Competing orders and quantum criticality in the high temperature superconductors

Eugene Demler (Harvard)
Kwon Park
Anatoli Polkovnikov
Subir Sachdev
Matthias Vojta (Augsburg)
Ying Zhang

Science **286**, 2479 (1999).

Transparencies online at
http://pantheon.yale.edu/~subir

Outline

I. Theoretical Models of Quantum Phase Transitions
  A. Quantum Ising Chain
  B. Coupled Ladder Antiferromagnet
  C. Square Lattice Antiferromagnet

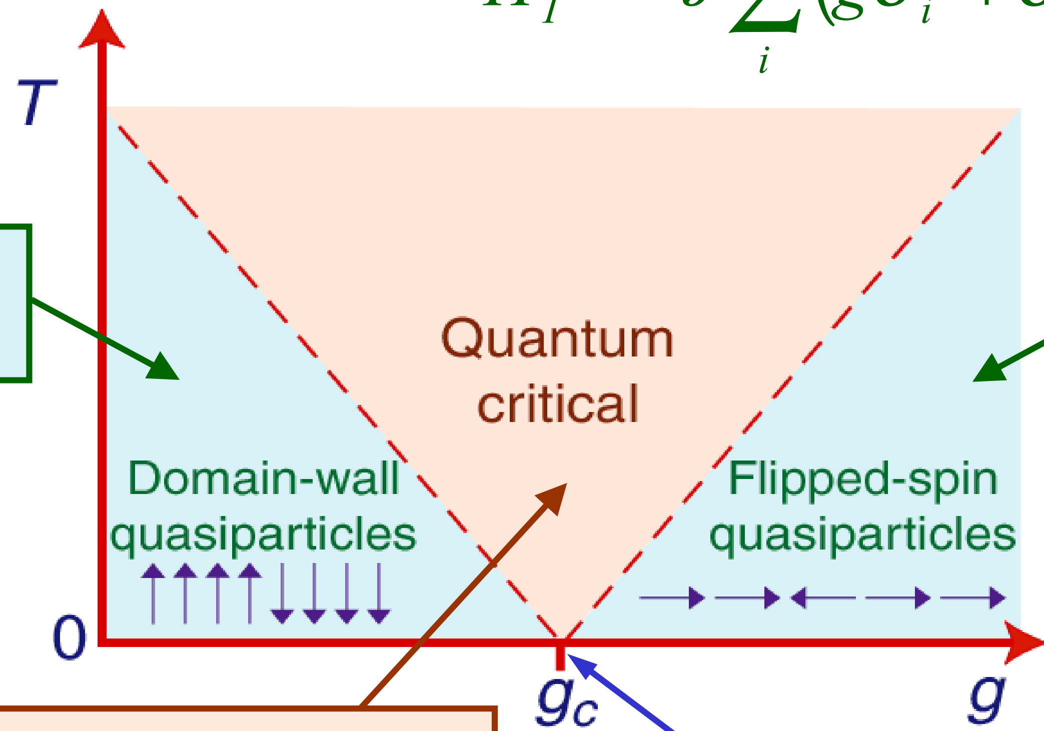
II. Magnetic transitions in a *d*-wave superconductor
  A. Survey of some recent experiments on the high temperature superconductors.
  B. Effect of an applied magnetic field

III. Conclusions

I.A Quantum Ising Chain

$$H_I = -J \sum_i \left(g\sigma_i^x + \sigma_i^z \sigma_{i+1}^z \right)$$

Quasiclassical dynamics



Quasiclassical dynamics

$$|\rightarrow\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)$$

$$|\leftarrow\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle - |\downarrow\rangle\right)$$

$$\chi(\omega) = \frac{i}{\hbar}\sum_k \int_0^\infty dt \left\langle \left[\sigma_j^z(t), \sigma_k^z(0) \right] \right\rangle e^{i\omega t}$$

$$= \frac{A}{T^{7/4}\left(1 - i\omega/\Gamma_R + \ldots\right)}$$

$$\Gamma_R = \left(2\tan\frac{\pi}{16} \right) \frac{k_B T}{\hbar}$$

$$\left\langle \sigma_j^z \sigma_k^z \right\rangle \sim \frac{1}{|j-k|^{1/4}}$$

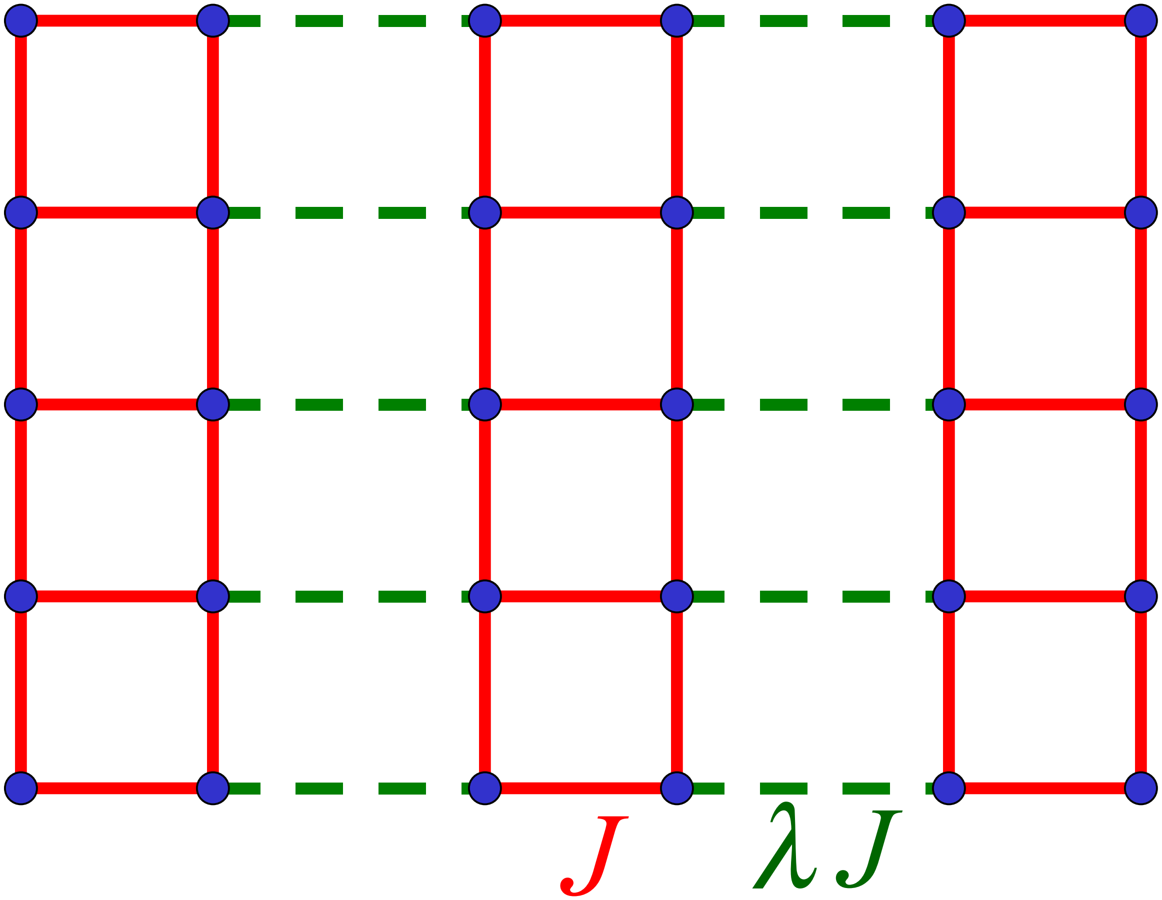
P. Pfeuty *Annals of Physics*, **57**, 79 (1970)

S. Sachdev and J. Ye, Phys. Rev. Lett. **69**, 2411 (1992).
S. Sachdev and A.P. Young, Phys. Rev. Lett. **78**, 2220 (1997).

I.B Coupled Ladder Antiferromagnet

N. Katoh and M. Imada, J. Phys. Soc. Jpn. **63**, 4529 (1994).
J. Tworzydlo, O. Y. Osman, C. N. A. van Duin, J. Zaanen, Phys. Rev. B **59**, 115 (1999).
M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, cond-mat/0107115.

S=1/2 spins on coupled 2-leg ladders



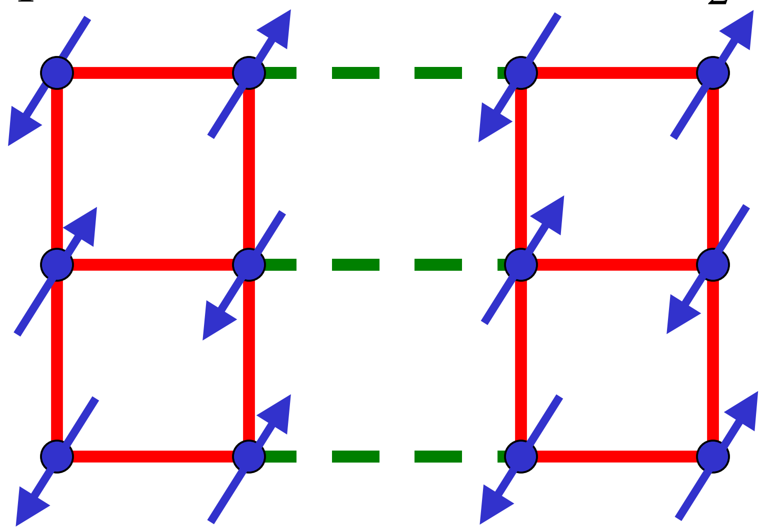
$$H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$0 \leq \lambda \leq 1$$

λ close to 1

Square lattice antiferromagnet

Experimental realization: La_2CuO_4



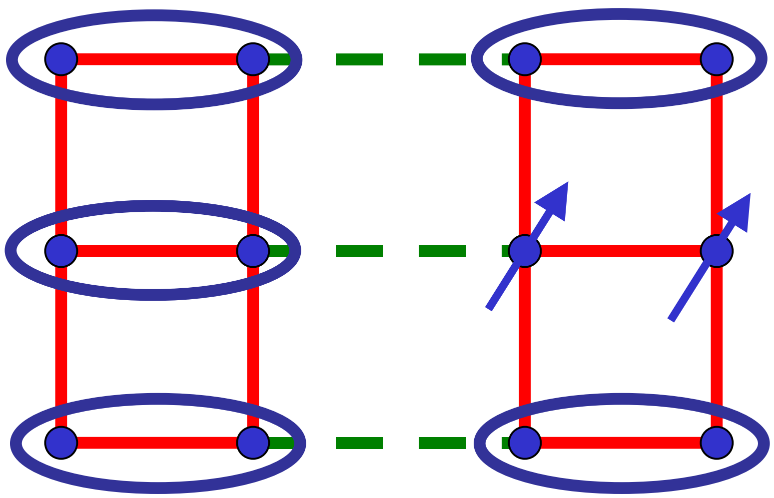
Ground state has long-range magnetic (Neel) order

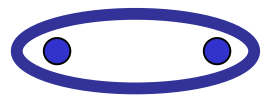
$$\left\langle \vec{S}_i \right\rangle = \left(-1\right)^{i_x + i_y} N_0 \neq 0$$

