Quantum criticality in the high temperature superconductors

University of Minnesota
October 2, 2013

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Talk online: sachdev.physics.harvard.edu
Sommerfeld-Bloch theory of metals, insulators, and superconductors: many-electron quantum states are adiabatically connected to independent electron states.
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**Band insulators**

An even number of electrons per unit cell.
Sommerfeld-Bloch theory of metals, insulators, and superconductors:
many-electron quantum states are adiabatically connected to independent electron states
Sommerfeld-Bloch theory of metals, insulators, and superconductors: many-electron quantum states are adiabatically connected to independent electron states.
Boltzmann-Landau theory of dynamics of metals: Long-lived quasiparticles (and quasiholes) have weak interactions which can be described by a Boltzmann equation.
Modern phases of quantum matter
Not adiabatically connected to independent electron states:

*many-particle quantum entanglement,*
Modern phases of quantum matter
Not adiabatically connected
to independent electron states:
many-particle
quantum entanglement,
and no quasiparticles
Iron pnictides:
a new class of high temperature superconductors
BaFe$_2$(As$_{1-x}$P$_x$)$_2$

Resistivity \sim \rho_0 + AT^\alpha


Superconductivity

Resistivity \sim \rho_0 + AT^\alpha


$\text{Resistivity } \sim \rho_0 + AT^\alpha$

$\text{BaFe}_2(\text{As}_{1-x}\text{P}_x)_2$

Superconductor

Bose condensate of pairs of electrons

Resistivity \sim \rho_0 + AT^\alpha

BaFe\(_2\)(As\(_{1-x}\)P\(_x\))\(_2\)

Ordinary metal (Fermi liquid)


Strange Metal
no quasiparticles,
Landau-Boltzmann theory does not apply

Resistivity \sim \rho_0 + AT^\alpha

High temperature superconductors

YBa$_2$Cu$_3$O$_{6+x}$
Strange Metal

K.M. Shen et al., Science 2005

M. Platé et al., PRL 2005

Smaller hole Fermi-pockets

Large hole Fermi surface

Thursday, October 3, 13
Strange Metal

K.M. Shen et al., Science 2005

M. Platé et al., PRL 2005

Smaller hole Fermi-pockets

Large hole Fermi surface
Strange Metal Pseudogap Thursday, October 3, 13
BaFe$_2$(As$_{1-x}$P$_x$)$_2$

Quantum critical point
(quantum phase transition involving loss of AF order)

Resistivity $\sim \rho_0 + AT^\alpha$


1. Quantum critical point in an insulator
   *Entanglement and non-quasiparticle dynamics*

2. Quantum critical point in a metal
   *The iron-based superconductors*

3. The pseudogap regime of the hole-doped cuprate superconductors
   *Angular fluctuations of a multicomponent order*
Outline

1. Quantum critical point in an insulator
   *Entanglement and non-quasiparticle dynamics*

2. Quantum critical point in a metal
   *The iron-based superconductors*

3. The pseudogap regime of the hole-doped cuprate superconductors
   *Angular fluctuations of a multicomponent order*
Square lattice antiferromagnet

\[ H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Examine ground state as a function of \( \lambda \)
At large $\lambda$ ground state is a “quantum paramagnet” with spins locked in valence bond singlets
For \( \lambda \approx 1 \), the ground state has antiferromagnetic ("Néel") order, and the spins align in a checkerboard pattern.
\[
\frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)
\]
Excitation spectrum in the paramagnetic phase

Spin $S = 1$

“triplon”
Excitation spectrum in the paramagnetic phase

Spin $S = 1$

“triplon”

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Excitation spectrum in the paramagnetic phase

Spin $S = 1$

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Excitation spectrum in the paramagnetic phase

Spin $S = 1$

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Excitation spectrum in the paramagnetic phase

Spin $S = 1$

“triplon”
Excitation spectrum in the Néel phase

Spin waves
Excitation spectrum in the Néel phase

Spin waves
Excitation spectrum in the Néel phase

Spin waves

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Excitations of TlCuCl₃ with varying pressure

Excitations of TlCuCl$_3$ with varying pressure

Broken valence bond excitations of the quantum paramagnet

Excitations of TlCuCl$_3$ with varying pressure

Spin wave and longitudinal excitations (similar to the Higgs particle) of the Néel state.

