Quantum matter without quasiparticles: SYK models, strange metals, and black holes

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Subir Sachdev

Talk online: sachdev.physics.harvard.edu
Quantum matter with quasiparticles:

- Landau quasi-particles & holes
- Phonon
- Magnon
- Roton
- Plasmon
- Polaron
- Exciton
- Laughlin quasiparticle
- Bogoliubovon
- Anderson-Higgs mode
- Massless Dirac Fermions
- Weyl fermions
- ....
Quantum matter with quasiparticles:

Most generally, a quasiparticle is an “additive” excitation:

Quasiparticles can be combined to yield additional excitations, with energy determined by the energies and densities of the constituents. Such a procedure yields all the low-lying excitations. Then we can apply the Boltzmann-Landau theory to make predictions for dynamics.
SDW Superconductivity

Resistivity \sim \rho_0 + AT^\alpha

\text{Strange Metal}

\text{BaFe}_2(\text{As}_{1-x}\text{P}_x)_2


*Physical Review B* 81, 184519 (2010)
Strange metals

\[ \frac{1}{\tau} = \alpha \frac{k_B T}{\hbar} \]

Quantum matter without quasiparticles:

No quasiparticle structure to excitations.

But how can we be sure that no quasiparticles exist in a given system? Perhaps there are some exotic quasiparticles inaccessible to current experiments........
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Consider how rapidly the system loses “phase coherence”, reaches local thermal equilibrium, or becomes “chaotic”
Local thermal equilibration or phase coherence time, $\tau_\phi$:

- There is an *lower bound* on $\tau_\phi$ in all many-body quantum systems as $T \to 0$,

  $$
  \tau_\phi > C \frac{\hbar}{k_B T},
  $$

  where $C$ is a $T$-independent constant. Systems *without* quasiparticles have $\tau_\phi \sim \hbar/(k_B T)$.

- In systems *with* quasiparticles, $\tau_\phi$ is parametrically larger at low $T$;
  e.g. in Fermi liquids $\tau_\phi \sim 1/T^2$,
  and in gapped insulators $\tau_\phi \sim e^\Delta/(k_B T)$ where $\Delta$ is the energy gap.
A bound on quantum chaos:

- In classical chaos, we measure the sensitivity of the position at time $t$, $q(t)$, to variations in the initial position, $q(0)$, i.e. we measure

$$
\left( \frac{\partial q(t)}{\partial q(0)} \right)^2 = (\{q(t), p(0)\}_\text{P.B.})^2
$$

- By analogy, we define $\tau_L$ as the Lyapunov time over which the wavefunction of a quantum system is scrambled by an initial perturbation. This scrambling can be measured by

$$
\left\langle \left| [\hat{A}(t), \hat{B}(0)] \right|^2 \right\rangle \sim e^{t/\tau_L}
$$

This quantum time was argued to obey lower bound

$$
\tau_L \geq \frac{1}{2\pi} \frac{\hbar}{k_B T}.
$$

There is no analogous bound in classical mechanics.

A. I. Larkin and Y. N. Ovchinnikov, JETP 28, 6 (1969)
A bound on quantum chaos:

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Quantum matter without quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models

Black holes with AdS$_2$ horizons

Fermi surface coupled to a gauge field

\[ \mathcal{L}[\Psi, a] = \Psi^\dagger \left( \partial_\tau - i a_\tau - \frac{(\nabla - i \vec{\alpha})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{\alpha})^2 \]
Quantum matter without quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models

Black holes with AdS$_2$ horizons

Same low energy theory

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$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left( \partial_\tau - ia_\tau - \frac{(\nabla - ia)^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$
Quantum matter without quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models

\[ \tau_L = \frac{\hbar}{2\pi k_B T} \]

Black holes with AdS\(_2\) horizons

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Fermi surface coupled to a gauge field

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\( \tau_L \): the Lyapunov time to reach quantum chaos
The Sachdev-Ye-Kitaev (SYK) model:

- A theory of a strange metal
- Dual theory of gravity on AdS$_2$
- Fastest possible quantum chaos

with $\tau_L = \frac{\hbar}{2\pi k_B T}$
Infinite-range model with quasiparticles

\[
H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \ldots
\]

\[
c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}
\]

\[
\frac{1}{N} \sum_i c_i^\dagger c_i = Q
\]

