Competing orders in a magnetic field:
spin and charge density waves in the cuprates

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Talk online at
http://pantheon.yale.edu/~subir
(Search for “Sachdev” on Google)
Zero temperature phases of the cuprate superconductors as a function of hole density

Neel LRO

SDW along (1,1) + insulator with localized holes

SDW oriented along (1,0)

Superconductor + Spin-density-wave
SC + SDW

~0.05 ~0.12

H

Theory for a system with strong interactions: describe SC and SC+SDW phases by expanding in the deviation from the quantum critical point between them.

J. E. Sonier et al., cond-mat/0108479.
Concentration of mobile carriers $\delta$ in, e.g., $\text{La}_2\text{CuO}_4$. Further neighbor magnetic couplings. Universal properties of magnetic quantum phase transition change little along this line. $T=0$, $\left\langle \vec{S} \right\rangle = 0$. Experiments. 

La$_2$CuO$_4$.

$\delta$ in, e.g., La$_{2-\delta}$Sr$_\delta$CuO$_4$.


Outline

I. Magnetic ordering transitions in the insulator ($\delta=0$).

II. Doping the Mott insulator
   Charge order nucleated by vortices

III. SC+SDW to SC transition: influence of an applied magnetic field.
   Neutron scattering measurements

IV. Conclusions
I. Magnetic ordering transitions in the insulator

\[ H = \sum_{i \neq j} J_{ij} \vec{S}_i \cdot \vec{S}_j \]

Action for collinear antiferromagnetic order parameter \( \phi_\alpha (\alpha=1,2,3) \):

\[ S_b = \int d^2 x d\tau \left[ \frac{1}{2} \left( (\nabla_x \phi_\alpha)^2 + c^2 (\partial_\tau \phi_\alpha)^2 \right) + V(\phi_\alpha^2) \right] \]


**Missing: Spin Berry Phases**

These are crucial in one dimension to obtain Bethe or Majumdar-Ghosh states for \( S=1/2 \) and the Haldane state for \( S=1 \).

In two dimensions, Berry phases induce **bond-centered charge order** in quantum “disordered” phase with \( \langle \phi_\alpha \rangle = 0 \).

Field theory of quantum “disordered” phase

Discretize spacetime into a cubic lattice:

\[ Z = \prod_j \int d{n}_j \delta(n_j^2 - 1) \exp \left( -\frac{1}{2g} \sum_{j,\mu} {n}_j \cdot {n}_{j+\mu} - \frac{i}{2} \sum_j \eta_j A_{j\tau} \right) \]

\( j \) → cubic lattice sites; \( \mu \) → \( x, y, \tau \);

\( \eta_j \) → ±1 on two square sublattices; \( n_j \sim \eta_j \vec{S}_j \) → Neel order parameter;

\( A_{j\mu} \) → oriented area of spherical triangle formed by \( n_j \), \( n_{j+\mu} \), and an arbitrary reference point \( n_0 \).

For large \( g \), perform a “high temperature” expansion to obtain an effective action for the \( A_{j\mu} \). This must be invariant under the “gauge transformation”

\[ A_{j\mu} \rightarrow A_{j\mu} - \gamma_{j+\mu} + \gamma_j \]

associated with a change in choice of \( n_0 \) (\( \gamma_j \) is the oriented area of the spherical triangle formed by \( n_j \) and the two choices for \( n_0 \)).

Also the area of the triangle is uncertain modulo \( 4\pi \), and so the effective action should be invariant under

\[ A_{j\mu} \rightarrow A_{j\mu} + 4\pi \]
Simplest large $g$ effective action for the $A_{j\mu}$

$$Z = \prod_{j, \mu} \int dA_{j\mu} \exp \left( -\frac{1}{2e^2} \sum \cos \left( \frac{1}{2} \epsilon_{\mu\nu\lambda} \Delta_\nu A_{j\lambda} \right) - \frac{i}{2} \sum \eta_j A_{j\tau} \right)$$

with $e^2 \sim g^2$

This compact QED in 2+1 dimensions with Berry phases.

This theory can be reliably analyzed by a duality mapping.

The gauge theory is \textit{always} in a \textit{confining} phase:

There is an energy gap and the ground state has \textbf{spontaneous bond order}.
Square lattice with first (J₁) and second (J₂) neighbor exchange interactions (say)

\[ H = \sum_{i<j} J_{ij} \vec{S}_i \cdot \vec{S}_j \]


See however L. Capriotti, F. Becca, A. Parola, S. Sorella, cond-mat/0107204.

Studies on the 2D pyrochlore lattice agree with related predictions of theory:

J.-B. Fouet, M. Mambrini, P. Sindzingre, C. Lhuillier, cond-mat/0108070.
R. Moessner, Oleg Tchernyshyov, S.L. Sondhi, cond-mat/0106286.
Framework for spin/charge order in cuprate superconductors

Further neighbor magnetic couplings

Insulator with localized holes

La$_2$CuO$_4$

Magnetic order

$\langle \bar{S} \rangle \neq 0$

T=0

Concentration of mobile carriers $\delta$

Confined, paramagnetic Mott insulator has

1. Stable $S=1$ spin exciton.
2. Broken translational symmetry:- bond-centered charge order.
3. $S=1/2$ moments near non-magnetic impurities
II. Doping the Mott insulator

“Large $N$” theory in region with preserved spin rotation symmetry

See also J. Zaanen, *Physica* C 217, 317 (1999),
Charge order nucleated by vortices

Memory of the Mott insulator should survive in and around vortices in superconducting order: superconductivity is suppressed in the vortex core, but the electrons should still strive to retain the exchange correlation energy of the Mott insulator. The vortex core is not a “normal Fermi liquid” as in BCS theory. This is the primary failure of BCS theory in the cuprate superconductors.


