Classifying two-dimensional superfluids: why there is more to cuprate superconductivity than the condensation of charge $-2e$ Cooper pairs

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Experiments on the cuprate superconductors show:

• Tendency to produce “density” wave order near wavevectors \( (2\pi/a)(1/4,0) \) and \( (2\pi/a)(0,1/4) \).

• Proximity to a Mott insulator at hole density \( \delta = 1/8 \) with long-range “density” wave order at wavevectors \( (2\pi/a)(1/4,0) \) and \( (2\pi/a)(0,1/4) \).

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Needed: A quantum theory of transitions between superfluid/supersolid/insulating phases at fractional filling, and a deeper understanding of the role of vortices
Outline

A. Superfluid-insulator transitions of bosons on the square lattice at fractional filling
   Quantum mechanics of vortices in a superfluid proximate to a commensurate Mott insulator at filling $f$

B. Extension to electronic models for the cuprate superconductors
   Dual vortex theories of the doped
   (1) Quantum dimer model
   (2) “Staggered flux” spin liquid
A. Superfluid-insulator transitions of bosons on the square lattice at fractional filling

Quantum mechanics of vortices in a superfluid proximate to a commensurate Mott insulator at filling $f$
Bosons at filling fraction $f = 1$

Weak interactions: superfluidity

Strong interactions: Mott insulator which preserves all lattice symmetries

Excitations of the superfluid: **Vortices**

Vortices proliferate as the superfluid approaches the insulator.

In two dimensions, we can view the vortices as point particle excitations of the superfluid. What is the quantum mechanics of these “particles”? 
In ordinary fluids, vortices experience the Magnus Force

\[ F_M = (\text{mass density of fluid}) \cdot (\text{velocity}) \cdot (\text{circulation}) \]
In a Galilean-invariant superfluid at $T = 0$, the Magnus force on a quantized vortex with vorticity $m$ ($m$ integer) is

$$F_M = m \hbar \rho \left( \mathbf{v} - \frac{d \mathbf{r}}{dt} \right) \times \hat{z}$$

where $\rho$ is the number density of bosons, $\mathbf{v}$ is local superfluid velocity, and $\mathbf{r}$ is the position of the vortex.
In the presence of a lattice, we must distinguish two physically distinct situations, and write

\[ F_M = F_M^{(E)} + F_M^{(B)} \]

with

1. A stationary vortex in a moving superfluid

\[ F_M^{(E)} = mE \text{ where } E = h\rho \mathbf{v} \times \hat{z} \]

2. A moving vortex in a stationary superfluid

\[ F_M^{(B)} = m \frac{d\mathbf{r}}{dt} \times \mathbf{B} \text{ where } \mathbf{B} = -h\rho \hat{z}. \]

The expression for \( F_M^{(E)} \) is basically correct (with \( \rho \to \rho_s \)), while that for \( F_M^{(B)} \) is *not* correct. The latter is modified by the periodic potential of the lattice close to a Mott insulator.................
$F^{(B)}_M$ can be re-interpreted as a Lorentz force on a vortex “particle” due to a “magnetic” field $B=\hbar \rho$

So we need to consider the quantum mechanics of a particle moving in a “magnetic” field $B$ and a periodic lattice potential --- the Hofstadter problem.

At filling fraction $f=1$, the $B$ field is such that there is exactly one flux quantum per unit cell. Such a $B$ field is “invisible”, and the vortex “particle” moves in conventional Bloch waves $\Rightarrow F^{(B)}_M = 0$.

At densities $\rho$ close to the Mott insulator density $\rho_{MI}$ the effective $B$ field is

$$B = -\hbar \left( \rho - \rho_{MI} \right) \hat{z}$$

where $\rho_{MI} = \frac{f}{a_0^2} = \frac{1}{a_0^2}$, and $a_0$ is the lattice spacing.
Bosons at filling fraction $f = 1/2$ (equivalent to $S=1/2$ AFMs)

Weak interactions: superfluidity

**Bosons at filling fraction** $f = 1/2$ (equivalent to $S=1/2$ AFMs)

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$\langle \psi \rangle \neq 0$

Weak interactions: superfluidity

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Weak interactions: superfluidity

Bosons at filling fraction $f = 1/2$ (equivalent to $S=1/2$ AFMs)

\[ \langle \psi \rangle = 0 \]

Strong interactions: insulator

Bosons at filling fraction $f = 1/2$ (equivalent to $S=1/2$ AFMs)

All insulating phases have "density" wave order $\rho(r) = \sum_q \rho_q e^{iQr}$ with $\langle \rho_q \rangle \neq 0$

Strong interactions: insulator $\langle \psi \rangle = 0$

Vortices in a superfluid near a Mott insulator at filling $f$.
Quantum mechanics of the vortex “particle” in a periodic potential with $f$ flux quanta per unit cell.

