Quantum criticality
and
high temperature superconductivity

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Foundations of quantum many body theory:

1. Ground states connected adiabatically to independent electron states

Metals
Foundations of quantum many body theory:

1. Ground states connected adiabatically to independent electron states
2. Quasiparticle structure of excited states
Modern phases of quantum matter:

1. Ground states disconnected from independent electron states: many-particle entanglement

2. Quasiparticle structure of excited states

Famous examples:

The fractional quantum Hall effect of electrons in two dimensions (e.g. in graphene) in the presence of a strong magnetic field. The ground state is described by Laughlin’s wavefunction, and the excitations are quasiparticles which carry fractional charge.
Modern phases of quantum matter:

1. Ground states disconnected from independent electron states: many-particle entanglement

2. Quasiparticle structure of excited states

Famous examples:

Electrons in one dimensional wires form the Luttinger liquid. The quanta of density oscillations ("phonons") are a quasiparticle basis of the low-energy Hilbert space. Similar comments apply to magnetic insulators in one dimension.
Modern phases of quantum matter:

1. Ground states disconnected from independent electron states: many-particle entanglement

2. No quasiparticles
Modern phases of quantum matter:

1. Ground states disconnected from independent electron states: many-particle entanglement

2. No quasiparticles

Only 2 examples:

1. Conformal field theories in spatial dimension $d > 1$
Modern phases of quantum matter:

1. Ground states disconnected from independent electron states: many-particle entanglement

2. No quasiparticles

Only 2 examples:

1. Conformal field theories in spatial dimension \( d > 1 \)

2. Quantum critical metals in dimension \( d = 2 \)
I. The simplest model without quasiparticles
   A. Superfluid-insulator transition of ultracold bosonic atoms in an optical lattice
   B. Conformal field theories in 2+1 dimensions and the AdS/CFT correspondence

2. Metals without quasiparticles
   A “non-Fermi” liquid in the high temperature superconductors: the Ising-nematic quantum critical point
Outline

1. The simplest model without quasiparticles
   A. Superfluid-insulator transition of ultracold bosonic atoms in an optical lattice
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Superfluid-insulator transition


Ultracold $^{87}$Rb atoms - bosons
$h_i^6 = 0$  \[ \Rightarrow \ U/t \]
\[ \Psi \rightarrow \text{a complex field representing the Bose-Einstein condensate of the superfluid} \]

\[ \langle \Psi \rangle \neq 0 \quad \text{Superfluid} \]

\[ \langle \Psi \rangle = 0 \quad \text{Insulator} \]

\[ \lambda \sim \frac{U}{t} \]
\[ S = \int d^2 r dt \left[ |\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right] \]

\[ V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u \left( |\Psi|^2 \right)^2 \]

\[ \lambda \sim U/t \]

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Particles and holes correspond to the 2 normal modes in the oscillation of \( \Psi \) about \( \Psi = 0 \).

\[ \langle \Psi \rangle \neq 0 \]

Superfluid

\[ \langle \Psi \rangle = 0 \]

Insulator

\[ \lambda \sim U/t \]

\[ \lambda_c \]

0

\[ \text{Re}(\Psi) \]

\[ \text{Im}(\Psi) \]

\[ V \]
\[ U \gg t \]

Insulator (the vacuum) at large repulsion between bosons

\[ |\text{Ground state}\rangle = \prod_i b_i^\dagger |0\rangle \]
$U \gg t$

Excitations of the insulator:

Particles $\sim \psi^\dagger$
Excitations of the insulator:

$U \gg t$

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Excitations of the insulator:

Particles $\sim \psi^\dagger$
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Excitations of the insulator:

Holes $\sim \psi$
Excitations of the insulator:

\[ U \gg t \]

Holes \( \sim \psi \)
$U \gg t$

Excitations of the insulator:

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$S = \int d^2r dt \left[ |\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]$

$V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u \left( |\Psi|^2 \right)^2$

Particles and holes correspond to the 2 normal modes in the oscillation of $\Psi$ about $\Psi = 0$. 