“Higgs” particle appears at theoretically predicted energy

S. Sachdev, arXiv:0901.4103

\[ = \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) \]

\[ \lambda_c \]

\[ \lambda \]
Quantum critical point: A new state of matter with long-range quantum entanglement and no quasiparticles.

Classical spin waves

Quantum critical

Dilute triplon gas

Neel order

Dynamics not described by quasiparticles: conformal field theory at $T>0$

Dynamics not described by quasiparticles: conformal field theory at $T>0$ with connections to gauge-gravity duality.

Quantum critical point
(quantum phase transition involving loss of AF order)

$\rho(T) \sim \rho_0 + AT^\alpha$

BaFe$_2$(As$_{1-x}$P$_x$)$_2$

Superconductivity

AF


Physical Review B 81, 184519 (2010)
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The electron spin polarization obeys

\[ \langle \vec{S}(r, \tau) \rangle = \varphi(r, \tau)e^{i\vec{K} \cdot \vec{r}} \]

where \( \vec{K} \) is the ordering wavevector.
Fluctuating Fermi pockets

Large Fermi surface

Strange Metal

Spin density wave (SDW)

Underlying SDW ordering quantum critical point in metal at $x = x_m$

Quantum Critical

Fermi surface + antiferromagnetism
Resistivity \sim \rho_0 + AT^\alpha

BaFe_2(As_{1-x}P_x)_2


Focus on points on the Fermi surface separated by $\mathbf{K}$.

Pairing “glue” from triplon (paramagnon) exchange


\[ \langle c_{\mathbf{k} \alpha}^\dagger c_{-\mathbf{k} \beta}^\dagger \rangle = \varepsilon_{\alpha \beta} \Delta_S (\cos k_x - \cos k_y) \]

d-wave superconductor: particle-particle pairing with sign-changing pairing amplitude
Near the antiferromagnetic critical point, the coupling becomes infinitely strong:
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- Pairing glue becomes stronger.
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- There is stronger fermion-boson scattering, and fermionic quasi-particles lose their integrity.
Near the antiferromagnetic critical point, the coupling becomes infinitely strong:

- Pairing glue becomes stronger.
- There is stronger fermion-boson scattering, and fermionic quasi-particles lose their integrity.
- Other instabilities can appear e.g. to charge density waves/striped order.
QMC for the onset of antiferromagnetism

Hot spots in a single band model
Hot spots in a two band model

QMC for the onset of antiferromagnetism

E. Berg, M. Metlitski, and S. Sachdev, arXiv:1206.0742
QMC for the onset of antiferromagnetism

Hot spots in a two band model

E. Berg, M. Metlitski, and S. Sachdev, arXiv:1206.0742
No sign problem in fermion determinant Monte Carlo!

Determinant is positive because of Kramer’s degeneracy, and no additional symmetries are needed; holds for arbitrary band structure and band filling, provided $K$ only connects hot spots in distinct bands.

E. Berg, M. Metlitski, and S. Sachdev, arXiv:1206.0742
Sign-problem-free Quantum Monte Carlo for antiferromagnetism in metals

d-wave superconducting survives in the strong-coupling region across the quantum critical point

\[ E. \text{ Berg, M. Metlitski, and S. Sachdev, Science 338, 1606 (2012).} \]
Fermi surface + antiferromagnetism

Spin density wave (SDW)

Underlying SDW ordering quantum critical point in metal at $x = x_m$
QCP for the onset of SDW order is actually within a superconductor.
Strange Metal

no quasiparticles, Landau-Boltzmann theory does not apply

\[ \text{Resistivity } \sim \rho_0 + AT^\alpha \]


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1. Quantum critical point in an insulator
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Strange Metal

Temperature [K]

AF insulator

Superconductor

$T_\text{N}$

$T^*$

$T_c$

Smaller hole Fermi-pockets

Large hole Fermi surface

K.M. Shen et al., Science 2005

M. Platé et al., PRL 2005
What about the pseudogap?
What about the pseudogap?
Fermi surface+antiferromagnetism

QCP for the onset of SDW order is actually within a superconductor
Theory of quantum criticality in the cuprates


Spin density wave (SDW)

Competition between SDW order and superconductivity moves the actual quantum critical point to \( x = x_s < x_m \).
The theory of quantum criticality in the cuprates involves the competition between SDW order and superconductivity. This competition moves the actual quantum critical point to $x = x_s < x_m$.

