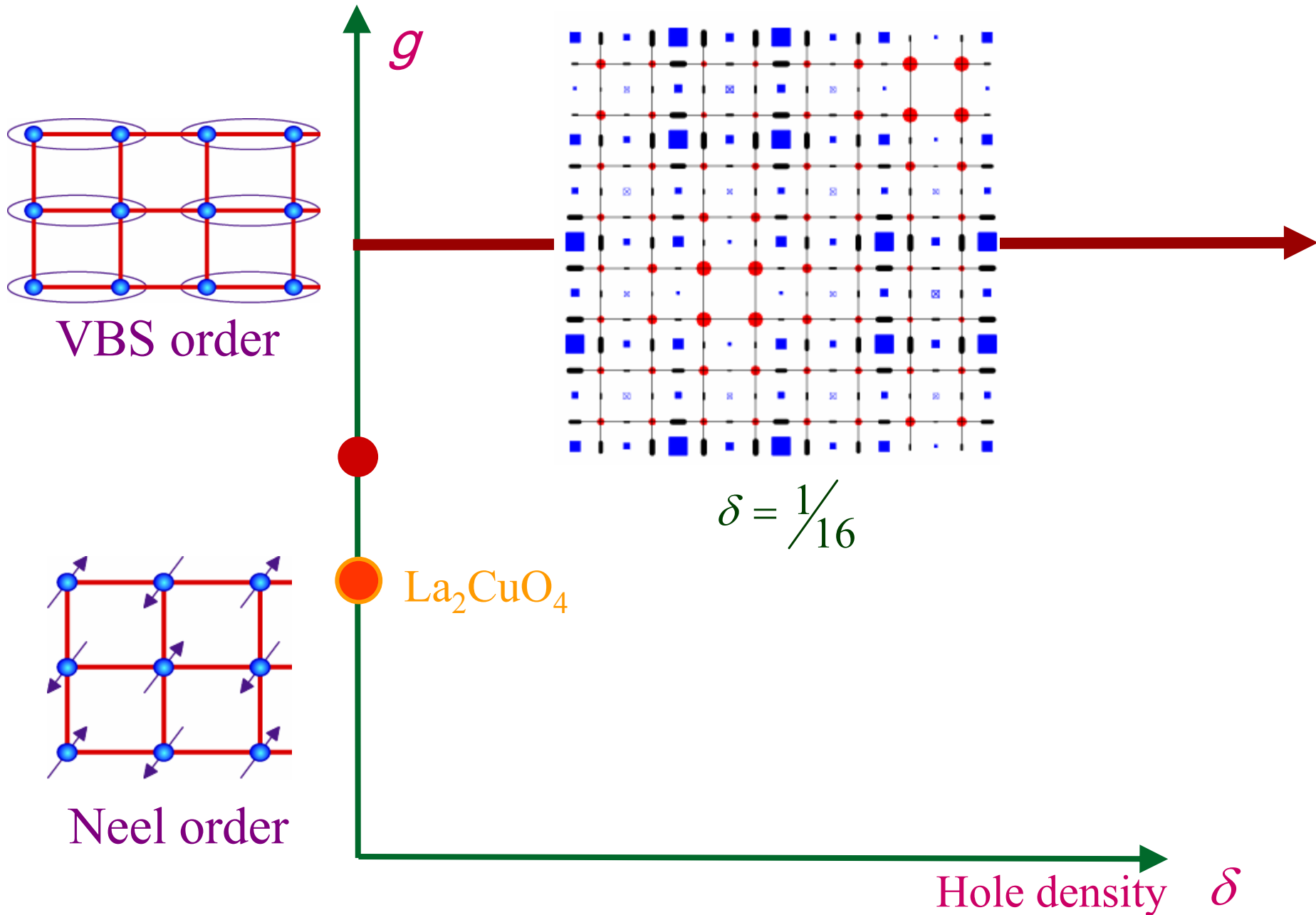
Most results of Part A on bosons can be applied unchanged with  $q$  as determined above

