Competing orders in a magnetic field:

spin and charge density waves in the cuprates

Eugene Demler (Harvard)

Kwon Park – see talk **L20.004**

Anatoli Polkovnikov

Subir Sachdev

Matthias Vojta (Augsburg)

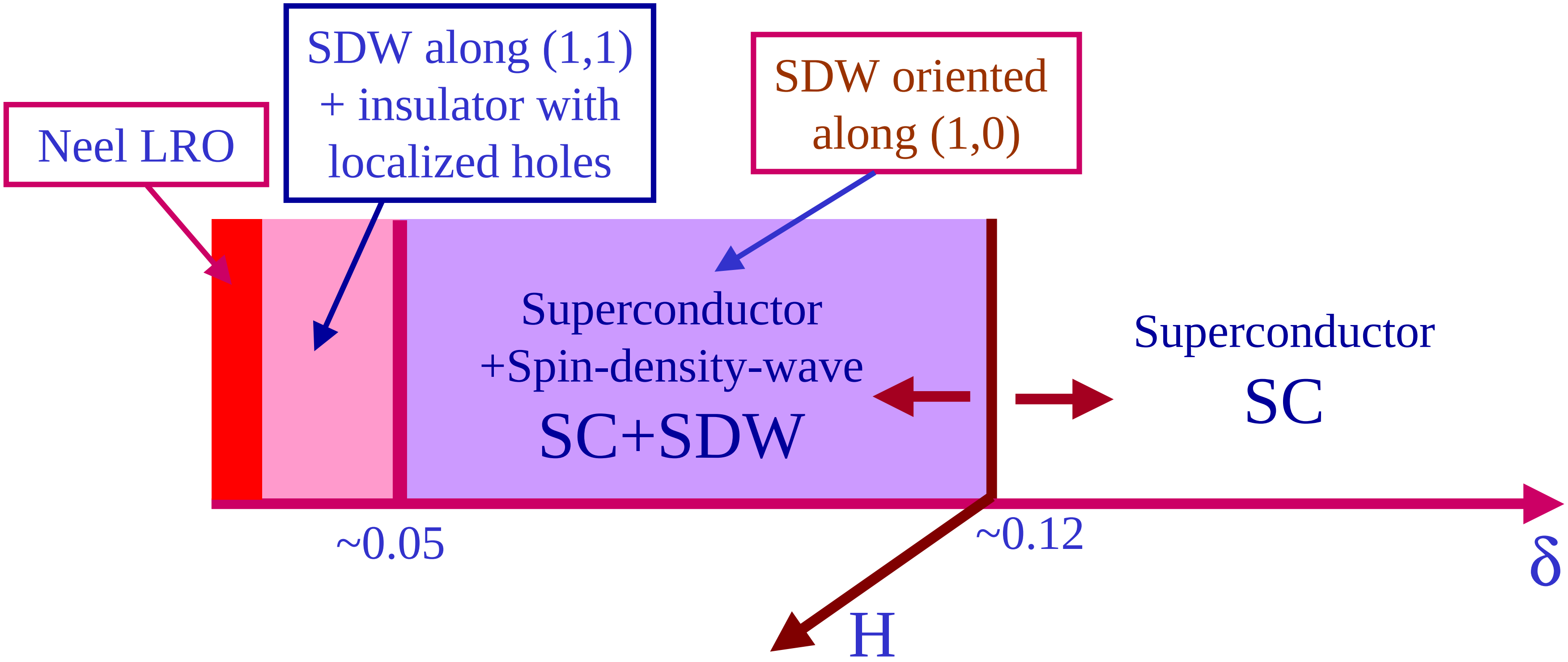
Ying Zhang – see talk **F13.014**

Talk online at

http://pantheon.yale.edu/~subir

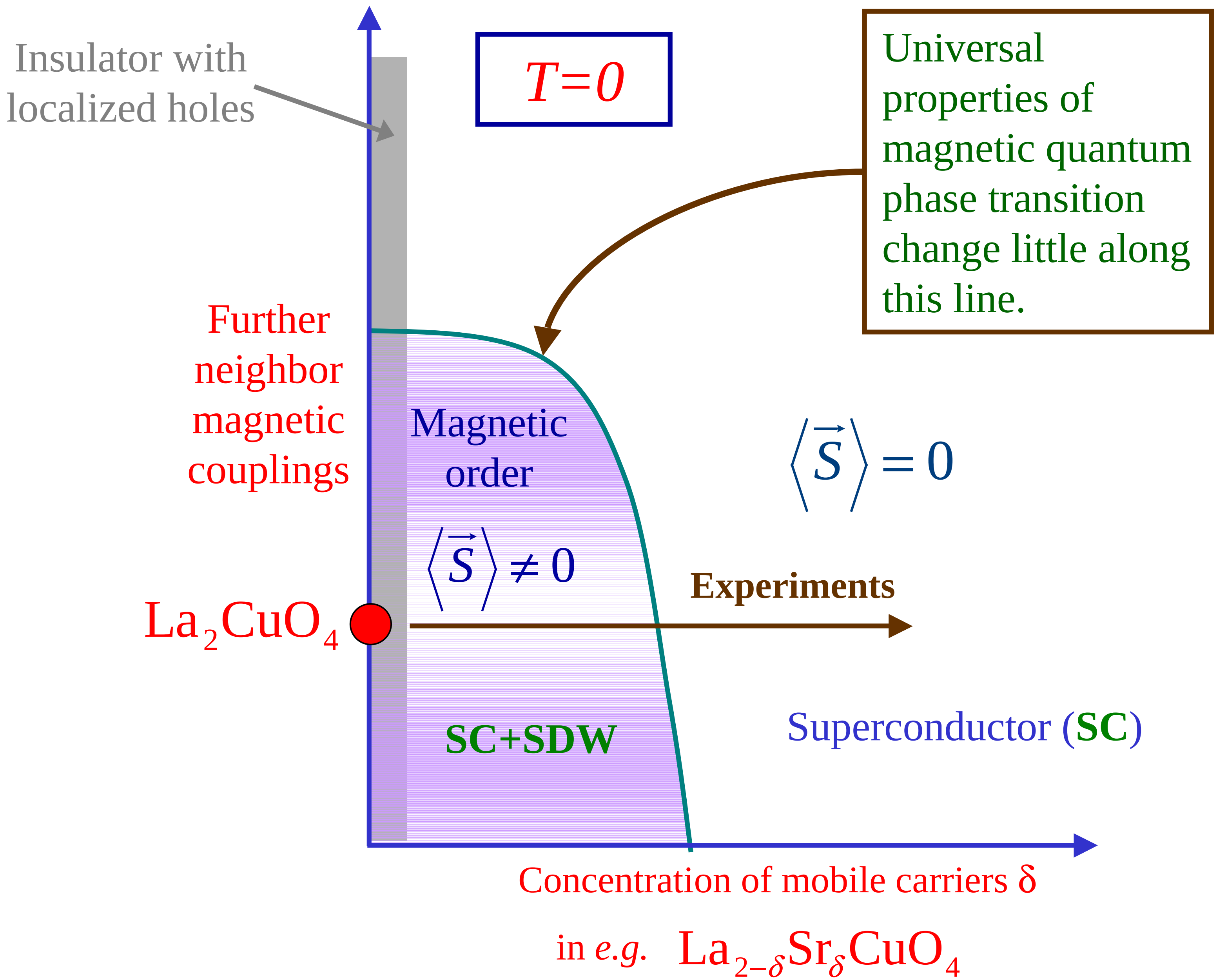
(Search for "*Sachdev*" on Google)

Zero temperature phases of the cuprate superconductors as a function of hole density



Theory for a system with strong interactions: describe SC and SC+SDW phases by expanding in the deviation from the quantum critical point between them.

B. Keimer *et al*. Phys. Rev. B **46**, 14034 (1992).
S. Wakimoto, G. Shirane *et al*., Phys. Rev. B **60**, R769 (1999).
G. Aeppli, T.E. Mason, S.M. Hayden, H.A. Mook, J. Kulda, Science **278**, 1432 (1997).
Y. S. Lee, R. J. Birgeneau, M. A. Kastner *et al*., Phys. Rev. B **60**, 3643 (1999).
J. E. Sonier *et al*., cond-mat/0108479.
C. Panagopoulos, B. D. Rainford, J. L. Tallon, T. Xiang, J. R. Cooper, and C. A. Scott, preprint.



S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).
A.V. Chubukov, S. Sachdev, and J. Ye, *Phys. Rev.* B **49**, 11919 (1994)

Outline

I. Magnetic ordering transitions in the insulator (δ=0).

II. Doping the Mott insulator
   Charge order nucleated by vortices

III. SC+SDW to SC transition: influence of an applied magnetic field.
   Neutron scattering measurements

IV. Conclusions

I. Magnetic ordering transitions in the insulator

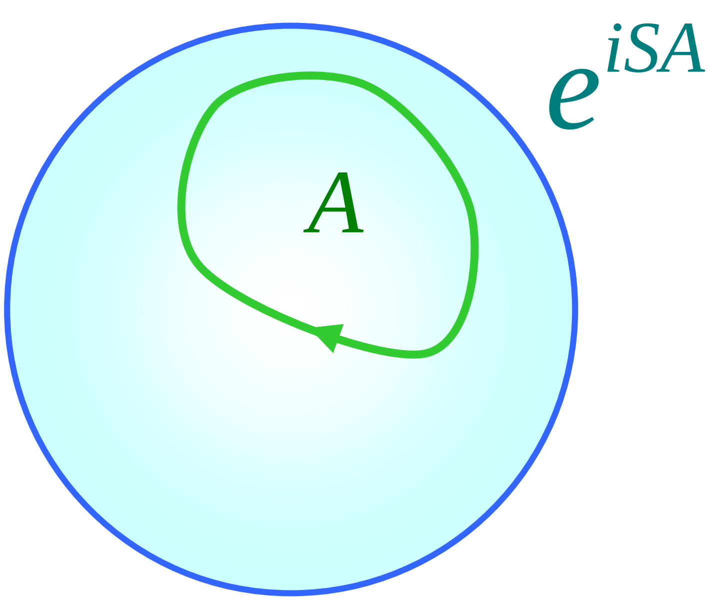
$$H = \sum_{i<j} J_{ij} \; \vec{S}_i \cdot \vec{S}_j$$

Action for collinear antiferromagnetic order parameter ϕ_α *(α=1,2,3)*:

$$S_b = \int d^2x d\tau \left[\frac{1}{2}\left(\left(\nabla_x \phi_\alpha\right)^2 + c^2 \left(\partial_\tau \phi_\alpha\right)^2 \right) + V\left(\phi_\alpha^2\right) \right]$$

S. Chakravarty, B.I. Halperin, and D.R. Nelson, Phys. Rev. B **39**, 2344 (1989).

Missing: Spin Berry Phases



These are crucial in one dimension to obtain Bethe or Majumdar-Ghosh states for *S*=1/2 and the Haldane state for *S*=1.