$\langle \Psi \rangle \neq 0$

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$\langle \Psi \rangle = 0$

Insulator

$\lambda_c$

$\lambda \sim U/t$
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Observation of Higgs quasi-normal mode across the superfluid-insulator transition of ultracold atoms in a 2-dimensional optical lattice:

Response to modulation of lattice depth scales as expected from the LHP pole

Quantum state with complex, many-body, “long-range” quantum entanglement

\[
S = \int d^2r dt \left[ |\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right]
\]

\[
V(\Psi) = (\lambda - \lambda_c)|\Psi|^2 + u (|\Psi|^2)^2
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\[
\langle \Psi \rangle \neq 0 \quad \text{Superfluid}
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\langle \Psi \rangle = 0 \quad \text{Insulator}
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\[
\lambda_c \sim U/t
\]
• Long-range entanglement
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- No quasiparticles - no simple description of excitations.
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- The low energy excitations are described by a theory which has the same structure as Einstein’s theory of special relativity, but with the spin-wave velocity playing the role of the velocity of light.
• Long-range entanglement

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• The low energy excitations are described by a theory which has the same structure as Einstein’s theory of special relativity, but with the spin-wave velocity playing the role of the velocity of light.

• The theory of the critical point is strongly-coupled because the quartic-coupling $u$ flows to a renormalization group fixed point (the Wilson-Fisher fixed point). This fixed point has an even larger symmetry corresponding to conformal transformations of spacetime: we refer to such a theory as a CFT3.
A conformal field theory in 2+1 spacetime dimensions: a CFT3

\begin{align*}
S &= \int d^2 r dt \left[ |\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi) \right] \\
V(\Psi) &= (\lambda - \lambda_c) |\Psi|^2 + u (|\Psi|^2)^2
\end{align*}
This diagram illustrates the phase diagram of a quantum system. The horizontal axis represents the coupling parameter $\lambda \sim U/t$, and the vertical axis represents the temperature $T$. The region labeled "Quantum critical" separates the "Superfluid" phase from the "Insulator" phase. The critical temperature $T_{KT}$ is indicated by a green line. The diagram shows the transition between these phases as a function of $T$ and $\lambda$. The crossing point $\lambda_c$ marks a critical point in the phase diagram.
"Boltzmann" theory of Nambu-Goldstone and vortices

Boltzmann theory of particles/holes

Superfluid

Insulator

Quantum critical

$T$

$\lambda_c$

$\lambda \sim U/t$
CFT3 at $T>0$

- Insulator
- Superfluid
- Quantum critical

Graph showing transitions: $T_{KT}$, $\lambda_c$, $\lambda \sim U/t$
CFT3 at $T>0$

Boltzmann theory of particles/holes/vortices does not apply
Electrical transport in a free quasiparticle CFT3 for $T > 0$

$\sigma$

$\sim T \delta(\omega)$

$\omega/T$
Quasiparticle view of quantum criticality (Boltzmann equation): Electrical transport for a (weakly) interacting CFT3

$\text{Re}[\sigma(\omega)]$

$O(1/(u^*)^2)$

$O((u^*)^2)$,
where $u^*$ is the fixed point interaction

Quasiparticle view of quantum criticality (Boltzmann equation): Electrical transport for a (weakly) interacting CFT3

\[ \sigma(\omega, T) = \frac{e^2}{\hbar} \sum \left( \frac{\hbar \omega}{k_B T} \right) \]

\[ \text{Re}[\sigma(\omega)] \]

\[ \mathcal{O}(1/(u^*)^2) \]

\[ \mathcal{O}((u^*)^2) \]

where \( u^* \) is the fixed point interaction

\[ \Sigma \to \text{a universal function} \]

Universal conductivity \( \sim e^2 / \hbar \)

Universal time scale \( \sim \hbar / k_B T \)

Quasiparticle view of quantum criticality (Boltzmann equation): Electrical transport for a (weakly) interacting CFT3

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$$\text{Re}[\sigma(\omega)] \sim O(1/(u^*)^2)$$

$$\Sigma \rightarrow \text{a universal function}$$

Universal conductivity $\sim e^2/\hbar$

Universal time scale $\sim \hbar/k_B T$

Needed: a method for computing the universal conductivity of strongly interacting CFT3s

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   A “non-Fermi” liquid in the high temperature superconductors: the Ising-nematic quantum critical point
Field theories in $d + 1$ spacetime dimensions are characterized by couplings $g$ which obey the renormalization group equation

$$u \frac{dg}{du} = \beta(g)$$

where $u$ is the energy scale. The RG equation is local in energy scale, i.e. the RHS does not depend upon $u$. 
Key idea: Implement $u$ as an extra dimension, and map to a local theory in $d+2$ spacetime dimensions.
Key idea: Implement $u$ as an extra dimension, and map to a local theory in $d+2$ spacetime dimensions. We identify the extra-dimensional co-ordinate $r = 1/u$. 