Theory of quantum criticality in the cuprates


Spin density wave (SDW)

**Competition between SDW order and superconductivity moves the actual quantum critical point to** $x = x_s < x_m$. 

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| Thursday, October 3, 13 |
The metal has an instability to both $d$-wave superconductivity and a $d$-wave charge density wave (bond order).


Magnetic–field–induced charge–stripe order in the high–temperature superconductor $\text{YBa}_2\text{Cu}_3\text{O}_y$

Tao Wu$^1$, Hadrien Mayaffre$^1$, Steffen Krämer$^1$, Mladen Horvatić$^1$, Claude Berthier$^1$, W. N. Hardy$^{2,3}$, Ruixing Liang$^{2,3}$, D. A. Bonn$^{2,3}$ & Marc–Henri Julien$^1$

8 September 2011 | Vol 477 | Nature | 191
An Intrinsic Bond-Centered Electronic Glass with Unidirectional Domains in Underdoped Cuprates

Y. Kohsaka,1 C. Taylor,1 K. Fujita,1,2 A. Schmidt,1 C. Lupien,3 T. Hanaguri,4 M. Azuma,5 M. Takano,5 H. Eisaki,6 H. Takagi,2,4 S. Uchida,2,7 J. C. Davis,1,8*

9 MARCH 2007 VOL 315 SCIENCE

Superconducting $d$-wave stripes in cuprates: Valence bond order coexisting with nodal quasiparticles

Matthias Vojta and Oliver Rösch

We point out that unidirectional bond-centered charge-density-wave states in cuprates involve electronic order in both $s$- and $d$-wave channels, with nonlocal Coulomb repulsion suppressing the $s$-wave component.
Pairing “glue” from triplon (paramagnon) exchange

"Same "glue" leads to bond/charge order!"

Magnetic-field-induced charge-stripe order in the high-temperature superconductor YBa$_2$Cu$_3$O$_y$

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8 SEPTEMBER 2011 | VOL 477 | NATURE | 191
Twofold twisted Fermi surface from staggered order in an underdoped high $T_c$ superconductor

Suchitra E. Sebastian,1* N. Harrison,2 F. F. Balakirev,2 M. M. Altarawneh,2,3 Ruixing Liang,4,5 D. A. Bonn,4,5 W. N. Hardy,4,5 G. G. Lonzarich,1

APS March meeting 2013
B2.00004
Direct observation of competition between superconductivity and charge density wave order in YBa$_2$Cu$_3$O$_6.67$

J. Chang$^{1,2,*}$, E. Blackburn$^3$, A. T. Holmes$^3$, N. B. Christensen$^4$, J. Larsen$^{4,5}$, J. Mesot$^{1,2}$, Ruixing Liang$^{6,7}$, D. A. Bonn$^{6,7}$, W. N. Hardy$^{6,7}$, A. Watenphul$^8$, M. v. Zimmermann$^8$, E. M. Forgan$^3$ and S. M. Hayden$^9$

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Figure 2

- Below 135 K, the intensity of the satellite in $\delta_1$ can be reversed by the application of a magnetic field.
- Scans along (0, c $\neq$ 0) at positions such as $T_3$, $T_6$ (with ortho-VIII oxygen ordering)
- The CDW environment. Our results provide a mechanism for the negative Hall and Seebeck effects.
FIG. 1: The temperature dependence of the CDW scattering intensity at $Q = [-0.31 \ 0 \ 1.48]$ in $\text{YBa}_2\text{Cu}_3\text{O}_{6.67}$ measured by resonant x-ray scattering in Ref. [4]. This sample has $T_c \approx 65.5\text{K}$.
Competing orders in thermally fluctuating superconductors in two dimensions

