# Quantum phase transitions in antiferromagnets and superfluids:

## Universal damping of quasiparticles and the spin collective mode in d-wave superconductors

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Transparencies on-line at
http://pantheon.yale.edu/~subir/mar00.pdf

Phys. Rev. Lett. **83**, 3916 (1999).
Science **286**, 2479 (1999).
cond-mat/9912020 (Phys. Rev. B in press)
cond-mat/0003163
Talk by M. Vojta (**E11.011)**

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1. Phase diagram of an extended t-J model.

2. Spin gaps and the magnetic ordering transition.

Comparison to neutron scattering on Zn-doped YBCO

(Fong et al PRL **82**, 1939 (1999))

Universal broadening of S=1 collective mode peak.

3. Time-reversal symmetry breaking and charge-ordering in a d-wave superconductor.

Quantum-critical relaxation of quasiparticles in photoemission

(Valla et al Science **285**, 2110 (1999))

and optical conductivity

(Corson et al cond-mat/0003243)

# 1. Phase diagram of an extended t-J model

Extended t-J model on the square lattice

$$H = \sum_{i>j} \left[ -t_{ij} c^{\dagger}_{i\alpha} c_{j\alpha} + J_{ij} \vec{S}_i \cdot \vec{S}_j + V_{ij} n_i n_j \right]$$

Plot phase diagram of stable ground states as a function of:

(1) Doping $\delta$

(2) Frustration $\dfrac{J_2 \text{ (second neighbor)}}{J_1 \text{(first neighbor)}}$

OR

$N$, where spin symmetry

$$SU(2) \rightarrow Sp(N)$$

Ground states are characterized by the manner in which symmetries are broken:

1. S - electromagnetic *U(1)*

2. M – magnetic *SU(2)*

3. C – lattice translations are reflections (must be broken by observables (like charge density) which are invariant under S and M)

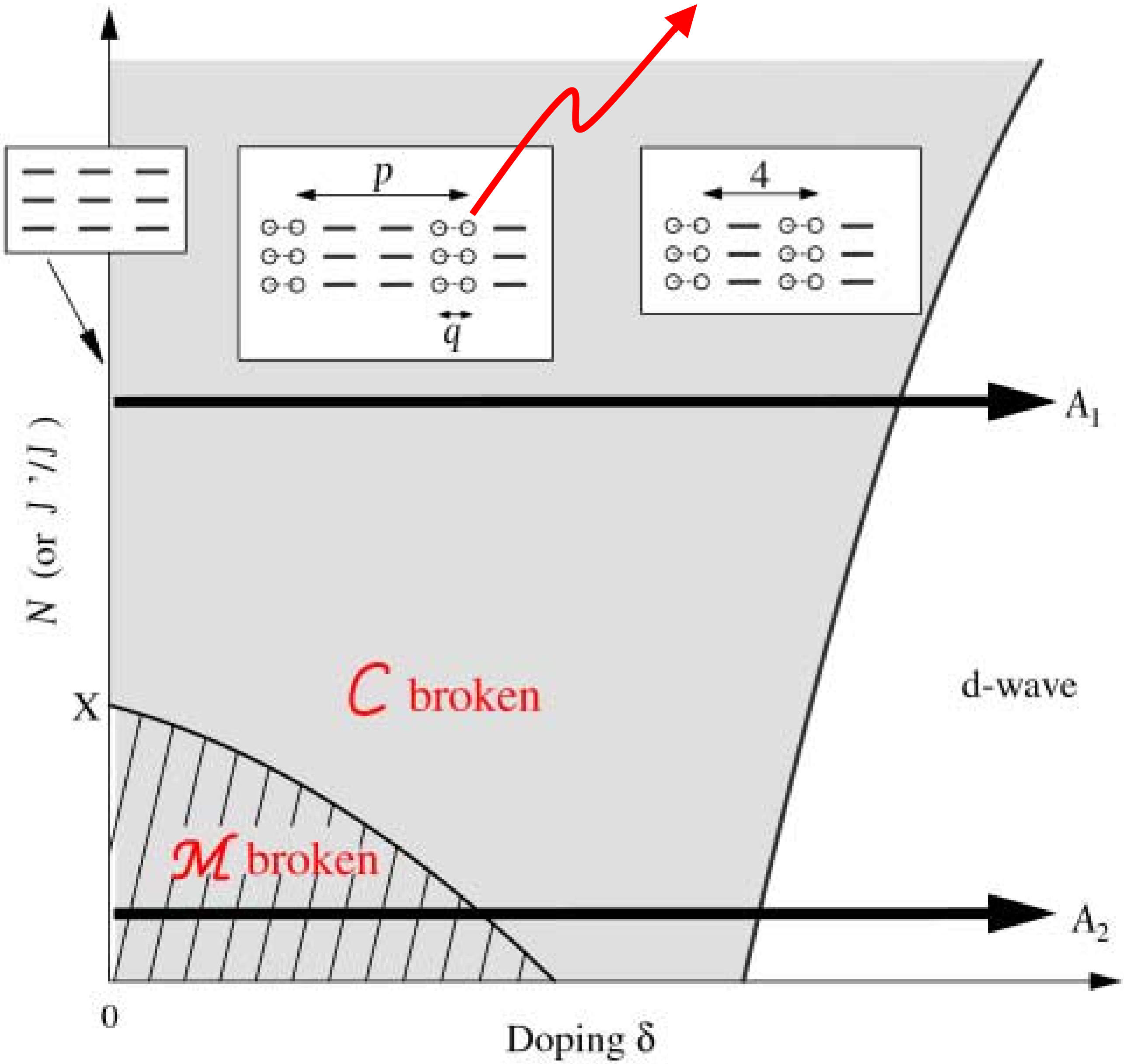
4. T – time-reversal symmetry.

