The quantum phases of matter and
gauge-gravity duality

University of Michigan, Ann Arbor, March 13, 2013

Subir Sachdev
Sommerfeld-Bloch theory of metals, insulators, and superconductors: many-electron quantum states are adiabatically connected to independent electron states
Modern phases of quantum matter
Not adiabatically connected
to independent electron states:

many-particle
quantum entanglement
**Quantum Entanglement:** quantum superposition with more than one particle

Hydrogen atom: \( |\uparrow\rangle \)

Hydrogen molecule:

\[
\begin{align*}
\text{Superposition of two electron states leads to non-local correlations between spins}
\end{align*}
\]
Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:

Hydrogen molecule:

\[ \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]

Einstein-Podolsky-Rosen “paradox”: Non-local correlations between observations arbitrarily far apart
Outline

1. $\mathbb{Z}_2$ Spin liquid in the kagome antiferromagnet

2. Superfluid-insulator transition of ultracold atoms in optical lattices:
   *Quantum criticality and conformal field theories*

3. Holography and the quasi-normal modes of black-hole horizons

4. Strange metals:
   *What lies beyond the horizon?*
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Crystals used in collaborations:
1) Polarized Raman scattering - Lemmens, Braunschweig
2) NMR – Takashi Imai, McMaster University
3) Thermal conductivity - Behnia, Ecole Superieure
4) µSR - Keren, Technion

ZnCu(OH)$_6$Cl$_2$
herbertsmithite single crystals

Our work at MIT:
1) The impurity question using x-rays
2) The spin excitations using neutrons

Wednesday, March 13, 13
Kagome antiferromagnet

\[ H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \]
Kagome antiferromagnet

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\[ \frac{1}{\sqrt{2}} (\uparrow\downarrow) - (\downarrow\uparrow) \]

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P. Fazekas and P. W. Anderson, 
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Kagome antiferromagnet

Alternative view

Pick a reference configuration
Kagome antiferromagnet

Alternative view

A nearby configuration
Kagome antiferromagnet

Alternative view

Difference: a closed loop
Kagome antiferromagnet

Alternative view

Ground state: sum over closed loops
Kagome antiferromagnet

Alternative view

Ground state: sum over closed loops
Mott insulator: Kagome antiferromagnet

Alternative view

Ground state: sum over closed loops
Kagome antiferromagnet

Alternative view

Ground state: sum over closed loops
\[ |\Psi\rangle \Rightarrow \text{Ground state of entire system,} \]
\[ \rho = |\Psi\rangle\langle \Psi| \]

\[ \rho_A = \text{Tr}_B \rho = \text{density matrix of region } A \]

Entanglement entropy \[ S_E = -\text{Tr} (\rho_A \ln \rho_A) \]
\[ |\Psi\rangle \Rightarrow \text{Ground state of entire system,} \]
\[ \rho = |\Psi\rangle \langle \Psi| \]

Take \[ |\Psi\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle_A \downarrow_B - |\downarrow\rangle_A |\uparrow\rangle_B) \]

Then \[ \rho_A = \text{Tr}_B \rho = \text{density matrix of region } A \]
\[ = \frac{1}{2} (|\uparrow\rangle_A \langle \uparrow|_A + |\downarrow\rangle_A \langle \downarrow|_A) \]

Entanglement entropy \[ S_E = -\text{Tr} (\rho_A \ln \rho_A) \]
\[ = \ln 2 \]
Entanglement entropy of a band insulator

Band insulators

An even number of electrons per unit cell
Entanglement entropy of a band insulator

$$S_E = aP - b \exp(-cP)$$

where $P$ is the surface area (perimeter) of the boundary between A and B.
Entanglement in the $\mathbb{Z}_2$ spin liquid

Entanglement in the $\mathbb{Z}_2$ spin liquid

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$$S_E = aP - \ln(2)$$

Mott insulator: Kagome antiferromagnet

Strong numerical evidence for a $\mathbb{Z}_2$ spin liquid


Fractionalized excitations in the spin-liquid state of a kagome-lattice antiferromagnet

Tian-Heng Han, Joel S. Helton, Shaoyan Chu, Daniel G. Nocera, Jose A. Rodriguez-Rivera, Collin Broholm & Young S. Lee

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Superfluid-insulator transition