In two dimensions, Berry phases induce *bond-centered charge order* in quantum "disordered" phase with $\langle \phi_\alpha \rangle = 0$

N. Read and S. Sachdev, Phys. Rev. Lett. **62**, 1694 (1989).

Field theory of quantum "disordered" phase

Discretize spacetime into a cubic lattice:

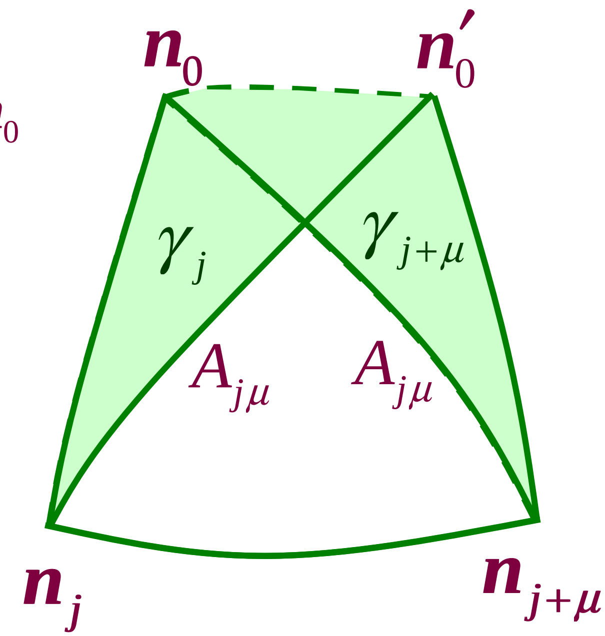
$$Z = \prod_j \int d\boldsymbol{n}_j \delta\left(\boldsymbol{n}_j^2 - 1\right) \exp\left(-\frac{1}{2g} \sum_{j,\mu} \boldsymbol{n}_j \cdot \boldsymbol{n}_{j+\mu} - \frac{i}{2} \sum_j \eta_j A_{j\tau} \right)$$

$j \rightarrow$ cubic lattice sites; $\mu \rightarrow x, y, \tau;$

$\eta_{\mathrm{j}} \rightarrow \pm 1$ on two square sublattices ; $\boldsymbol{n}_j \sim \eta_j \vec{S}_j \rightarrow$ Neel order parameter;

$A_{j\mu} \rightarrow$ oriented area of spherical triangle

formed by $\boldsymbol{n}_j$, $\boldsymbol{n}_{j+\mu}$, and an arbitrary reference point $\boldsymbol{n}_0$



For large g, perform a "high temperature" expansion to obtain an effective action for the $A_{j\mu}$. This must be invariant under the "gauge transformation"

$$A_{j\mu} \rightarrow A_{j\mu} - \gamma_{j+\mu} + \gamma_j$$

associated with a change in choice of $\boldsymbol{n}_0$ (γ_j is the oriented area of the spherical triangle formed by $\boldsymbol{n}_j$ and the two choices for $\boldsymbol{n}_0$).

Also the area of the triangle is uncertain modulo 4π, and so the effective action should be invariant under

$$A_{j\mu} \rightarrow A_{j\mu} + 4\pi$$

Simplest large g effective action for the $A_{j\mu}$

$$Z = \prod_{j,\mu} \int dA_{j\mu} \exp\left(-\frac{1}{2e^2} \sum_{\square} \cos\left(\frac{1}{2} \varepsilon_{\mu\nu\lambda} \Delta_\nu A_{j\lambda} \right) - \frac{i}{2} \sum_j \eta_j A_{j\tau} \right)$$

with $e^2 \sim g^2$

This compact QED in 2+1 dimensions with Berry phases.

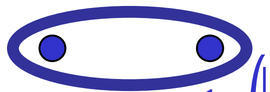
This theory can be reliably analyzed by a duality mapping.

The gauge theory is *<u>always</u>* in a *confining* phase:

There is an energy gap and the ground state has **<u>spontaneous bond order</u>**.

Square lattice with first(J_1) and second (J_2) neighbor exchange interactions (say)

$$H = \sum_{i<j} J_{ij} \; \vec{S}_i \cdot \vec{S}_j$$



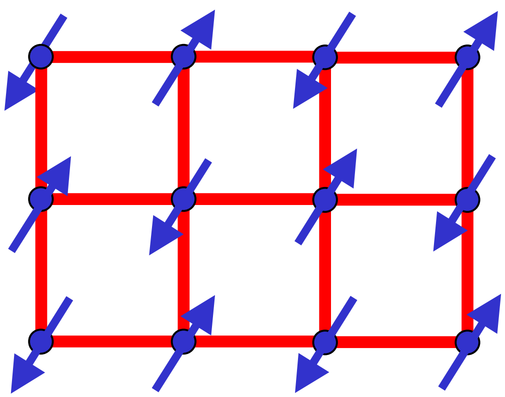
$$= \frac{1}{\sqrt{2}}\Big(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle \Big)$$

N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).

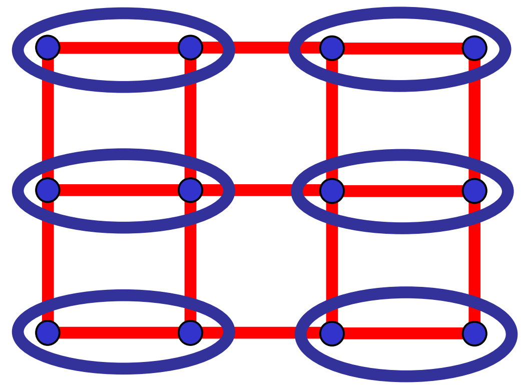
O. P. Sushkov, J. Oitmaa, and Z. Weihong, *Phys. Rev*. B **63**, 104420 (2001).

M.S.L. du Croo de Jongh, J.M.J. van Leeuwen, W. van Saarloos, *Phys. Rev*. B **62**, 14844 (2000).

See however L. Capriotti, F. Becca, A. Parola, S. Sorella, cond-mat/0107204 .



Neel state



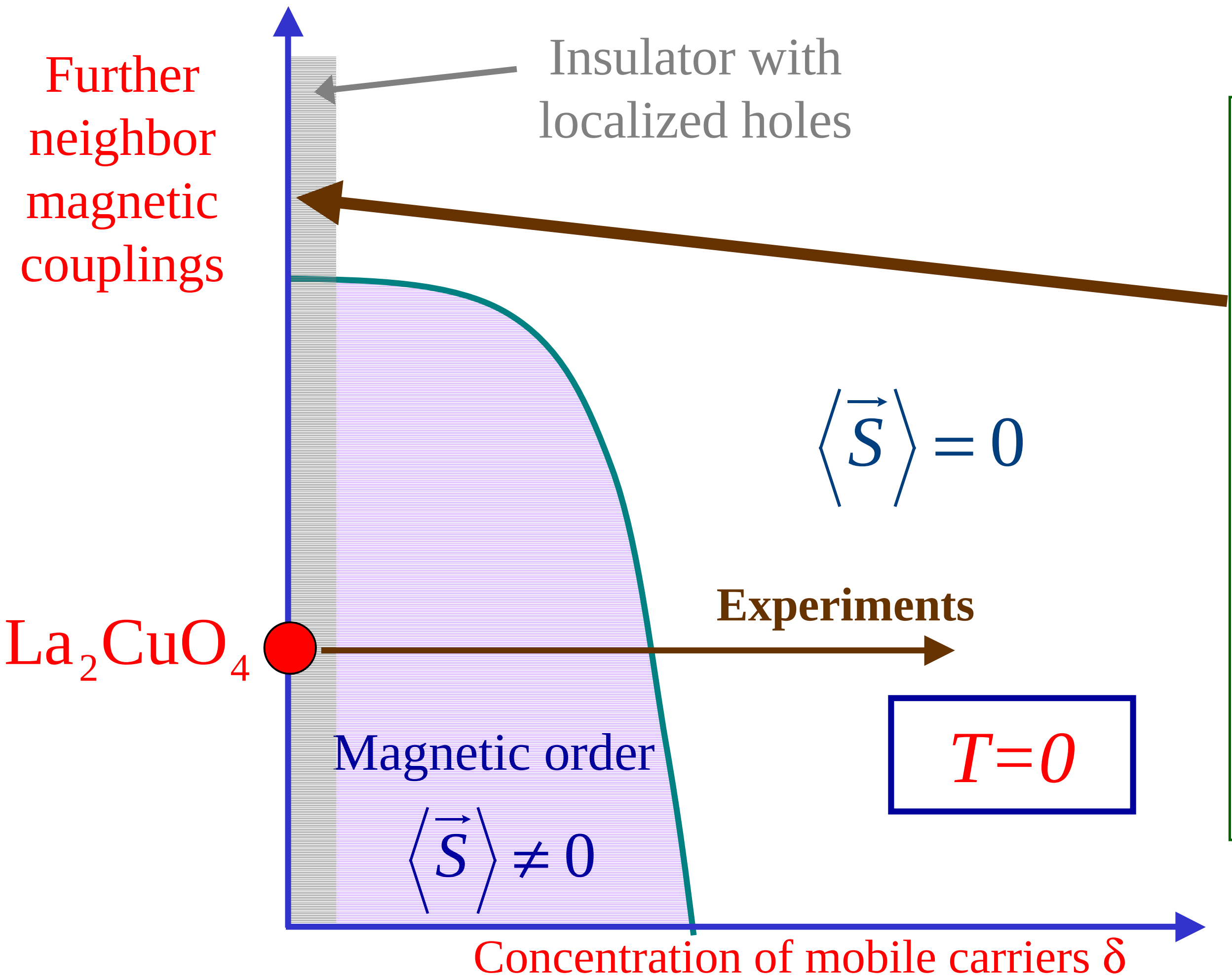
Spin-Peierls (or plaquette) state
"Bond-centered charge order"

J_2 / J_1

Studies on the 2D pyrochlore lattice agree with related predictions of theory:

J.-B. Fouet, M. Mambrini, P. Sindzingre, C. Lhuillier, cond-mat/0108070.
R. Moessner, Oleg Tchernyshyov, S.L. Sondhi, cond-mat/0106286.