Excitations: 2 spin waves

λ close to 0

Weakly coupled ladders





$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle \right)$$

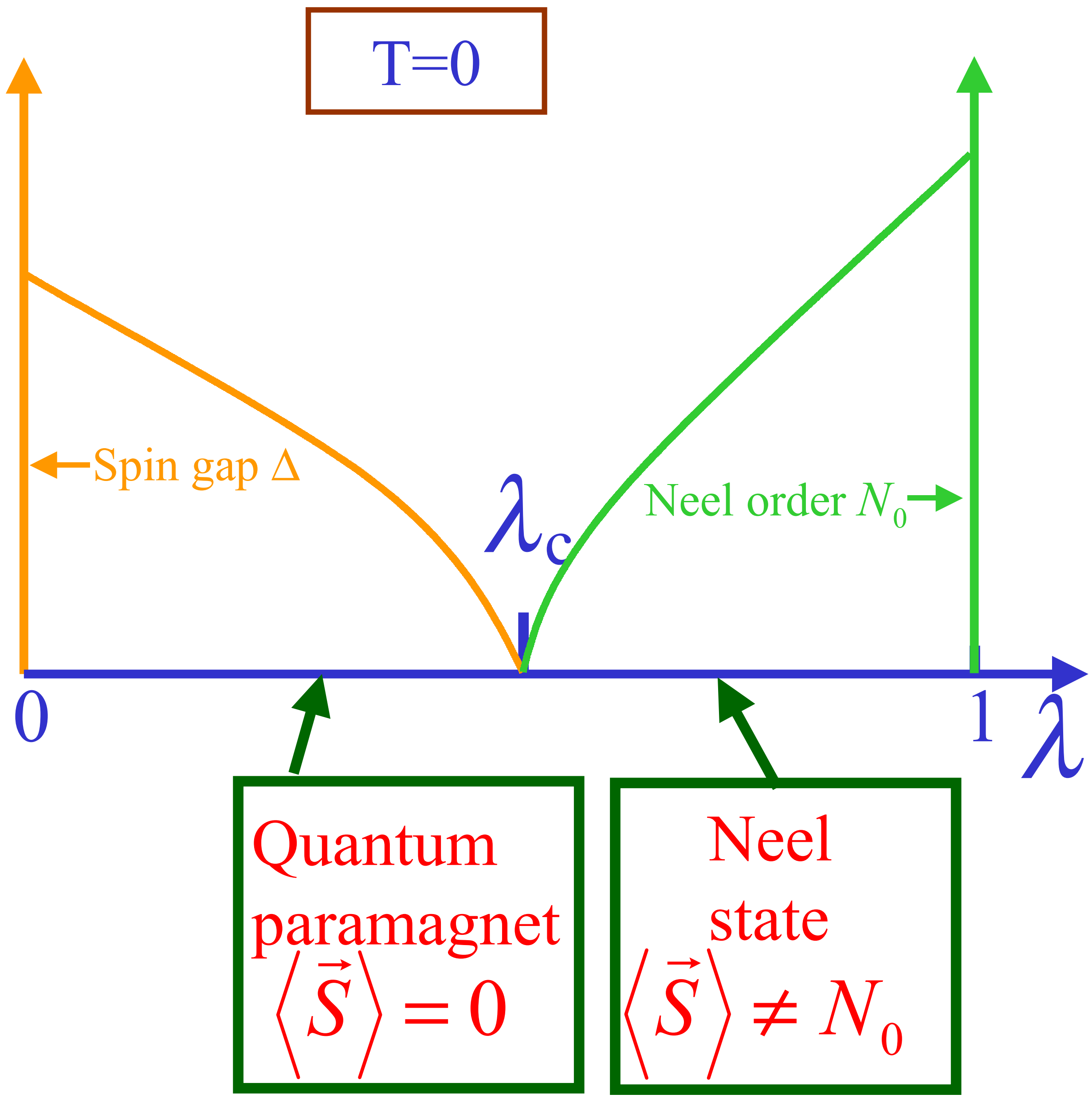
Paramagnetic ground state

$$\left\langle \vec{S}_i \right\rangle = 0$$

Excitation: S=1 ***exciton*** (spin collective mode)

Energy dispersion away from antiferromagnetic wavevector

$$\varepsilon = \Delta + \frac{c^2 k^2}{2\Delta}$$


T=0
Spin gap Δ
λ_c
Neel order N_0
0
1
λ
Quantum paramagnet
$\langle \vec{S} \rangle = 0$
Neel state
$\langle \vec{S} \rangle \neq N_0$

Quantum field theory:

λ close to λ_c

$$S_b = \int d^2x d\tau \left[\frac{1}{2} \left(\left(\nabla_x \phi_\alpha \right)^2 + c^2 \left(\partial_\tau \phi_\alpha \right)^2 + r\phi_\alpha^2 \right) + \frac{g}{4!} \left(\phi_\alpha^2 \right)^2 \right]$$

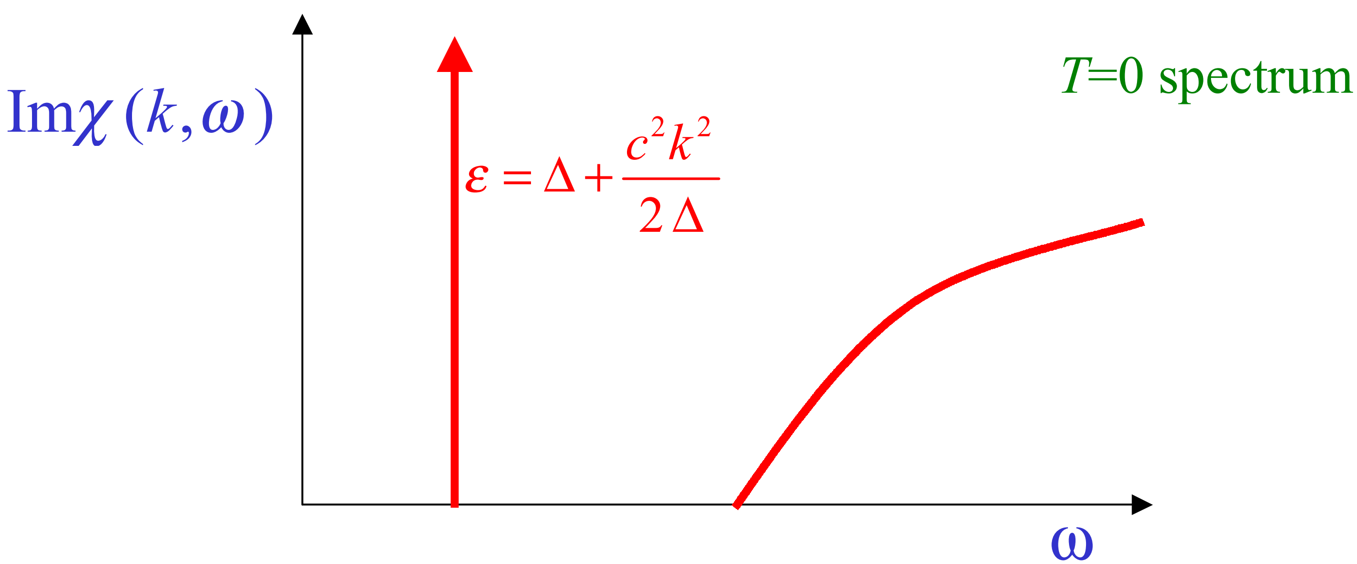
ϕ_α → 3-component antiferromagnetic order parameter

$r > 0$ → $\lambda < \lambda_c$

$r < 0$ → $\lambda > \lambda_c$

Oscillations of ϕ_α about zero (for $r > 0$)

→ spin-1 collective mode



I.C Square Lattice Antiferromagnet

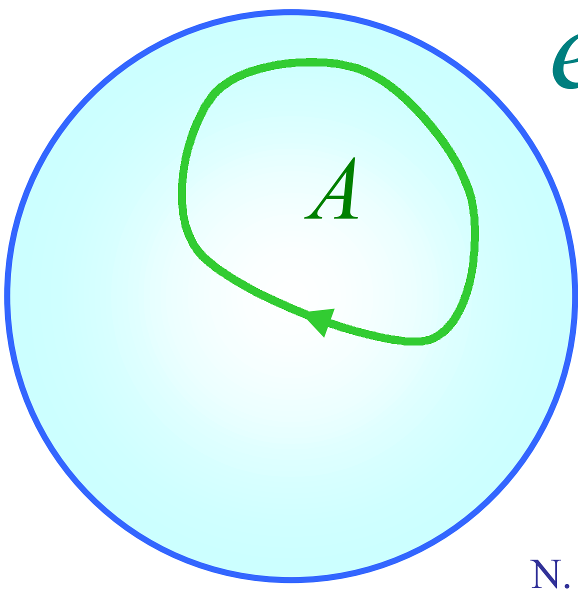
$$H = \sum_{i<j} J_{ij} \; \vec{S}_i \cdot \vec{S}_j$$

Action:

$$S_b = \int d^2x d\tau \left[\frac{1}{2} \left(\left(\nabla_x \phi_\alpha \right)^2 + c^2 \left(\partial_\tau \phi_\alpha \right)^2 \right) + V\left(\phi_\alpha^2 \right) \right]$$

S. Chakravarty, B.I. Halperin, and D.R. Nelson, Phys. Rev. B **39**, 2344 (1989).

Missing: Spin Berry Phases



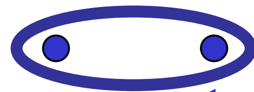
$$e^{iSA}$$

Berry phases induce
bond charge order
in quantum "disordered"
phase with $\langle \phi_\alpha \rangle = 0$;
"Dual order parameter"

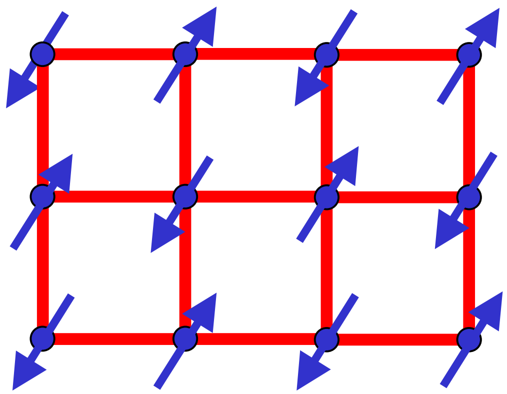
N. Read and S. Sachdev, Phys. Rev. Lett. **62**, 1694 (1989).

Square lattice with first(J_1) and second (J_2) neighbor exchange interactions

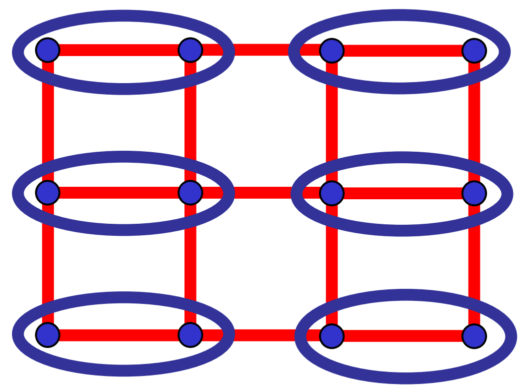
$$H = \sum_{i<j} J_{ij} \; \vec{S}_i \cdot \vec{S}_j$$



$$= \frac{1}{\sqrt{2}}\left(\left| \uparrow\downarrow \right\rangle - \left| \downarrow\uparrow \right\rangle \right)$$



Neel state



Spin-Peierls (or plaquette) state
"Bond-centered charge order"

J_2 / J_1

Onset of bond-charge order: Z_4 order parameter

Co-existence

Vanishing of spin gap and Neel order: ϕ^4 field theory

N. Read and S. Sachdev, Phys. Rev. Lett. **62**, 1694 (1989).

O. P. Sushkov, J. Oitmaa, and Z. Weihong, Phys. Rev. B **63**, 104420 (2001).

M.S.L. du Croo de Jongh, J.M.J. van Leeuwen, W. van Saarloos, Phys. Rev. B **62**, 14844 (2000).

