Spin ordering quantum transitions in two-dimensional superconductors

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Ying Zhang

Science **286**, 2479 (1999).

Transparencies on-line at
http://pantheon.yale.edu/~subir

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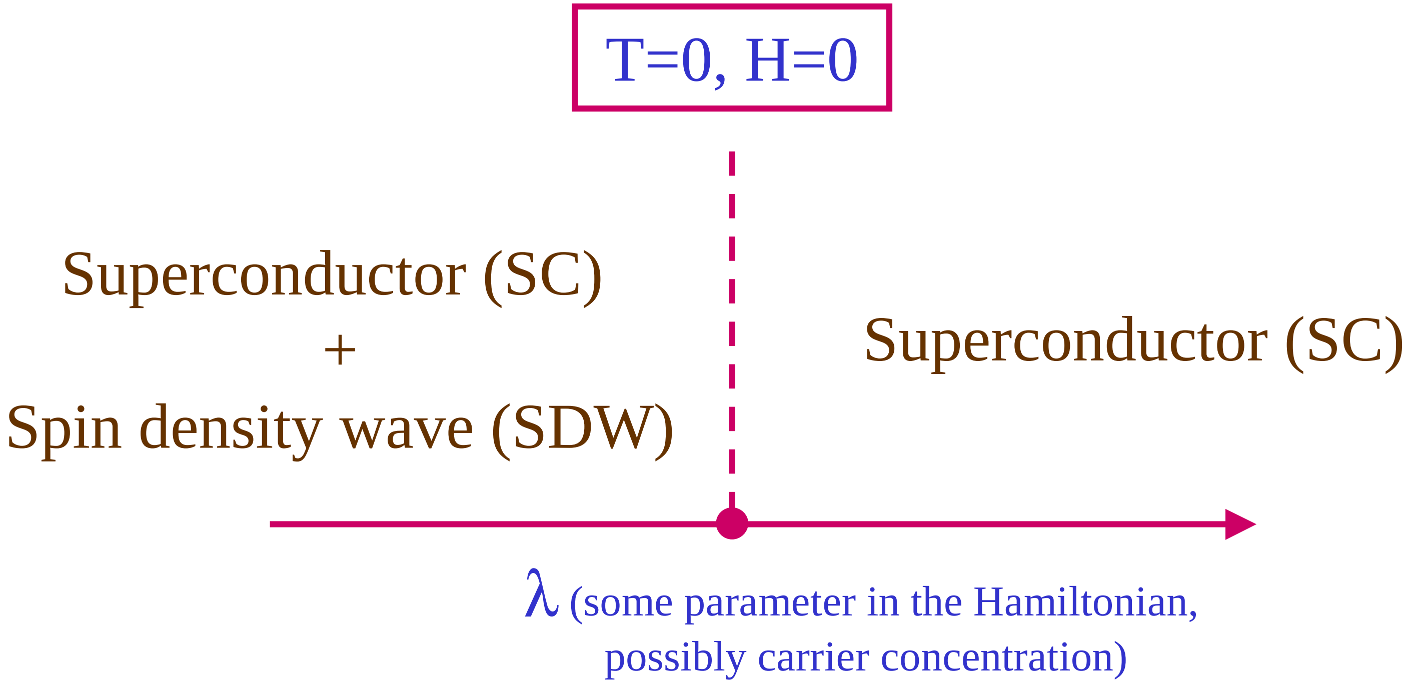


Many experimental indications that the cuprate superconductors are not too far from such a quantum phase transition:

G. Aeppli, T.E. Mason, S.M. Hayden, H.A. Mook, J. Kulda, Science **278**, 1432 (1997).
Y. S. Lee, R. J. Birgeneau, M. A. Kastner *et al.*, Phys. Rev. B **60**, 3643 (1999).
S. Katano, M. Sato, K. Yamada, T. Suzuki, and T. Fukase Phys. Rev. B **62**, 14677 (2000).
B. Lake, G. Aeppli *et al.*, Science to appear.
Y. Sidis, C. Ulrich, P. Bourges, *et al.*, cond-mat/0101095.
H. Mook, P. Dai, F. Dogan, cond-mat/0102047.
J.E. Sonier *et al.*, preprint.

S=1 resonance mode in YBCO

$YBa_2Cu_3O_7$

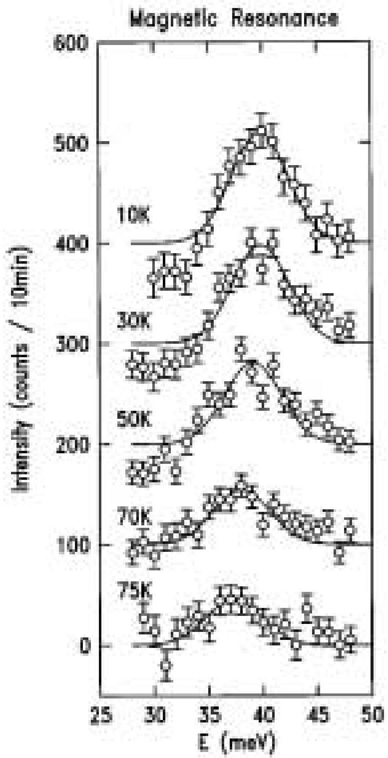
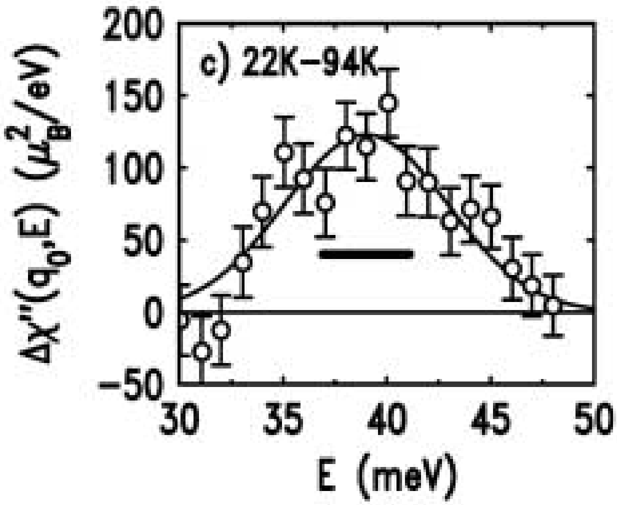


FIG. 8. Unpolarized beam, constant-**Q** data [**Q**=(3/2,1/2,−1.7)] of the 40 meV magnetic resonance obtained by subtracting the signal below T_c from the T=100 K background. The lines are fits to Gaussians, as described in the text. For clarity successive scans are offset by 100.

Resolution limited width

H.F. Fong, B. Keimer, D. Reznik, D.L. Milius, and I.A. Aksay, Phys. Rev. B **54**, 6708 (1996)

$YBa_2Cu_3O_7 + 0.5\%$ Zn



Zn induced half-width = 4.25 meV

H. F. Fong, P. Bourges, Y. Sidis, L. P. Regnault, J. Bossy, A. Ivanov, D.L. Milius, I. A. Aksay, and B. Keimer, Phys. Rev. Lett. **82**, 1939 (1999)

Main result (I)

View spin resonance as a collective "soft-mode" (*S=1* exciton) associated with the onset of SDW order in superconductor.

Energy scale at which the collective spin resonance is broadened by Zn impurities:

"Swiss cheese" model

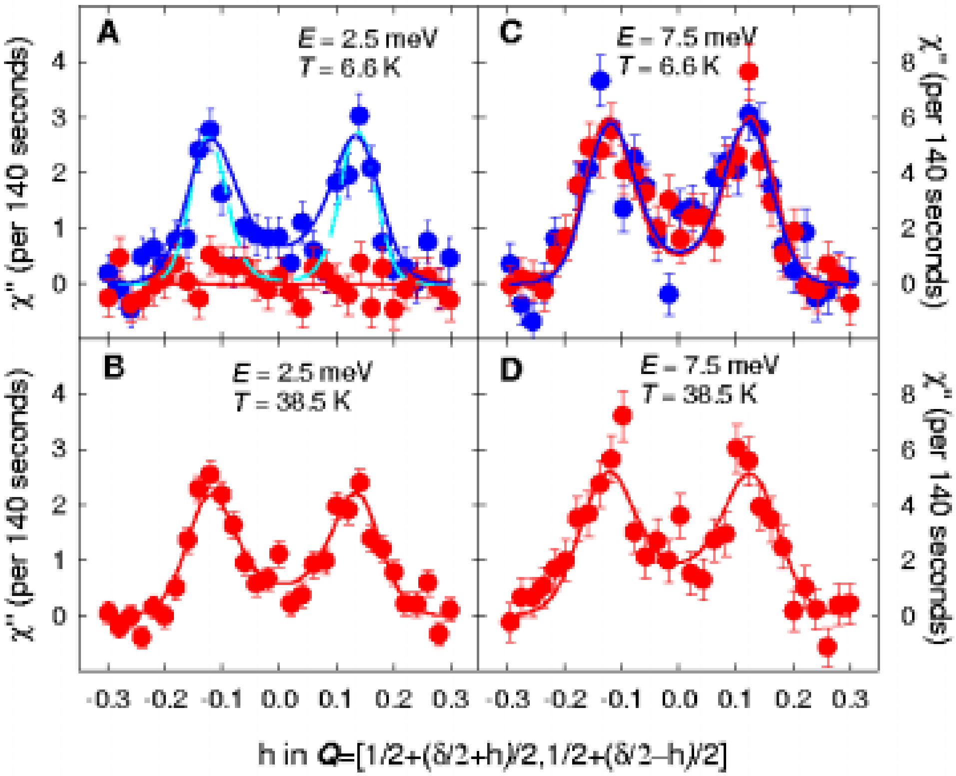
Inverse Q of resonance = C $n_{\text{imp}}\xi^2$