Framework for spin/charge order in cuprate superconductors



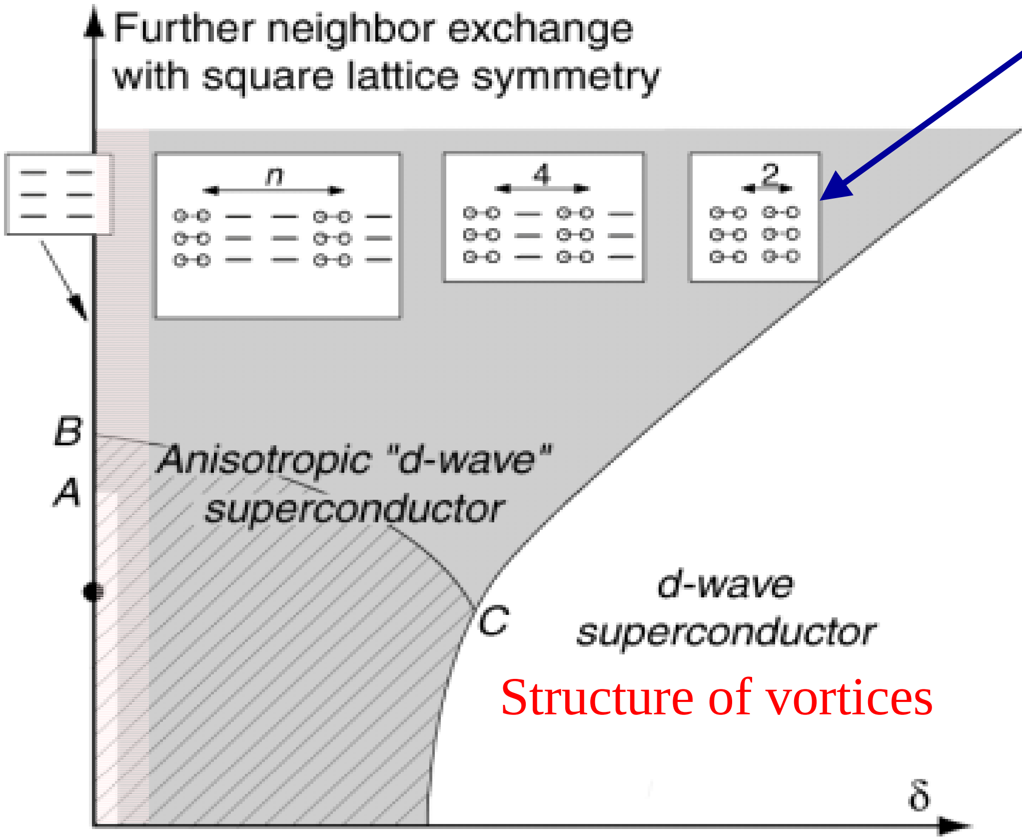
Confined, paramagnetic Mott insulator has

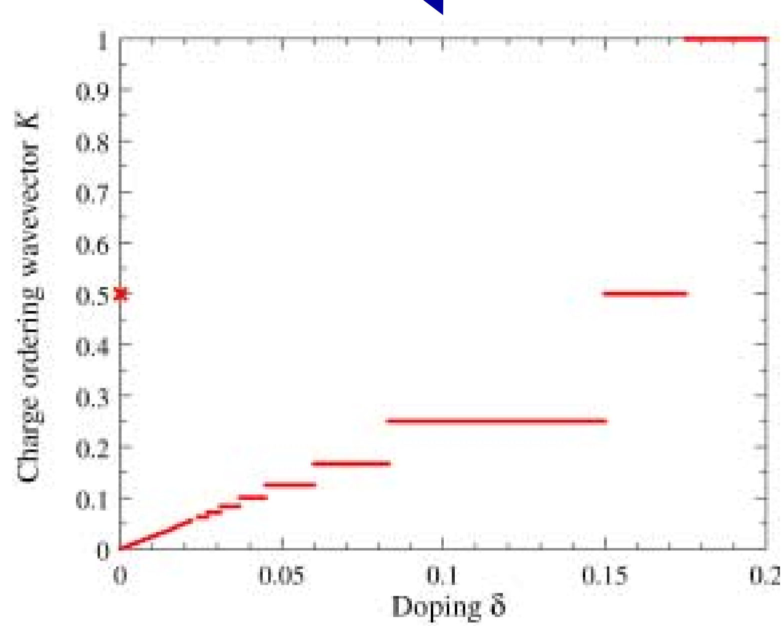
1. Stable *S*=1 spin exciton.
2. Broken translational symmetry:- bond-centered charge order.
3. *S*=1/2 moments near non-magnetic impurities

II. Doping the Mott insulator

“Large *N*” theory in region with preserved spin rotation symmetry

S. Sachdev and N. Read, *Int. J. Mod. Phys.* B **5**, 219 (1991).
M. Vojta and S. Sachdev, *Phys. Rev. Lett.* **83**, 3916 (1999).
M. Vojta, Y. Zhang, and S. Sachdev, *Phys. Rev.* B **62**, 6721 (2000).





Hatched region --- spin order
Shaded region ---- charge order

See also J. Zaanen, *Physica* C **217**, 317 (1999),
S. Kivelson, E. Fradkin and V. Emery, *Nature* **393**, 550 (1998),
S. White and D. Scalapino, *Phys. Rev. Lett.* **80**, 1272 (1998).

Charge order nucleated by vortices

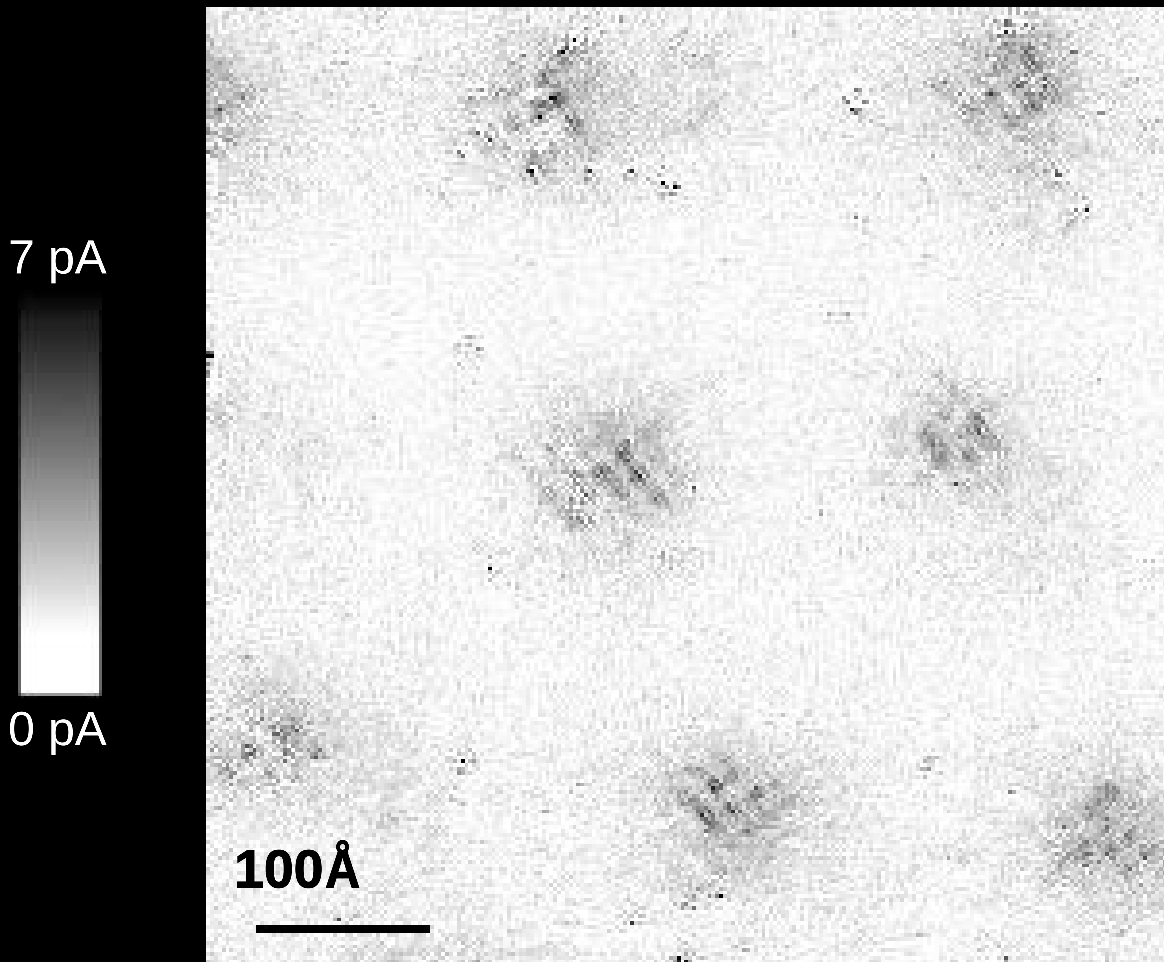
Memory of the Mott insulator should survive in and around vortices in superconducting order: superconductivity is suppressed in the vortex core, but the electrons should still strive to retain the exchange correlation energy of the Mott insulator. The vortex core is *not* a "normal Fermi liquid" as in BCS theory. This is the primary failure of BCS theory in the cuprate superconductors.

S. Sachdev, *Phys. Rev.* B **45**, 389 (1992);
N. Nagaosa and P.A. Lee, *Phys. Rev.* B **45**, 966 (1992);
D.P. Arovas, A. J. Berlinsky, C. Kallin, and S.-C. Zhang *Phys. Rev. Lett.* **79**, 2871 (1997).
J. H. Han and D. H. Lee, *Phys. Rev. Lett.* **85**, 1100 (2000).
M. Franz and Z. Tesanovic, *Phys. Rev.* B **63**, 064516 (2001);
J. I. Kishine, P.A. Lee, and X.-G. Wen, *Phys. Rev. Lett.* **86**, 5365 (2001).