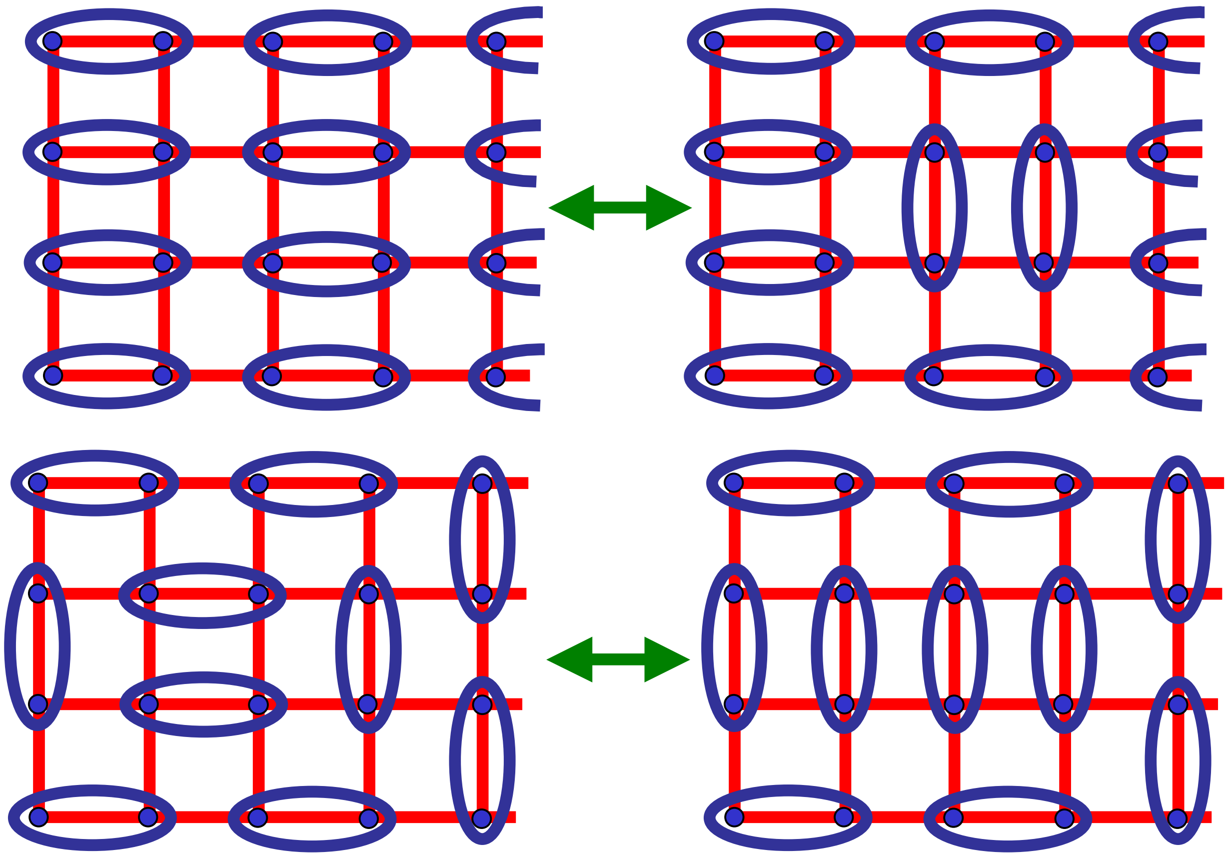
K.Park and S.Sachdev, cond-mat/0108214.

See however L. Capriotti, F. Becca, A. Parola, S. Sorella, cond-mat/0107204 .

YALE UNIVERSITY TERCENTENNIAL 1701 2001

Quantum dimer model –

D. Rokhsar and S. Kivelson Phys. Rev. Lett. **61**, 2376 (1988)



Quantum "entropic" effects prefer one-dimensional striped structures in which the largest number of singlet pairs can resonate. The state on the upper left has more flippable pairs of singlets than the one on the lower left.

These effects lead to a broken square lattice symmetry near the transition to the Neel state.

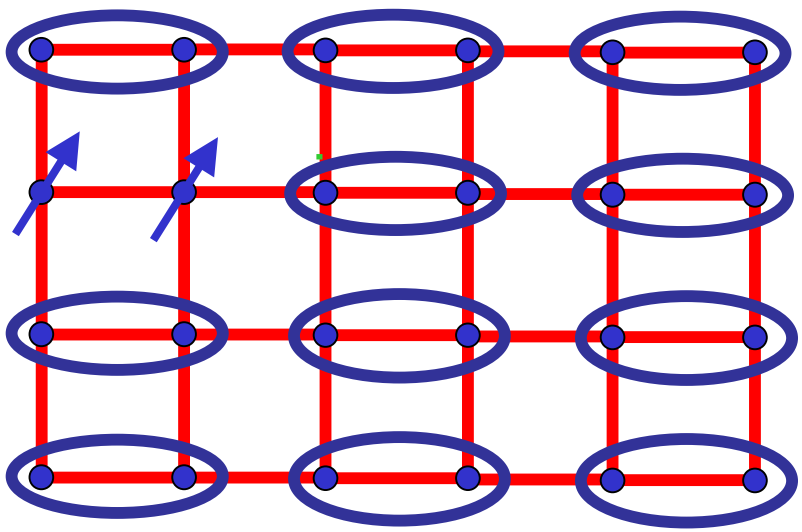
N. Read and S. Sachdev Phys. Rev. B **42**, 4568 (1990).

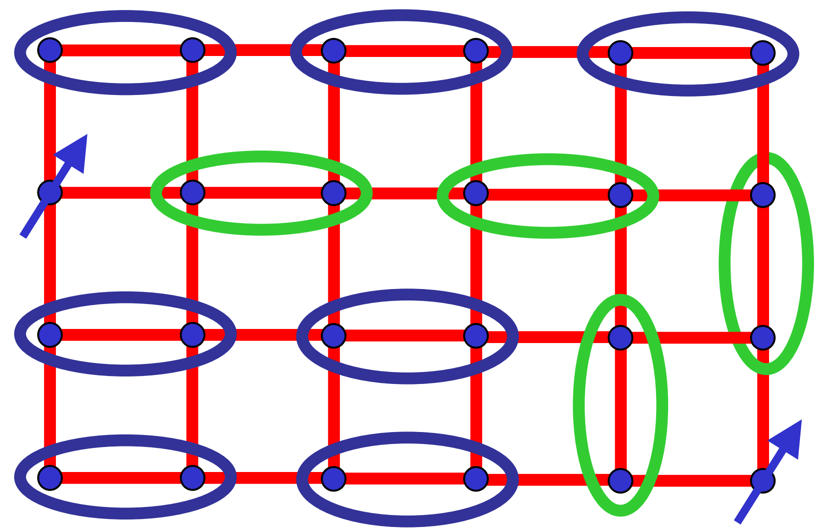
Properties of paramagnet with bond-charge-order

Stable S=1 spin exciton – quanta of 3-component ϕ_α

$$\varepsilon_k = \Delta + \frac{c_x^2 k_x^2 + c_y^2 k_y^2}{2\Delta}$$

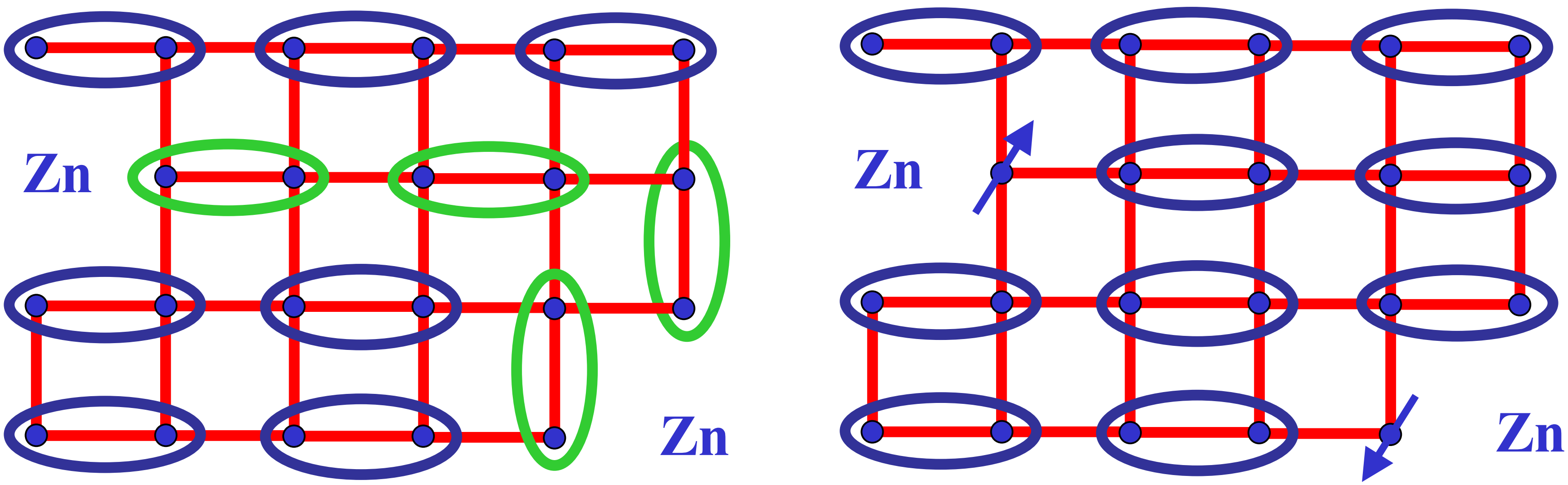
$\Delta \rightarrow$ Spin gap





S=1/2 spinons are ***confined***
by a linear potential.

Effect of static non-magnetic impurities (Zn or Li)