e.g.
Neel state - breaks only M

Incommensurate, collinear spin-density-wave - breaks M and C

# Schematic phase diagram

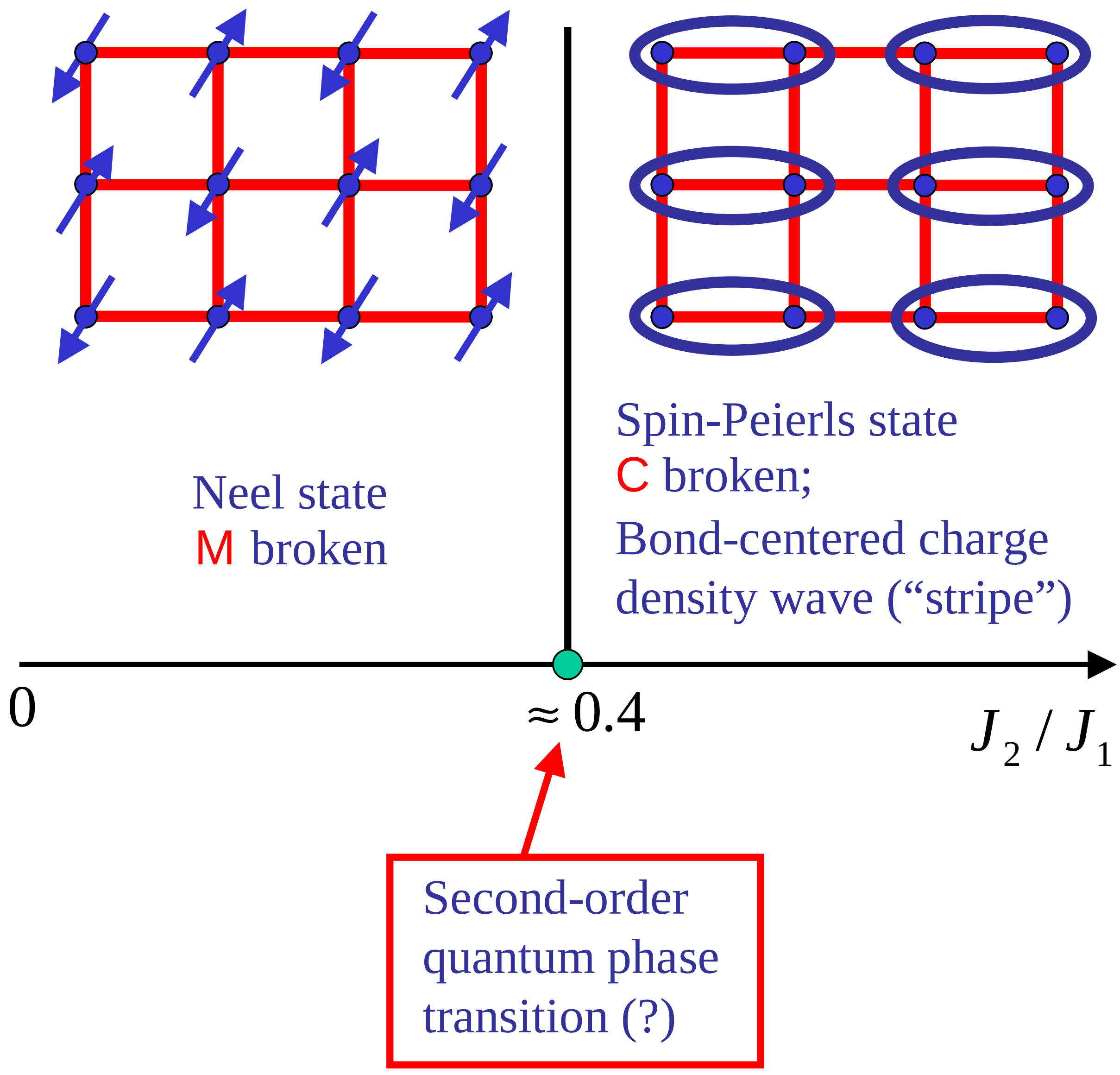
hole density ≈ 0.5/unit length




S broken for all δ>0 , large *N*

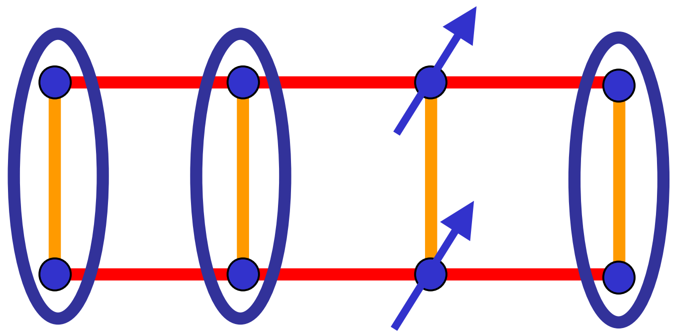
# 2. Spin gaps and the magnetic ordering transition

$\delta=0$




(Read and Sachdev, PRL **62**, 1694 (1989)
Kotov et al, PRB **60**, 14613 (1999)
Singh et al, PRB **60**, 7278 (1999))