C $\rightarrow$ universal number

$\xi \rightarrow$ spin correlation length which diverges at the onset of SDW order

Result also holds near SDW transitions in Mott insulators

M. Vojta, C. Buragohain, and S. Sachdev, Phys. Rev. B **61**, 15152 (2000)

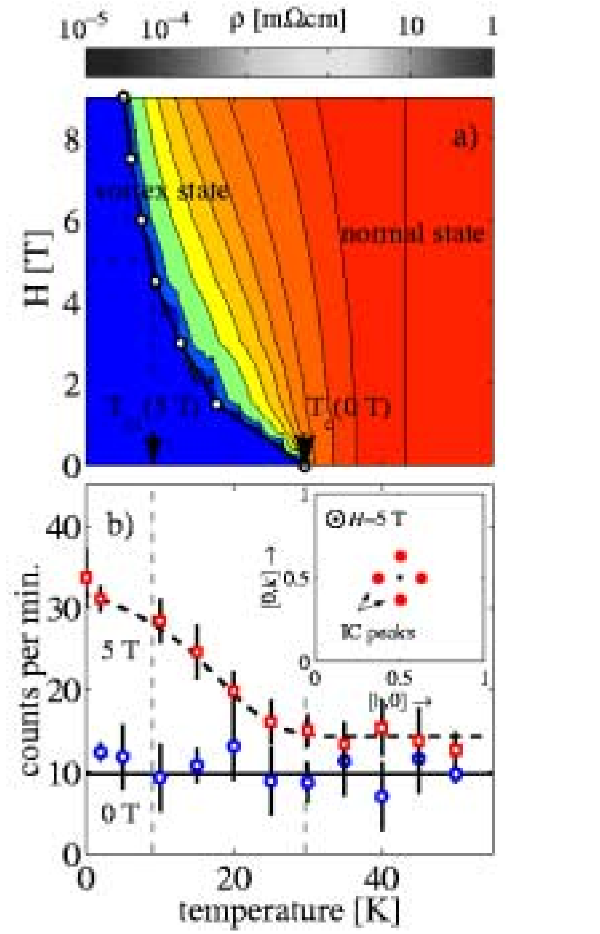


Neutron scattering off $La_{2-x}Sr_xCuO_4$

($x = 0.163$, ***SC phase***)

in $H = 0$ (red dots) and $H = 7.5$T (blue dots).

B. Lake, G. Aeppli *et al*., Science **291**, 1759 (2001)

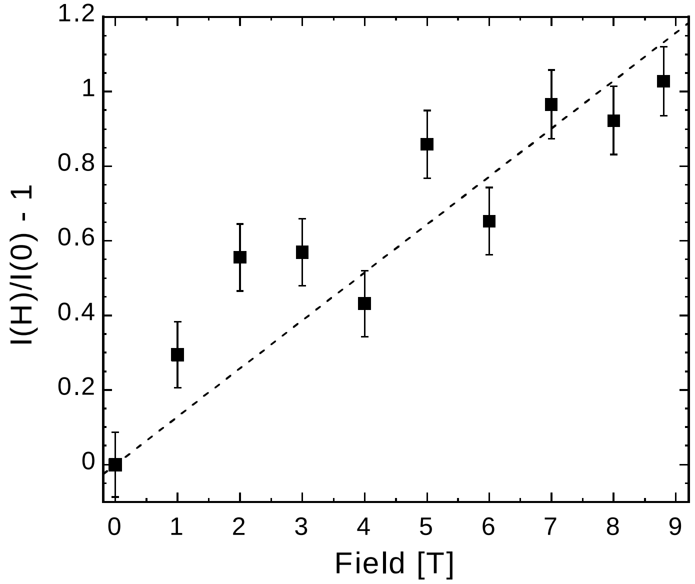


Elastic neutron scattering off $La_{2-x}Sr_xCuO_4$

($x = 0.10$, ***SC + SDW phase***)

in $H = 0$ (blue dots) and $H = 5$T (red dots).

B. Lake, H. Ronnow *et al*., preprint



Elastic neutron scattering off La_2CuO_{4+y}

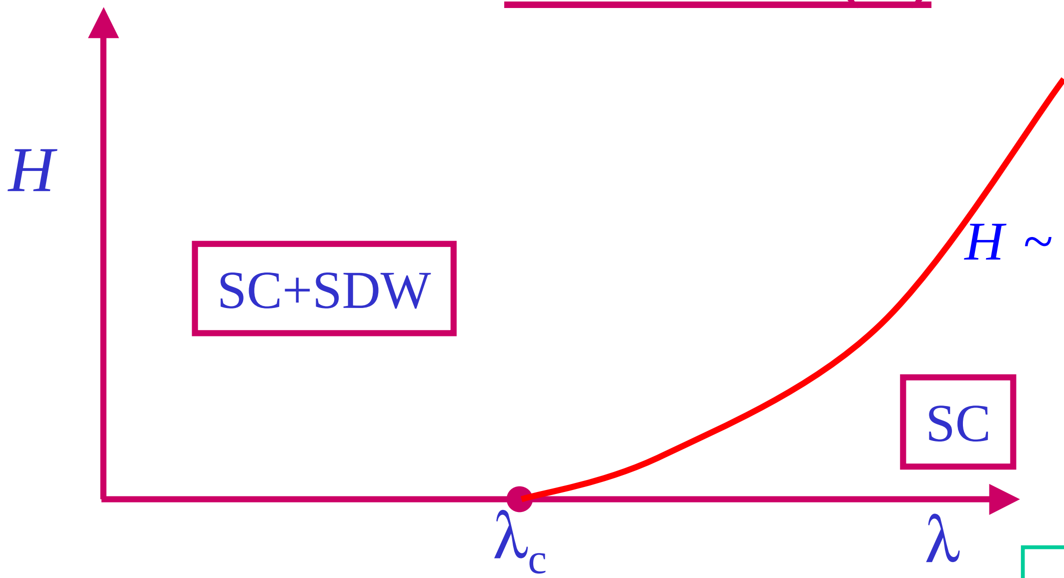
B. Khaykovich, Y. S. Lee, S. Wakimoto,

K. J. Thomas, M. A. Kastner,

and R.J. Birgeneau, to appear

Main result (II)