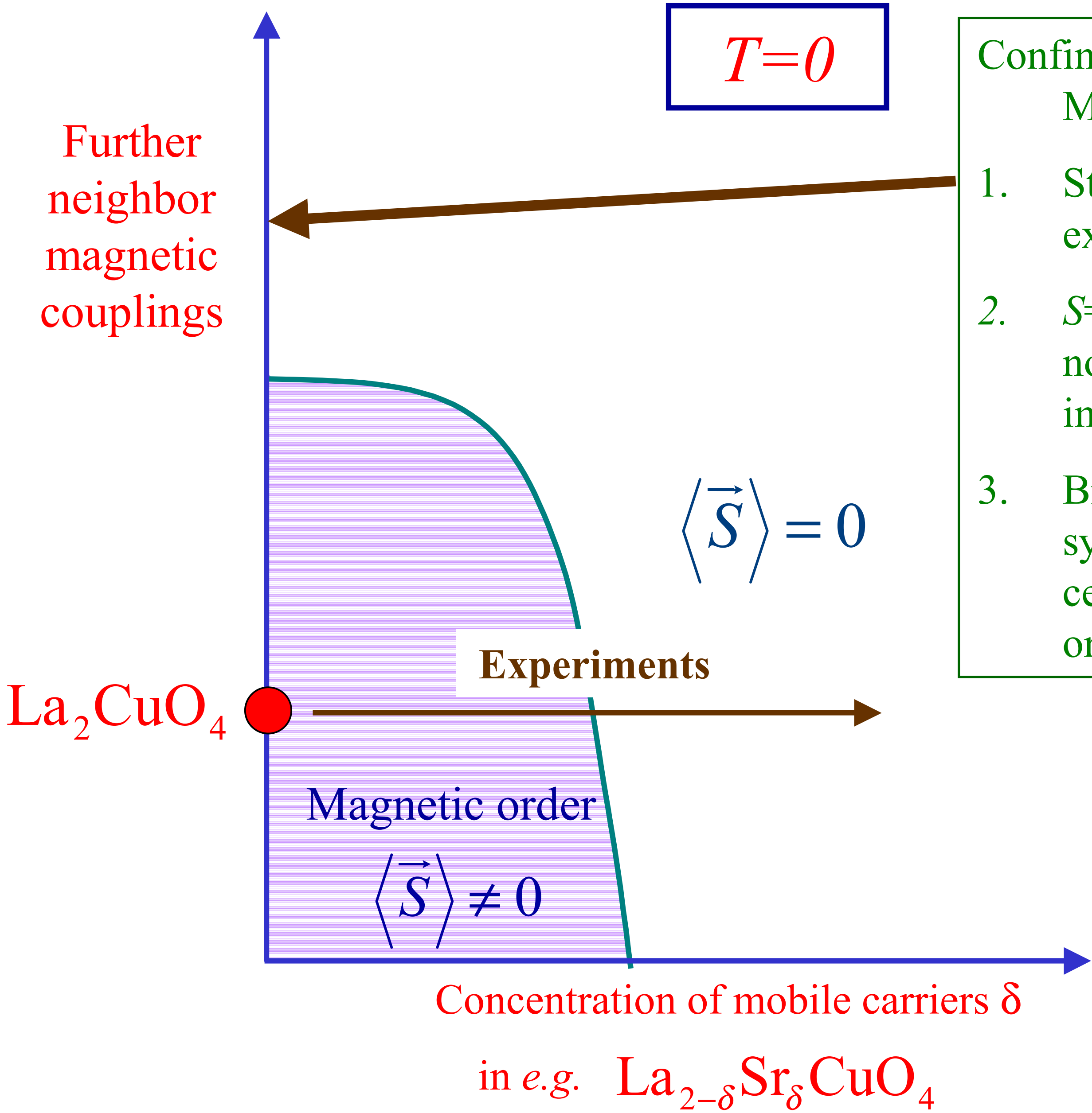


Spinon confinement implies that free S=1/2 moments form near each impurity

$$\chi_{\text{impurity}}(T \to 0) = \frac{S(S+1)}{3k_B T}$$

A.M Finkelstein, V.E. Kataev, E.F. Kukovitskii, G.B. Teitel'baum, Physica C **168**, 370 (1990).

1701 YALE UNIVERSITY TERCENTENNIAL 2001



Confined, paramagnetic Mott insulator has

1. Stable S=1 spin exciton .
2. S=1/2 moments near non-magnetic impurities
3. Broken translational symmetry:- bond-centered charge order.

S. Sachdev and J. Ye, Phys. Rev. Lett. **69**, 2411 (1992).
A.V. Chubukov, S. Sachdev, and J. Ye, Phys. Rev. B **49**, 11919 (1994)

Neutron scattering in YBCO

$YBa_2Cu_3O_7$

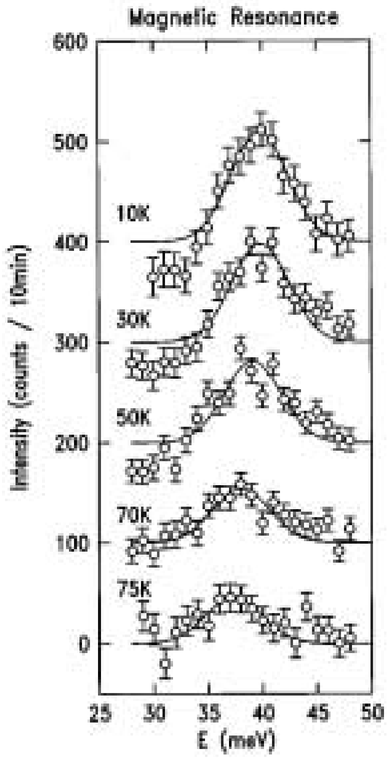


FIG. 8. Unpolarized beam, constant-**Q** data [**Q**=(3/2,1/2,−1.7)] of the 40 meV magnetic resonance obtained by subtracting the signal below T_c from the T=100 K background. The lines are fits to Gaussians, as described in the text. For clarity successive scans are offset by 100.

S=1 exciton near antiferromagnetic ordering wavevector **Q** = (π,π)

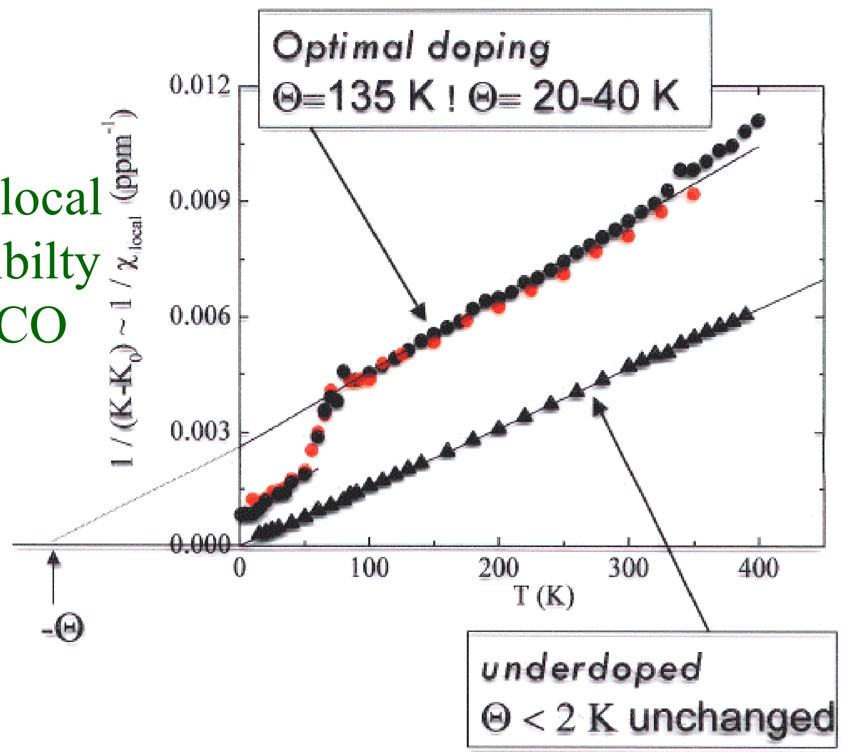
Resolution limited width

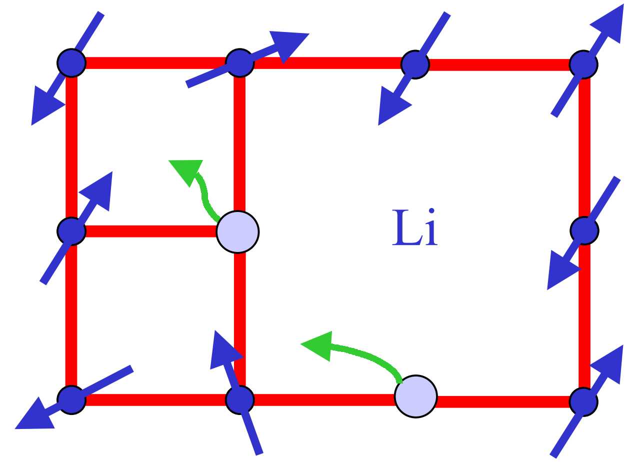
H.F. Fong, B. Keimer, D. Reznik, D.L. Milius, and I.A. Aksay, Phys. Rev. B **54**, 6708 (1996)

1701 YALE UNIVERSITY TERCENTENNIAL 2001

Measurement of spin susceptibility near non-magnetic (Zn/Li) impurities

Inverse local susceptibilty in YBCO





^{7}Li NMR below T_c

J. Bobroff, H. Alloul, W.A. MacFarlane, P. Mendels, N. Blanchard, G. Collin, and J.-F. Marucco, Phys. Rev. Lett. **86**, 4116 (2001)

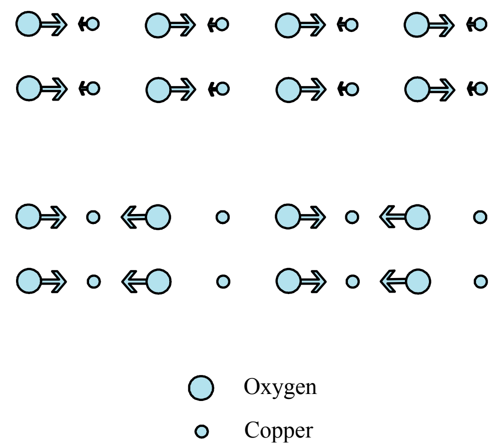
See also D. L. Sisson, S. G. Doettinger, A. Kapitulnik, R. Liang, D. A. Bonn, and W. N. Hardy, Phys. Rev. B **61**, 3604 (2000).

Measured $\chi_{\text{impurity}}(T \to 0) = \dfrac{S(S+1)}{3k_B T}$ with $S = 1/2$ in underdoped sample.

Not expected from BCS theory, which predicts $\chi_{\text{impurity}}(T \to 0) \neq \infty$ for a non-magnetic impurity with strong potential scattering.

Neutron scattering measurements of phonon spectra

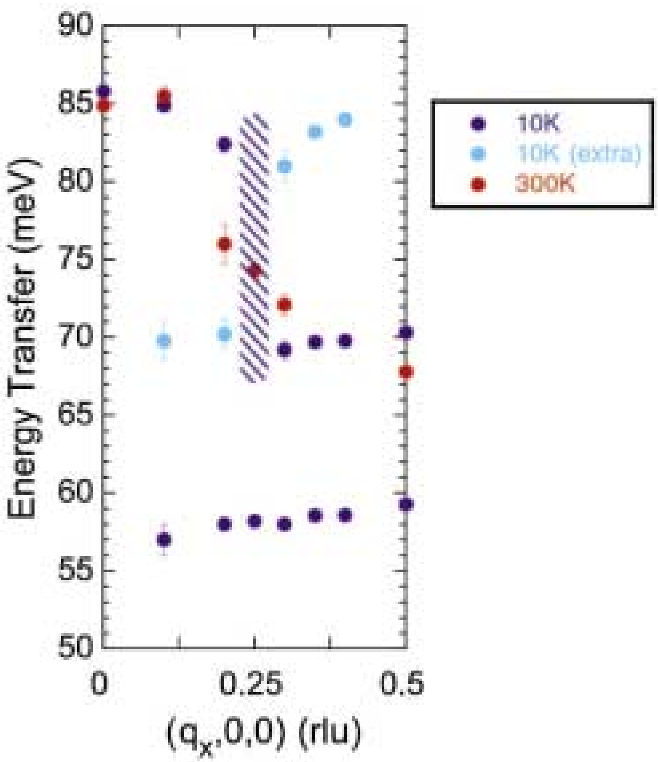
Discontinuity in the dispersion of a LO phonon of the O ions at wavevector $k = \pi/2$: evidence for bond-charge order with period *2a*