\(t_{ij}\) are independent random variables with \(\overline{t_{ij}} = 0\) and \(|t_{ij}|^2 = t^2\)

Fermions occupying the eigenstates of a \(N \times N\) random matrix
Infinite-range model with quasiparticles

Feynman graph expansion in \( t_{ij} \), and graph-by-graph average, yields exact equations in the large \( N \) limit:

\[
G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} , \quad \Sigma(\tau) = t^2 G(\tau)
\]

\[
G(\tau = 0^-) = Q.
\]

\( G(\omega) \) can be determined by solving a quadratic equation.

\[
-\text{Im} \ G(\omega)
\]
Infinite-range model with quasiparticles

Now add weak interactions

\[ H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;\ell} c_i^\dagger c_j^\dagger c_k c_\ell \]

\( J_{ij;\ell} \) are independent random variables with \( \overline{J_{ij;\ell}} = 0 \) and \( \overline{|J_{ij;\ell}|^2} = J^2 \). We compute the lifetime of a quasiparticle, \( \tau_\alpha \), in an exact eigenstate \( \psi_\alpha(i) \) of the free particle Hamiltonian with energy \( E_\alpha \). By Fermi’s Golden rule, for \( E_\alpha \) at the Fermi energy

\[
\frac{1}{\tau_\alpha} = \pi J^2 \rho_0^2 \int dE_\beta dE_\gamma dE_\delta f(E_\beta)(1 - f(E_\gamma))(1 - f(E_\delta)) \delta(E_\alpha + E_\beta - E_\gamma - E_\delta)
= \pi^3 J^2 \rho_0^2 \frac{T^2}{4}
\]

where \( \rho_0 \) is the density of states at the Fermi energy.

**Fermi liquid state:** Two-body interactions lead to a scattering time of quasiparticle excitations from in (random) single-particle eigenstates which diverges as \( \sim T^{-2} \) at the Fermi level.
SYK model

\[ H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,l=1}^{N} J_{ij;kl} c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i \]

\[ c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij} \]

\[ Q = \frac{1}{N} \sum_i c_i^\dagger c_i \]

\( J_{ij;kl} \) are independent random variables with \( \overline{J_{ij;kl}} = 0 \) and \( \overline{|J_{ij;kl}|^2} = J^2 \)

\( N \rightarrow \infty \) yields critical strange metal.

S. Sachdev and J. Ye, PRL 70, 3339 (1993)
Feynman graph expansion in $J_{ij}$, and graph-by-graph average, yields exact equations in the large $N$ limit:

\[
G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} , \quad \Sigma(\tau) = -J^2G^2(\tau)G(-\tau) \]

\[
G(\tau = 0^-) = Q .
\]

Low frequency analysis shows that the solutions must be gapless and obey

\[
\Sigma(z) = \mu - \frac{1}{A} \sqrt{z} + \ldots , \quad G(z) = \frac{A}{\sqrt{z}}
\]

for some complex $A$. The ground state is a non-Fermi liquid, with a continuously variable density $Q$. 

SYK and AdS$_2$

- Non-zero GPS entropy as $T \to 0$, $S(T \to 0) = NS_0 + \ldots$

Not a ground state degeneracy: due to an exponentially small (in $N$) many-body level spacing at all energies down to the ground state energy.

A. Georges, O. Parcollet, and S. Sachdev, PRB 63, 134406 (2001)
SYK and AdS$^2$

- Non-zero GPS entropy as $T \to 0$, $S(T \to 0) = NS_0 + \ldots$

  Not a ground state degeneracy: due to an exponentially small (in $N$) many-body level spacing at all energies down to the ground state energy.