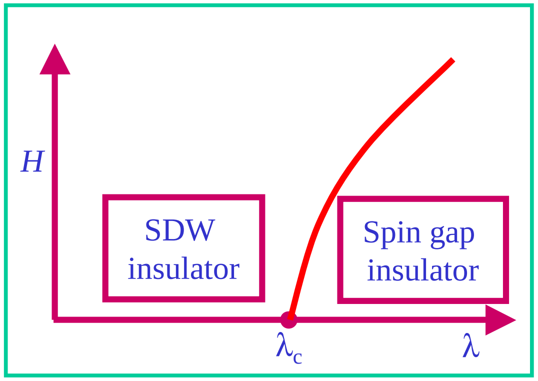
(Extreme Type II superconductor)



$$H \sim \frac{(\lambda-\lambda_c)}{\ln\left(1/(\lambda-\lambda_c)\right)}$$

Naïve expectation by analyticity in *H*: $\lambda-\lambda_c \sim H^2$

Holds in a Mott insulator



Similar results hold for other ordering transitions in a SC,

e.g. onset of orbital antiferromagnet or CDW order

E. Demler, S. Sachdev, and Y. Zhang, cond-mat/0103192.

Outline

I. Onset of SDW order in a Mott insulator--the coupled ladder antiferromagnet.

  A. Field theory for quantum phase transition.

  B. General theory of localized defects across the quantum critical point.

II. Onset of SDW order in a superconductor.

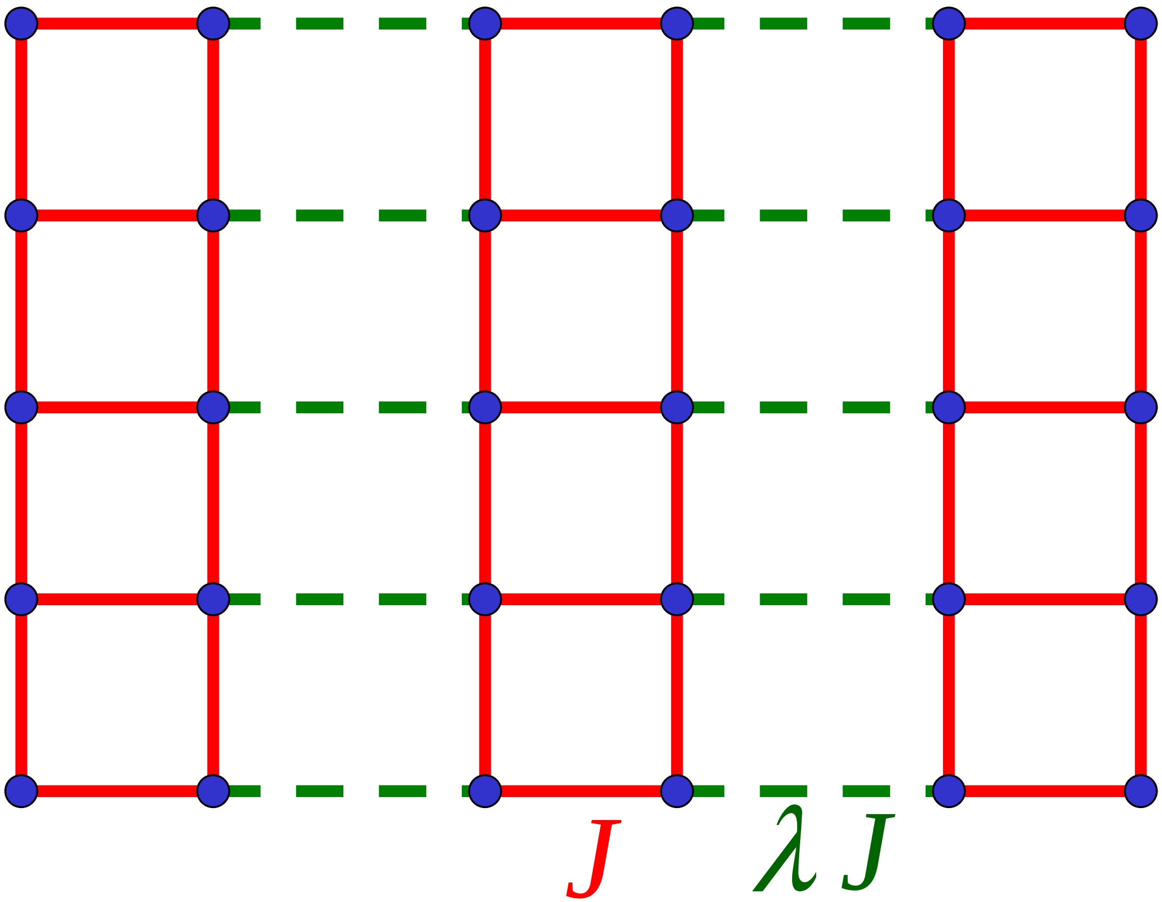
  A. Field theory for quantum phase transition and localized defects

  B. Effect of external magnetic field.

I. Neel and paramagnetic states of the coupled ladder antiferromagnet

N. Katoh and M. Imada, J. Phys. Soc. Jpn. **63**, 4529 (1994)
J. Tworzydlo, O. Y. Osman, C. N. A. van Duin, J. Zaanen, Phys. Rev. B **59**, 115 (1999).

S=1/2 spins on coupled 2-leg ladders



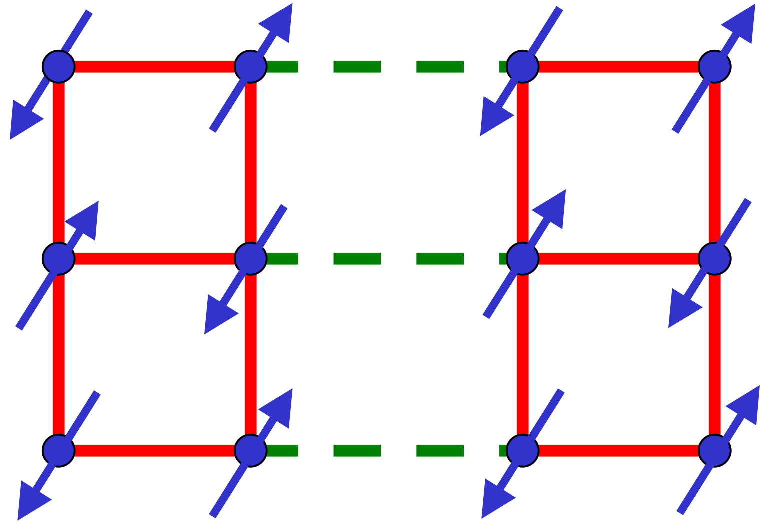
$$H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$0 \le \lambda \le 1$$

λ close to 1

Square lattice antiferromagnet

Experimental realization: La_2CuO_4



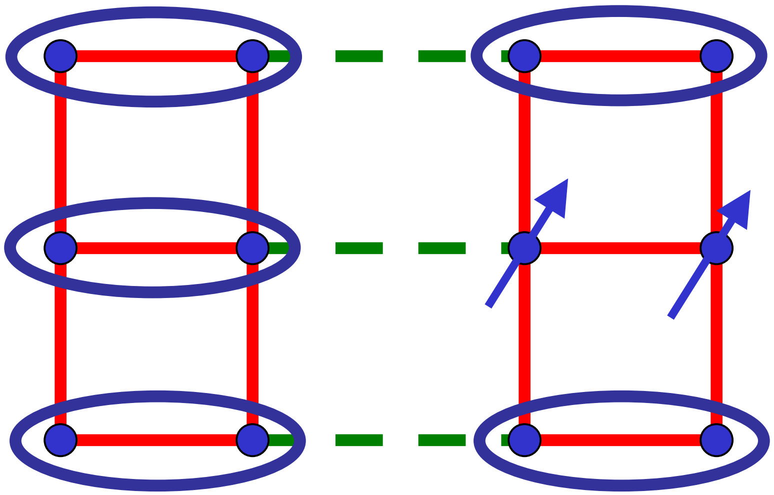
Ground state has long-range magnetic (Neel) order

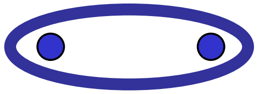
$$\left\langle \vec{S}_i \right\rangle = \left(-1\right)^{i_x+i_y} N_0 \neq 0$$