Local magnetic order in the vortex core is "quantum-disordered": so there is a spin gap and charge order should appear, as in the doped paramagnetic Mott insulator.

K. Park and S. Sachdev, *Phys. Rev.* B **64**, 184510 (2001).

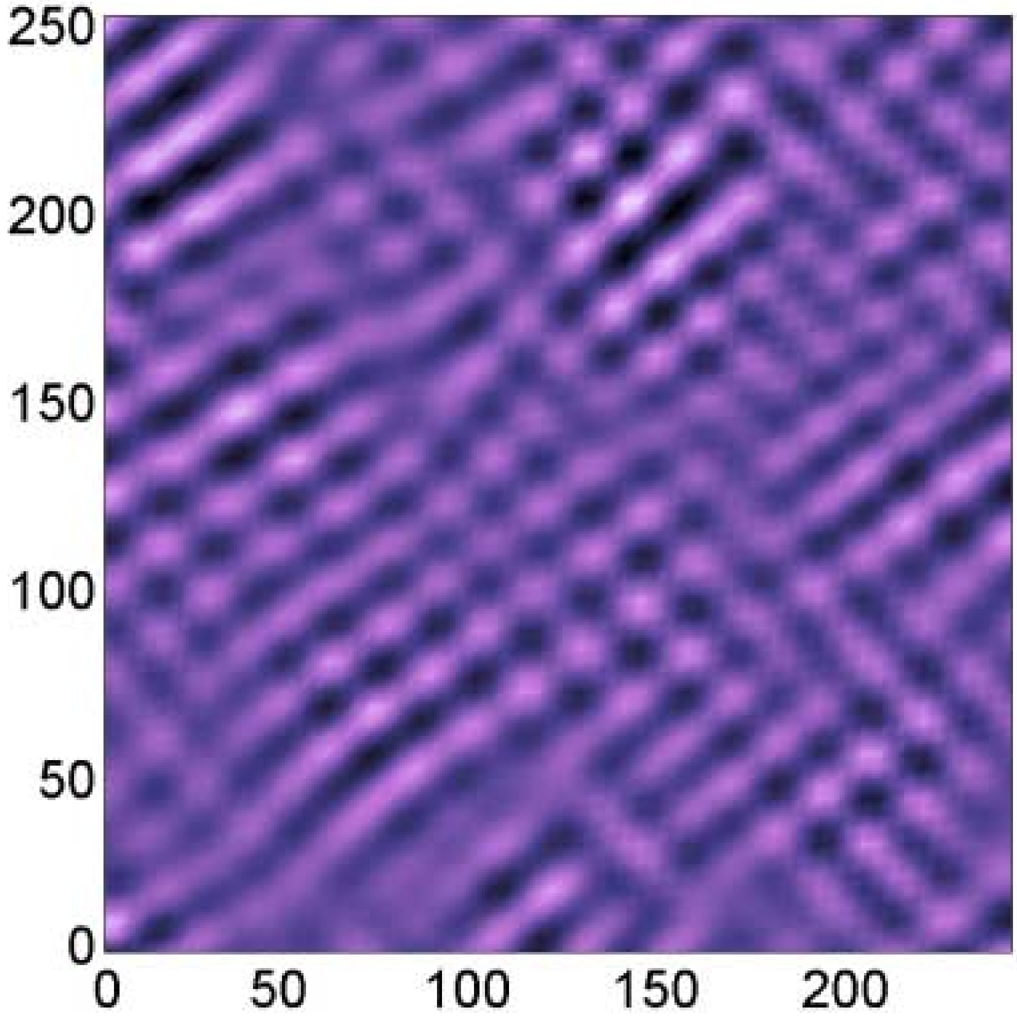
Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV



J. Hoffman E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* 295, 466 (2002).

Talk T14.008

STM image of pinned charge order in $Bi_2Sr_2CaCu_2O_{8+\delta}$ in zero magnetic field



Charge order period
= 4 lattice spacings

C. Howald, H. Eisaki, N. Kaneko, and A. Kapitulnik, cond-mat/0201546

Observation of static charge order in $YBa_2Cu_3O_{6.35}$
(spin correlations are dynamic)

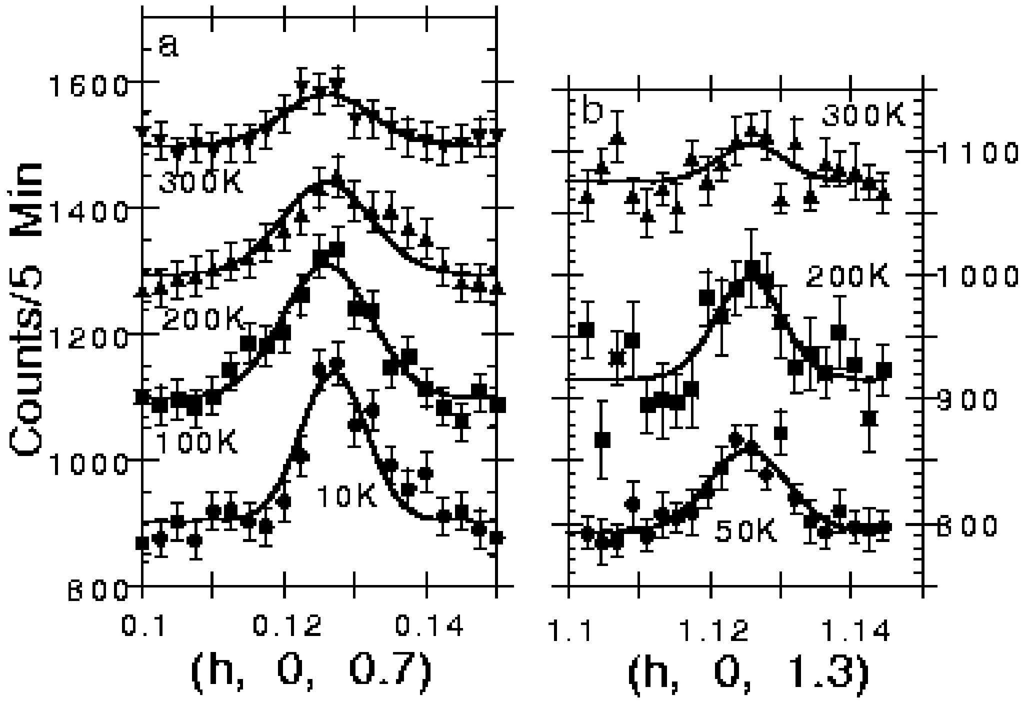
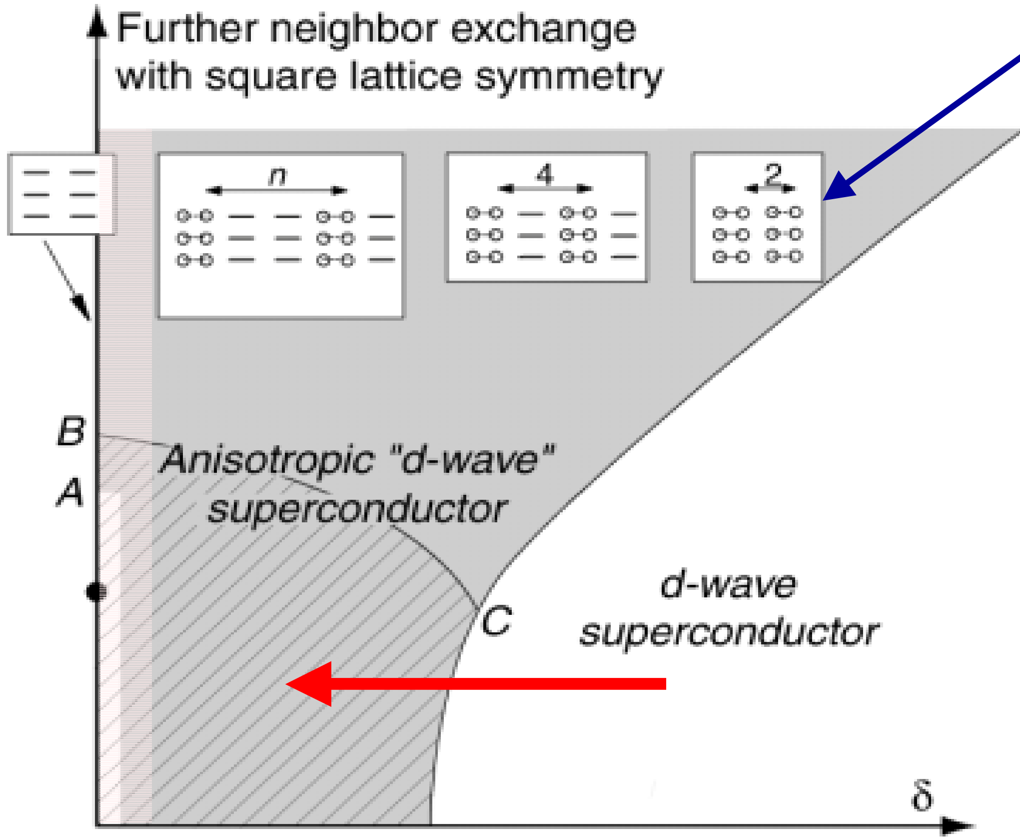


FIG. 1. Measurements of the charge order for YBCO6.35. (a) Measurements obtained at a small momentum transfer so the results are not affected by impurity powder lines. Powder lines were also avoided around the (1.125, 0, 1.3) r.l.u. position shown in (b). The lines are Gaussian fits to the data. In (a) 200 and (b) 100 additional counts were added onto successive scans so the data could be presented on the same plot. The scattering broadens at higher temperatures.