Holography
For a relativistic CFT in $d$ spatial dimensions, the proper length, $ds$, in the holographic space is fixed by demanding the scale transformation ($i = 1 \ldots d$)

$$x_i \rightarrow \zeta x_i \quad , \quad t \rightarrow \zeta t \quad , \quad ds \rightarrow ds$$
This gives the unique metric

$$ds^2 = \frac{1}{r^2} \left( -dt^2 + dr^2 + dx_i^2 \right)$$

This is the metric of anti-de Sitter space $\text{AdS}_{d+2}$. 
This emergent spacetime is a solution of Einstein gravity with a negative cosmological constant

\[ S_E = \int d^4 x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) \right] \]
There is a family of solutions of Einstein gravity which describe non-zero temperatures.

\[ S_E = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) \right] \]
Gauge-gravity duality at non-zero temperatures

A 2+1 dimensional system at $T > 0$ with couplings at its quantum critical point

$$S_E = \int d^4 x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) \right]$$
A “horizon”, similar to the surface of a black hole!

\[ S_E = \int d^4 x \sqrt{-g} \left[ \frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) \right] \]

A 2+1 dimensional system at \( T > 0 \) with couplings at its quantum critical point.
Gauge-gravity duality at non-zero temperatures

A 2+1 dimensional system at $T > 0$ with couplings at its quantum critical point

The temperature and entropy of the horizon equal those of the quantum critical point
Gauge-gravity duality at non-zero temperatures

The temperature and entropy of the horizon equal those of the quantum critical point.

Quasi-normal modes of quantum criticality = waves falling into black hole

A 2+1 dimensional system at $T > 0$ with couplings at its quantum critical point.
Gauge-gravity duality at non-zero temperatures

A 2+1 dimensional system at $T > 0$ with couplings at its quantum critical point

Characteristic damping time of quasi-normal modes:

$\left( \frac{k_B}{\hbar} \right) \times$ Hawking temperature

The temperature and entropy of the horizon equal those of the quantum critical point
Traditional CMT

- Identify quasiparticles and their dispersions
- Compute scattering matrix elements of quasiparticles (or of collective modes)
- These parameters are input into a quantum Boltzmann equation
- Deduce dissipative and dynamic properties at non-zero temperatures
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**Holography and black-branes**

- Start with strongly interacting CFT without particle- or wave-like excitations
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### Traditional CMT

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### Holography and black-branes

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AdS$_4$ theory of quantum criticality

Most general effective holographic theory for linear charge transport with 4 spatial derivatives:

\[
S_{\text{bulk}} = \frac{1}{g^2_M} \int d^4 x \sqrt{g} \left[ \frac{1}{4} F_{ab} F^{ab} + \gamma L^2 C_{abcd} F^{ab} F^{cd} \right] \\
+ \int d^4 x \sqrt{g} \left[ -\frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) \right],
\]

This action is characterized by 3 dimensionless parameters, which can be linked to data of the CFT (OPE coefficients): 2-point correlators of the conserved current $J_\mu$ and the stress energy tensor $T_{\mu\nu}$, and a 3-point $T, J, J$ correlator. Constraints from both the CFT and the gravitational theory bound $|\gamma| \leq 1/12 = 0.0833..$

AdS$_4$ theory of quantum criticality

The $\gamma > 0$ result has similarities to the quantum-Boltzmann result for transport of particle-like excitations.

The $\gamma < 0$ result can be interpreted as the transport of vortex-like excitations.

The $\gamma = 0$ case is the exact result for the large $N$ limit of SU($N$) gauge theory with $\mathcal{N} = 8$ supersymmetry (the ABJM model). The $\omega$-independence is a consequence of self-duality under particle-vortex duality ($S$-duality).