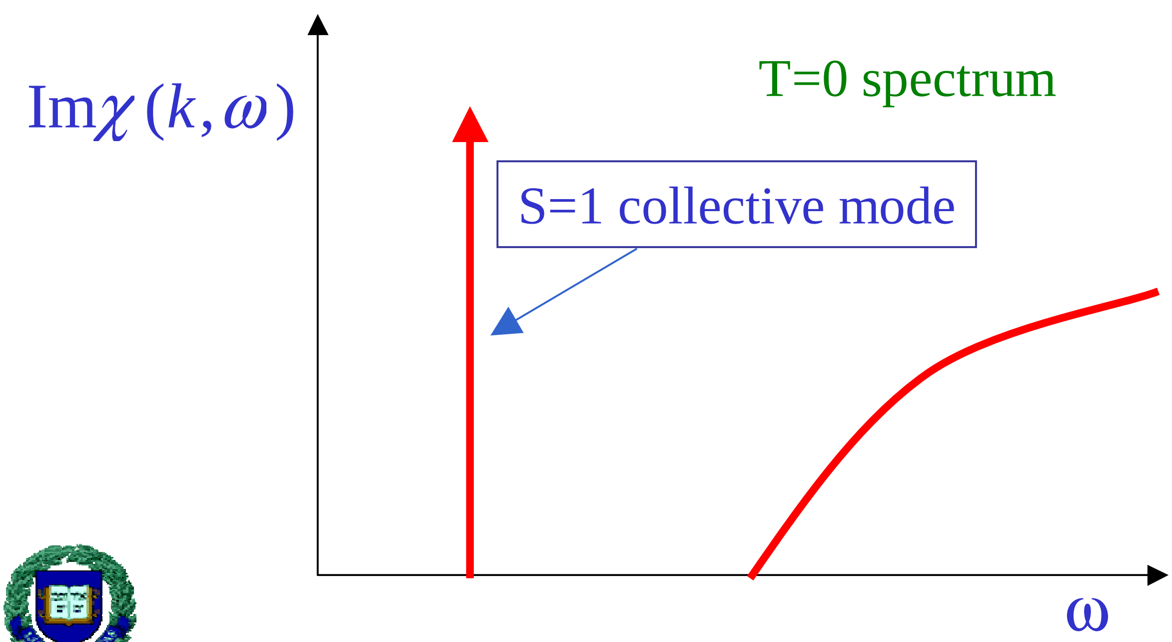
# Excited states in the paramagnet



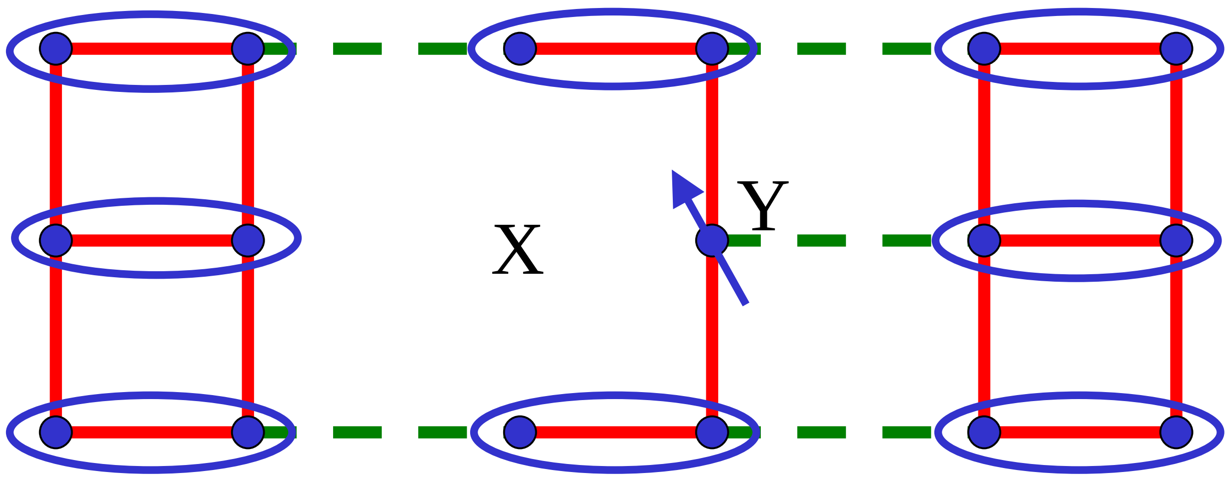
Triplet (S=1) particle (collective mode)
Energy dispersion away from antiferromagnetic wavevector

$$\varepsilon = \Delta + \frac{c^2 k^2}{2\Delta}$$

$\Delta \rightarrow$ Spin gap



# Zn impurity in the paramagnet (or d-wave superconductor)




Susceptibility $\chi = A\chi_b + \chi_{imp}$

($A$ = area of system)

In paramagnetic phase as $T \to 0$

$$\chi_b = \left(\frac{\Delta}{\hbar^2 c^2 \pi}\right) e^{-\Delta/k_B T} \quad ; \chi_{imp} = \frac{S(S+1)}{3k_B T}$$

$S = 1/2$

$\phi_\alpha$ → 3-component antiferromagnetic order parameter

Orientation of "impurity" spin → $n_\alpha(\tau)$ (unit vector)

Action of "impurity" spin

$$\mathcal{S}_{\text{imp}} = \int d\tau \left[ iSA_\alpha(n)\frac{dn_\alpha}{d\tau} - \gamma S n_\alpha(\tau)\phi_\alpha(x=0,\tau) \right]$$

$A_\alpha(n)$ → Dirac monopole function

Boundary quantum field theory: $\mathcal{S}_b + \mathcal{S}_{\text{imp}}$

Action for magnetic ordering transition in the bulk

$$\mathcal{S}_b = \int d^dx d\tau \left[ \frac{1}{2}\left( (\nabla_x \phi_\alpha)^2 + c^2(\partial_\tau \phi_\alpha)^2 + r\phi_\alpha^2 \right) + \frac{g}{4!}(\phi_\alpha^2)^2 \right]$$