Charge order period
= 8 lattice spacings

H. A. Mook, Pengcheng Dai, and F. Dogan
Phys. Rev. Lett. **88**, 097004 (2002).

II. Doping the Mott insulator



Hatched region --- spin order
Shaded region ---- charge order

“Large *N*” theory in region with preserved spin rotation symmetry

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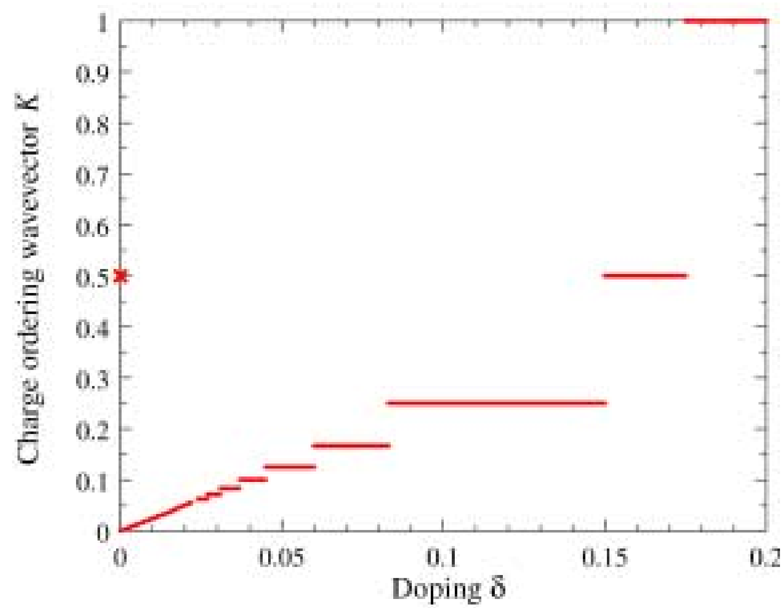
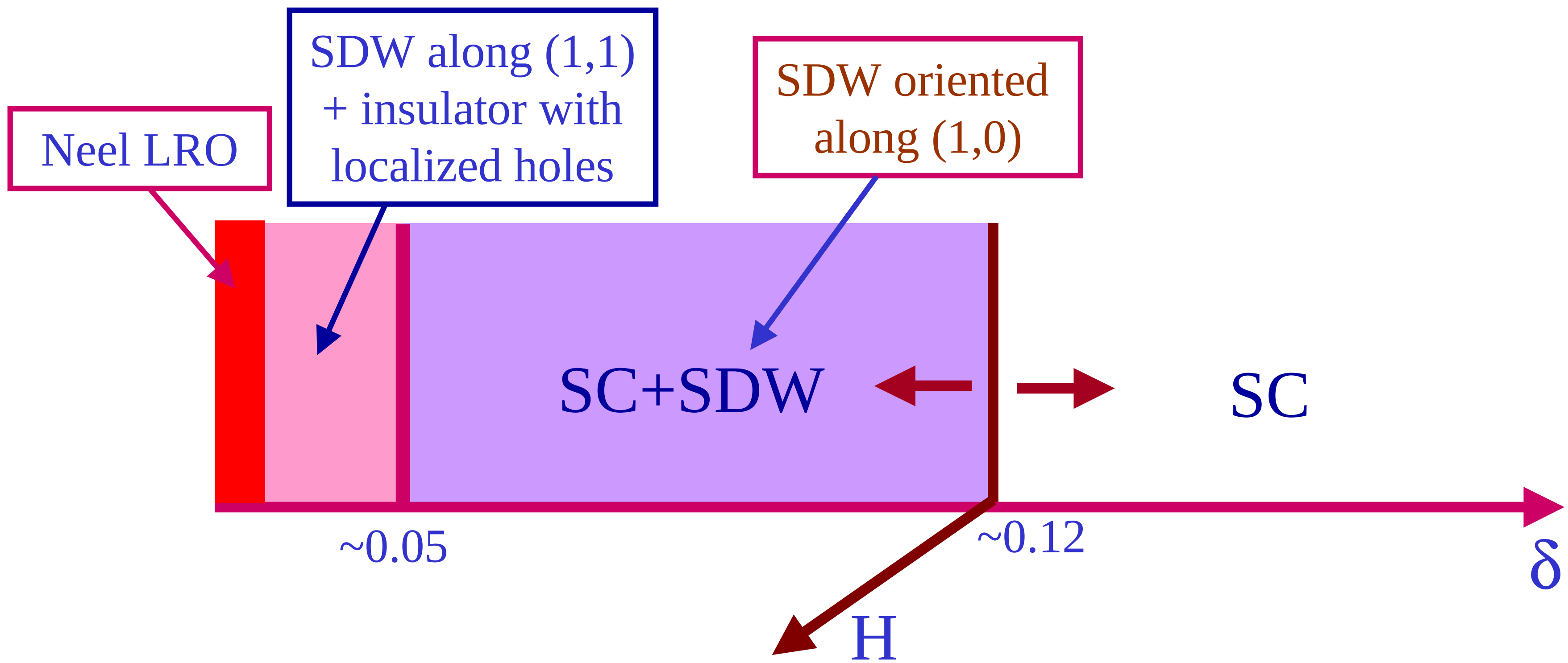
M. Vojta and S. Sachdev, *Phys. Rev. Lett*. **83**, 3916 (1999).
M. Vojta, Y. Zhang, and S. Sachdev, *Phys. Rev*. B **62**, 6721 (2000).



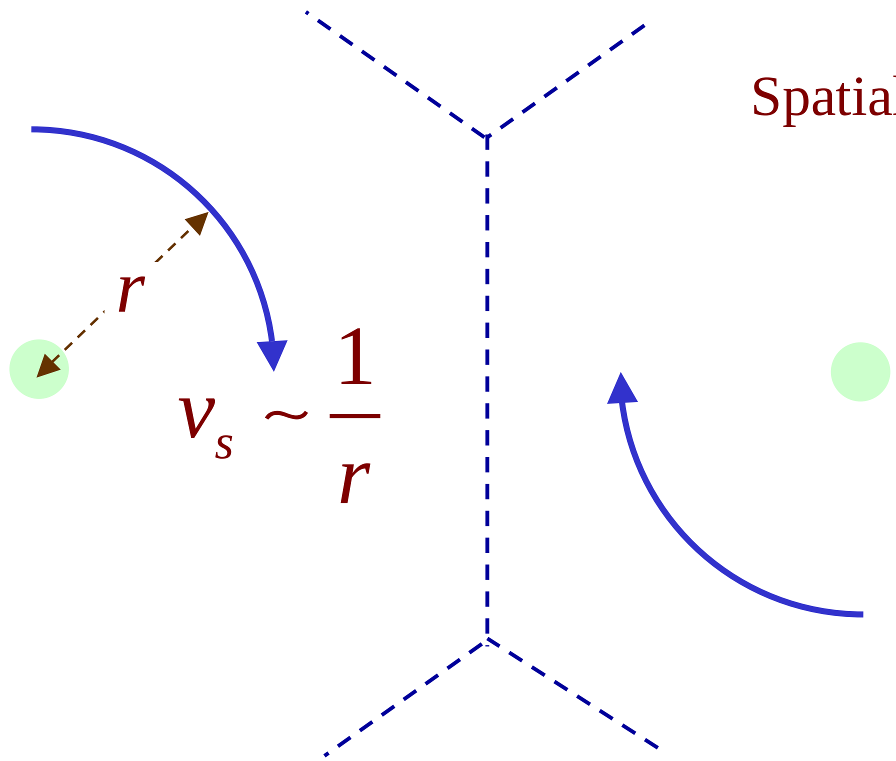
See also J. Zaanen, *Physica* C **217**, 317 (1999),
S. Kivelson, E. Fradkin and V. Emery, *Nature* **393**, 550 (1998),
S. White and D. Scalapino, *Phys. Rev. Lett*. **80**, 1272 (1998).

III. SC+SDW to SC transition: influence of an applied magnetic field



B. Keimer *et al*. Phys. Rev. B **46**, 14034 (1992).
S. Wakimoto, G. Shirane *et al*., Phys. Rev. B **60**, R769 (1999).
G. Aeppli, T.E. Mason, S.M. Hayden, H.A. Mook, J. Kulda, Science **278**, 1432 (1997).
Y. S. Lee, R. J. Birgeneau, M. A. Kastner *et al*., Phys. Rev. B **60**, 3643 (1999).
J. E. Sonier *et al*., cond-mat/0108479.
C. Panagopoulos, B. D. Rainford, J. L. Tallon, T. Xiang, J. R. Cooper, and C. A. Scott, preprint.

Dominant effect: **uniform** softening of spin excitations by superflow kinetic energy



Spatially averaged superflow kinetic energy

$$\sim \left\langle v_s^2 \right\rangle \sim \frac{H}{H_{c2}} \ln \frac{3H_{c2}}{H}$$

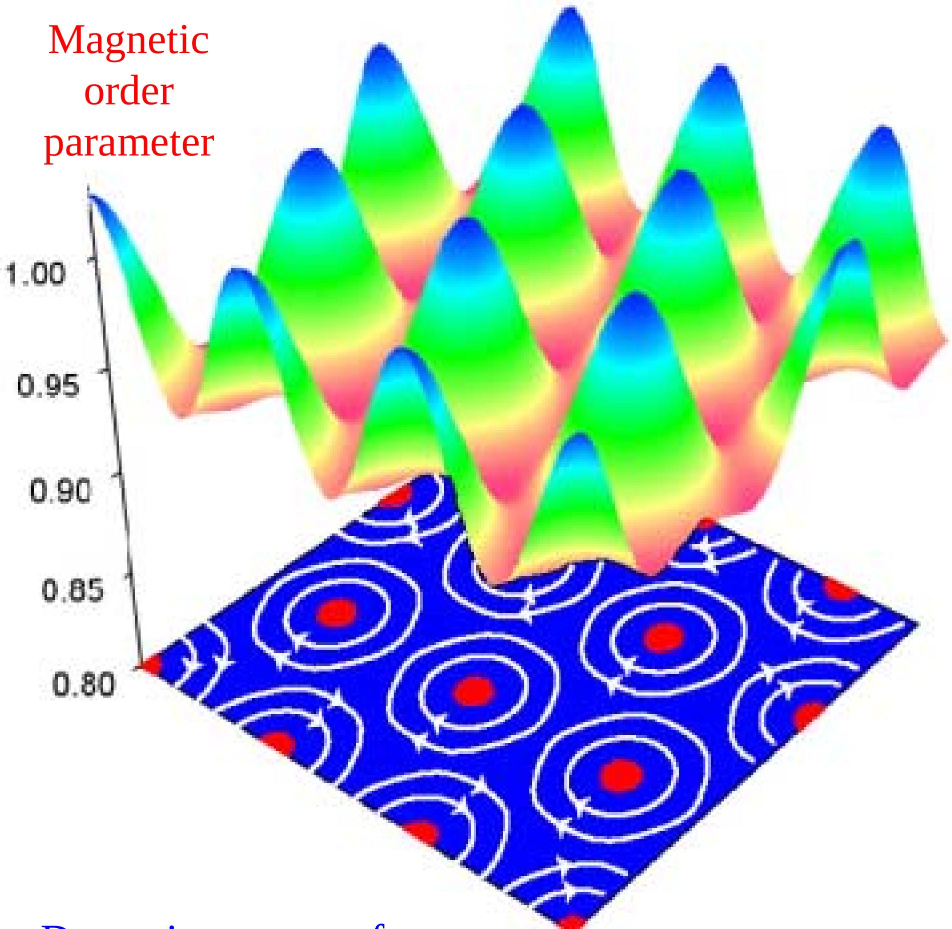
Coupling determining spin excitation energy, *s*,

replaced by $s_{eff}\left(H\right) = s - C\frac{H}{H_{c2}}\ln\left(\frac{3H_{c2}}{H}\right)$