Excitations: 2 spin waves

λ close to 0

Weakly coupled ladders



 $= \frac{1}{\sqrt{2}}\left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$

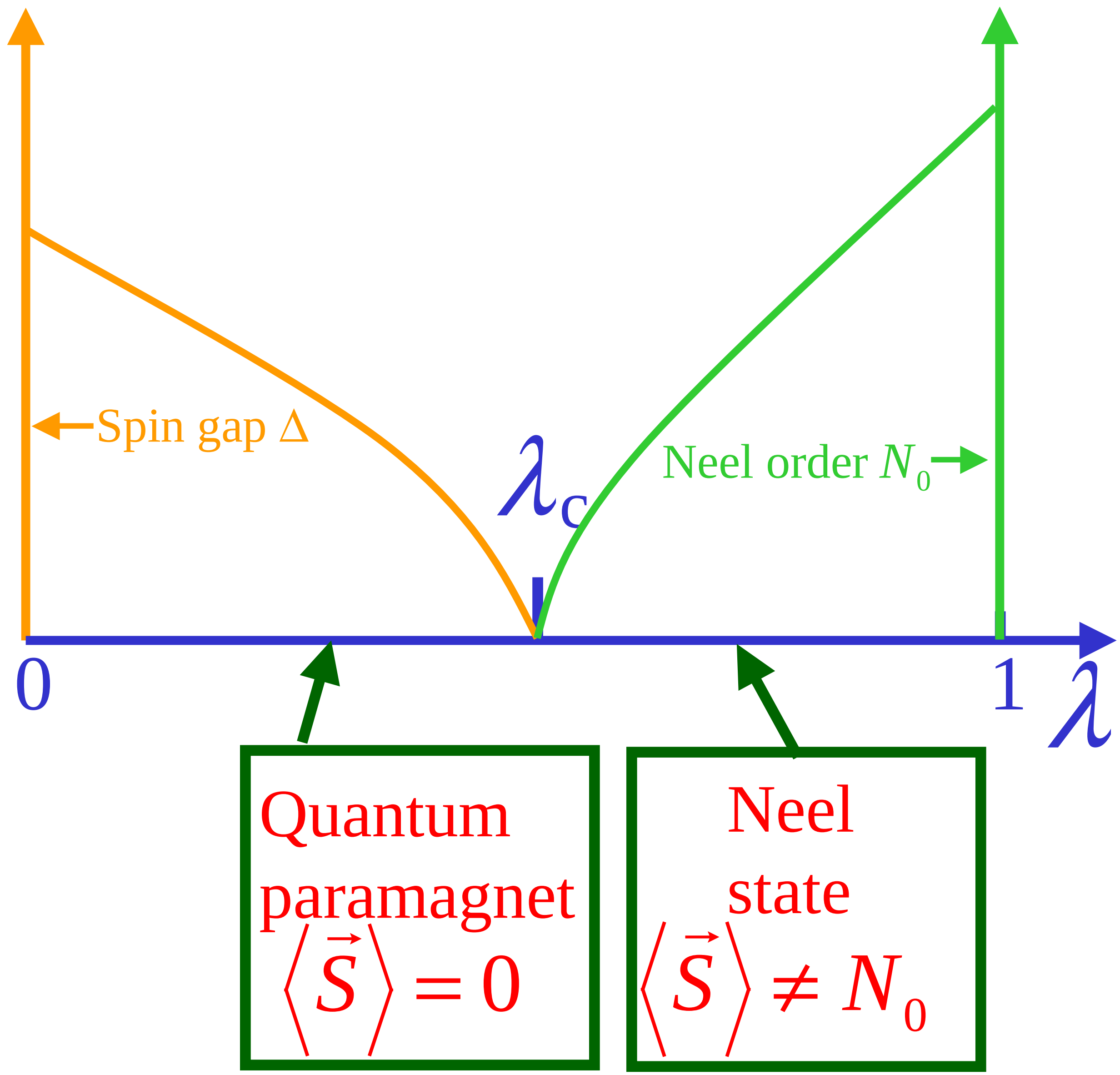
Paramagnetic ground state

$$\left\langle \vec{S}_i \right\rangle = 0$$

Excitation: S=1 particle (collective mode)

Energy dispersion away from antiferromagnetic wavevector

$$\varepsilon = \Delta + \frac{c^2 k^2}{2\Delta}$$



Spin gap Δ
λ_c
Neel order N_0
0
1
λ
Quantum paramagnet
$\langle \vec{S} \rangle = 0$
Neel state
$\langle \vec{S} \rangle \neq N_0$

I.A Quantum field theory:

λ close to λ_c

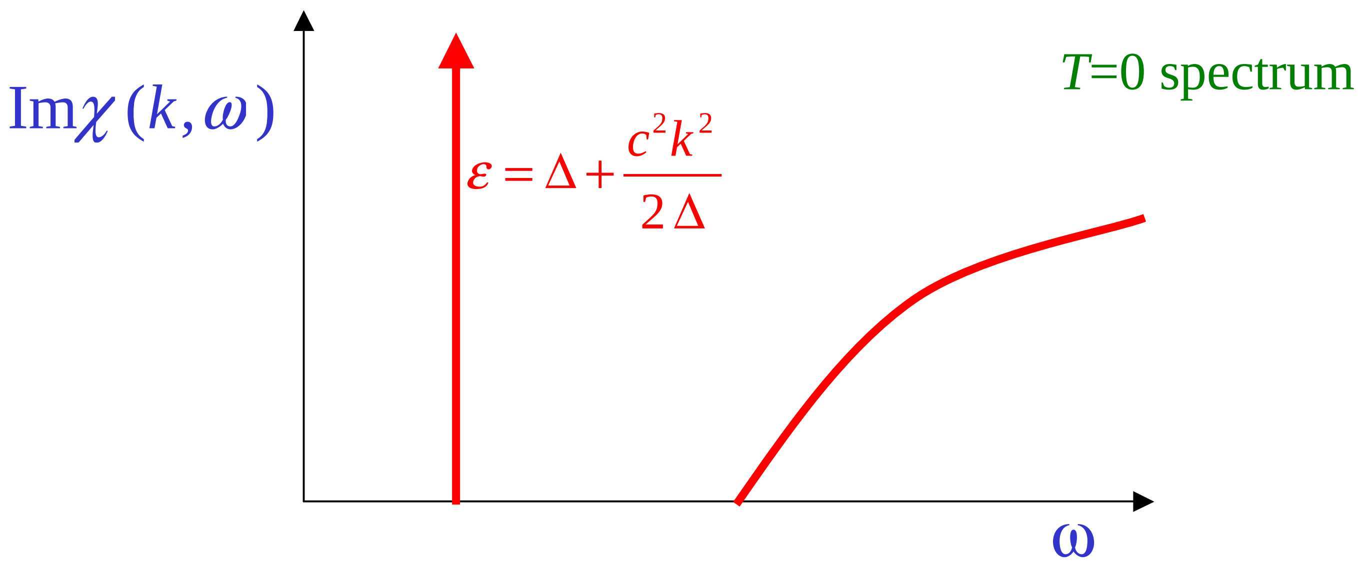
$$S_b = \int d^2x d\tau \left[\frac{1}{2}\left(\left(\nabla_x \phi_\alpha\right)^2 + c^2 \left(\partial_\tau \phi_\alpha\right)^2 + r\phi_\alpha^2 \right) + \frac{g}{4!}\left(\phi_\alpha^2\right)^2 \right]$$

ϕ_α → 3-component antiferromagnetic order parameter

$r > 0$	→	$\lambda < \lambda_c$
$r < 0$	→	$\lambda > \lambda_c$

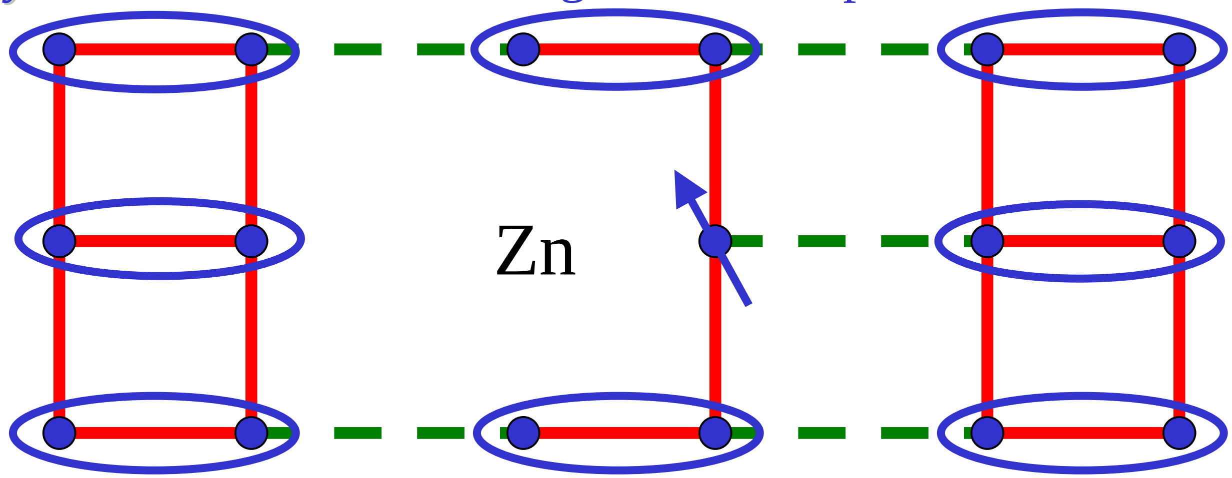
Oscillations of ϕ_α about zero (for $r > 0$)

→ spin-1 collective mode



I.B Impurities in the coupled-ladder antiferromagnet

Make *any* localized deformation e.g. remove a spin



Susceptibility $\chi = A\chi_b + \chi_{imp}$ (A = area of system)

In paramagnetic phase as $T \to 0$

$$\chi_b = \left(\frac{\Delta}{\hbar^2 c^2 \pi}\right) e^{-\Delta/k_B T} \quad ; \chi_{imp} = \frac{S(S+1)}{3k_B T}$$