$k = 0$

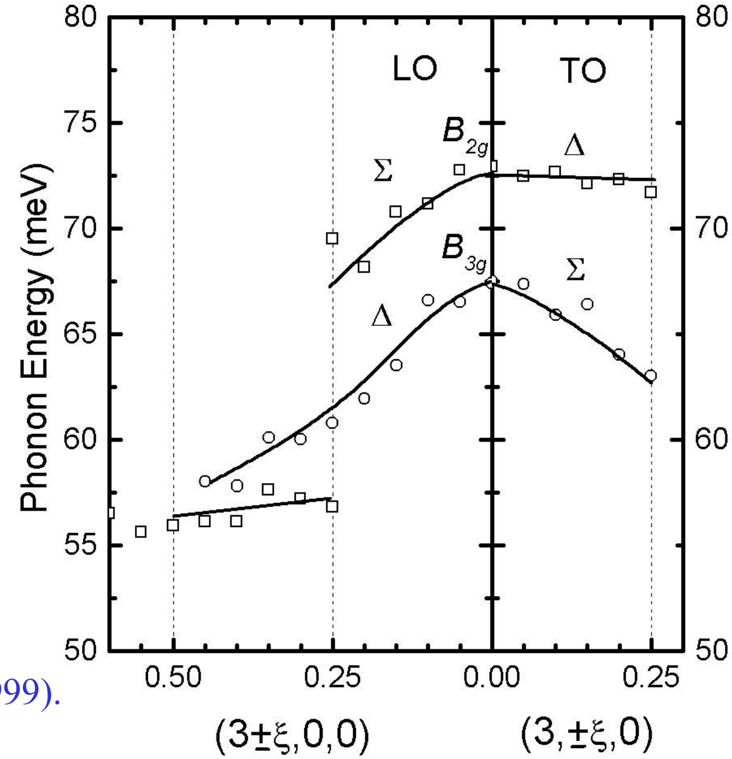
$k = \pi$

$La_{1.85}Sr_{0.15}CuO_4$



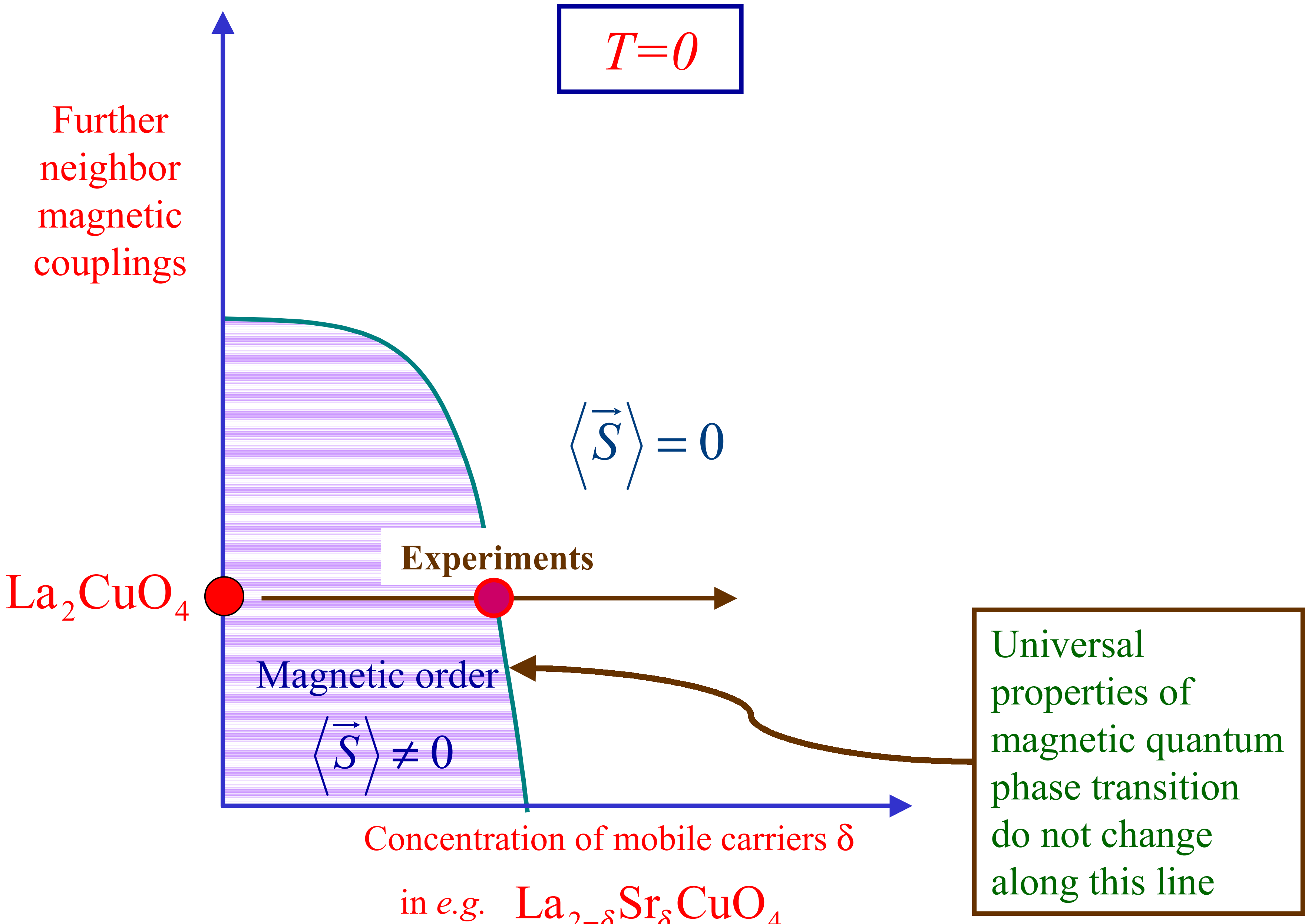
R. J. McQueeney, Y. Petrov, T. Egami, M. Yethiraj, G. Shirane, and Y. Endoh, Phys. Rev. Lett. **82**, 628 (1999).
L. Pintschovius and M. Braden, Phys. Rev. B **60**, R15039 (1999).

$YBa_2Cu_3CuO_{6.95}$



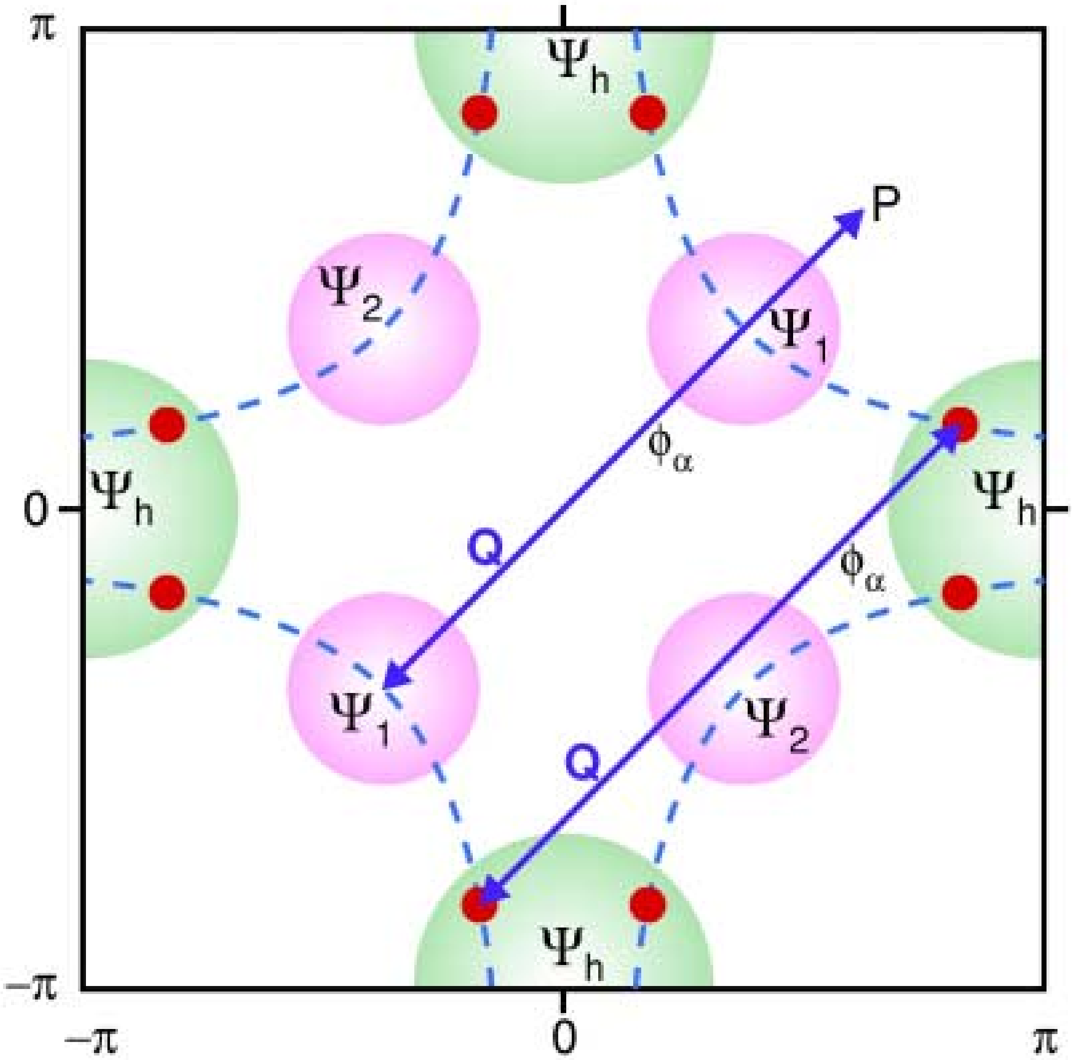
R. J. McQueeney, T. Egami, J.-H. Chung, Y. Petrov, M. Yethiraj, M. Arai, Y. Inamura, Y. Endoh, C. Frost and F. Dogan, cond-mat/0105593.

1701 2001 YALE UNIVERSITY TERCENTENNIAL



S. Sachdev and J. Ye, Phys. Rev. Lett. **69**, 2411 (1992).
A.V. Chubukov, S. Sachdev, and J. Ye, Phys. Rev. B **49**, 11919 (1994)

II.A Magnetic transitions in a d-wave superconductor

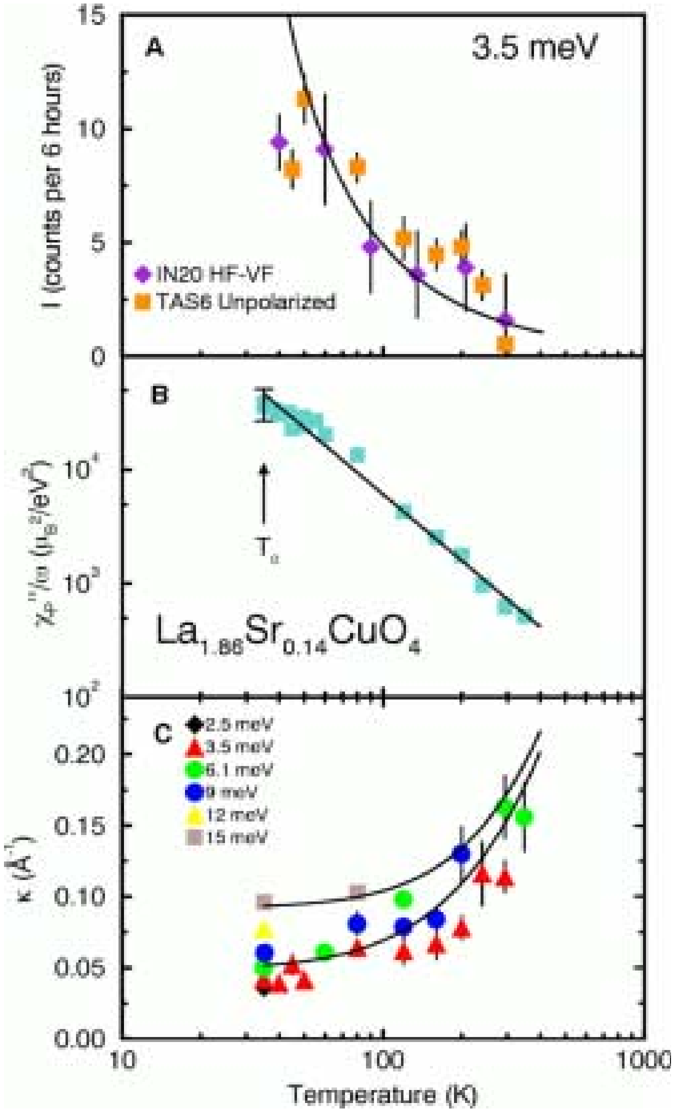


Ψ_h : *strongly coupled* to ϕ_α, but do not damp ϕ_α as long as $\Delta < 2\,\Delta_h$