Fundamental property: couplings $g$ and $\gamma$ approach universal values near quantum critical point

Finite density of impurities $n_{\text{imp}}$

Relevant perturbation – strength determined by only energy scale that is linear in $n_{\text{imp}}$ and contains only bulk parameters

Damping occurs on the energy scale

$$\Gamma \equiv \frac{n_{\text{imp}} (\hbar c)^2}{\Delta}$$

## Fate of collective mode peak

Without impurities $$\chi(G,\omega)=\frac{A}{\Delta^2-\omega^2}$$

With impurities $$\chi(G,\omega)=\frac{A}{\Delta^2}\Phi\left(\frac{\hbar\omega}{\Delta},\frac{\Gamma}{\Delta}\right)$$

$\Phi$ → *Universal* scaling function. We computed it in a "self-consistent, non-crossing" approximation

Predictions: Half-width of line $\approx \Gamma$

Universal asymmetric lineshape

$YBa_2Cu_3O_7 + 0.5\%$ Zn

$n_{imp} = 0.005$

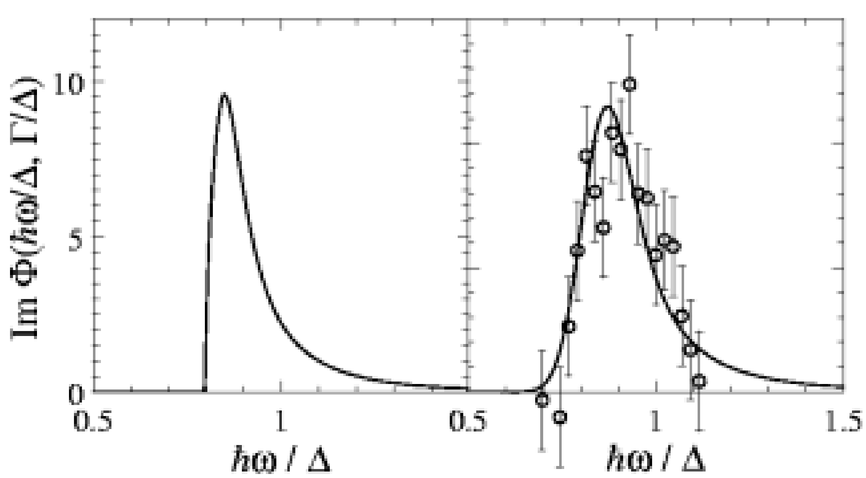
$\Delta = 40$ meV

$\hbar c = 0.2$ eV

$\Rightarrow \quad \Gamma = 5$ meV

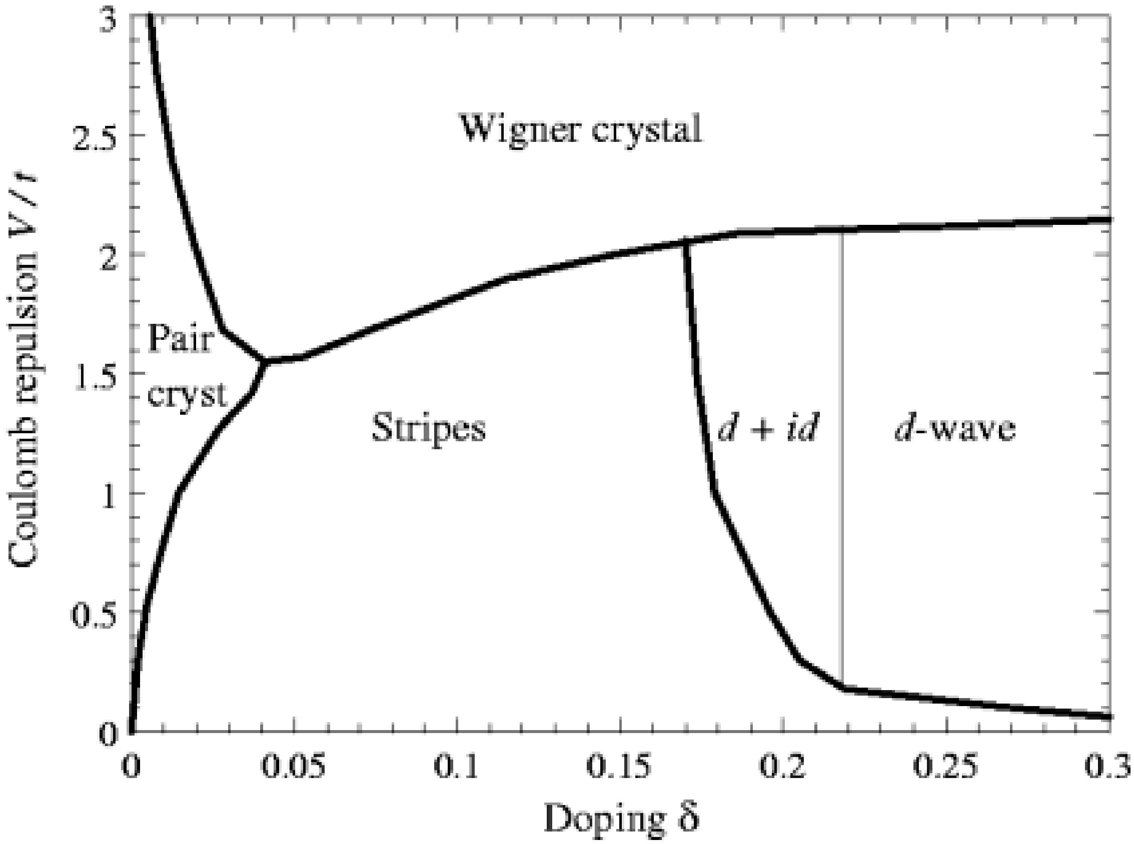
H. F. Fong et al. Phys. Rev. Lett. **82**, 1939 (1999)