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(Received 6 August 2003; revised manuscript received 24 November 2003; published 6 April 2004)

We extend recent low-temperature analyses of competing orders in the cuprate superconductors to the pseudogap regime where all orders are fluctuating. A universal continuum limit of a classical Ginzburg-Landau functional is used to characterize fluctuations of the superconducting order: this describes the crossover from Gaussian fluctuations at high temperatures to the vortex-binding physics near the onset of global phase coherence. These fluctuations induce affiliated corrections in the correlations of other orders, and in particular, in the different realizations of charge order. Implications for scanning tunneling spectroscopy and neutron-scattering experiments are noted: there may be a regime of temperatures near the onset of superconductivity where the charge order is enhanced with increasing temperatures.
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The observed decrease in charge order with decreasing $T$ in $YBa_{2}Cu_{3}O_{6.67}$ at low $T$ was predicted in Ref. [9] using a Landau theory framework [10] to describe competition between superconductivity and charge density wave order. Here we will extend the theory to a much wider regime of temperatures. The Landau theory introduces a complex field $\phi$ to represent the superconductivity, and two complex fields $\chi(x,y)$ for representing the charge order. The latter can represent modulations at the wavevectors $Q_{x,y}$ in not only the site onsets is unlike an arrested ordering transition, or precursor critical fluctuations.
Key idea: analogy with the onset of antiferromagnetism in the insulator $\text{La}_2\text{CuO}_4$

Below $T_{\mathrm{Néel}}$

$$T_{\mathrm{Néel}} = 325\text{K}$$

B. Keimer et al.,

D. Vaknin et al.,

Above $T_{\mathrm{Néel}}$
**Key idea:** analogy with the onset of antiferromagnetism in the *insulator* La$_2$CuO$_4$  

Gradual onset of intensity over a wide range of $T$ is a consequence of *angular thermal* fluctuations of an order parameter with 3 or more components in 2 spatial dimensions  

$T_{\text{Néel}} = 325$K  


Polyakov, 1975  
Chakravarty, Halperin, Nelson 1989
Multi-component order parameter for the pseudogap

Superconducting order $\Psi(\mathbf{r})$:

$$\langle c_{i\alpha}^{\dagger} c_{j\beta}^{\dagger} \rangle = \varepsilon_{\alpha\beta} \left[ \sum_{\mathbf{k}} \Delta_S(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] \Psi((\mathbf{r}_i + \mathbf{r}_j)/2)$$

Charge/bond order $\Phi_{x,y}(\mathbf{r})$ at wavevectors $\mathbf{Q}_{x,y}$:

$$\langle c_{i\alpha}^{\dagger} c_{j\beta}^{\dagger} \rangle = \delta_{\alpha\beta} \left[ \sum_{\mathbf{k}} P_{Q_x}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{iQ_x \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} \Phi_x((\mathbf{r}_i + \mathbf{r}_j)/2)$$
$$+ \delta_{\alpha\beta} \left[ \sum_{\mathbf{k}} P_{Q_y}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} \right] e^{iQ_y \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} \Phi_y((\mathbf{r}_i + \mathbf{r}_j)/2)$$
Multi-component order parameter
Support from theory of antiferromagnetic quantum criticality
Multi-component order parameter

Support from theory of antiferromagnetic quantum criticality
Multi-component order parameter

Label order parameter by a 6-component unit vector \( n_\alpha \) with \( \sum_\alpha n_\alpha^2 = 1 \)
O(6) non-linear sigma model

\[ Z = \int \mathcal{D}n_\alpha(r) \delta \left( \sum_{\alpha=1}^{6} n_\alpha^2(r) - 1 \right) \exp \left( - \frac{\rho_s}{2T} \int d^2r \left[ \sum_{\alpha=1}^{2} (\nabla n_\alpha)^2 
+ \lambda \sum_{\alpha=3}^{6} (\nabla n_\alpha)^2 
+ g \sum_{\alpha=3}^{6} n_\alpha^2 
+ w \left[ (n_3^2 + n_4^2)^2 + (n_5^2 + n_6^2)^2 \right] \right) \right). \]

where \( \Psi \propto n_1 + in_2, \Phi_x \propto n_3 + in_4, \Phi_y \propto n_5 + in_6. \)

Describes \( O(6) \Rightarrow O(2) \times O(2) \times O(2) \times \mathbb{Z}_2. \) The coupling \( g \) determines the anisotropy between superconductivity and charge order.