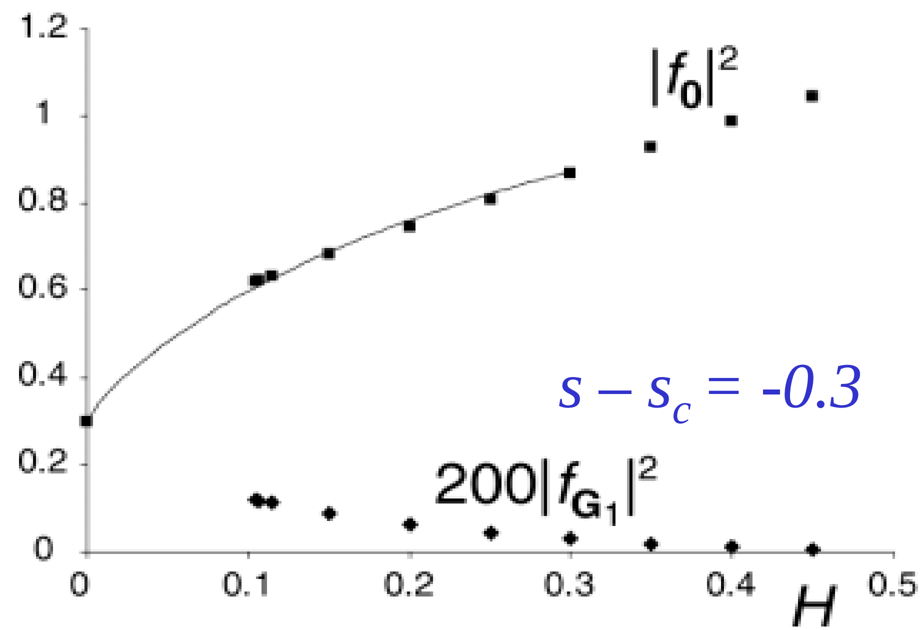
E. Demler, S. Sachdev, and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Structure of *long-range* SDW order in SC+SDW phase

E. Demler, S. Sachdev, and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).



$$\delta \left| f_0 \right|^2 \propto H \ln(1/H)$$



Dynamic structure factor

$$S(\boldsymbol{k},\omega) = (2\pi)^3 \delta(\omega) \sum_{\boldsymbol{G}} \left| f_{\boldsymbol{G}} \right|^2 \delta(\boldsymbol{k}-\boldsymbol{G}) + \cdots$$

$\boldsymbol{G} \rightarrow$ reciprocal lattice vectors of vortex lattice.

$\boldsymbol{k}$ measures deviation from SDW ordering wavevector $\boldsymbol{K}$

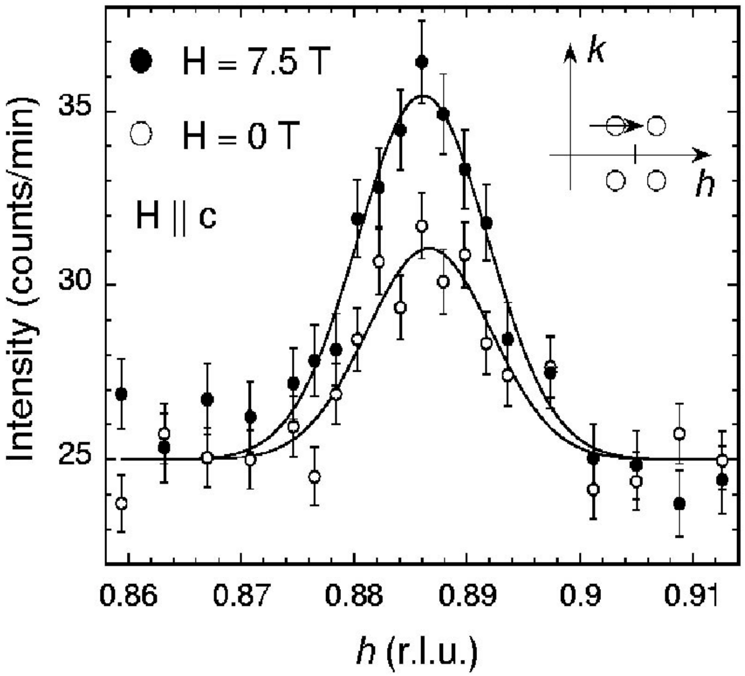
D. P. Arovas, A. J. Berlinsky, C. Kallin, and S.-C. Zhang, *Phys. Rev. Lett.* **79**, 2871 (1997) discussed static magnetism within the vortex cores in the SC phase.Their model implies a ~*H* dependence of the intensity

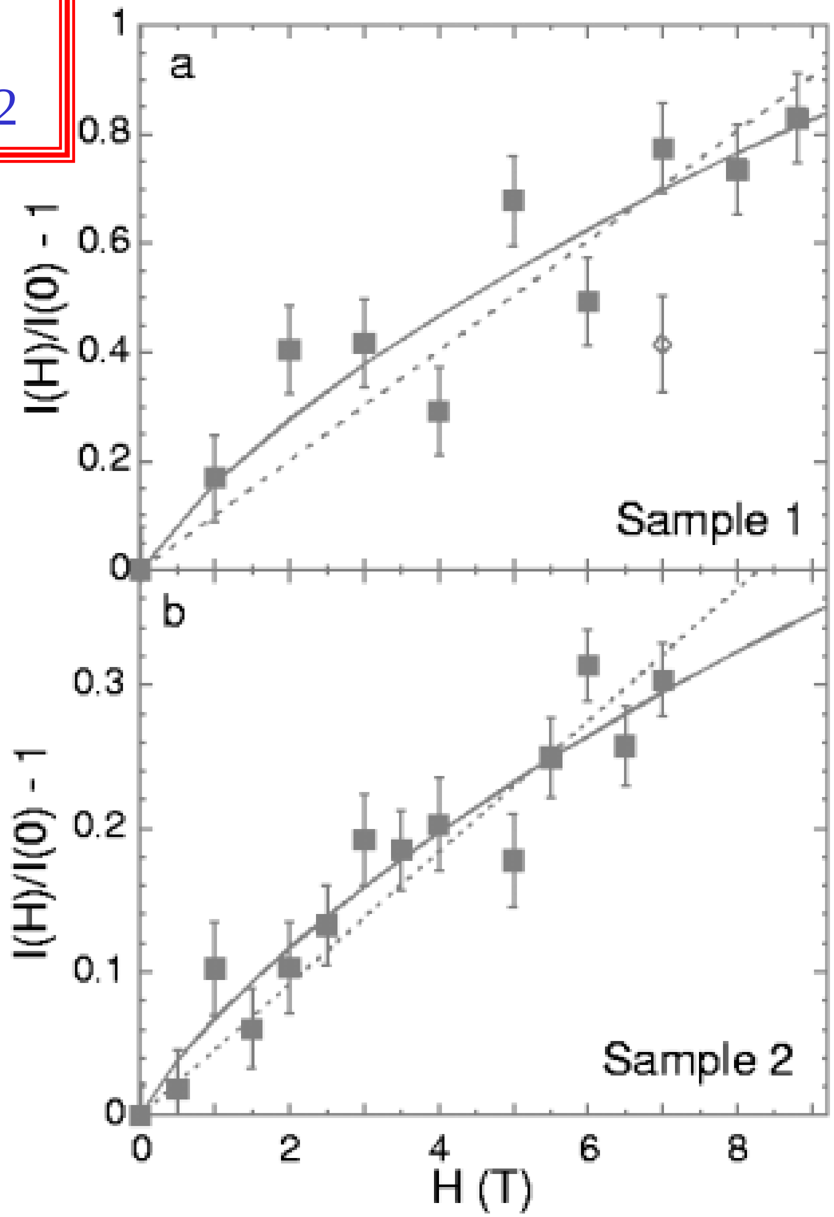
Neutron scattering measurements of static spin correlations of the superconductor+spin-density-wave (SC+SDW) in a magnetic field