Measured half-width = 4.25 meV

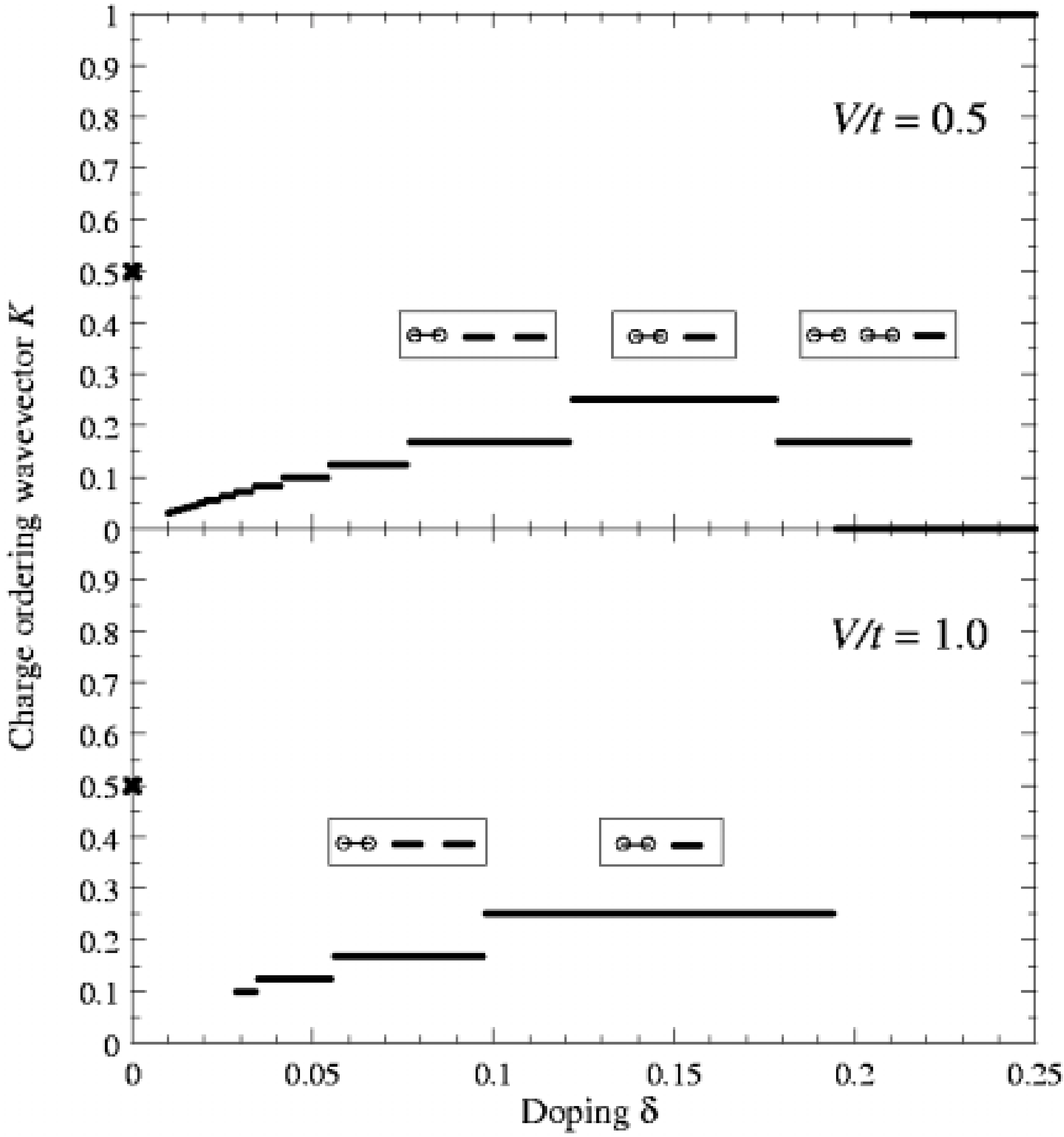


# 3. Time-reversal symmetry breaking and charge-ordering in a d-wave superconductor.

Sample phase diagram along $A_1$

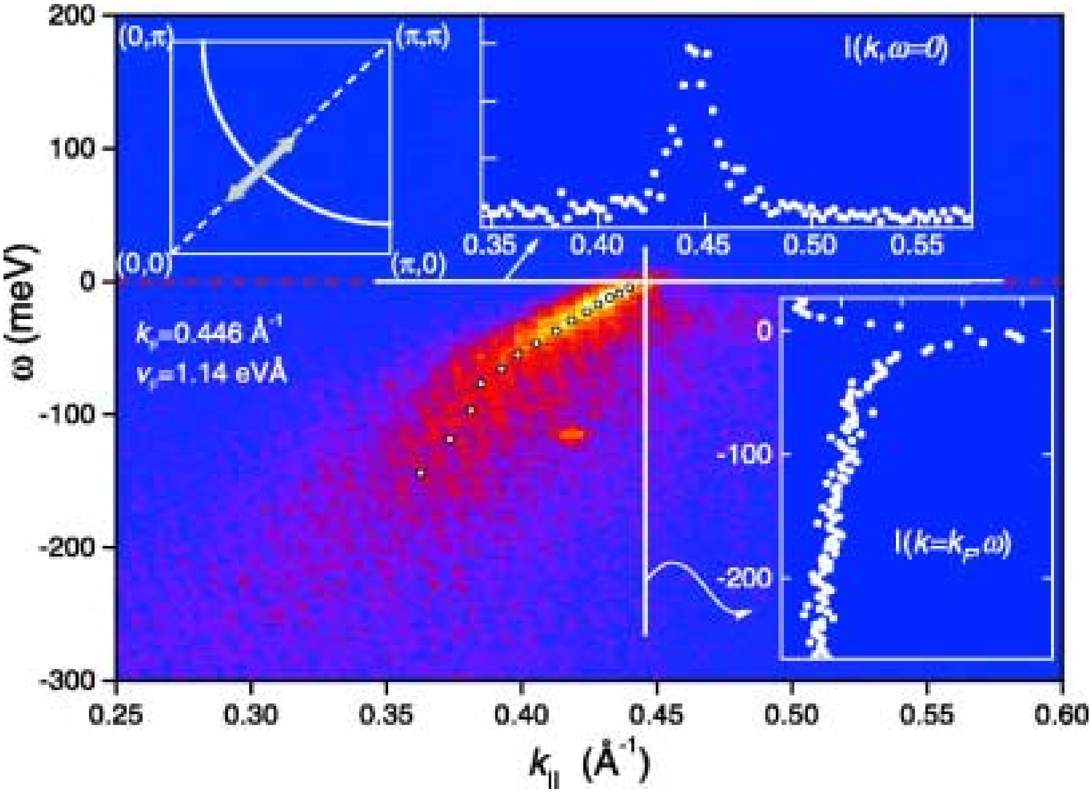


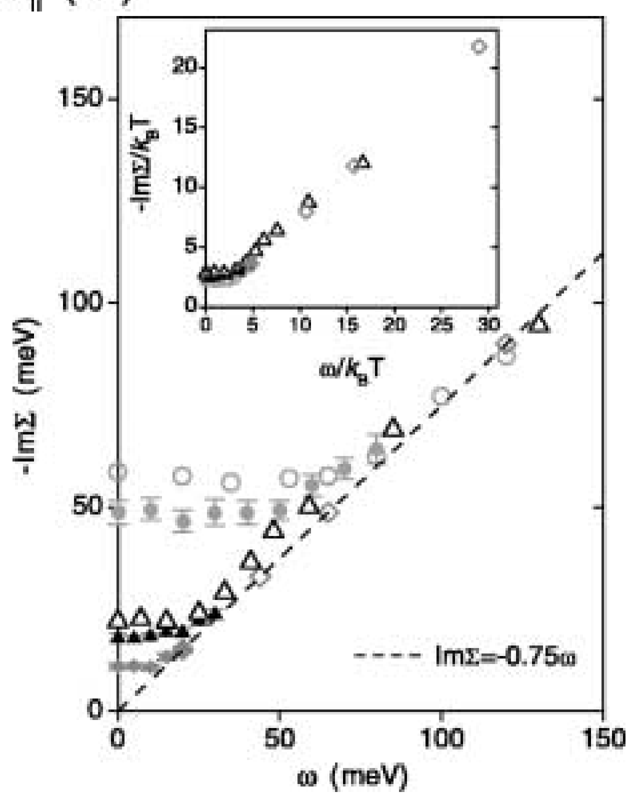
# Ordering wavevector of bond-centered stripes



# Photoemission on BSSCO

(Valla et al Science **285**, 2110 (1999))