Ultracold $^{87}\text{Rb}$ atoms - bosons

$\text{Superfluid}$

$\text{Insulator}$

$0 \quad \lambda_c \quad \lambda$
$\Psi \rightarrow \text{a complex field representing the Bose-Einstein condensate of the superfluid}$
\[ S = \int d^2 r d t \left[ |\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right] \]

\[ V(\Psi) = (\lambda - \lambda_c) |\Psi|^2 + u (|\Psi|^2)^2 \]

\[ \langle \Psi \rangle \neq 0 \quad \text{Superfluid} \]

\[ \langle \Psi \rangle = 0 \quad \text{Insulator} \]
\[ S = \int d^2 r d t \left[ |\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right] \]

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Particles and holes correspond to the 2 normal modes in the oscillation of \( \Psi \) about \( \Psi = 0 \).

Superfluid

\[ \langle \Psi \rangle \neq 0 \]

Insulator

\[ \langle \Psi \rangle = 0 \]
Insulator (the vacuum)
at large repulsion between bosons
Excitations of the insulator:

Particles $\sim \Psi^\dagger$
Excitations of the insulator:

Holes $\sim \Psi$
\[ S = \int d^2r dt \left[ |\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right] \]

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Nambu-Goldstone mode is the oscillation in the phase \( \Psi \) at a constant non-zero \( |\Psi| \).

\[ \langle \Psi \rangle \neq 0 \quad \text{Superfluid} \]
\[ \langle \Psi \rangle = 0 \quad \text{Insulator} \]
A conformal field theory in 2+1 spacetime dimensions: a CFT3

\[ S = \int d^2r dt \left[ |\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right] \]

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Quantum state with complex, many-body, “long-range” quantum entanglement

\[ \langle \Psi \rangle \neq 0 \]

Superfluid

\[ \langle \Psi \rangle = 0 \]

Insulator

0 \[ \lambda_c \]
\[ \lambda \]
\[ S = \int d^2r dt \left[ |\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right] \]

\[ V(\Psi) = (\lambda - \lambda_c) |\Psi|^2 + u (|\Psi|^2)^2 \]

No well-defined normal modes, or particle-like excitations

\[ \langle \Psi \rangle \neq 0 \quad \text{Superfluid} \]

\[ \langle \Psi \rangle = 0 \quad \text{Insulator} \]

\( \lambda_c \)
\[
S = \int d^2r dt \left[ |\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]
\]

\[
V(\Psi) = (\lambda - \lambda_c) |\Psi|^2 + u (|\Psi|^2)^2
\]

Higgs mode is the oscillation in the amplitude \(|\Psi|\). This decays rapidly by emitting pairs of Nambu-Goldstone modes.

\[
\langle \Psi \rangle \neq 0 \quad \text{Superfluid}
\]

\[
\langle \Psi \rangle = 0 \quad \text{Insulator}
\]

\[\lambda_c\]
Despite rapid decay, there is a well-defined Higgs “quasi-normal mode”. This is associated with a pole in the lower-half of the complex frequency plane.

\[
S = \int d^2r dt \left[ |\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]
\]

\[
V(\Psi) = (\lambda - \lambda_c) |\Psi|^2 + u (|\Psi|^2)^2
\]

D. Podolsky, A. Auerbach, and D. P. Arovas, PRB 84, 174522 (2011).
\[ S = \int d^2 r dt \left[ |\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right] \]

\[ V(\Psi) = (\lambda - \lambda_c) |\Psi|^2 + u (|\Psi|^2)^2 \]

The Higgs quasi-normal mode is at the frequency

\[
\frac{\omega_{\text{pole}}}{\Delta} = -i \frac{4}{\pi} + \frac{1}{N} \left( \frac{16 (4 + \sqrt{2} \log (3 - 2\sqrt{2}))}{\pi^2} + 2.46531203396 i \right) + O \left( \frac{1}{N^2} \right)
\]

where \(\Delta\) is the particle gap at the complementary point in the “paramagnetic” state with the same value of \(|\lambda - \lambda_c|\), and \(N = 2\) is the number of vector components of \(\Psi\). The universal answer is a consequence of the strong interactions in the CFT3.
Observation of Higgs quasi-normal mode across the superfluid-insulator transition of ultracold atoms in a 2-dimensional optical lattice:

Response to modulation of lattice depth scales as expected from the LHP pole

Observation of Higgs quasi-normal mode across the superfluid-insulator transition of ultracold atoms in a 2-dimensional optical lattice: Response to modulation of lattice depth scales as expected from the LHP pole

Figure 4 | Scaling of the low-frequency response. The low-frequency response in the superfluid regime shows a scaling compatible with the prediction \((1 - j/j_c)^{-2} v^3\) (Methods). Shown is the temperature response rescaled with \((1 - j/j_c)^2\) for \(V_0 = 10E_r\) (grey), \(9.5E_r\) (black), \(9E_r\) (green), \(8.5E_r\) (blue) and \(8E_r\) (red) as a function of the modulation frequency. The black line is a fit of the form \(av^b\) with a fitted exponent \(b = 2.9(5)\). The inset shows the same data points without rescaling, for comparison. Error bars, s.e.m.


\[ S = \int d^2r dt \left[ |\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right] \]

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A conformal field theory in 2+1 spacetime dimensions:

a CFT3

\[ \langle \Psi \rangle \neq 0 \]

Superfluid

\[ \langle \Psi \rangle = 0 \]

Insulator

\[ \lambda \]

\[ \lambda_c \]

0
CFT3 at $T > 0$

Superfluid

Quantum critical

Insulator

$T_{KT}$

$T$

$\lambda$

$\lambda_c$

0
Quantum critical dynamics

Quantum “nearly perfect fluid”
with shortest possible local equilibration time, $\tau_{eq}$

$$\tau_{eq} = C \frac{\hbar}{k_B T}$$

where $C$ is a universal constant.

Response functions are characterized by poles in LHP
with $\omega \sim k_B T/\hbar$.
These poles (quasi-normal modes) appear naturally in
the holographic theory.
(Analogs of Higgs quasi-normal mode.)

Quantum critical dynamics

Transport co-efficients not determined by collision rate of quasiparticles, but by fundamental constants of nature

Conductivity

\[ \sigma = \frac{Q^2}{\hbar} \times \text{[Universal constant } O(1) \text{]} \]

\(Q\) is the “charge” of one boson


CFT3 at $T>0$

- **Superfluid**
- **Insulator**
- **Quantum critical**

$T$ vs $\lambda$

$T_{KT}$
CFT3 at $T>0$

Boltzmann theory of particles/holes/vortices does not apply
CFT3 at $T>0$

Needed: Accurate theory of quantum critical dynamics
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Renormalization group:  \Rightarrow  Follow coupling constants of quantum many body theory as a function of length scale $r$
Renormalization group: \( \Rightarrow \) Follow coupling constants of quantum many body theory as a function of length scale \( r \)

\[ r \quad \rightarrow \quad x_i \]

Key idea: \( \Rightarrow \) Implement \( r \) as an extra dimension, and map to a local theory in \( d + 2 \) spacetime dimensions.

J. McGreevy, arXiv0909.0518
For a relativistic CFT in $d$ spatial dimensions, the metric in the holographic space is fixed by demanding the scale transformation ($i = 1 \ldots d$)

$$x_i \rightarrow \zeta x_i \quad , \quad t \rightarrow \zeta t \quad , \quad ds \rightarrow ds$$
This gives the unique metric

\[ ds^2 = \frac{1}{r^2} (-dt^2 + dr^2 + dx_i^2) \]

This is the metric of anti-de Sitter space \( \text{AdS}_{d+2} \).
AdS/CFT correspondence

$\text{AdS}_4 \quad \mathcal{R}^{2,1} \quad \text{Minkowski}$

$CFT_3$

$r \quad x^i$
Holography and Entanglement

AdS$_4$ \hspace{2cm} R$^{2,1}$ \hspace{2cm} Minkowski

CFT$_3$
Associate entanglement entropy with an observer in the enclosed spacetime region, who cannot observe “outside” : i.e. the region is surrounded by an imaginary horizon.

Holography and Entanglement

AdS$_4$

$R^{2,1}$

Minkowski

CFT3

Minimal surface area measures entanglement entropy

Computation of minimal surface area yields
\[ S_E = aP - \gamma, \]
where \( \gamma \) is a shape-dependent universal number.