Local magnetic order in the vortex core is “quantum-disordered”: so there is a spin gap and charge order should appear, as in the doped paramagnetic Mott insulator.

Vortex-induced LDOS of Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ integrated from 1meV to 12meV

STM image of pinned charge order in Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ in zero magnetic field

Charge order period = 4 lattice spacings

C. Howald, H. Eisaki, N. Kaneko, and A. Kapitulnik, cond-mat/0201546
Observation of static charge order in YBa$_2$Cu$_3$O$_{6.35}$

(spinn correlations are dynamic)

Charge order period = 8 lattice spacings

**FIG. 1.** Measurements of the charge order for YBCO6.35. (a) Measurements obtained at a small momentum transfer so the results are not affected by impurity powder lines. Powder lines were also avoided around the (1.125, 0, 1.3) c.l.u. position shown in (b). The lines are Gaussian fits to the data. In (a) 200 and (b) 100 additional counts were added onto successive scans so the data could be presented on the same plot. The scattering broadens at higher temperatures.

H. A. Mook, Pengcheng Dai, and F. Dogan
II. Doping the Mott insulator

“Large N” theory in region with preserved spin rotation symmetry

See also J. Zaanen, *Physica C* 217, 317 (1999),
III. SC+SDW to SC transition: influence of an applied magnetic field

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SC+SDW

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J. E. Sonier et al., cond-mat/0108479.
Dominant effect: **uniform** softening of spin excitations by superflow kinetic energy

Spatially averaged superflow kinetic energy

\[ \sim \left< v_s^2 \right> \sim \frac{H}{H_{c2}} \ln \frac{3H_{c2}}{H} \]

Coupling determining spin excitation energy, \( s \),

replaced by \( s_{\text{eff}}(H) = s - C \frac{H}{H_{c2}} \ln \left( \frac{3H_{c2}}{H} \right) \)

**Structure of long-range SDW order in SC+SDW phase**


Dynamic structure factor

\[ S(k, \omega) = (2\pi)^3 \delta(\omega) \sum_G |f_G|^2 \delta(k - G) + \cdots \]

\[ G \rightarrow \text{reciprocal lattice vectors of vortex lattice.} \]

\[ k \text{ measures deviation from SDW ordering wavevector } K \]

\[ \delta |f_0|^2 \propto H \ln(1/H) \]

\[ s - s_c = -0.3 \]

Neutron scattering measurements of static spin correlations of the superconductor+spin-density-wave (SC+SDW) in a magnetic field

Elastic neutron scattering off La$_2$CuO$_{4+y}$


Solid line --- fit to:

\[
\frac{I(H)}{I(0)} = 1 + a \frac{H}{H_{c2}} \ln \left( \frac{3.0H_{c2}}{H} \right)
\]

\(a\) is the only fitting parameter

Best fit value - \(a = 2.4\) with \(H_{c2} = 60\) T
Neutron scattering of La$_{2-x}$Sr$_x$CuO$_4$ at $x=0.1$


**Solid line - fit to:**

$$I(H) = a \frac{H}{H_{c2}} \ln \left( \frac{H_{c2}}{H} \right)$$
Main results

Neutron scattering observation of SDW order enhanced by superflow.

STM observation of CDW fluctuations enhanced by superflow and pinned by vortex cores.

Effect of magnetic field on SDW+SC to SC transition (extreme Type II superconductivity)

\[ T = 0 \]

Quantitative connection between the two experiments?

Spin density wave order parameter for general ordering wavevector

\[ S_{\alpha}(r) = \Phi_{\alpha}(r)e^{iK \cdot r} + \text{c.c.} \]

\[ \Phi_{\alpha}(r) \text{ is a complex field (except for } K=(\pi,\pi) \text{ when } e^{iK \cdot r} = (-1)^{r_x+r_y} \]  

Wavevector \( K=(3\pi/4,\pi) \)

Exciton wavefunction \( \Phi_{\alpha}(r) \) describes envelope of this order. Phase of \( \Phi_{\alpha}(r) \) represents sliding degree of freedom

Associated “charge” density wave order

\[ \delta \rho(r) \propto S_{\alpha}^2(r) = \sum_{\alpha} \Phi_{\alpha}^2(r)e^{i2K \cdot r} + \text{c.c.} \]

Pinning of CDW order by vortex cores in SC phase

Y. Zhang, E. Demler, and S. Sachdev, cond-mat/0112343.

$$\langle \Phi_\alpha^2 (r, \tau) \rangle \propto \zeta \int d \tau_1 \langle \Phi_\alpha (r, \tau) \Phi^*_\alpha (r_v, \tau_1) \rangle^2$$
Vortex-induced LDOS of Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ integrated from 1 meV to 12 meV

Conclusions

I. Cuprate superconductivity is associated with doping Mott insulators with charge carriers

II. The correct paramagnetic Mott insulator has charge-order and confinement of spinons

III. Mott insulator reveals itself vortices and near impurities. Predicted effects seen recently in STM and NMR experiments.

IV. Semi-quantitative predictions for neutron scattering measurements of spin-density-wave order in superconductors; theory also establishes connection to STM experiments.

V. Future experiments should search for SC+SDW to SC quantum transition driven by a magnetic field.

VI. Major open question: how does understanding of low temperature order parameters help explain anomalous behavior at high temperatures?