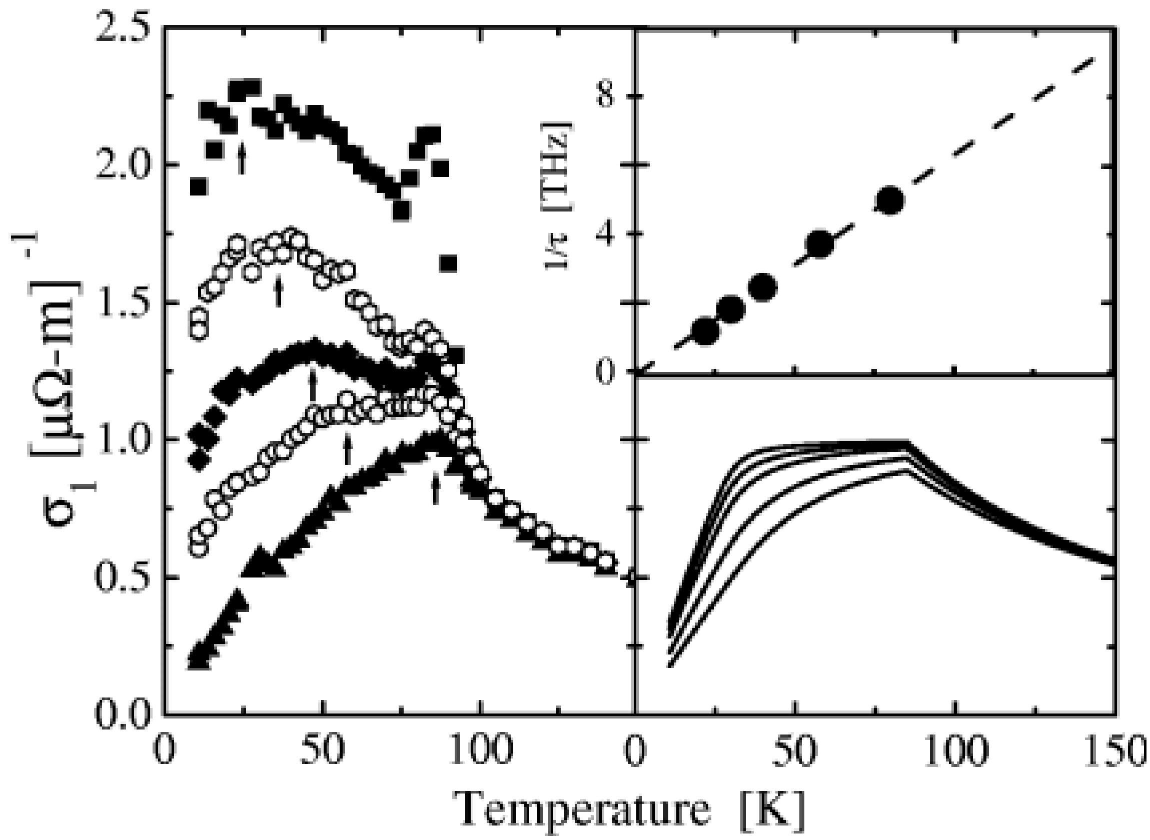




Quantum-critical damping of quasi-particles along (1,1)

Quasi-particles sharp along (1,0)

# THz conductivity of BSCCO

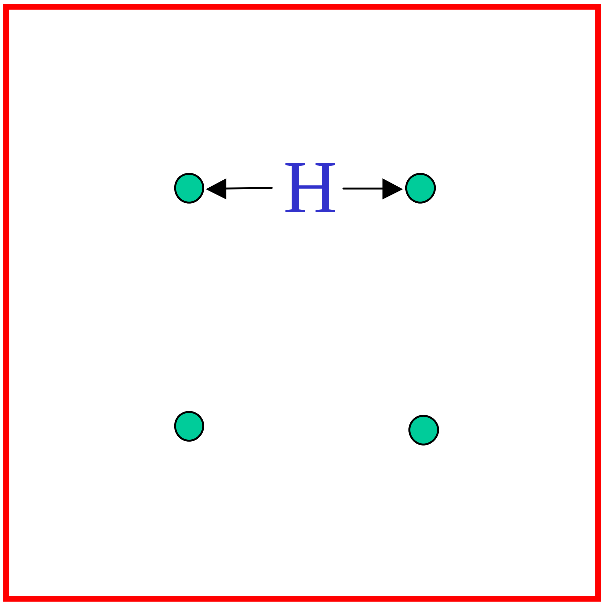
(Corson et al  cond-mat/0003243)




Quantum-critical damping of quasi-particles

# Candidate quantum-critical points

## A. Onset of C ordering




Gapless Fermi Points in a d-wave superconductor

Quantum-critical damping of fermions only if H = K (charge-orderingwavevector)