Elastic neutron scattering off La_2CuO_{4+y}

B. Khaykovich, Y. S. Lee, S. Wakimoto, K. J. Thomas, M. A. Kastner, and R.J. Birgeneau, cond-mat/0112505.

Talk T14.002





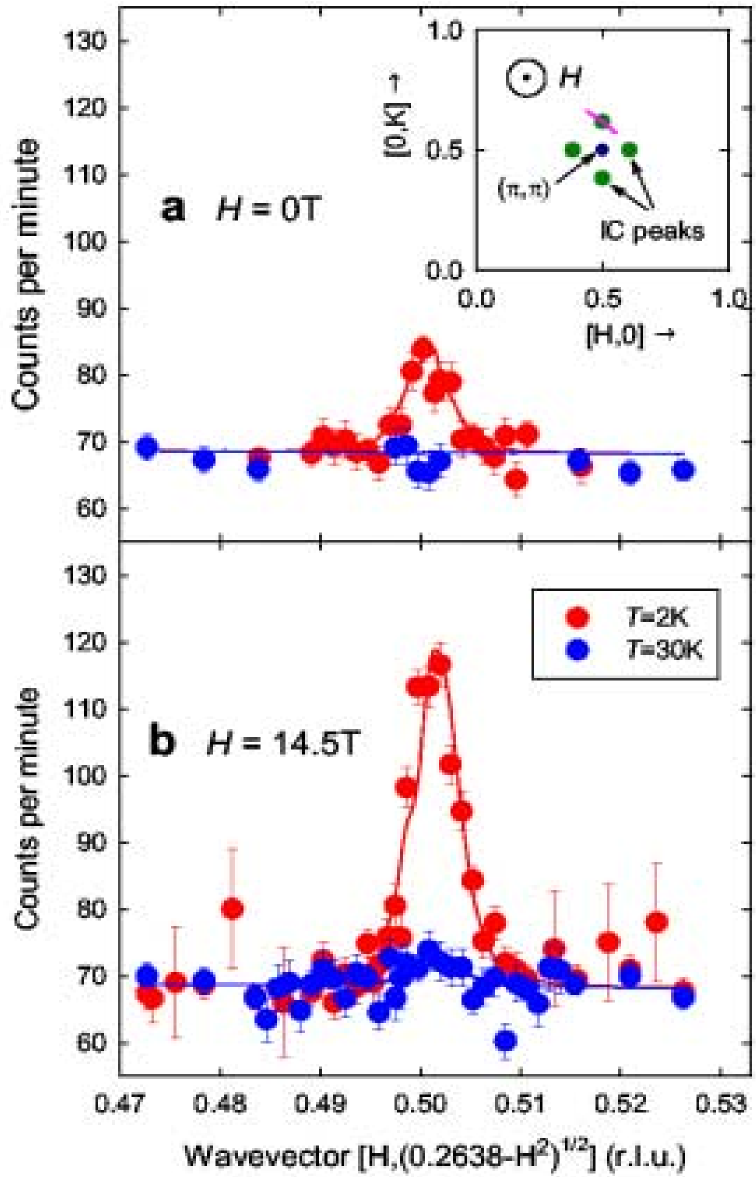
Solid line --- fit to : $\frac{I(H)}{I(0)} = 1 + a\frac{H}{H_{c2}}\ln\left(\frac{3.0H_{c2}}{H}\right)$

a is the only fitting parameter

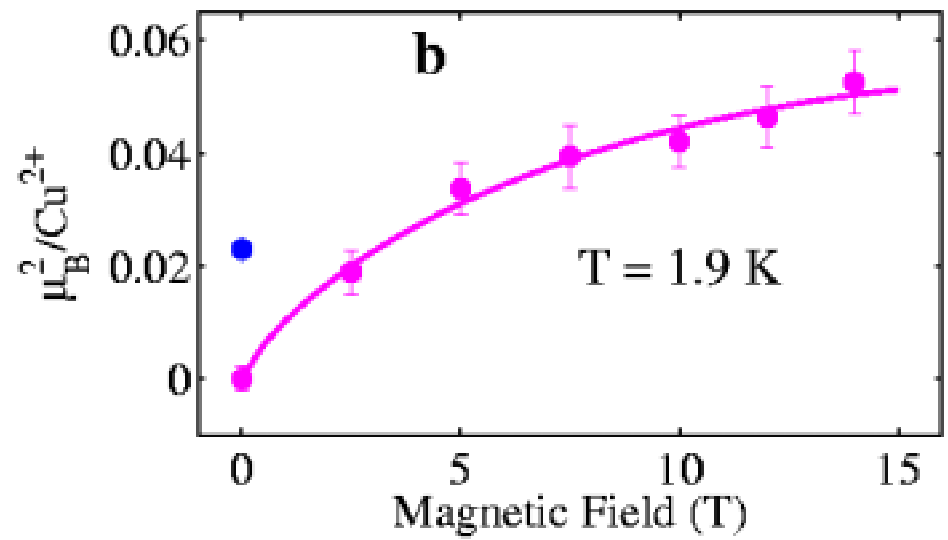
Best fit value - a = 2.4 with H_{c2} = 60 T

Neutron scattering of $La_{2-x}Sr_xCuO_4$ at x=0.1

Talk
T14.001



B. Lake, H. M. Rønnow, N. B. Christensen,
G. Aeppli, K. Lefmann, D. F. McMorrow,
P. Vorderwisch, P. Smeibidl, N. Mangkorntong,
T. Sasagawa, M. Nohara, H. Takagi, T. E. Mason,
Nature, **415**, 299 (2002).



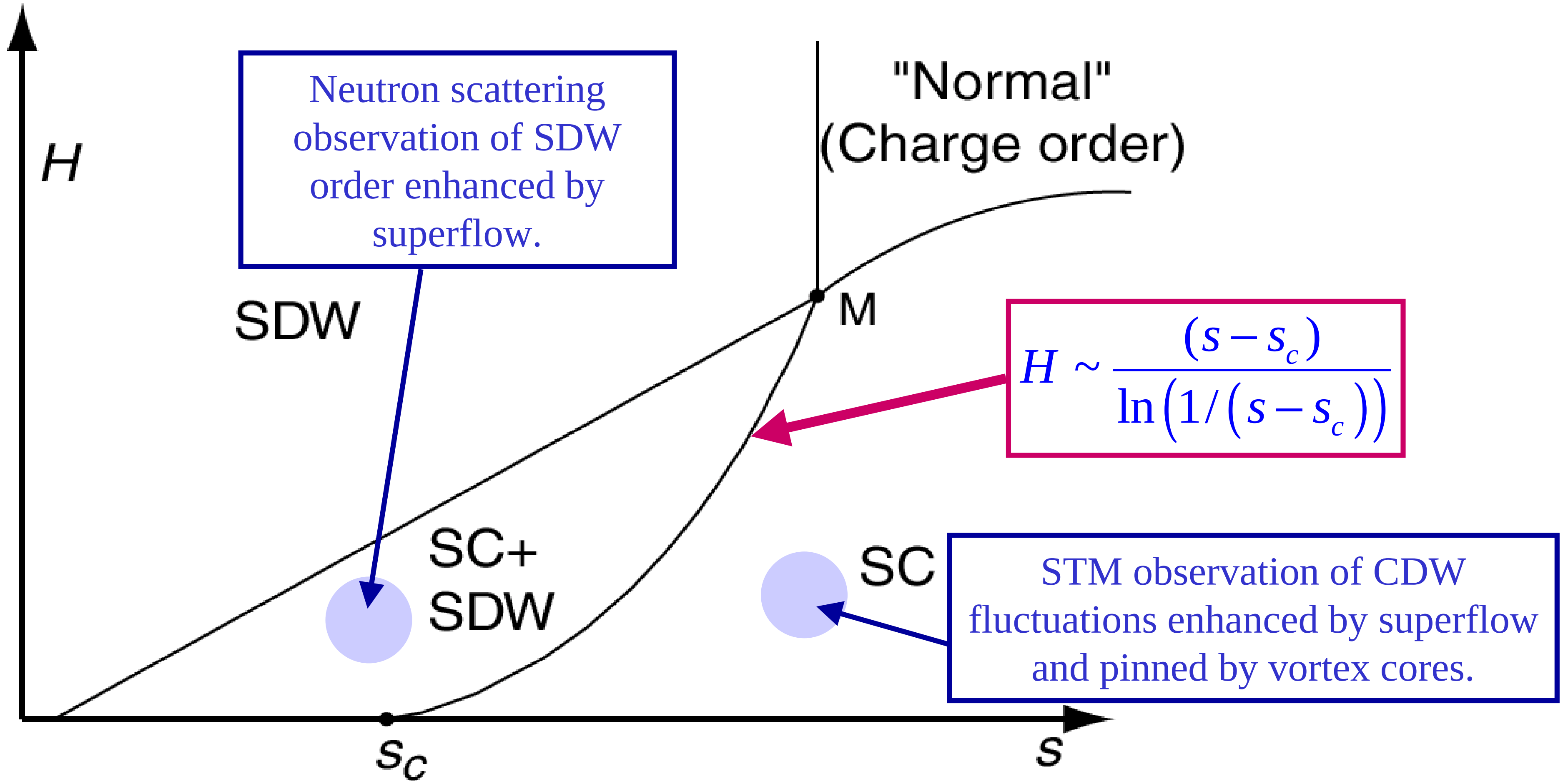
Solid line - fit to : $I(H) = a\frac{H}{H_{c2}}\ln\left(\frac{H_{c2}}{H}\right)$

Effect of magnetic field on SDW+SC to SC transition

(extreme Type II superconductivity)

$T=0$

Main results



E. Demler, S. Sachdev, and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

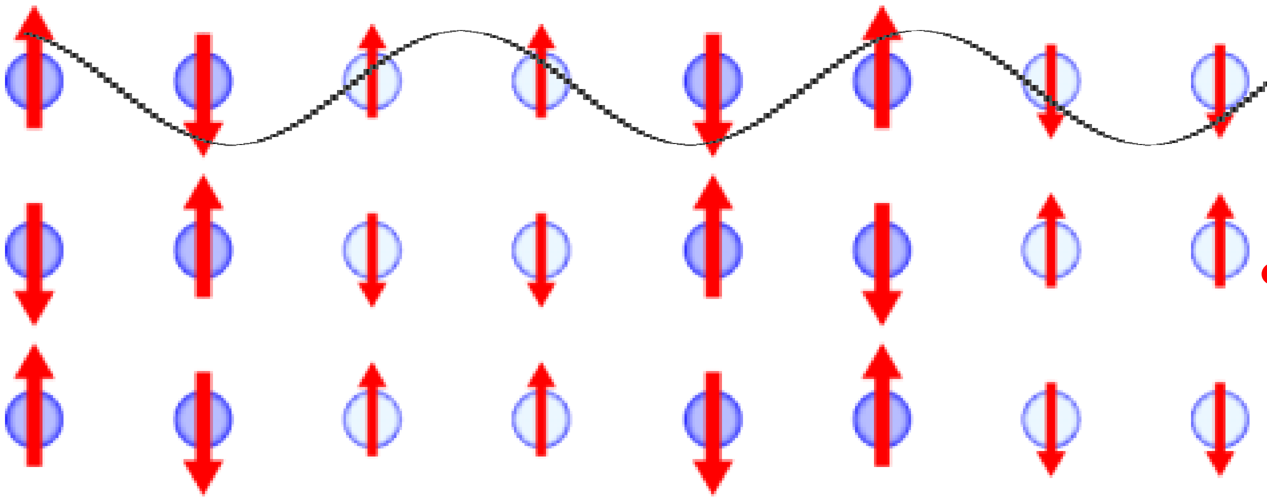
Quantitative connection between the two experiments ?