This emergent spacetime is a solution of Einstein gravity with a negative cosmological constant.

\[
S_E = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) \right]
\]
AdS/CFT correspondence at non-zero temperatures

AdS$_4$-Schwarzschild black-brane

There is a family of solutions of Einstein gravity which describe non-zero temperatures

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) \right]$$
AdS/CFT correspondence at non-zero temperatures

AdS$_4$-Schwarzschild black-brane

Black-brane (horizon) at temperature of 2+1 dimensional quantum critical system

$$ds^2 = \left( \frac{L}{r} \right)^2 \left[ \frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2 \right]$$

with $f(r) = 1 - (r/R_h)^3$
AdS/CFT correspondence at non-zero temperatures

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Black-brane (horizon) at temperature of 2+1 dimensional quantum critical system

A 2+1 dimensional system at its quantum critical point:

$$k_B T = \frac{3\hbar}{4\pi R_h}.$$
AdS/CFT correspondence at non-zero temperatures

AdS$_4$-Schwarzschild black-brane

Black-brane (horizon) at temperature of 2+1 dimensional quantum critical system

Friction of quantum criticality = waves falling past the horizon

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**AdS$_4$-Schwarzschild black-brane**

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Black-brane (horizon) at temperature of 2+1 dimensional quantum critical system

Quasi-normal modes of waves near horizon -- quasi-normal modes of quantum criticality (and Higgs)
AdS$_4$ theory of quantum criticality

Most general effective holographic theory for linear charge transport with 4 spatial derivatives:

$$S_{\text{bulk}} = \frac{1}{g_M^2} \int d^4x \sqrt{g} \left[ \frac{1}{4} F_{ab} F^{ab} + \gamma L^2 C_{abcd} F^{ab} F^{cd} \right]$$

$$+ \int d^4x \sqrt{g} \left[ -\frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) \right],$$

Here $F_{ab}$ is a 4-dimensional gauge field strength, which is “dual” to a conserved U(1) current of the CFT. $C_{abcd}$ is the Weyl tensor.


**AdS$_4$ theory of quantum criticality**

Most general effective holographic theory for linear charge transport with 4 spatial derivatives:

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+ \int d^4 x \sqrt{g} \left[ -\frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) \right],
\]

This action is characterized by 3 dimensionless parameters, which can be linked to data of the CFT (OPE coefficients): 2-point correlators of the conserved current $J_\mu$ and the stress energy tensor $T_{\mu\nu}$, and a 3-point $T, J, J$ correlator.

AdS\textsubscript{4} theory of quantum criticality

Most general effective holographic theory for linear charge transport with 4 spatial derivatives:

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\end{equation}

Boundary and bulk methods both show that $|\gamma| \leq 1/12$, and the bound is saturated by free fields.

AdS$_4$ theory of quantum criticality

The $\gamma > 0$ result has similarities to the quantum-Boltzmann result for transport of particle-like excitations.

AdS$_4$ theory of quantum criticality

The $\gamma < 0$ result can be interpreted as the transport of vortex-like excitations

Stability constraints on the effective theory ($|\gamma| < 1/12$) allow only a limited $\omega$-dependence in the conductivity. This contrasts with the Boltzmann theory in which $\sigma(\omega)/\sigma_{\infty}$ becomes very large in the regime of its validity.

**AdS\(_4\) theory of quantum criticality**

Poles in LHP of conductivity at \(\omega \sim k_B T / \hbar\); analog of Higgs quasinormal mode–quasinormal modes of black brane.

AdS$_4$ theory of quantum criticality

Zeros in LHP of conductivity — quasinormal modes of S-dual theory

AdS$_4$ theory of quantum criticality

It can be shown that the conductivity of any CFT3 must satisfy two sum rules

$$\int_0^\infty d\omega \text{Re} [\sigma(\omega) - \sigma(\infty)] = 0$$

$$\int_0^\infty d\omega \text{Re} \left[ \frac{1}{\sigma(\omega)} - \frac{1}{\sigma(\infty)} \right] = 0$$

- The AdS$_4$ theory satisfies both sum rules exactly.

- The Boltzmann theory must make a choice between the “particle” or “vortex” basis, and so satisfies only one of the sum rules.