# Phase diagram of doped antiferromagnets

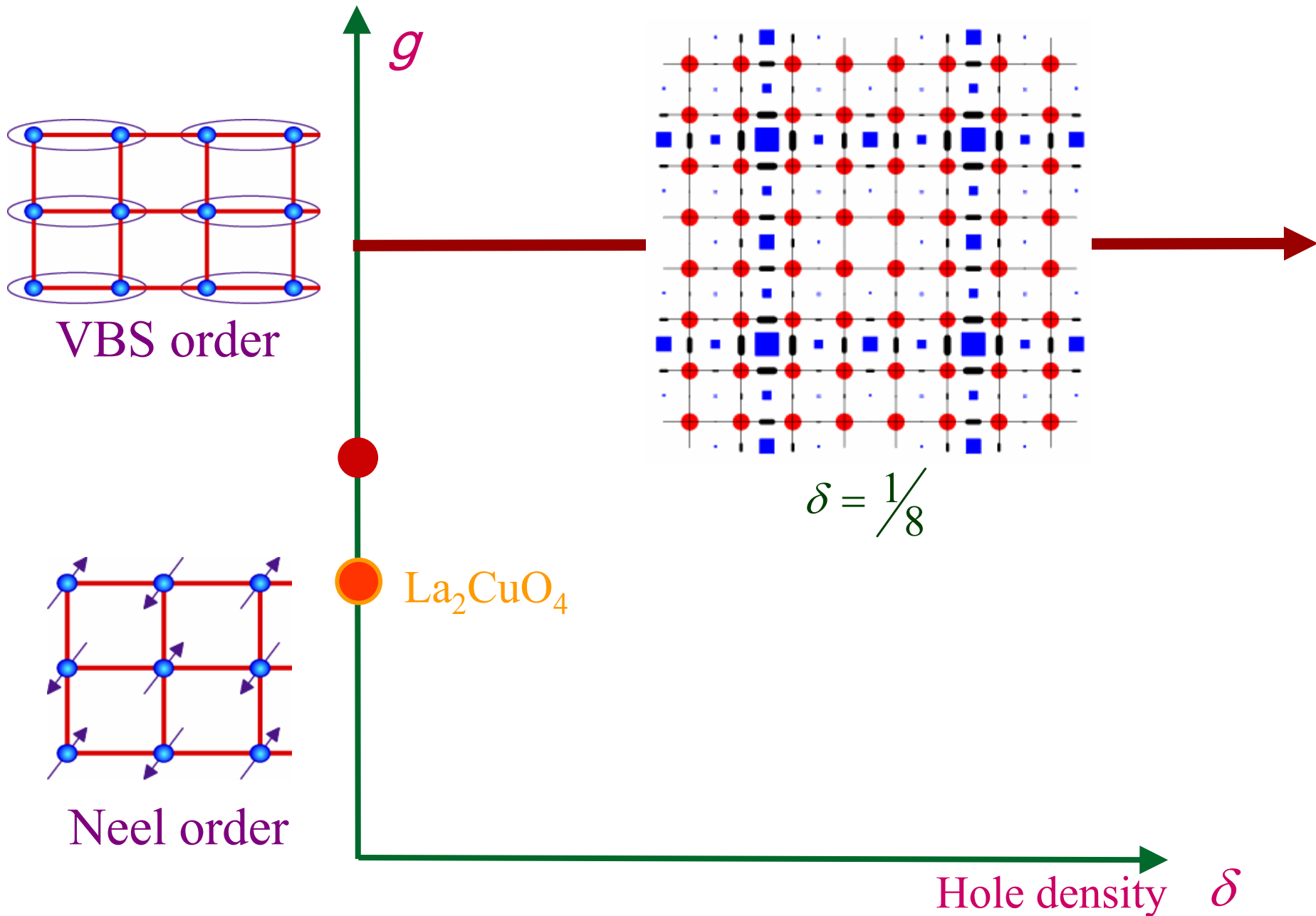




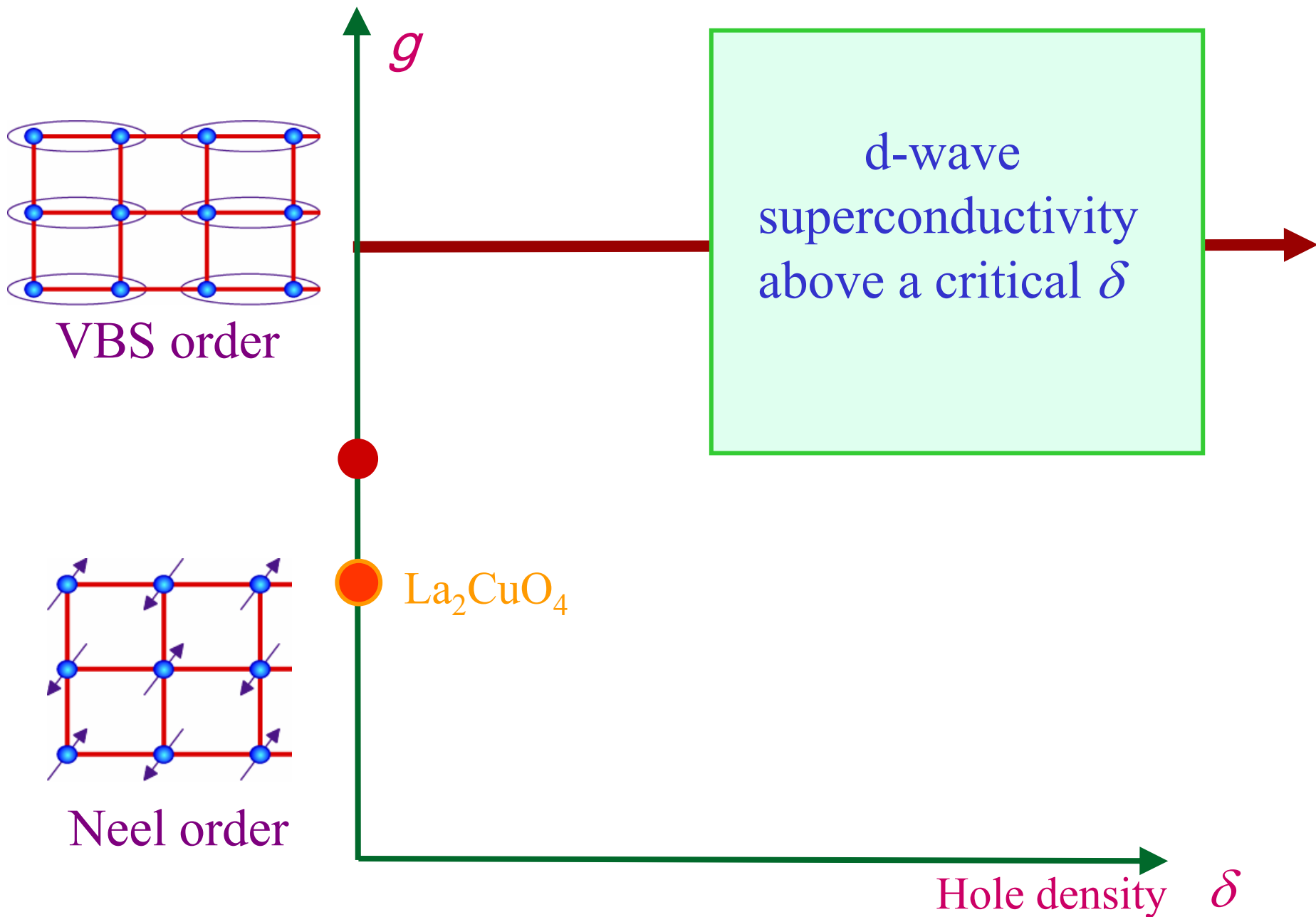
# Phase diagram of doped antiferromagnets



# Phase diagram of doped antiferromagnets



# Phase diagram of doped antiferromagnets



## Conclusions

- I. Description of the competition between superconductivity and density wave order in term of defects (vortices). Theory naturally excludes “disordered” phase with no order.
- II. Vortices carry the quantum numbers of *both* superconductivity *and* the square lattice space group (in a projective representation).
- III. Vortices carry halo of charge order, and pinning of vortices/anti-vortices leads to a unified theory of STM modulations in zero and finite magnetic fields.
- IV. **Conventional (LGW) picture:** density wave order causes the transport energy gap, the appearance of the Mott insulator.  
**Present picture:** Mott localization of charge carriers is more fundamental, and (weak) density wave order emerges naturally in theory of the Mott transition.