Theory of SC+SDW to SC quantum transition

Spin density wave order parameter for general ordering wavevector

$$S_\alpha(\boldsymbol{r}) = \Phi_\alpha(\boldsymbol{r}) e^{i\boldsymbol{K}\cdot\boldsymbol{r}} + \text{c.c.}$$

$\Phi_\alpha(\boldsymbol{r})$ is a complex field $\left(\text{except for } \boldsymbol{K}=(\pi,\pi) \text{ when } e^{i\boldsymbol{K}\cdot\boldsymbol{r}} = (-1)^{r_x+r_y}\right)$



Wavevector $\boldsymbol{K}=(3\pi/4,\pi)$

Exciton wavefunction $\Phi_\alpha(\boldsymbol{r})$ describes *envelope* of this order. Phase of $\Phi_\alpha(\boldsymbol{r})$ represents sliding degree of freedom

Associated "charge" density wave order

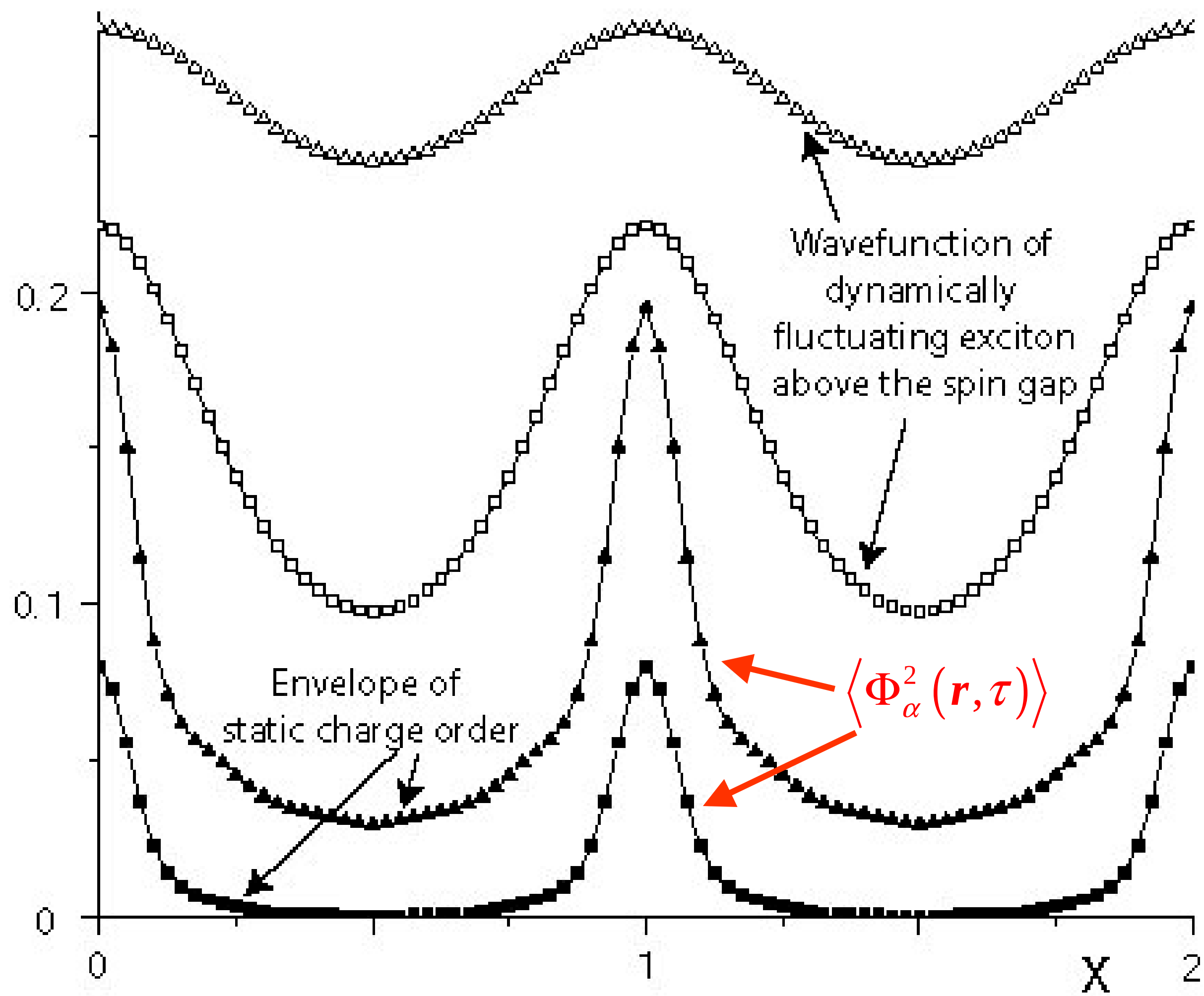
$$\delta\rho(\boldsymbol{r}) \propto S_\alpha^2(\boldsymbol{r}) = \sum_\alpha \Phi_\alpha^2(\boldsymbol{r}) e^{i2\boldsymbol{K}\cdot\boldsymbol{r}} + \text{c.c.}$$

J. Zaanen and O. Gunnarsson, *Phys. Rev.* B **40**, 7391 (1989).
H. Schulz, *J. de Physique* **50**, 2833 (1989).
O. Zachar, S. A. Kivelson, and V. J. Emery, *Phys. Rev.* B **57**, 1422 (1998).

Pinning of CDW order by vortex cores in SC phase

Y. Zhang, E. Demler, and S. Sachdev, cond-mat/0112343.

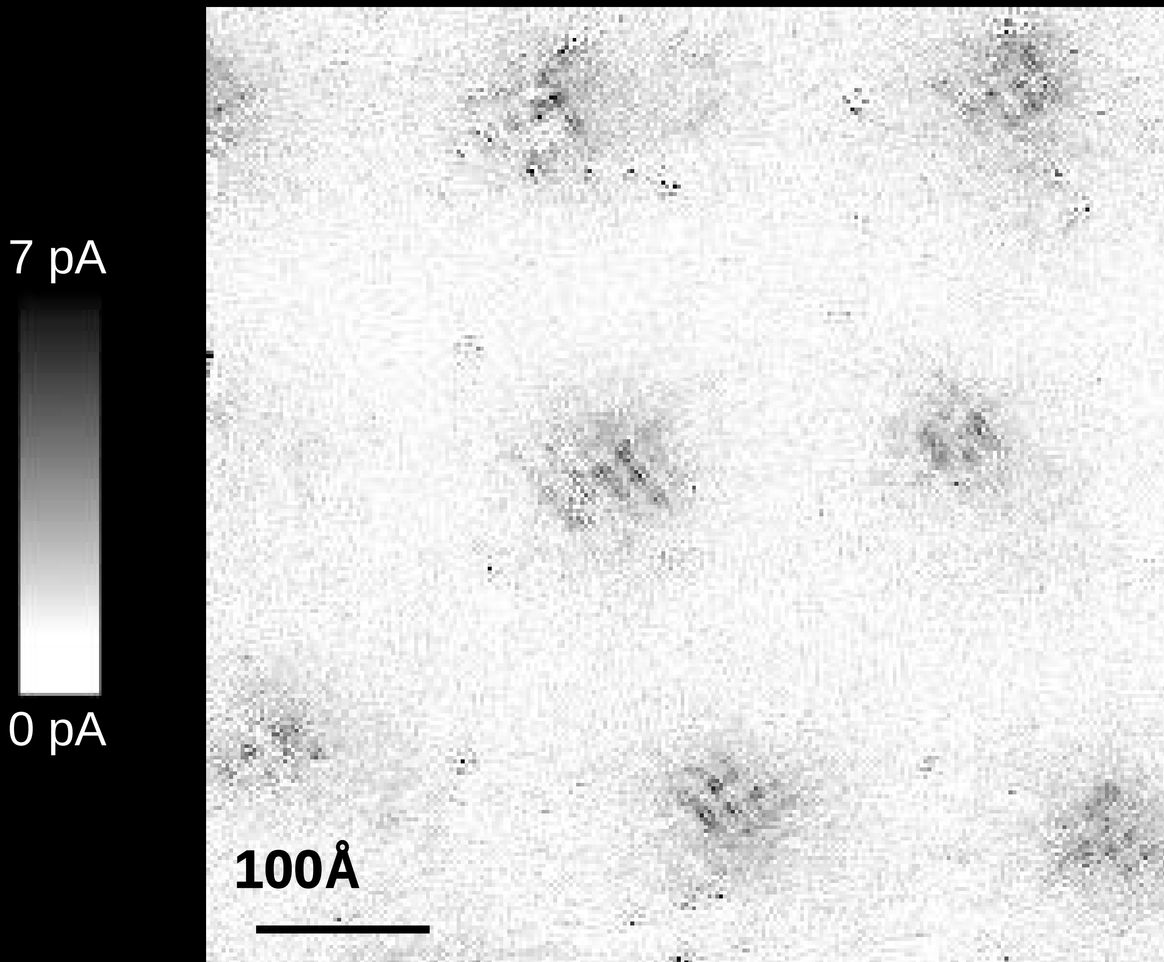
$$\left\langle \Phi_\alpha^2(\boldsymbol{r},\tau)\right\rangle \propto \zeta \int d\tau_1 \left\langle \Phi_\alpha(\boldsymbol{r},\tau)\Phi_\alpha^*(\boldsymbol{r}_v,\tau_1)\right\rangle^2$$



□ → low magnetic field

△ → high magnetic field near the boundary to the SC+SDW phase

Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV



J. Hoffman E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* 295, 466 (2002).

Talk T14.008

Conclusions

I. Cuprate superconductivity is associated with doping Mott insulators with charge carriers

II. The correct paramagnetic Mott insulator has charge-order and confinement of spinons

III. Mott insulator reveals itself vortices and near impurities. Predicted effects seen recently in STM and NMR experiments.

IV. Semi-quantitative predictions for neutron scattering measurements of spin-density-wave order in superconductors; theory also establishes connection to STM experiments.

V. Future experiments should search for SC+SDW to SC quantum transition driven by a magnetic field.

VI. Major open question: how does understanding of low temperature order parameters help explain anomalous behavior at high temperatures ?