For a general impurity χ_{imp} *defines* the value of S

In Neel phase as $T \to 0$

$$\chi_{b\perp} = \rho_s / \hbar^2 c^2 \quad ; \chi_{imp\perp} = \text{finite}$$ (no free moment)

Quantum field theory for *S=1* resonance in the presence of a non-magnetic impurity

Orientation of "impurity" spin -- $n_\alpha(\tau)$ (unit vector)

Action of "impurity" spin

$$S_{\text{imp}} = \int d\tau \left[iSA_\alpha(n)\frac{dn_\alpha}{d\tau} - \gamma S n_\alpha(\tau)\phi_\alpha(x=0,\tau) \right]$$

$A_\alpha(n) \rightarrow$ Dirac monopole function

Boundary quantum field theory: $S_b + S_{\text{imp}}$

Recall -

$$S_b = \int d^2x d\tau \left[\frac{1}{2}\left((\nabla_x \phi_\alpha)^2 + c^2 (\partial_\tau \phi_\alpha)^2 + r\phi_\alpha^2 \right) + \frac{g}{4!}\left(\phi_\alpha^2\right)^2 \right]$$

Renormalization group analysis:
g and γ reach non-zero fixed point values

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993);
S. Sachdev, C. Buragohain, and M. Vojta, Science, **286**, 2479 (1999).
J.L. Smith and Q. Si, Europhys. Lett. **45**, 228 (1999).
A.M. Sengupta, Phys. Rev. B **61**, 4041 (2000);

***β* functions** *(ε=3-d)*:

$$\beta(g) = -\varepsilon g + \frac{11g^2}{6} - \frac{23g^3}{12} + O(g^4)$$

$$\beta(\gamma) = -\frac{\varepsilon\gamma}{2} + \gamma^3 - \gamma^5 + \frac{5g^2\gamma}{144} + \pi^2\left(S(S+1) - \frac{1}{3}\right)g\gamma^3 + O\left(\left(\gamma, \sqrt{g}\right)^7\right)$$

No new relevant perturbations near the impurity;
All other boundary perturbations are irrelevant –
e.g. $\lambda \int d\tau \phi_\alpha^2(x=0,\tau)$

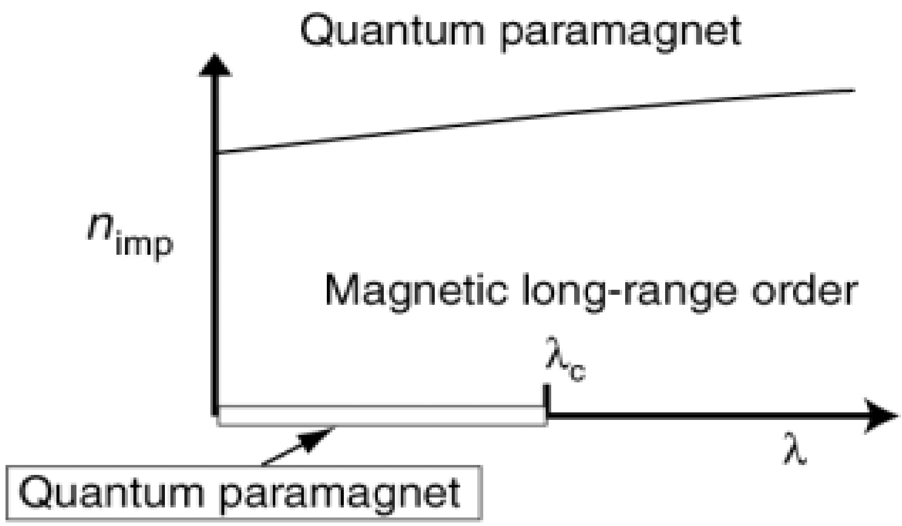
Δ and *c* completely determine spin dynamics near an impurity –
No new parameters are necessary !

Finite density of impurities n_{imp}

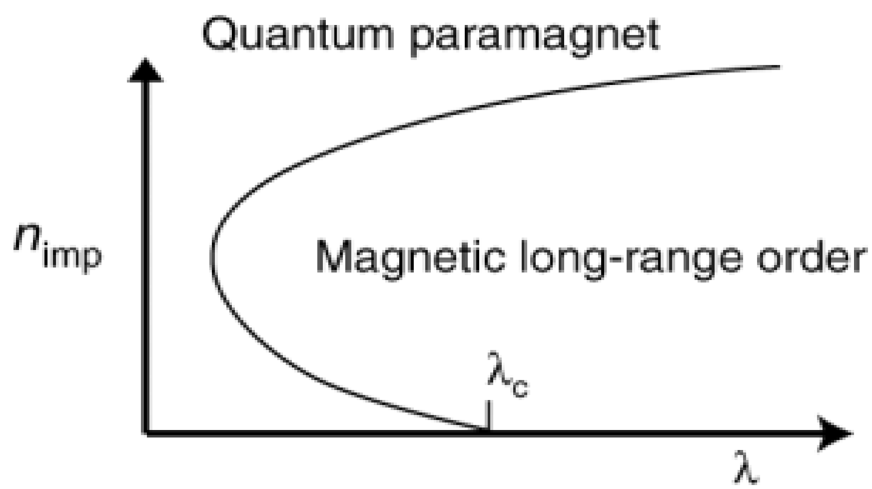
Relevant perturbation – strength determined by only energy scale Γ that is linear in n_{imp} and contains only bulk parameters

$$\Gamma \equiv \frac{n_{\text{imp}} (\hbar c)^2}{\Delta} \left(1 + O\left(\frac{\Delta}{J} \right) \right) \quad ; \quad \frac{1}{Q} \equiv \frac{\Gamma}{\Delta} = n_{\text{imp}} \left(\frac{\hbar c}{\Delta} \right)^2 = n_{\text{imp}} \xi^2$$

Possible phase diagrams



$\Gamma_c / \Delta = 0^+$



$\Gamma_c / \Delta = \text{constant}$

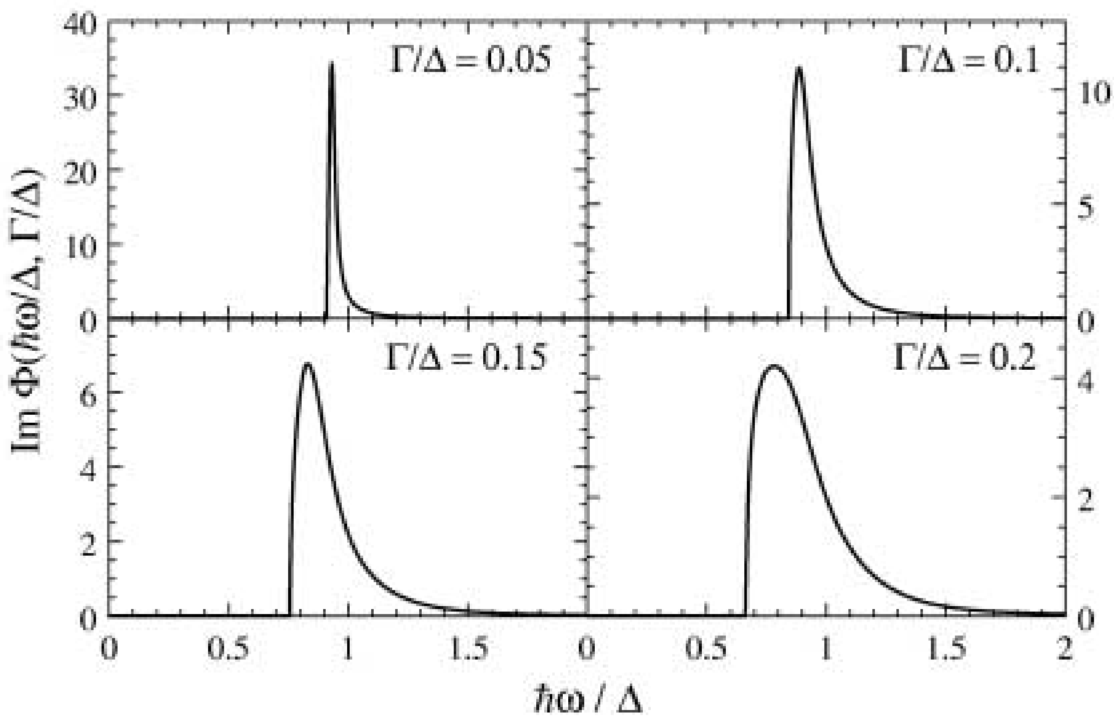
Dynamic susceptibility at ω of order Δ

Without impurities $$\chi(G,\omega) = \frac{A}{\Delta^2 - \omega^2}$$

With impurities $$\chi(G,\omega) = \frac{A}{\Delta^2}\Phi\left(\frac{\hbar\omega}{\Delta}, \frac{\Gamma}{\Delta}\right)$$

$$\Gamma \equiv \frac{n_{\text{imp}}(\hbar c)^2}{\Delta}$$

Φ → *Universal* scaling function. We computed it in a "self-consistent, non-crossing" approximation