*T>0* fermion Green's function:

$$G_F(k,\omega) = \frac{\Lambda^{-\eta_F}}{(k_B T)^{1-\eta_F}} \mathrm{X}_1\left(\frac{ck}{k_B T}, \frac{\hbar\omega}{k_B T}\right)$$

## B. T breaking transition from a $d_{x^2-y^2}$ superconductor to a $d_{x^2-y^2}+id_{xy}$ superconductor

Attractive features:

1. No special commensuration required

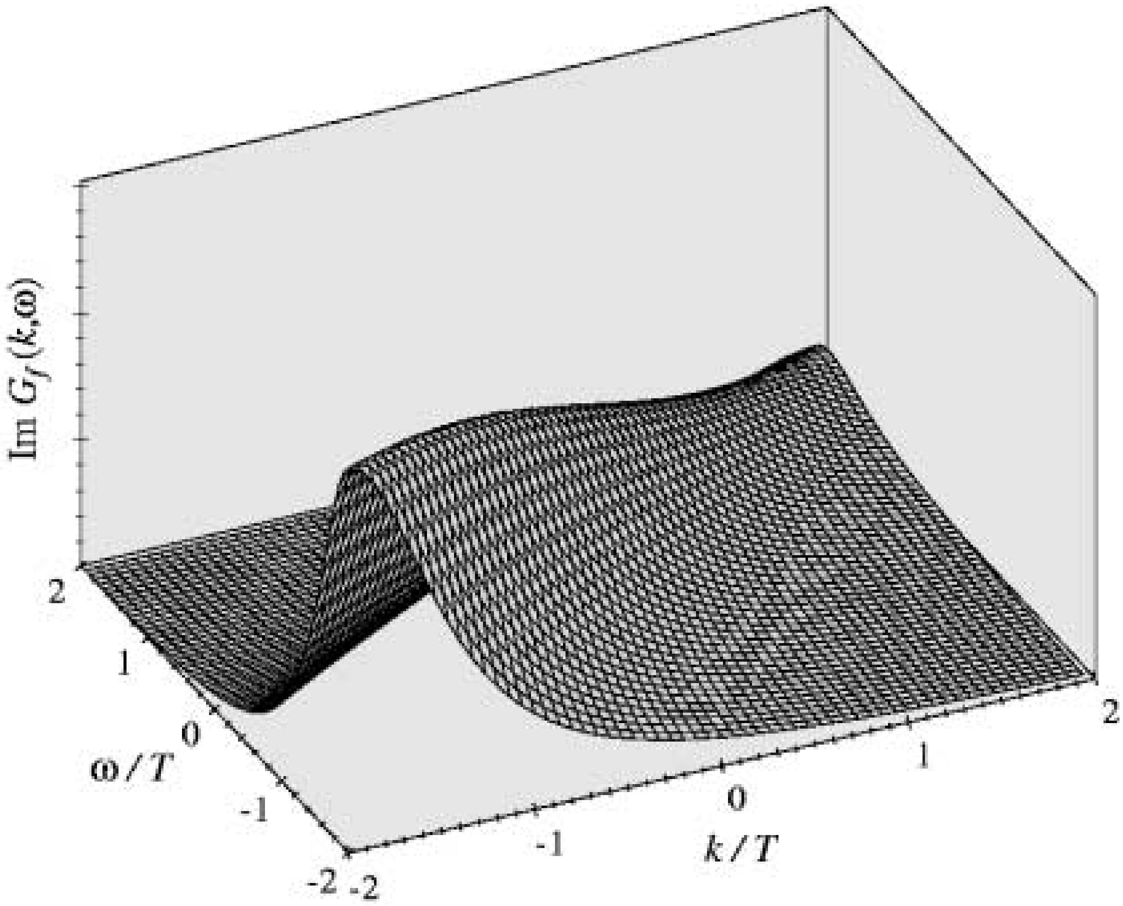
2. Order parameter ~ $\sin(k_x)\sin(k_y)$