Space group symmetries of Hamiltonian:

$T_x, T_y$: Translations by a lattice spacing in the $x, y$ directions.

$R$: Rotation by 90 degrees.

Magnetic space group:

$$T_x T_y = e^{2\pi i f} T_y T_x;$$

$$R^{-1} T_y R = T_x; \quad R^{-1} T_x R = T_y^{-1}; \quad R^4 = 1$$
Vortices in a superfluid near a Mott insulator at filling $f=p/q$

Hofstadter spectrum of the quantum vortex “particle” with field operator $\varphi$

At filling $f=p/q$ ($p, q$ relatively prime integers) there are $q$ species of vortices, $\varphi_\ell$ (with $\ell=1\ldots q$), associated with $q$ gauge-equivalent regions of the Brillouin zone

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At filling \( f = p/q \) \((p, q \) relatively prime integers\) there are \( q \) species of vortices, \( \phi_\ell \) \((with \ell = 1 \ldots q)\), associated with \( q \) gauge-equivalent regions of the Brillouin zone.

The \( q \) vortices form a *projective* representation of the space group

\[
T_x : \phi_\ell \rightarrow \phi_{\ell+1} ; \quad T_y : \phi_\ell \rightarrow e^{2\pi i \ell f} \phi_\ell
\]

\[
R : \phi_\ell \rightarrow \frac{1}{\sqrt{q}} \sum_{m=1}^{q} \phi_m e^{2\pi i \ell mf}
\]

Vortices in a superfluid near a Mott insulator at filling $f=p/q$

The $q \varphi_\ell$ vortices characterize both superconducting and density wave orders

Superconductor/insulator: $\langle \varphi_\ell \rangle = 0 \neq \langle \varphi_\ell \rangle \neq 0$
Vortices in a superfluid near a Mott insulator at filling $f=p/q$

The $q \varphi_\ell$ vortices characterize both superconducting and density wave orders

Density wave order:
Status of space group symmetry determined by density operators $\rho_Q$ at wavevectors $Q_{mn} = \frac{2\pi p}{q}(m,n)$

$$\rho_{mn} = e^{i\pi mnf} \sum_{\ell=1}^{q} \varphi_\ell^* \varphi_{\ell+n} e^{2\pi i \ell mf}$$

$$T_x : \rho_Q \rightarrow \rho_Q e^{iQ \cdot \hat{x}} ; \quad T_y : \rho_Q \rightarrow \rho_Q e^{iQ \cdot \hat{y}}$$

$$R : \rho(Q) \rightarrow \rho(RQ)$$
Vortices in a superfluid near a Mott insulator at filling $f=p/q$

The $q \varphi_\ell$ vortices characterize both superconducting and density wave orders.

Vorticity modulations:
In the presence of an applied magnetic field, there are also modulations in the vorticity at the same wavevectors $Q_{mn} = \frac{2\pi p}{q} (m, n)$

$$V_{mn} = e^{i\pi mnf} \sum_{\ell=1}^{q} \left( \varphi_\ell^* \frac{\partial \varphi_{\ell+n}}{\partial \tau} - \frac{\partial \varphi_\ell^*}{\partial \tau} \varphi_{\ell+n} \right) e^{2\pi i \ell m f}$$
Field theory with projective symmetry

Degrees of freedom:

- $q$ complex $\varphi_\ell$ vortex fields
- 1 non-compact U(1) gauge field $A_\mu$ which mediates $F_M^{(E)}$ and $F_M^{(B)}$

\[
S = \int d^2x d\tau \left[ \sum_\ell \left\{ |(\partial_\mu - iA_\mu)\varphi_\ell|^2 + s|\varphi_\ell|^2 \right\} + \frac{K_\mu}{2} (\epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda - B \delta_{\mu\tau})^2 + \sum_{\ell mn} \gamma_{mn} \varphi_\ell^* \varphi_{\ell+m}^* \varphi_{\ell+n} \varphi_{\ell+m-n} \right]
\]

where $K_{x,y} = 1/(4\pi^2 \rho_s)$ and $K_\tau^{-1} = dn/d\mu$, and

\[
B = -h(\rho - \rho_{MI}) \quad \text{with} \quad \rho_{MI} = \frac{f}{a_0^2} = \frac{p}{qa_0^2}.
\]
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The projective symmetries constrain the couplings $\gamma_{mn}$ to obey

$$\gamma_{mn} = \gamma_{-m,-n} ; \quad \gamma_{mn} = \gamma_{m,m-n} ; \quad \gamma_{mn} = \gamma_{m-2n,-n}$$

$$\gamma_{\bar{m}\bar{n}} = \frac{1}{q} \sum_{mn} \gamma_{mn} e^{-2\pi if[n(\bar{m}-\bar{n})+\bar{n}(m-n)]}$$
Field theory with projective symmetry