Solve by cluster Monte Carlo and \( 1/N \) expansion.

Comparison of Monte Carlo with experiments

Charge order structure factor $S_{\Phi_x}$

$$S_{\Phi_x} = \int d^2r \langle \Phi_x(r)\Phi_x(0) \rangle$$

For $ga^2 = 0.30$ and $wa^2 = 0.0$ we have $\rho_s = 160$K.
The height was also rescaled to make the peak heights match.

Comparison of Monte Carlo with experiments

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Temperature applies in the immediate vicinity of $\phi_{0}$ with a vertical dashed line. The prediction of Ref. [9] of increasing charge order with increasing $\chi$, showing a peak at around $T = 0.39$. A finite-size scaling analysis estimates $L = 0.15$ to determine the Kosterlitz-Thouless temperature for each system size. Note that $\chi = 0$, $\alpha = 0$, and use the relation helicity modulus $= \frac{1}{2}
abla^2 \phi_{0}$, to the left of the peak.

Other experiments in the pseudogap

- The same set of parameters used to describe X-ray scattering, also predict the strength of superconducting fluctuations above $T_c$. Indeed YBa$_2$Cu$_3$O$_{6+x}$ shows significant fluctuation diamagnetism over the same range of temperatures. (S. Chatterjee et al, in progress).

PHYSICAL REVIEW B 88, 060505(R) (2013)

I. Kokanović, 1,2,* D. J. Hills, 1 M. L. Sutherland, 1 R. Liang, 3 and J. R. Cooper 1

Diamagnetism of YBa$_2$Cu$_3$O$_{6+x}$ crystals above $T_c$: Evidence for Gaussian fluctuations
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- Charge order was originally observed around vortex cores, indicating its competition with superconductivity.

---

**A Four Unit Cell Periodic Pattern of Quasi-Particle States Surrounding Vortex Cores in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$**

J. E. Hoffman,¹ E. W. Hudson,¹,²,* K. M. Lang,¹ V. Madhavan,¹ H. Eisaki,³† S. Uchida,³ J. C. Davis¹,²†

*Present address: Department of Applied Physics, California Institute of Technology, Pasadena, CA 91125 – 1001.*

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- Charge order was originally observed around vortex cores, indicating its competition with superconductivity.

- The charge order becomes long-ranged in high magnetic fields, and can explain the observed quantum oscillations (Taillefer, Sebastian).
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• Charge order was originally observed around vortex cores, indicating its competition with superconductivity.

• The charge order becomes long-ranged in high magnetic fields, and can explain the observed quantum oscillations (Taillefer, Sebastian).

• Fluctuating 6-component order can explain “Fermi arc” photoemission spectra (Randeria; D. Chowdhury et al, in progress)
Antiferromagnetic interactions induce $d$-wave superconductivity in metals. This has now been established in the strong-coupling region near an antiferromagnetic critical point.
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At strong coupling, the same interactions can also induce an incommensurate $d$-wave bond order.
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There is an approximate “rotation” symmetry between $d$-wave superconductivity and the bond (charge) order.
Antiferromagnetic interactions induce \textit{d-wave} superconductivity in metals. This has now been established in the strong-coupling region near an antiferromagnetic critical point.

At strong coupling, the same interactions can also induce an incommensurate \textit{d-wave bond order}.

There is an approximate “rotation” symmetry between \textit{d-wave superconductivity} and the bond (charge) order.

The “pseudogap” phase is described by the angular fluctuations of a 6-component vector representing the superconducting and bond orders.