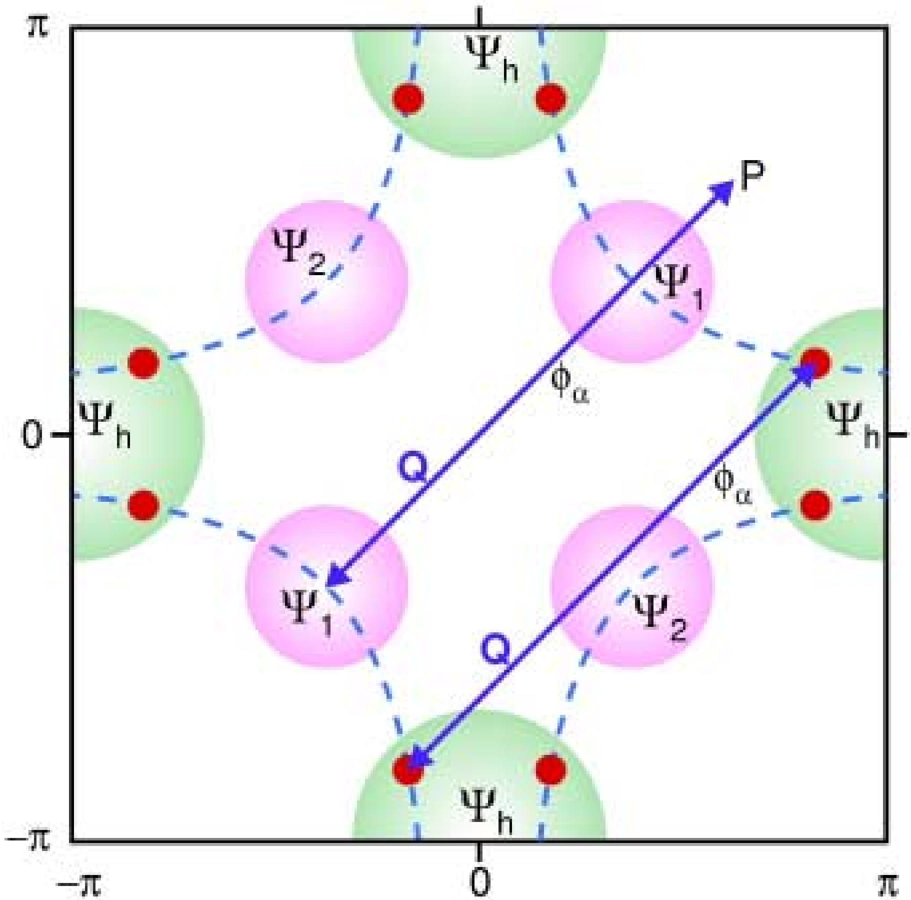
Predictions:
Half-width of line ≈ Γ

Universal asymmetric lineshape

S. Sachdev, C. Buragohain, M. Vojta, Science **286**, 2479 (1999).

M. Vojta, C. Buragohain, and S. Sachdev, Phys. Rev. B **61**, 15152 (2000).

II.A SDW ordering transition in a d-wave superconductor



Ψ_h : *strongly coupled* to ϕ_α, but do not damp ϕ_α as long as $\Delta < 2\,\Delta_h$

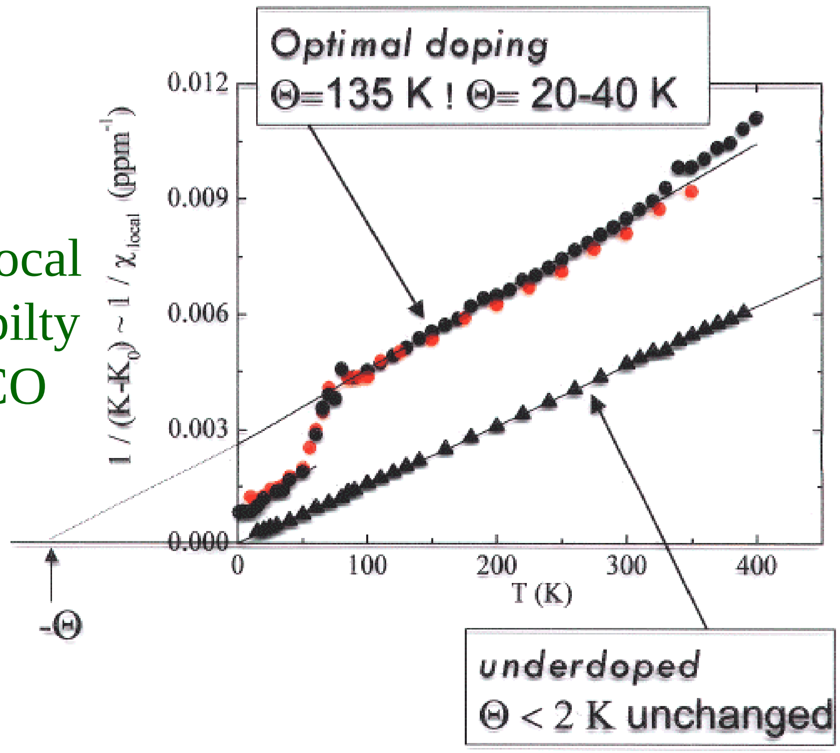
$\Psi_{1,2}$: *decoupled* from ϕ_α

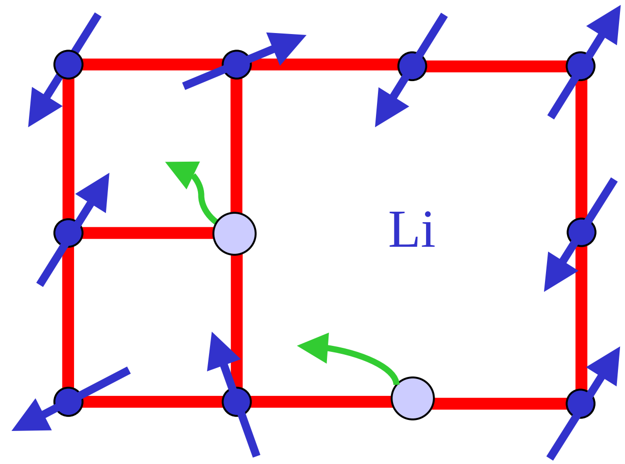
Leading universal properties of transition are identical to those in Mott insulator

If Q connects nodal points, a new theory of ϕ_α coupled to $\Psi_{1,2}$ is needed: this theory obeys identical scaling relations.

Measurement of spin susceptibility near non-magnetic (Zn/Li) impurities

Inverse local susceptibilty in YBCO





^{7}Li NMR below T_c

J. Bobroff, H. Alloul, W.A. MacFarlane, P. Mendels, N. Blanchard, G. Collin, and J.-F. Marucco, cond-mat/0010234.

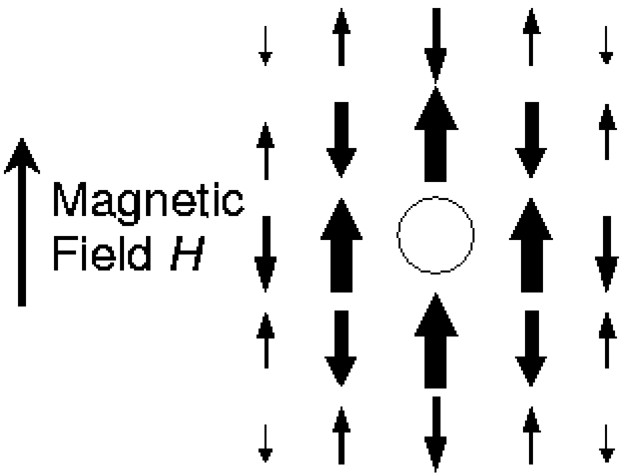
Measured $\chi_{\text{impurity}}(T \to 0) = \dfrac{S(S+1)}{3k_B T}$ with $S = 1/2$ in underdoped sample.

Not expected from BCS theory, which predicts $\chi_{\text{impurity}}(T \to 0) \neq \infty$ for a non-magnetic impurity with strong potential scattering.