AdS$_4$ theory of quantum criticality

![Graph showing conductivity as a function of $\omega$](graph.png)

- Stability constraints on the effective theory ($|\gamma| < 1/12$) allow only a limited $\omega$-dependence in the conductivity

The dynamics of quantum criticality via Quantum Monte Carlo and holography

William Witczak-Krempa, Erik Sorensen, Subir Sachdev

(Submitted on 11 Sep 2013 (v1), last revised 29 Nov 2013 (this version, v2))

Understanding the real time dynamics of quantum systems without quasiparticles constitutes an important yet challenging problem. We study the superfluid-insulator quantum-critical point of bosons on a two-dimensional lattice, a system whose excitations cannot be described in a quasiparticle basis. We present detailed quantum Monte Carlo results for two separate lattice realizations: their low-frequency conductivities are found to have the same universal dependence on imaginary frequency and temperature. We then use the structure of the real time dynamics of conformal field theories described by the holographic gauge/gravity duality to make progress on the difficult problem of analytically continuing the Monte Carlo data to real time. Our method yields quantitative and experimentally testable results on the frequency-dependent conductivity near the quantum critical point, and on the spectrum of quasinormal modes in the vicinity of the superfluid-insulator quantum phase transition. Extensions to other observables and universality classes are discussed.

Universal Conductivity in a Two-dimensional Superfluid-to-Insulator Quantum Critical System

Kun Chen, Longxiang Liu, Youjin Deng, Lode Pollet, Nikolay Prokof'ev

(Submitted on 22 Sep 2013)

We compute the universal conductivity of the (2+1)-dimensional XY universality class, which is realized for a superfluid-to-Mott insulator quantum phase transition at constant density. Based on large-scale Monte Carlo simulations of the classical (2+1)-dimensional $J$-current model and the two-dimensional Bose-Hubbard model, we can precisely determine the conductivity on the quantum critical plateau, $\sigma(\infty) = 0.359(4)\sigma_Q$ with $\sigma_Q$ the conductivity quantum. The universal conductivity is the schoolbook example of where the AdS/CFT correspondence from string theory can be tested and made to use. The shape of our $\sigma(i\omega_n) - \sigma(\infty)$ function in the Matsubara representation is accurate enough for a conclusive comparison and establishes the particle-like nature of charge transport. We find that the holographic gauge/gravity duality theory for transport properties can be made compatible with the data if temperature of the horizon of the black brane is different from the temperature of the conformal field theory. The requirements for measuring the universal conductivity in a cold gas experiment are also determined by our calculation.
Quantum Monte Carlo for lattice bosons


FIG. 1. Probing quantum critical dynamics (a) Phase diagram of the superfluid-insulator quantum phase transition as a function of $t/U$ (hopping amplitude relative to the onsite repulsion) and temperature $T$ at integer filling of the bosons. The conformal QCP at $T = 0$ is indicated by a blue disk. (b) Quantum Monte Carlo data for the frequency-dependent conductivity, $\sigma(i \omega_n)/\sigma_Q$, near the QCP along the imaginary frequency axis, for both the quantum rotor and Villain models. The data has been extrapolated to the thermodynamic limit and zero temperature. The error bars are statistical, and do not include systematic errors arising from the assumed forms of the fitting functions, which we estimate to be 5–10%.

FIG. 2. Quantum Monte Carlo data (a) Finite-temperature conductivity for a range of $\beta U$ in the $L \rightarrow \infty$ limit for the quantum rotor model at $(t/U)_c$. The solid blue squares indicate the final $T \rightarrow 0$ extrapolated data. (b) Finite-temperature conductivity in the $L \rightarrow \infty$ limit for a range of $L_\tau$ for the Villain model at the QCP. The solid red circles indicate the final $T \rightarrow 0$ extrapolated data. The inset illustrates the extrapolation to $T = 0$ for $\omega_n/(2\pi T) = 7$. The error bars are statistical for both a) and b).
Good agreement between high precision Monte Carlo for imaginary frequencies, and holographic theory after rescaling effective $T$ and taking $\sigma_Q = 1/g_M^2$.

AdS$_4$ theory of quantum criticality

Predictions of holographic theory, after analytic continuation to real frequencies

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Iron pnictides:
a new class of high temperature superconductors
Superconductivity

\[ \text{BaFe}_2(\text{As}_{1-x}\text{P}_x)_2 \]

Resistivity
\[ \rho \sim \rho_0 + AT^\alpha \]


*Physical Review B* 81, 184519 (2010)
Superconductivity

Resistivity \sim \rho_0 + A T^\alpha

Strange Metal
no quasiparticles, Landau-Boltzmann theory does not apply

BaFe$_2$(As$_{1-x}$P$_x$)$_2$

AF + nematic

Superconductivity


*Physical Review B* 81, 184519 (2010)
The upper panels of Fig. 1d. The two- and four-fold oscillations appear at low temperatures, whereas the middle and lower panels of Fig. 1d. The distinct two-fold oscillations. The phase diagram is displayed in Fig. 1c. The temperature evolution of the regime, resulting in a phase diagram similar to the pseudogap and the other at.