This entropy, and other dynamic correlators of the SYK models, imply that the SYK model is holographically dual to black holes with an AdS$_2$ horizon. The Bekenstein-Hawking entropy of the black hole equals $NS_0$:

$$\text{GPS} = \text{BH}.$$
Einstein-Maxwell theory + cosmological constant

\[ ds^2 = \left( \frac{d\zeta^2 - dt^2}{\zeta^2} \right) + d\vec{x}^2 \]

Gauge field: \( A = \left( \frac{\mathcal{E}}{\zeta} \right) dt \)

SYK and AdS\(_2\)

Mapping to SYK applies when temperature \( \ll 1/(\text{size of } \mathbb{T}^2) \)

S. Sachdev, PRL 105, 151602 (2010)
\[ G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \ , \ \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau) \]

\[ \Sigma(z) = \mu - \frac{1}{A} \sqrt{z} + \ldots \ , \ \ G(z) = \frac{A}{\sqrt{z}} \]

At frequencies $\ll J$, the $i\omega + \mu$ can be dropped, and without it equations are invariant under the reparametrization and gauge transformations

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = [f'(\sigma_1)f'(\sigma_2)]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} G(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = [f'(\sigma_1)f'(\sigma_2)]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \Sigma(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.
Let us write the large $N$ saddle point solutions of $S$ as

$$G_s(\tau_1 - \tau_2) \sim (\tau_1 - \tau_2)^{-1/2}$$
$$\Sigma_s(\tau_1 - \tau_2) \sim (\tau_1 - \tau_2)^{-3/2}.$$

These are not invariant under the reparametrization symmetry but are invariant only under a $\text{SL}(2,\mathbb{R})$ subgroup under which

$$f(\tau) = \frac{a\tau + b}{c\tau + d}, \quad ad - bc = 1.$$

So the (approximate) reparametrization symmetry is spontaneously broken.
Connections of SYK to gravity and AdS$_2$ horizons

- Reparameterization and gauge invariance are the ‘symmetries’ of the Einstein-Maxwell theory of gravity and electromagnetism

- SL(2,R) is the isometry group of AdS$_2$. 
Reparametrization and phase zero modes

We can write the path integral for the SYK model as

\[ Z = \int \mathcal{D}G(\tau_1, \tau_2) \mathcal{D}\Sigma(\tau_1, \tau_2) e^{-NS[G, \Sigma]} \]

for a known action \( S[G, \Sigma] \). We find the saddle point, \( G_s, \Sigma_s \), and only focus on the “Nambu-Goldstone” modes associated with breaking reparameterization and U(1) gauge symmetries by writing

\[ G(\tau_1, \tau_2) = [f'(\tau_1)f'(\tau_2)]^{1/4} G_s(f(\tau_1) - f(\tau_2)) e^{i\phi(\tau_1) - i\phi(\tau_2)} \]

(and similarly for \( \Sigma \)). Then the path integral is approximated by

\[ Z = \int \mathcal{D}f(\tau) \mathcal{D}\phi(\tau) e^{-NS_{\text{eff}}[f, \phi]}. \]
SYK and AdS

\[ Z = \int \mathcal{D}f(\tau)\mathcal{D}\phi(\tau)e^{-NS_{\text{eff}}[f,\phi]} . \]

Symmetry arguments, and explicit computations, show that the effective action is

\[ S_{\text{eff}}[f, \phi] = \frac{K}{2} \int_0^{1/T} d\tau (\partial_\tau \phi + i(2\pi E T)\partial_\tau \epsilon)^2 - \frac{\gamma}{4\pi^2} \int_0^{1/T} d\tau \{ \tan(\pi T(\tau + \epsilon(\tau), \tau) \}, \]

where \( f(\tau) \equiv \tau + \epsilon(\tau) \), the couplings \( K, \gamma, \) and \( E \) can be related to thermodynamic derivatives and we have used the Schwarzian:

\[ \{ g, \tau \} \equiv \frac{g^{''''}}{g'} - \frac{3}{2} \left( \frac{g''}{g'} \right)^2 . \]
Einstein-Maxwell theory + cosmological constant

\[ \text{AdS}_2 \times \mathbb{T}^2 \]

\[ ds^2 = \left( \frac{d\zeta^2}{\zeta^2} - dt^2 \right) + d\vec{x}^2 \]

Gauge field: \( A = (\mathcal{E}/\zeta) dt \)

Mapping to SYK applies when temperature \( \ll \frac{1}{\text{(size of } \mathbb{T}^2)} \)

S. Sachdev, PRL 105, 151602 (2010)
Einstein-Maxwell theory + cosmological constant

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Gauge field: \[ A = (E/\zeta)dt \]

\[ \mathbb{T}^2 \]