Spatial structure of insulators for \( q=2 \) (\( f=1/2 \))

All insulating phases have density-wave order \( \rho(r) = \sum \rho_Q e^{iQ \cdot r} \) with \( \langle \rho_Q \rangle \neq 0 \)
Field theory with projective symmetry
Spatial structure of insulators for $q=4$ ($f=1/4$ or $3/4$)

$a \times b$ unit cells;
$q/a', q/b', ab/q'$
all integers
Field theory with projective symmetry

Pinned vortices in the superfluid

Any pinned vortex must choose an orientation in flavor space. This necessarily leads to modulations in the local density of states over the spatial region where the vortex executes its quantum zero point motion.

Density operators $\rho_q$ at wavevectors $Q_{mn} = \frac{2\pi p}{q} (m, n)$

$$\rho_{mn} = e^{i\pi mnf} \sum_{\ell=1}^{q} \Phi_{\ell}^* \Phi_{\ell+n} e^{2\pi i \ell mf}$$

In the cuprates, assuming boson density=density of Cooper pairs we have $\rho_{MI} = 7/16$, and $q = 16$ (both models in part B yield this value of $q$). So modulation must have period $a \times b$ with $16/a$, $16/b$, and $ab/16$ all integers.
Vortex-induced LDOS of Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ integrated from 1meV to 12meV at 4K

Vortices have halos with LDOS modulations at a period $\approx$ 4 lattice spacings


Measuring the inertial mass of a vortex

Solve the equations of motion

\[ m_v \frac{d^2 \mathbf{r}}{dt^2} = F_M^{(E)} + F_M^{(B)} \]

for a triangular lattice of vortices in the harmonic approximation. We estimate that retardation effects can be neglected (the ‘electric’ interactions are instantaneous and obey the ‘Coulomb’ law). We define

\[ u_{\text{rms}} = \text{rms displacement of vortex from its equilibrium position}, \]

which can be determined from the LDOS modulations in the STM measurement. Then we find from the vortex ‘magnetophonon’ spectrum

\[ m_v = 0.0419 \frac{\hbar^2 A_0}{\rho_s u_{\text{rms}}^4} F \left( \frac{u_{\text{rms}}^2 B}{\hbar} \right) \]

\[ F(x) \approx 0.5039 + \sqrt{0.2461 + 0.4147x^2} \]

where \( A_0 \) is the area of a vortex lattice unit cell, and \( B = -\hbar(\rho - \rho_{MI}) \).
Measuring the inertial mass of a vortex

Preliminary estimates for the BSCCO experiment:

Inertial vortex mass \(m_v \approx 10m_e\)

Vortex magnetoplasmon frequency \(\nu_p \approx 1\) THz = 4 meV

Large uncertainty due to uncertainty in value of \(u_{rms}\)

Note: With nodal fermionic quasiparticles, \(m_v\) is expected to be dependent on the magnetic field \(i.e.\) vortex density

G. E. Volovik, JETP Lett. 65, 217 (1997);
B. Extension to electronic models for the cuprate superconductors

_Dual vortex theories of the doped_
(1) _Quantum dimer model_
(2) “Staggered flux” _spin liquid_
$g$ = parameter controlling strength of quantum fluctuations in a semiclassical theory of the destruction of Neel order

**La$_2$CuO$_4$**

Neel order
(B.1) Phase diagram of doped antiferromagnets

(B.1) Phase diagram of doped antiferromagnets

Dual vortex theory of doped dimer model for interplay between VBS order and $d$-wave superconductivity

VBS order

Neel order

$\text{La}_2\text{CuO}_4$

Hole density $\delta$
\( H_{\text{dqd}} = J \sum (| \bullet \bullet \rangle \langle \bullet \bullet | + | \bullet \bullet \rangle \langle \bullet \bullet |) \)

\[ -t \sum (| \circ \rangle \langle \circ | + | \circ \rangle \langle \circ |) - \cdots \]

Density of holes = \( \delta \)

Duality mapping of doped quantum dimer model shows:

Vortices in the superconducting state obey the magnetic translation algebra

\[ T_x T_y = e^{2\pi i f} T_y T_x \]

with \[ f = \frac{p}{q} = \frac{1 - \delta_{MI}}{2} \]

where \( \delta_{MI} \) is the density of holes in the proximate Mott insulator (for \( \delta_{MI} = 1/8, f = 7/16 \Rightarrow q = 16 \))

Note: \( f \) = density of Cooper pairs

Most results of Part A on bosons can be applied unchanged with \( q \) as determined above
(B.1) Phase diagram of doped antiferromagnets
(B.1) Phase diagram of doped antiferromagnets

VBS order

Neel order

La$_2$CuO$_4$

$g$

Hole density

$\delta = \frac{1}{16}$
(B.1) Phase diagram of doped antiferromagnets

VBS order

La$_2$CuO$_4$

Neel order

Hole density $\delta$

$\delta = \frac{1}{8}$
**B.1** Phase diagram of doped antiferromagnets

- **VBS order**
- **Neel order**
- **La$_2$CuO$_4$**

- **Hole density** $\delta$
- **d-wave superconductivity above a critical $\delta$**
(B.2) Dual vortex theory of doped “staggered flux” spin liquid


The theory is expressed in terms of neutral fermionic spinons \( \psi \) which are charged under a U(1) gauge field \( C_\mu \). The electrically charged carriers are represented by 2 boson species, \( b_1 \) and \( b_2 \), which carry opposite charges under \( C_\mu \).

We wish to describe quantum fluctuations in such a superconductor near a transition to a Mott insulator. The Mott insulator has hole density \( \delta_{MI} \), with

\[
\frac{\delta_{MI}}{2} = \frac{p}{q'},
\]

with \( p, q \) relatively prime integers.
(B.2) Dual vortex theory of doped “staggered flux” spin liquid

It is essential to account for the quantum dynamics of the bosons $b_1$ and $b_2$, each near density $p/q$. We do this by applying the methods developed earlier for boson systems separately to both $b_1$ and $b_2$.

This yields a theory with a pair of $q$ complex vortex fields $\varphi_{1\ell}$ and $\varphi_{2\ell}$, which are dual to the two species of bosons, $b_1$, $b_2$ of the SU(2) gauge theory. Each vortex carries physical magnetic flux $h/(2e)$. These vortices are coupled to 2 non-compact U(1) gauge fields: $A_\mu$ (responsible for the Magnus forces), and $B_\mu$ (whose Chern-Simons dual is coupled to the nodal fermions).
(B.2) Dual vortex theory of doped “staggered flux” spin liquid

The effective action for the theory is:

\[
S_{sf} = S_v + S_A
\]

\[
S_v = \int d^2r d\tau \sum_{\ell=0}^{q-1} \left[ h_s (-1)^{\ell} \left\{ \varphi_{1,\ell+q/2}^* \left( \frac{\partial}{\partial \tau} - iA_\tau - iB_\tau \right) \varphi_{1\ell} \right. \\
\left. - \varphi_{2,\ell+q/2}^* \left( \frac{\partial}{\partial \tau} - iA_\tau + iB_\tau \right) \varphi_{2\ell} \right\} \\
+ |(\partial_i - iA_i - iB_i)\varphi_{1\ell}|^2 + s|\varphi_{1\ell}|^2 \\
+ |(\partial_i - iA_i + iB_i)\varphi_{2\ell}|^2 + s|\varphi_{2\ell}|^2 \right]
\]

\[
S_A = \int d^2r d\tau \left[ \frac{K_\mu}{2} \left( \epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda \right)^2 + \frac{i}{\pi} \epsilon_{\mu\nu\lambda} B_\mu \partial_\nu C_\lambda + \overline{\psi} \gamma_\mu (\partial_\mu - iC_\mu) \psi \right]
\]

There are also additional “monopole” terms which are not shown.
Main results:

- Presence of the staggered flux makes the vortices “non-relativistic” and allows a theory of a dilute gas of vortices and anti-vortices.

- As the superfluid approaches the Mott insulator, the vortices and anti-vortices form “excitonic” bound states which condense first.

- This implies that a supersolid intervenes between the superfluid and the insulator.
Superfluids near Mott insulators

The Mott insulator has average Cooper pair density, $f = p/q$ per site, while the density of the superfluid is close (but need not be identical) to this value.

- Vortices with flux $\hbar/(2e)$ come in multiple (usually $q$) “flavors”
- The lattice space group acts in a projective representation on the vortex flavor space.
- These flavor quantum numbers provide a distinction between superfluids: they constitute a “quantum order”
- Any pinned vortex must chose an orientation in flavor space. This necessarily leads to modulations in the local density of states over the spatial region where the vortex executes its quantum zero point motion.