The tetragonal-to-orthorhombic structural transition at temperature. Here, although recent experiments have provided hints of angular displacement. Here, for systems with tetragonal symmetry, has defined by the magnetization induced in the magnetic field. By combining these results, has equally important role in these two systems is highly controversial.

High temperature superconductors

\[ \text{YBa}_2\text{Cu}_3\text{O}_{6+x} \]
Visualization of the emergence of the pseudogap state and the evolution to superconductivity in a lightly hole-doped Mott insulator

Y. Kohsaka, T. Hanaguri, M. Azuma, M. Takano, J. C. Davis, and H. Takagi


Evidence for “nematic” order (i.e. breaking of 90° rotation symmetry) in Ca$_{1.88}$Na$_{0.12}$CuO$_2$Cl$_2$. 
Quantum criticality of Ising-nematic ordering in a metal

A metal with a Fermi surface with full square lattice symmetry
Quantum criticality of Ising-nematic ordering in a metal

<\phi> \neq 0

or

<\phi> = 0

Pomeranchuk instability as a function of coupling \lambda
Quantum criticality of Ising-nematic ordering in a metal

Phase diagram as a function of $T$ and $\lambda$

$T_{I-n}$

$\langle \phi \rangle \neq 0$

$\langle \phi \rangle = 0$

Fermi liquid

Fermi liquid

$\lambda_c$
Quantum criticality of Ising-nematic ordering in a metal

Phase diagram as a function of $T$ and $\lambda$
Quantum criticality of Ising-nematic ordering in a metal

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Phase diagram as a function of $T$ and $\lambda$
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Phase diagram as a function of $T$ and $\lambda$

Strongly-coupled “non-Fermi liquid” metal with no quasiparticles

$\langle \phi \rangle \neq 0$

$\langle \phi \rangle = 0$
Quantum criticality of Ising-nematic ordering in a metal

Phase diagram as a function of $T$ and $\lambda$

- Strongly-coupled “non-Fermi liquid” metal with no quasiparticles

- Fermi liquid: $\langle \phi \rangle \neq 0$

- Fermi liquid: $\langle \phi \rangle = 0$

$T_{l-n}$
Quantum criticality of Ising-nematic ordering in a metal

Strongly-coupled “non-Fermi liquid” metal with no quasiparticles

Common theoretical belief from a quasiparticle-like analysis: resistivity of strange metal $\rho(T) \sim T^{4/3}$. 
Quantum criticality of Ising-nematic ordering in a metal

Common theoretical belief from a quasiparticle-like analysis: resistivity of strange metal $\rho(T) \sim T^{4/3}$.

Ignores conservation of total momentum (analog of “phonon drag”)
Effective action for Ising order parameter

\[ S_\phi = \int d^2r d\tau \left[ (\partial_\tau \phi)^2 + c^2(\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right] \]
Quantum criticality of Ising-nematic ordering in a metal

Effective action for Ising order parameter

\[ S_\phi = \int d^2r d\tau \left[ (\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c) \phi^2 + u \phi^4 \right] \]

Effective action for electrons, with Fermi surface:

\[ S_c = \sum_{\alpha=1}^{N_f} \sum_k \int d\tau c_{k\alpha}^\dagger (\partial_\tau + \varepsilon_k) c_{k\alpha} \]

where \( \varepsilon_k = -\mu - 2t(\cos k_x + \cos k_y) + \ldots \)
Quantum criticality of Ising-nematic ordering in a metal

“Yukawa” coupling between Ising order and Fermi surface

\[ S_{\phi c} = -g \int d\tau \sum_{\alpha=1}^{N_f} \sum_{k,q} \phi_q (\cos k_x - \cos k_y) c_{k+q/2,\alpha}^{\dagger} c_{k-q/2,\alpha} \]

for spatially dependent \( \phi \)

\[ \langle \phi \rangle > 0 \quad \text{and} \quad \langle \phi \rangle < 0 \]
Quantum criticality of Ising-nematic ordering in a metal