SYK and AdS

Same long-time effective action

\[ \zeta = \infty \]

\[ \zeta \]

\[ \vec{x} \]

Mapping to SYK applies when temperature \( \ll 1/(\text{size of } \mathbb{T}^2) \)
One can also derive the thermodynamic properties from the large-\(N\) saddle point free energy:

\[
F_N = \frac{1}{\beta} \log P_f (\beta) + \frac{1}{2} \int d\beta_1 d\beta_2 \beta_1 \beta_2 G(\beta_1, \beta_2) \quad \Box (8)
\]

In the second line we write the free energy in a low temperature expansion, where \(U\approx 0.0406 J\) is the ground state energy, \(S_0\approx 0.232\) is the zero temperature entropy [32, 4], and \(T = c/v = \frac{\epsilon}{J}\). The entropy term can be derived by inserting the conformal saddle point solution (2) in the effective action. The specific heat can be derived from knowledge of the leading (in \(1/J\)) correction to the conformal saddle, but the energy requires the exact (numerical) finite \(J\) solution.

### 3 The generalized SYK model

In this section, we will present a simple way to generalize the SYK model to higher dimensions while keeping the solvable properties of the model in the large-\(N\) limit. For concreteness of the presentation, in this section we focus on a (1 + 1)-dimensional example, which describes a one-dimensional array of SYK models with coupling between neighboring sites. It should be clear how to generalize, and we will discuss more details of the generalization to arbitrary dimensions and generic graphs in section 6.

#### 3.1 Definition of the chain model

Figure 1: A chain of coupled SYK sites: each site contains \(N \gg 1\) fermion with SYK interaction. The coupling between nearest neighbor sites are four fermion interaction with two from each site.

Yingfei Gu, Xiao-Liang Qi, and D. Stanford, arXiv:1609.07832
Einstein-Maxwell theory + cosmological constant

$ds^2 = \frac{(d\zeta^2 - dt^2)}{\zeta^2} + d\vec{x}^2$

Gauge field: $A = (\mathcal{E}/\zeta)dt$

$\zeta = \infty$

Mapping to SYK applies when temperature $\ll 1/(\text{size of } \mathbb{T}^2)$

S. Sachdev, PRL 105, 151602 (2010)
Coupled SYK and AdS$_4$

AdS$_2 \times R^2$

$ds^2 = (d\zeta^2 - dt^2)/\zeta^2 + d\vec{x}^2$

Gauge field: $A = (\mathcal{E}/\zeta)dt$

Charge density $Q$

$S = \int d^4x \sqrt{-\hat{g}} \left( \hat{R} + 6/L^2 - \frac{1}{2} \sum_{i=1}^{2} (\partial \hat{\varphi}_i)^2 - \frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \right)$,

Einstein-Maxwell-axion theory with saddle point $\hat{\varphi}_i = kx_i$

leading to momentum dissipation
The coupled-SYK and AdS\(_4\) models realize a disordered metal with no quasiparticle excitations. (a “strange metal”)

R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849
Quantum chaos:

• In both the SYK and holographic models, the growth of chaos is characterized by

\[ \langle \{ c(x,t), c^\dagger(0,0) \}^2 \rangle \sim \exp \left( \frac{1}{\tau_L} \left( t - \frac{|x|}{v_B} \right) \right) \]

where the Lyapunov time saturates the lower bound \( \hbar/(2\pi k_B T) \) and the butterfly velocity \( v_B \sim T^{1/2} \).

• The thermal diffusivity, \( D_E \) is given exactly by

\[ D_E = v_b^2 \tau_L. \]

There is no universal relationship between the charge diffusivity, \( D_c \), and \( v_B \).
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There is no universal relationship between the charge diffusivity, \( D_c \), and \( v_B \).