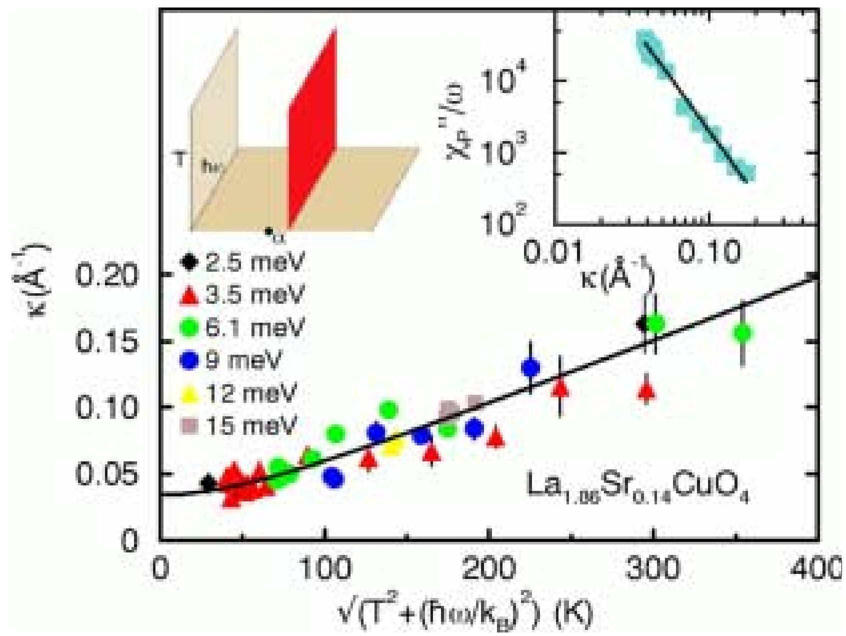
$\Psi_{1,2}$: *decoupled* from ϕ_α

Leading universal properties of transition are identical to that in an insulator

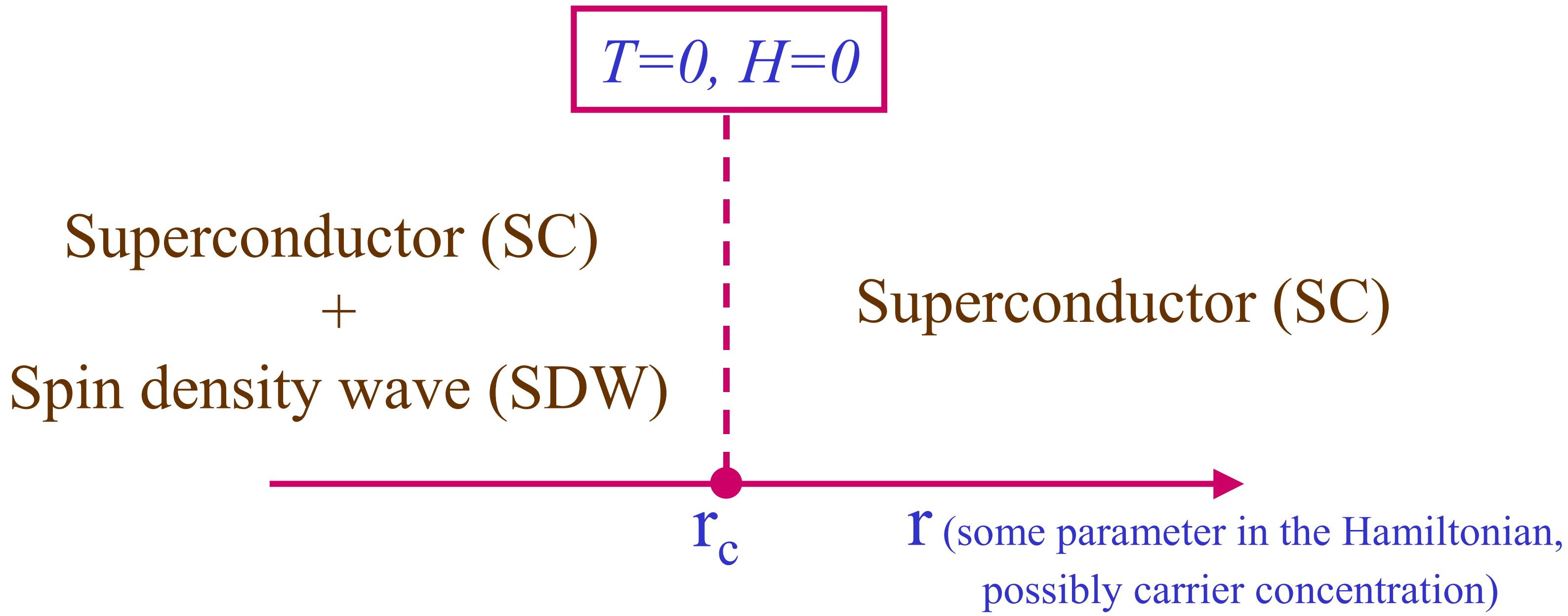
Neutron scattering measurements of dynamic spin susceptibility at an incommensurate wavevector: T and ω dependent divergence scaling as a function of $\hbar\omega/k_BT$



G. Aeppli, T.E. Mason, S,M. Hayden, H.A. Mook, and J. Kulda, *Science* **278**, 1432 (1998).



II.B Effect of magnetic field on SDW order in SC phase



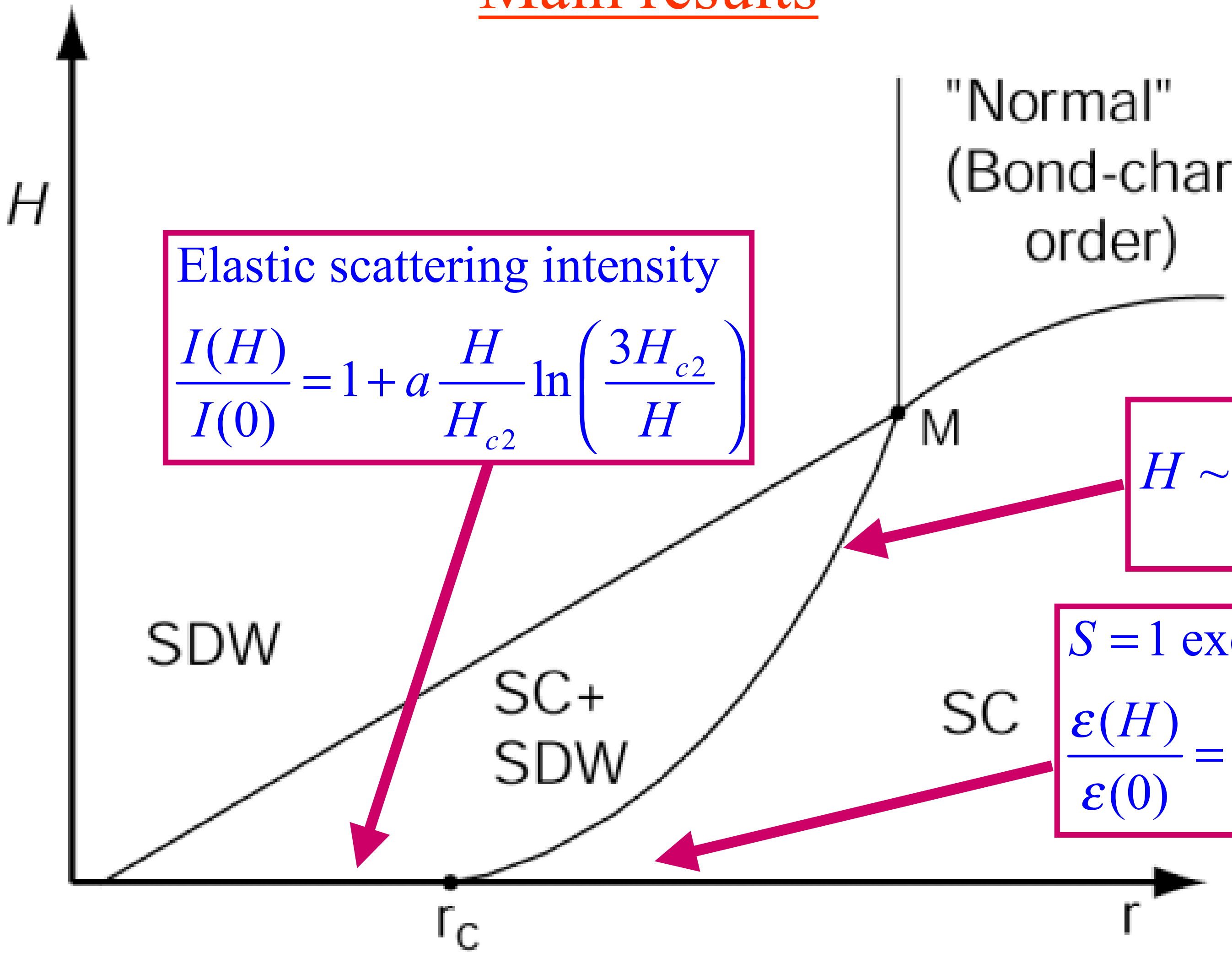
Many experimental indications that the cuprate superconductors are not too far from such a quantum phase transition:

G. Aeppli, T.E. Mason, S.M. Hayden, H.A. Mook, J. Kulda, Science **278**, 1432 (1997).
Y. S. Lee, R. J. Birgeneau, M. A. Kastner *et al.,* Phys. Rev. B **60**, 3643 (1999).
S. Katano, M. Sato, K. Yamada, T. Suzuki, and T. Fukase Phys. Rev. B **62**, 14677 (2000).
B. Lake, G. Aeppli *et al*., Science **291**, 1759 (2001).
Y. Sidis, C. Ulrich, P. Bourges, *et al*., cond-mat/0101095.
H. Mook, P. Dai, F. Dogan, cond-mat/0102047.
J.E. Sonier *et al*., preprint.

Main results

(extreme Type II superconductivity)

T=0



Elastic scattering intensity

$$\frac{I(H)}{I(0)} = 1 + a\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$$

$$H \sim \frac{(r - r_c)}{\ln\left(1/(r - r_c)\right)}$$

$S = 1$ exciton energy

$$\frac{\varepsilon(H)}{\varepsilon(0)} = 1 - b\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$$

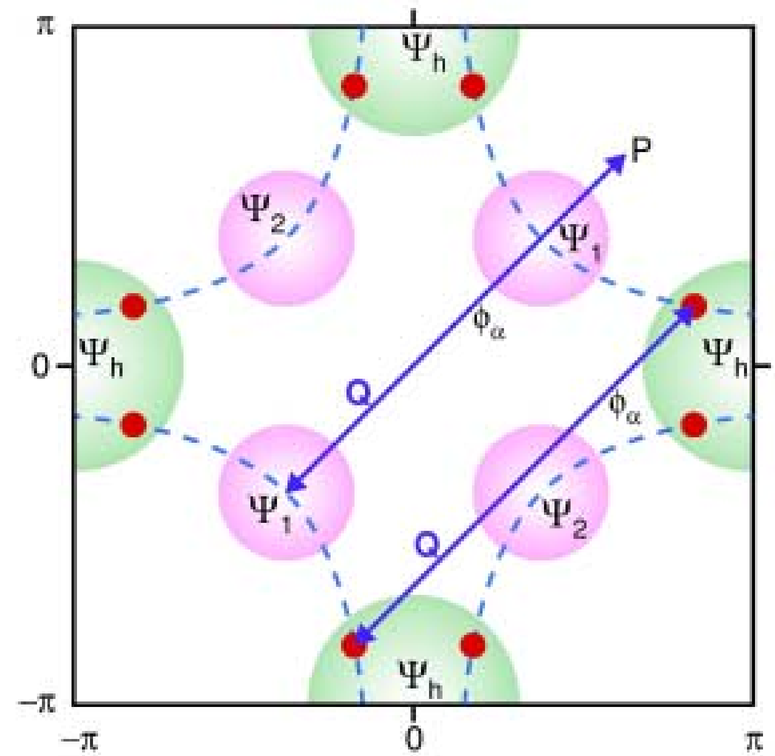
- All functional forms are **exact**.
- Similar results apply to other competing orders *e.g.* SC + staggered flux

E. Demler, S. Sachdev, and Y. Zhang, Phys. Rev. Lett. **87**, 067202 (2001)

YALE UNIVERSITY TERCENTENNIAL 1701 2001

Structure of quantum theory

- Charge-order is not critical: can neglect Berry phases.