Traditional CMT

- Identify quasiparticles and their dispersions
- Compute scattering matrix elements of quasiparticles (or of collective modes)
- These parameters are input into a quantum Boltzmann equation
- Deduce dissipative and dynamic properties at non-zero temperatures
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<thead>
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**Holography and black-branes**

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Universal Scaling of the Conductivity at the Superfluid-Insulator Phase Transition

Jurij Šmakov and Erik Sørensen

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(Received 30 May 2005; published 27 October 2005)

The scaling of the conductivity at the superfluid-insulator quantum phase transition in two dimensions is studied by numerical simulations of the Bose-Hubbard model. In contrast to previous studies, we focus on properties of this model in the experimentally relevant thermodynamic limit at finite temperature $T$. We find clear evidence for deviations from $\omega_k$ scaling of the conductivity towards $\omega_k/T$ scaling at low Matsubara frequencies $\omega_k$. By careful analytic continuation using Padé approximants we show that this behavior carries over to the real frequency axis where the conductivity scales with $\omega/T$ at small frequencies and low temperatures. We estimate the universal dc conductivity to be $\sigma^* = 0.45(5)Q^2/h$, distinct from previous estimates in the $T = 0$, $\omega/T \gg 1$ limit.
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QMC yields $\sigma(0)/\sigma_\infty \approx 1.36$

Holography yields $\sigma(0)/\sigma_\infty = 1 + 4\gamma$ with $|\gamma| \leq 1/12$.

Maximum possible holographic value $\sigma(0)/\sigma_\infty = 1.33$

http://dx.doi.org/10.1103/PhysRevLett.95.180603

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2. Superfluid-insulator transition of ultracold atoms in optical lattices: 
   Quantum criticality and conformal field theories

3. Holography and the quasi-normal modes of black-hole horizons

4. Strange metals: 
   What lies beyond the horizon?
Resistivity $\sim \rho_0 + AT^n$

Electrons (fermions) occupy states inside a Fermi “surface” (circle) of radius $k_F$ which is determined by the density of electrons, $Q$. 
A Strange Metal

Can bosons form a metal?
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Yes, if each boson, \( b \), *fractionalizes* into 2 fermions (‘quarks’) \( b = f_1 f_2 \)!

S. Sachdev, arXiv:1209.1637
Can bosons form a metal?

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- Each quark is charged under an emergent gauge force, which encapsulates the entanglement in the ground state.
Can bosons form a metal?

Yes, if each boson, \( b \), \textit{fractionalizes} into 2 fermions (‘quarks’)
\[
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\]

- Each quark is charged under an \textit{emergent} gauge force, which encapsulates the entanglement in the ground state.
- The quarks have “hidden” Fermi surfaces of radius \( k_F \).
The density of particles $Q$ creates an electric flux $\epsilon_r$ which modifies the metric of the emergent spacetime.
The density of particles $Q$ creates an electric flux $\mathcal{E}_r$ which modifies the metric of the emergent spacetime.
The general metric transforms under rescaling as

\[ x_i \rightarrow \zeta x_i, \quad t \rightarrow \zeta^z t, \quad ds \rightarrow \zeta^{\theta/d} ds. \]

Recall: conformal matter has \( \theta = 0, z = 1 \), and the metric is anti-de Sitter
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The value \( \theta = d - 1 \) reproduces all the essential characteristics of the entropy and entanglement entropy of a strange metal.
The general metric transforms under rescaling as

\[ x_i \rightarrow \zeta x_i, \quad t \rightarrow \zeta^z t, \quad ds \rightarrow \zeta^{\theta/d} ds. \]

The null-energy condition of gravity yields \( z \geq 1 + \theta/d \). In \( d = 2 \), this leads to \( z \geq 3/2 \). Field theory on strange metal yields \( z = 3/2 \) to 3 loops!


Conclusions

Realizations of many-particle entanglement: $\mathbb{Z}_2$ spin liquids and conformal quantum critical points
Conclusions

Conformal quantum matter

- New insights and solvable models for diffusion and transport of strongly interacting systems near quantum critical points
- The description is far removed from, and complementary to, that of the quantum Boltzmann equation which builds on the quasiparticle/vortex picture.
- Good prospects for experimental tests of frequency-dependent, non-linear, and non-equilibrium transport
Conclusions

More complex examples in metallic states are experimentally ubiquitous, but pose difficult strong-coupling problems to conventional methods of field theory.
Conclusions

String theory and gravity in emergent dimensions offer a remarkable new approach to describing states with many-particle quantum entanglement.

Much recent progress offers hope of a holographic description of “strange metals”