- Quantum chaos is intimately linked to the loss of phase coherence from electron-electron interactions. As the time derivative of the local phase is determined by the local energy, phase fluctuations and chaos are linked to interaction-induced energy fluctuations, and hence thermal diffusivity.
\[ S = \int d^4 x \sqrt{-\hat{g}} \left( \hat{\mathcal{R}} + 6 / L^2 - \frac{1}{2} \sum_{i=1}^{2} (\partial \hat{\varphi}_i)^2 - \frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \right), \]

Einstein-Maxwell-axion theory with saddle point \( \hat{\varphi}_i = k x_i \)

leading to momentum dissipation
Coupled SYK and AdS$_4$

Matching correlators for thermoelectric diffusion, and quantum chaos

AdS$_2 \times \mathbb{R}^2$

$$ds^2 = \frac{(d\zeta^2 - dt^2)}{\zeta^2} + d\vec{x}^2$$

Gauge field: $A = (\mathcal{E}/\zeta)dt$

$$\zeta = \infty$$

$S = \int d^4x \sqrt{-\hat{g}} \left( \hat{R} + \frac{6}{L^2} - \frac{1}{2} \sum_{i=1}^{2} (\partial \hat{\varphi}_i)^2 - \frac{1}{4} \hat{F}_{\mu \nu} \hat{F}^{\mu \nu} \right)$

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leading to momentum dissipation
Quantum matter without quasiparticles:

The Sachdev-Ye-Kitaev (SYK) models

Black holes with AdS$_2$ horizons

Fermi surface coupled to a gauge field

$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left( \partial_\tau - ia_\tau - \frac{(\nabla - i\tilde{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \tilde{a})^2$$
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\[ \tau_L = \frac{\hbar}{2\pi k_B T} \]

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\(\tau_L\): the Lyapunov time to reach quantum chaos
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\( \tau_L \): the Lyapunov time to reach quantum chaos

\( v_B \): the “butterfly velocity” for the spatial propagation of chaos
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Fermi surface coupled to a gauge field

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Fermi surface coupled to a gauge field

\[ \mathcal{L}[\psi \pm, a] = \]

\[
\psi_+^\dagger \left( \partial_\tau - i \partial_x - \partial_y^2 \right) \psi_+ + \psi_-^\dagger \left( \partial_\tau + i \partial_x - \partial_y^2 \right) \psi_-
\]

\[
- a \left( \psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_- \right) + \frac{1}{2 g^2} \left( \partial_y a \right)^2
\]

Fermi surface coupled to a gauge field

Compute out-of-time-order correlator to diagnose quantum chaos

\[ f(t) = \frac{1}{N^2} \theta(t) \sum_{i,j=1}^{N} \int d^2x \quad \text{Tr} \left[ e^{-\beta H/2} \{ \psi_i(x, t), \psi_j^\dagger(0) \} \right] \]

\[ \times e^{-\beta H/2} \{ \psi_i(x, t), \psi_j^\dagger(0) \}^\dagger \]

\[ \sim \exp \left( \frac{(t - x/v_B)}{\tau_L} \right) \]
Fermi surface coupled to a gauge field

Compute out-of-time-order correlator to diagnose quantum chaos

Strongly-coupled theory with no quasiparticles and fast scrambling:

\[
\tau_L \approx \frac{\hbar}{2.48 \, k_B T} \quad , \quad v_B \approx 4.1 \, \frac{N T^{1/3} \, v_F^{5/3}}{e^{4/3} \, \gamma^{1/3}} \quad , \quad D_E \approx 0.42 \, v_B^2 \tau_L
\]

\(N\) is the number of fermion flavors, \(v_F\) is the Fermi velocity, \(\gamma\) is the Fermi surface curvature, \(e\) is the gauge coupling constant.
Quantum matter **without quasiparticles**:

The Sachdev-Ye-Kitaev (SYK) models

Black holes with AdS\(_2\) horizons

Fermi surface coupled to a gauge field

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\[ \tau_L: \text{ the Lyapunov time to reach quantum chaos} \]
\[ v_B: \text{ the “butterfly velocity” for the spatial propagation of chaos} \]
Quantum matter *without* quasiparticles:

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Thermal diffusivity, $D_E$:

$$D_E = (\text{universal number}) \times v_B^2 \tau_L$$

in all three models

\[ \mathcal{L}[\Psi, a] = \Psi^\dagger \left( \partial_\tau - ia_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2 \]

$\tau_L$: the Lyapunov time to reach quantum chaos

$v_B$: the “butterfly velocity” for the spatial propagation of chaos