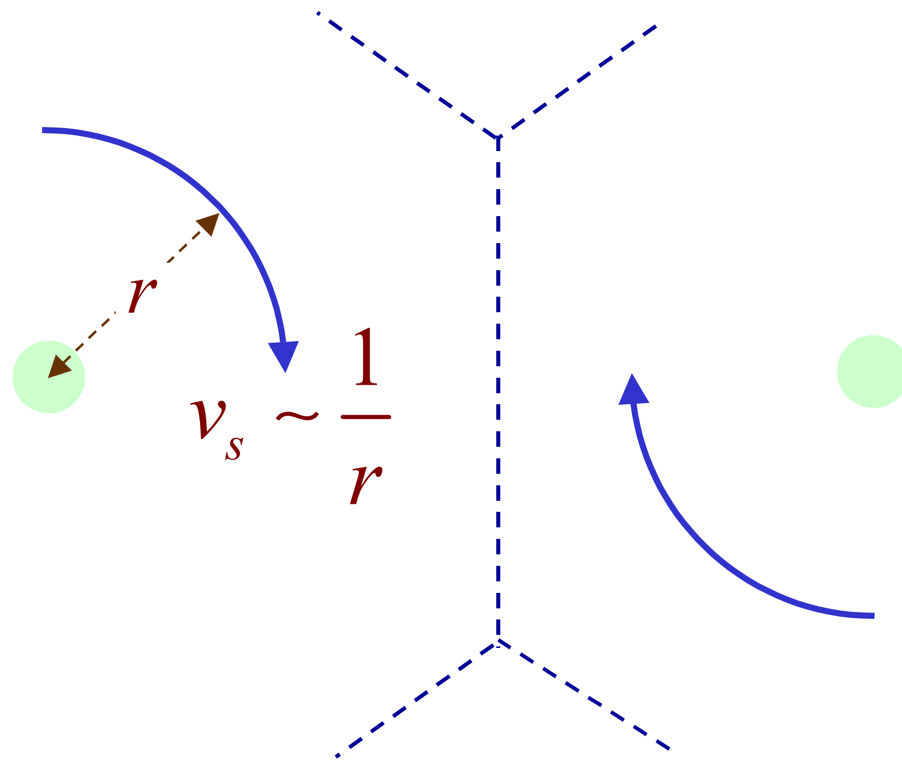


- Generically, momentum conservation prohibits decay of $S=1$ exciton ϕ_α into $S=1/2$ fermionic excitations at low energies. Virtual pairs of fermions only renormalize parameters in the effective action for ϕ_α .
- Zeeman coupling only leads to corrections at order H^2
- Simple Landau theory couplings between ϕ_α and superconducting order ψ are allowed (S.-C. Zhang, Science **275**, 1089 (1997)), *e.g.*:

$$V\left(\phi_\alpha^2\right) \rightarrow V\left(\phi_\alpha^2\right) + \lambda \phi_\alpha^2 \left|\psi\right|^2$$

$$S_b = \int d^2 x d\tau \left[\frac{1}{2} \left(\left(\nabla_x \phi_\alpha\right)^2 + c^2 \left(\partial_\tau \phi_\alpha\right)^2 \right) + V\left(\phi_\alpha^2\right) \right]$$

Dominant effect: **uniform** softening of spin excitations by superflow kinetic energy



$$v_s \sim \frac{1}{r}$$

Spatially averaged superflow kinetic energy

$$\sim \left\langle v_s^2 \right\rangle \sim \frac{H}{H_{c2}} \ln \frac{3H_{c2}}{H}$$

See D. P. Arovas *et al*., Phys. Rev. Lett. **79**, 2871 (1997)
for a different viewpoint.

Influence of $\psi(x)$ on extended spin eigenmodes:

$$|\psi(x)| = 1 - \frac{1}{2x^2}$$ outside each vortex core because of superflow kinetic energy

$$\left\langle |\psi(x)|^2 \right\rangle = 1 - \frac{H}{2H_{c2}} \ln\left(\frac{3.0 H_{c2}}{H}\right)$$

In SC phase, spin gap obeys:

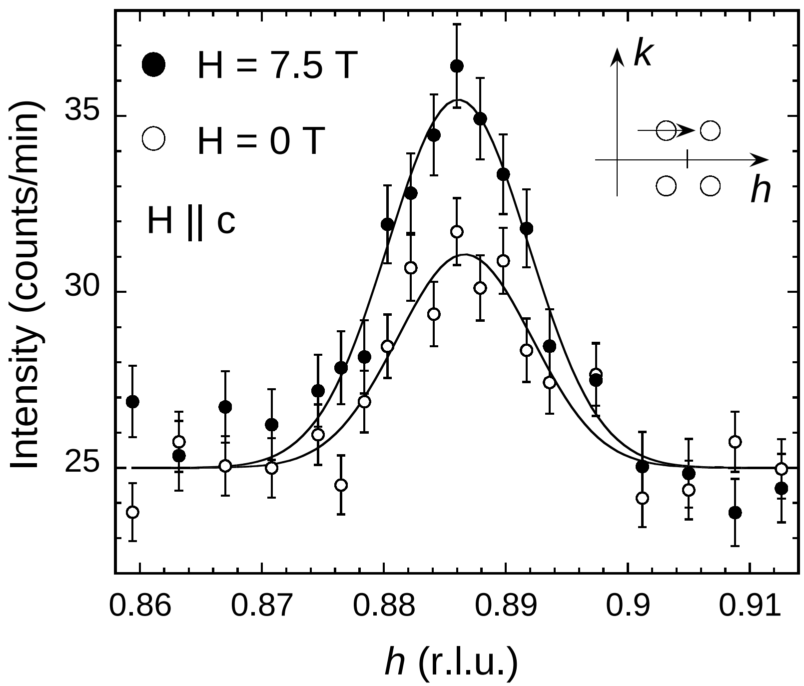
$$\Delta(H) = \Delta(0) - \frac{24\pi c^2 v}{Ng\left(1 - \frac{3v^2}{g}\right)} \frac{H}{2H_{c2}} \ln\left(\frac{3.0 H_{c2}}{H}\right)$$

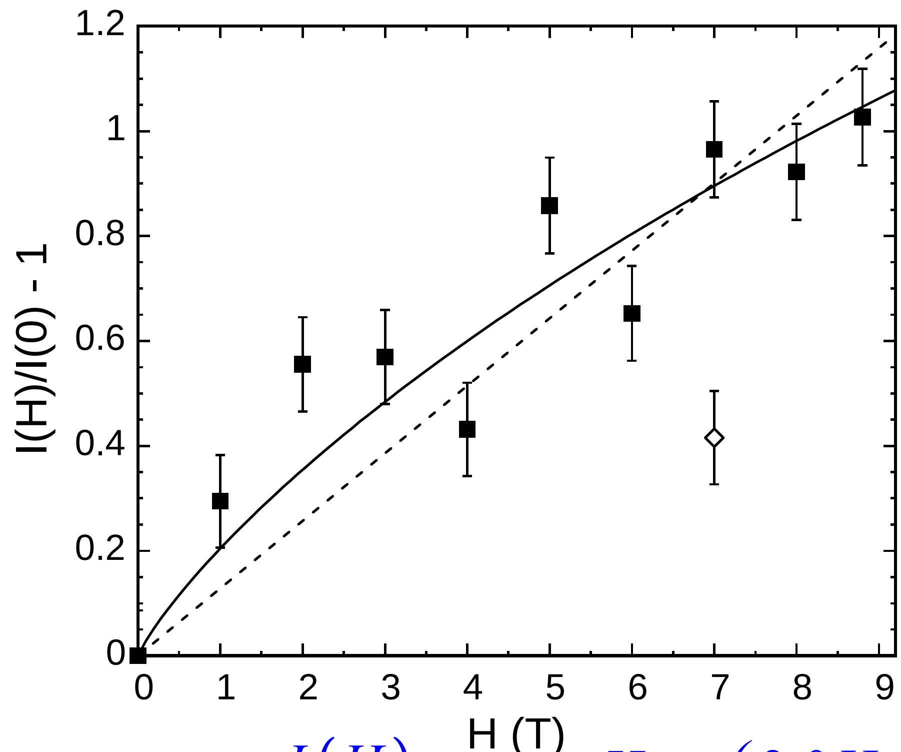
In SC+SDW phase, intensity of elastic scattering obeys:

$$I(H) = I(0) + \frac{6v}{g\left(1 - \frac{3v^2}{g}\right)} \frac{H}{2H_{c2}} \ln\left(\frac{3.0 H_{c2}}{H}\right)$$

Elastic neutron scattering off La_2CuO_{4+y}

B. Khaykovich, Y. S. Lee, S. Wakimoto, K. J. Thomas,
M. A. Kastner, and R.J. Birgeneau, preprint.





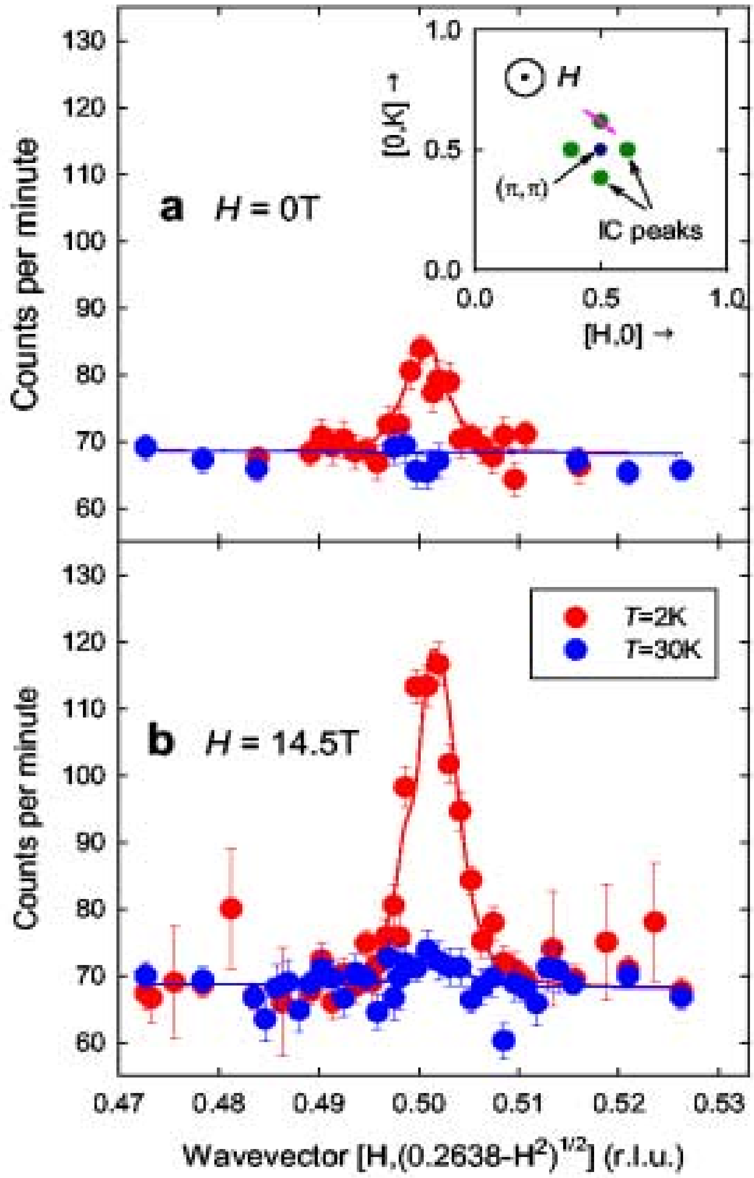
Solid line --- fit to : $\frac{I(H)}{I(0)} = 1 + a\frac{H}{H_{c2}}\ln\left(\frac{3.0H_{c2}}{H}\right)$