Zn impurity in $YBa_2Cu_3O_{6.7}$

Moments measured by
analysis of Knight shifts

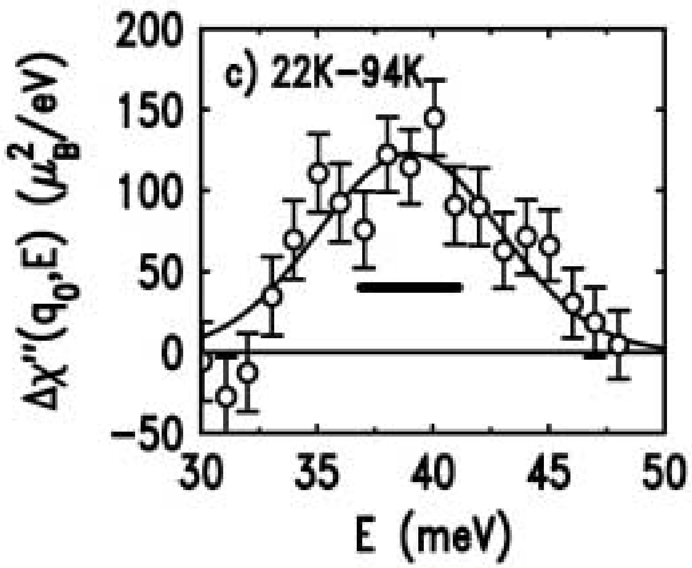
M.-H. Julien, T. Feher,
M. Horvatic, C. Berthier,
O. N. Bakharev, P. Segransan,
G. Collin, and J.-F. Marucco,
Phys. Rev. Lett. **84**, 3422
(2000); also earlier work of
the group of H. Alloul and the
original experiment of
A.M Finkelstein, V.E. Kataev,
E.F. Kukovitskii, and
G.B. Teitel'baum, Physica C
168, 370 (1990).



Berry phases of precessing spins do not cancel
between the sublattices in the vicinity of the
impurity: net uncancelled phase of S=1/2

$YBa_2Cu_3O_7 + 0.5\%$ Zn

H. F. Fong, P. Bourges, Y. Sidis, L. P. Regnault, J. Bossy, A. Ivanov, D.L. Milius, I. A. Aksay, and B. Keimer, Phys. Rev. Lett. **82**, 1939 (1999)



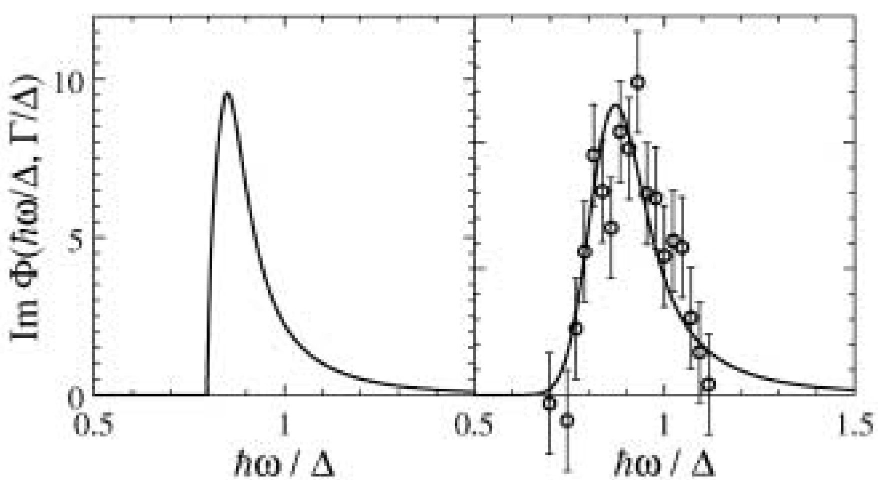
$$n_{\mathrm{imp}} = 0.005$$

$$\Delta = 40\ \mathrm{meV}$$

$$\hbar c = 0.2\ \mathrm{eV}$$

$$\Rightarrow \quad \Gamma = 5\ \mathrm{meV},\ \Gamma/\Delta = 0.125$$

Quoted half-width = 4.25 meV



II.B Effect of external magnetic field

- Theory should account for quantum spin fluctuations
- All effects are ~ H^2 except those associated with H induced superflow.
- Can treat SC order in a static Ginzburg-Landau theory

Action $F_{GL}/T + S_b + S_c$

$$F_{GL} = \int d^2x \left[-|\psi|^2 + \frac{|\psi|^4}{2} + |(\nabla_x - iA)\psi|^2 \right]$$

$$S_c = \int d^2x d\tau \left[\frac{v}{2} \phi_\alpha^2 |\psi|^2 \right]$$

See also S.C. Zhang, Science, **275**, 1089 (1997); D. P. Arovas *et al.*, Phys. Rev. Lett. **79**, 2871 (1997).

$$S_b = \int d^2x \int_0^{1/T} d\tau \left[\frac{1}{2} \left((\nabla_x \phi_\alpha)^2 + c^2 (\partial_\tau \phi_\alpha)^2 + r\phi_\alpha^2 \right) + \frac{g}{4!} (\phi_\alpha^2)^2 \right]$$

Self-consistent Hartree theory of quantum spin fluctuations (large N limit)

$$\chi(x,x',\omega_n)\,\delta_{\alpha\beta} = \left\langle \phi_\alpha(x,\omega_n)\,\phi_\beta(x',-\omega_n)\right\rangle$$

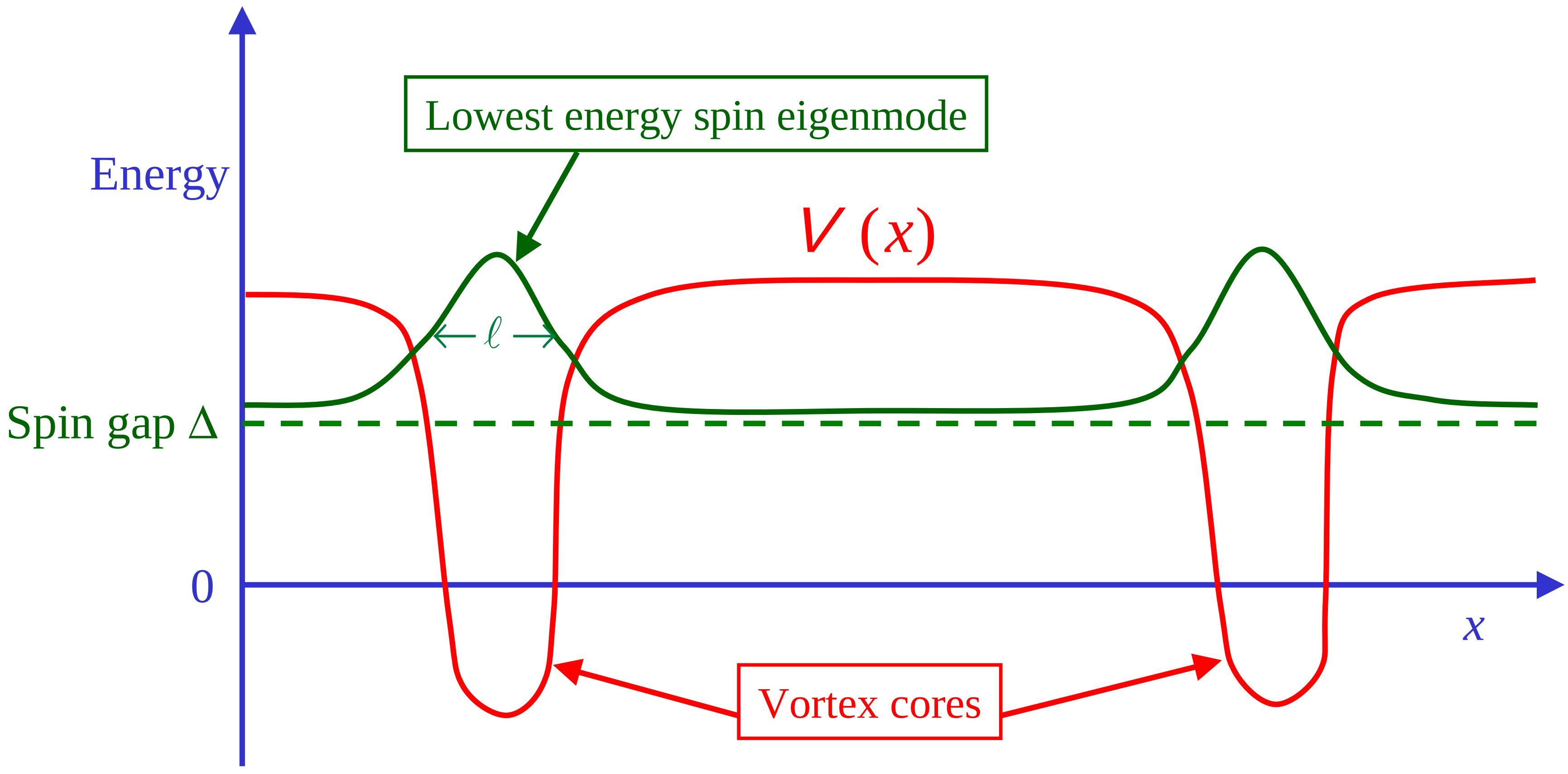
$$\left(\omega_n{}^2 - c^2\nabla_x^2 + V(x)\right)\chi(x,x',\omega_n) = \delta(x-x')$$

$$V(x) = r + v\left|\psi(x)\right|^2 + (NgT/6)\sum_{\omega_n}\chi(x,x,\omega_n)$$

$$\left[-1+\left|\psi(x)\right|^2 - \left(\nabla_x - i\vec{A}\right)^2\right]\psi(x) + (NvT/2)\sum_{\omega_n}\chi(x,x,\omega_n)\,\psi(x) = 0$$

$V(x) \rightarrow$ local classical energy of spin fluctuations; can become negative in vortex cores for $v > 0$.