The “standard model”:

\[
\mathcal{S}_\phi = \int d^2r d\tau \left[ (\partial_\tau \phi)^2 + c^2(\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right]
\]

\[
\mathcal{S}_c = \sum_{\alpha=1}^{N_f} \sum_k \int d\tau c_{k\alpha}^\dagger \left( \partial_\tau + \varepsilon_k \right) c_{k\alpha}
\]

\[
\mathcal{S}_{\phi c} = -g \int d\tau \sum_{\alpha=1}^{N_f} \sum_{k,q} \phi_q \left( \cos k_x - \cos k_y \right) c_{k+q/2,\alpha}^\dagger c_{k-q/2,\alpha}
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Quantum criticality of Ising-nematic ordering in a metal

The “standard model”:

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\]

\[
S_c = \sum_{\alpha=1}^{N_f} \int d^2r d\tau c^\dagger_\alpha \left( \partial_\tau - \frac{\nabla^2}{2m} - \mu \right) c_\alpha
\]

\[
S_{\phi c} = -g \int d^2r d\tau \sum_{\alpha=1}^{N_f} \phi \left[ c^\dagger_\alpha \left\{ (\partial_x^2 - \partial_y^2) c_\alpha \right\} + \left\{ (\partial_x^2 - \partial_y^2) c^\dagger_\alpha \right\} c_\alpha \right]
\]

This continuum theory has a conserved momentum \( P \), and \( \chi_{J,P} \neq 0 \), and so the resistivity \( \rho(T) = 0 \)
**Resistivity of strange metal**

In the presence of weak disorder of quenched Gaussian random fields

\[
S_{\text{dis}} = \int d^2r d\tau \left[ V(\mathbf{r}) c_\mathbf{c}^\dagger + h(\mathbf{r}) \phi \right],
\]

\[
\overline{V(\mathbf{r})} = 0 \quad ; \quad \overline{V(\mathbf{r})V(\mathbf{r}')} = V_0^2 \delta(\mathbf{r} - \mathbf{r}'),
\]

\[
\overline{h(\mathbf{r})} = 0 \quad ; \quad \overline{h(\mathbf{r})h(\mathbf{r}')} = h_0^2 \delta(\mathbf{r} - \mathbf{r}'),
\]

we obtain the resistivity for current along angle \( \vartheta \)

\[
\rho(T) = \frac{1}{\chi_{J,P}^2} \lim_{\omega \to 0} \int \frac{d^2k}{(2\pi)^2} k^2 \cos^2(\theta_k - \vartheta) \left( V_0^2 \frac{\text{Im } \Pi_{c_\mathbf{c}^\dagger}^R(\omega, \mathbf{k})}{\omega} + h_0^2 \frac{\text{Im } D_\phi^R(\omega, \mathbf{k})}{\omega} \right).
\]

Resistivity of strange metal

In the presence of weak disorder of quenched Gaussian random fields

\[ S_{\text{dis}} = \int d^2r d\tau \left[ V(\mathbf{r}) c^\dagger c + h(\mathbf{r}) \phi \right], \]

\[ \overline{V(\mathbf{r})} = 0 ; \quad \overline{V(\mathbf{r})V(\mathbf{r}')} = V_0^2 \delta(\mathbf{r} - \mathbf{r}'), \]

\[ \overline{h(\mathbf{r})} = 0 ; \quad \overline{h(\mathbf{r})h(\mathbf{r}')} = h_0^2 \delta(\mathbf{r} - \mathbf{r}'), \]

we obtain the resistivity for current along angle \( \vartheta \)

\[ \rho(T) = \frac{1}{\chi_{J,P}} \lim_{\omega \to 0} \int \frac{d^2k}{(2\pi)^2} k^2 \cos^2(\theta_k - \vartheta) \left( V_0^2 \frac{\text{Im} \Pi^R_{c^\dagger c}(\omega, \mathbf{k})}{\omega} + h_0^2 \frac{\text{Im} D^R_\phi(\omega, \mathbf{k})}{\omega} \right) \]

Fermi surface term: Obtain \( T \)-dependent corrections to residual resistivity similar to earlier work


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**Bosonic term: Dominant contribution:**

\[ \rho(T) \sim T^{(d-z+\eta)/z} \]

Crosses over from the “relativistic” form \((z = 1, \eta \approx 0)\) with \( \rho(T) \sim T \) at higher \( T \),

to the “Landau-damped” form \((z = 3, \eta = 0)\) with \( \rho(T) \sim (T \ln(1/T))^{-1/2} \)
at lower \( T \) (subtle corrections to scaling specific to this field theory).

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Exciting recent progress on the description of transport in metallic states without quasiparticles, via field theory and holography.