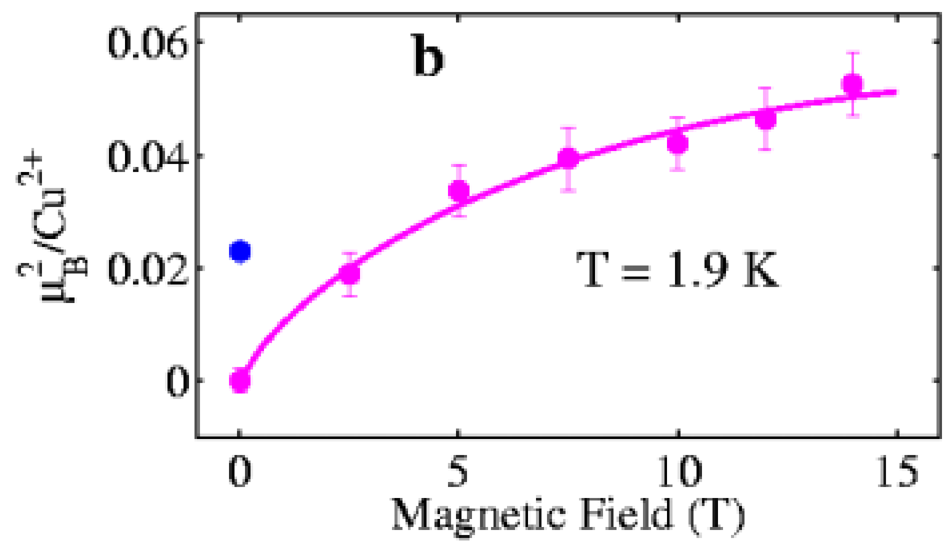
a is the only fitting parameter

Best fit value - a = 2.4 with H_{c2}= 60 T

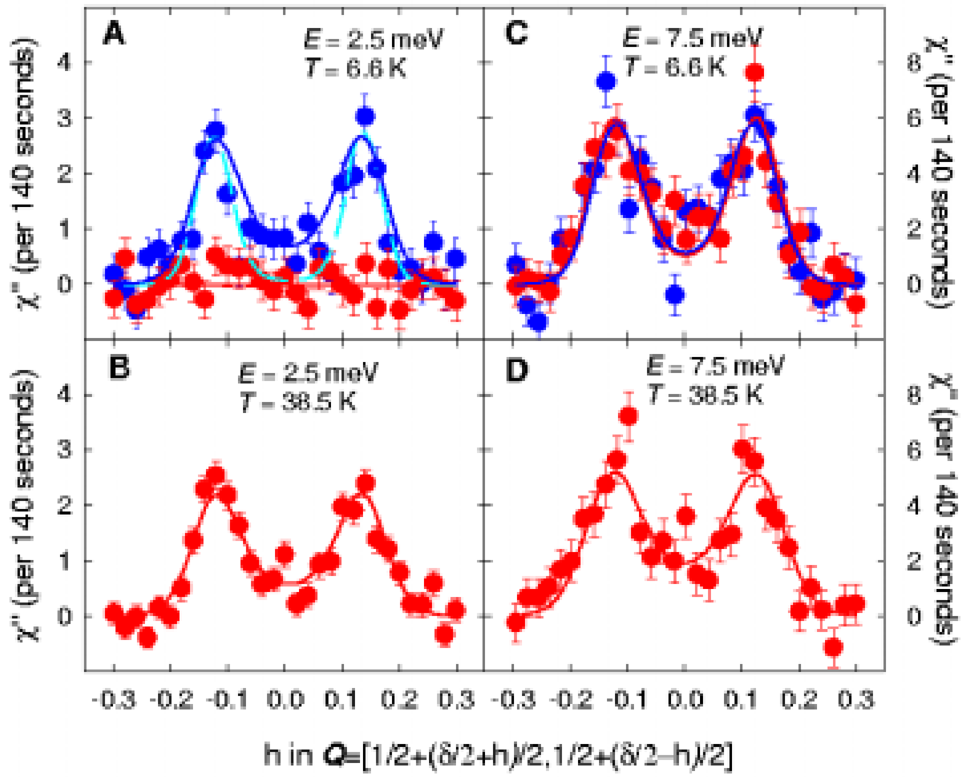
Neutron scattering of $La_{2-x}Sr_xCuO_4$ at x=0.1

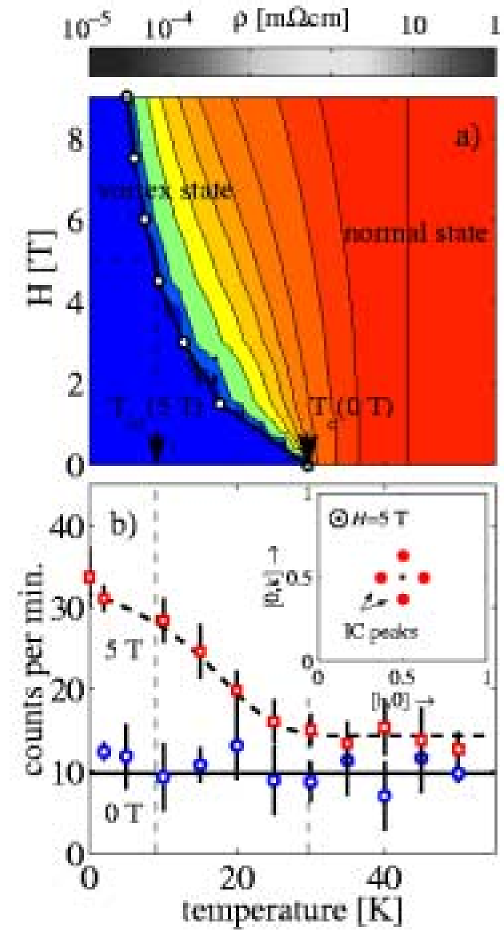
B. Lake, G. Aeppli, *et al.*





Solid line - fit to : $I(H) = a\frac{H}{H_{c2}}\ln\left(\frac{H_{c2}}{H}\right)$





Neutron scattering off $La_{2-x}Sr_xCuO_4$

($x = 0.163$, ***SC phase***)

in H=0 (red dots) and H=7.5T (blue dots).

B. Lake, G. Aeppli *et al*., Science **291**, 1759 (2001)

Elastic neutron scattering off $La_{2-x}Sr_xCuO_4$

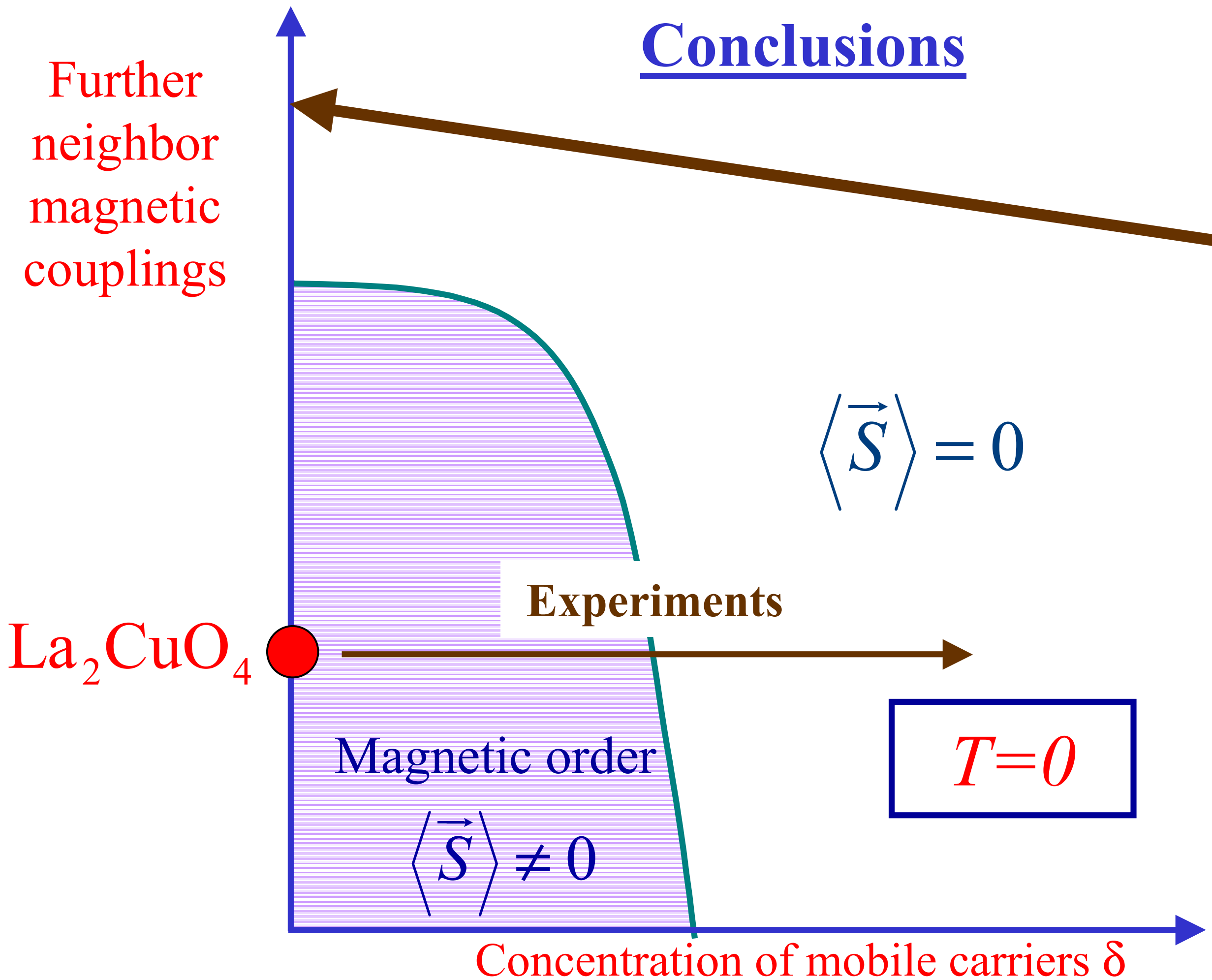
($x = 0.10$, ***SC + SDW phase***)

in H=0 (blue dots) and H=5T (red dots).

B. Lake, H. Ronnow *et al*., cond-mat/0104026

1701 2001 YALE UNIVERSITY TERCENTENNIAL

Conclusions



Confined, paramagnetic Mott insulator has

1. Stable S=1 spin exciton ϕ_α .
2. Broken translational symmetry:- bond-centered charge order.
3. S=1/2 moments near non-magnetic impurities

Theory of magnetic ordering quantum transitions in antiferromagnets and superconductors leads to quantitative theories for

- Spin correlations in a magnetic field
- Effect of Zn/Li impurities on collective spin excitations