However, spin gap remains finite because of quantum fluctuations



As $\Delta \rightarrow 0$, $\ell \rightarrow \infty$, because of self interaction, g, of spin excitations.

A.J. Bray and M.A. Moore, J. Phys. C **15**, L765 (1982).
J.A. Hertz, A. Fleishman, and P.W. Anderson, Phys. Rev. Lett. **43**, 942 (1979).

Influence of $\psi(x)$ on extended spin eigenmodes:

$$\left|\psi(x)\right| = 1 - \frac{1}{2x^2}$$ outside each vortex core because of superflow kinetic energy

$$\left\langle \left|\psi(x)\right|^2 \right\rangle = 1 - \frac{H}{2H_{c2}} \ln\left(\frac{3.0H_{c2}}{H}\right)$$

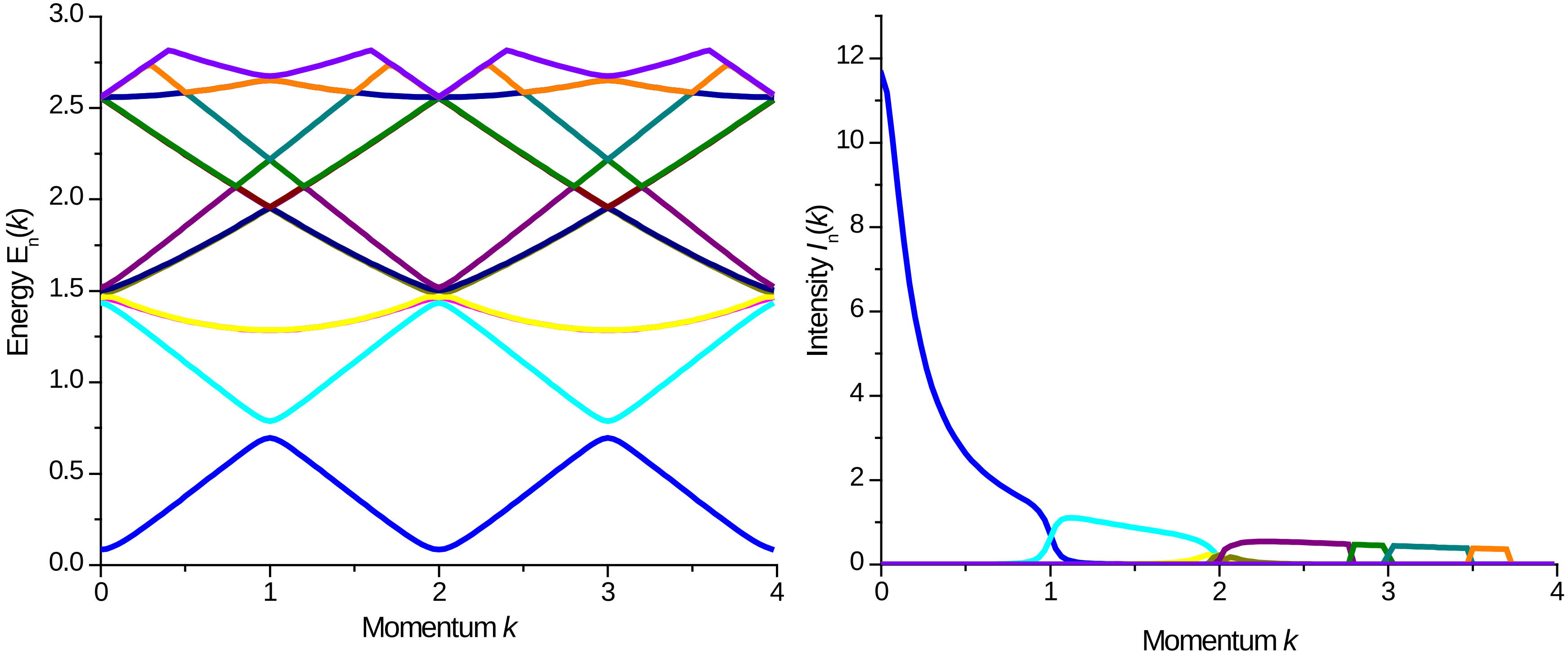
In SC phase, spin gap obeys:

$$\Delta(H) = \Delta(0) - \frac{24\pi c^2 v}{Ng\left(1 - \frac{3v^2}{g}\right)} \frac{H}{2H_{c2}} \ln\left(\frac{3.0H_{c2}}{H}\right)$$

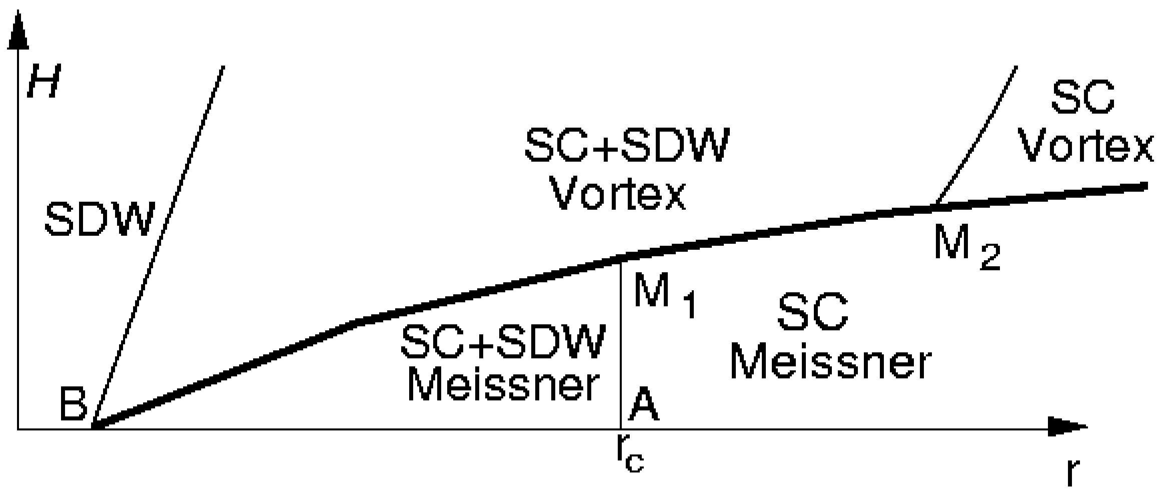
In SC+SDW phase, intensity of elastic scattering obeys:

$$I(H) = I(0) + \frac{6v}{g\left(1 - \frac{3v^2}{g}\right)} \frac{H}{2H_{c2}} \ln\left(\frac{3.0H_{c2}}{H}\right)$$

$$\overline{\chi''}(k,\omega) = \sum_n I_n(k)\,\delta\big(\omega - E_n(k)\big)$$



Consequences of a finite London penetration depth (finite κ)



Conclusions

1. Strong experimental evidence for *S=1/2* moment near Zn and Li impurities in the underdoped high temperature superconductor.

2. New boundary conformal quantum field theory in 2+1 dimensions describes scattering of spin resonance mode off “non-magnetic” impurities.

3. This, and other properties of the high temperature superconductors (existence of *S=1* spin resonance mode, possible bond-centered charge order) are naturally understood by a theory of doping paramagnetic Mott insulators with confinement.

4. Singular response of superflow to an applied magnetic field leads to strong field dependence in quantum phase transitions associated with other order parameters (SDW, CDW, dDW…).