so *maximal* damping along (1,1);

*no* damping along (1,0)

*T>0* fermion Green's function:

$$G_F(k,\omega) = \frac{\Lambda^{-\eta_F}}{(k_B T)^{1-\eta_F}} \mathrm{X}_1\left(\frac{ck}{k_B T}, \frac{\hbar\omega}{k_B T}\right)$$



*T>0* fermion Green's function:

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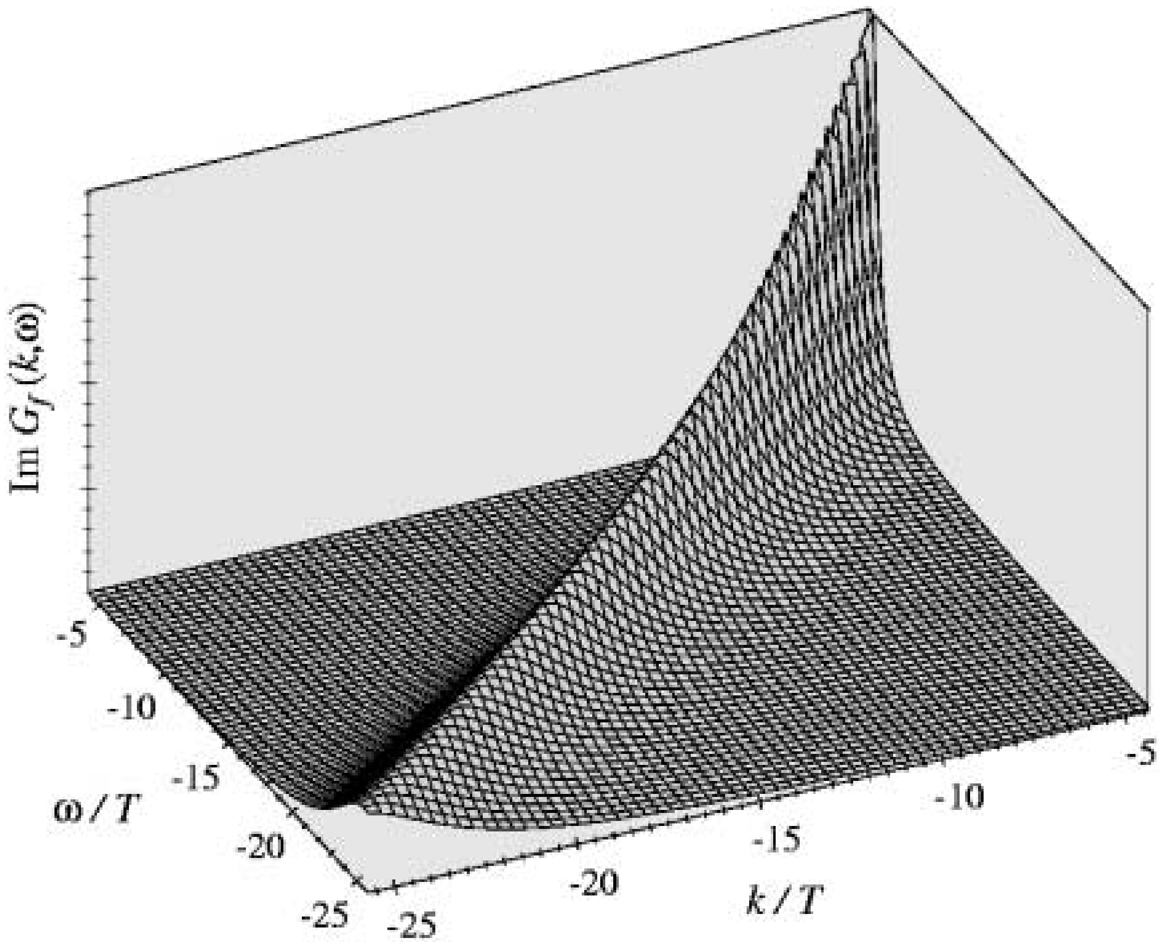

## Conclusions

1. Universal T=0 damping of S=1 collective mode by non-magnetic impurities.

   Linewidth: $$\Gamma \equiv \frac{n_{\text{imp}}(\hbar c)^2}{\Delta}$$

   independent of impurity parameters.

2. Classification of quantum-critical points leading to critical damping of quasiparticles in superconductor

   Most attractive possibility: T breaking transition from a $d_{x^2-y^2}$ superconductor to a $d_{x^2-y^2} + id_{xy}$ superconductor

   Leads to quantum-critical damping along (1,1), and no damping along (1,0), with no unnatural fine-tuning.

*Note*: stable ground state of cuprates can always be a $d_{x^2-y^2}$ superconductor; only need thermal/quantum fluctuations to $d_{x^2-y^2} + id_{xy}$ order in quantum-critical region.