Quantum phases and critical points of correlated metals

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Outline

I. **Kondo lattice models**
   Doniach's phase diagram and its quantum critical point

II. Paramagnetic states of quantum antiferromagnets:
   (A) Confinement of spinons and bond order
   (B) Spin liquids with deconfined spinons: Z_2 and U(1) gauge theories

III. A new phase: a fractionalized Fermi liquid (FL*)

IV. Extended phase diagram and its critical points

V. Conclusions

I. Doniach's $T=0$ phase diagram for the Kondo lattice

$$H = \sum_{i<j} t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + \sum_{i} \left(J_K c_{i\sigma}^{\dagger} \vec{\tau}_{\sigma\sigma'} c_{i\sigma} \cdot \vec{S}_{fi} \right)$$

$c_{i\sigma} \rightarrow$ Conduction electrons;

$\vec{S}_{fi} \rightarrow$ localized $f_{i\sigma}$ moments (assumed S=1/2, for specificity)

Local moments choose some static spin arrangement

$$J_{RKKY} \sim J_K^2 / t \gg T_K \sim \exp\left(-t / J_K\right)$$

"Heavy" Fermi liquid with moments Kondo screened by conduction electrons. Fermi surface obeys Luttinger's theorem.

SDW

FL

J_K / t

Luttinger's theorem on a d-dimensional lattice for the FL phase

Let v_0 be the volume of the unit cell of the ground state,
n_T be the total number density of electrons per volume v_0.
(need not be an integer)

$$n_T = n_f + n_c = 1 + n_c$$

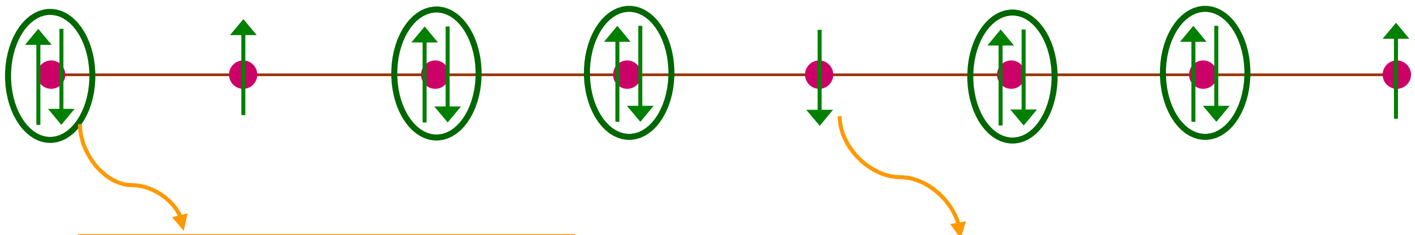
$$2 \times \frac{v_0}{(2\pi)^d} \left(\text{Volume enclosed by Fermi surface} \right)$$

$$= n_T \,(\text{mod } 2)$$

A "large" Fermi surface

Arguments for the Fermi surface volume of the FL phase

Single ion Kondo effect implies $J_K \to \infty$ at low energies



$$\left(c_{i\uparrow}^{\dagger} f_{i\downarrow}^{\dagger} - c_{i\downarrow}^{\dagger} f_{i\uparrow}^{\dagger}\right)\left|0\right\rangle$$

$f_{i\downarrow}^{\dagger}\left|0\right\rangle$, S=1/2 hole

Fermi liquid of S=1/2 holes with hard-core repulsion

$$\begin{aligned}\text{Fermi surface volume } &= -\left(\text{density of holes}\right) \bmod 2 \\ &= -\left(1 - n_c\right) = \left(1 + n_c\right) \bmod 2\end{aligned}$$

Arguments for the Fermi surface volume of the FL phase

Alternatively:

Formulate Kondo lattice as the large U limit of the Anderson model

$$H = \sum_{i<j} t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + \sum_{i} \left(V c_{i\sigma}^{\dagger} f_{i\sigma} + V f_{i\sigma}^{\dagger} c_{i\sigma} + \varepsilon_f \left(n_{fi\uparrow} + n_{fi\downarrow} \right) + U n_{fi\uparrow} n_{fi\downarrow} \right) + \cdots$$

$$n_T = n_f + n_c$$

For small U, Fermi surface volume $= \left(n_f + n_c \right) \bmod 2.$

This is adiabatically connected to the large U limit where $n_f = 1$

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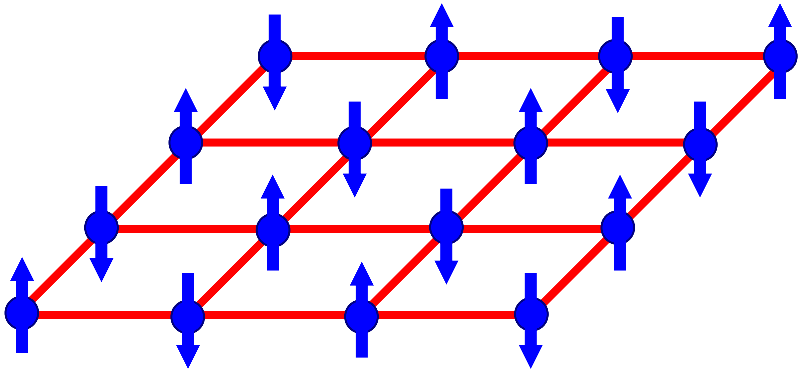
Ground states of quantum antiferromagnets

Begin with magnetically ordered states, and consider quantum transitions which restore spin rotation invariance

Two classes of ordered states:

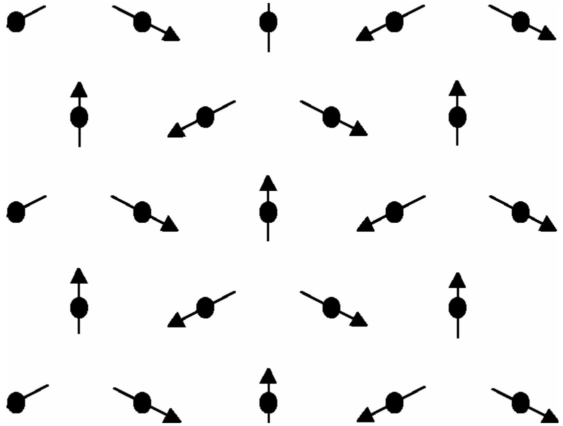
(α) Collinear spins



$$\left\langle \vec{S}(\boldsymbol{r}) \right\rangle \propto \vec{N} \cos(\boldsymbol{Q} \bullet \boldsymbol{r})$$

$$\boldsymbol{Q} = (\pi, \pi); \ \vec{N}^2 = 1$$

(β) Non-collinear spins



$$\left\langle \vec{S}(\boldsymbol{r}) \right\rangle \propto \vec{N}_1 \cos(\boldsymbol{Q} \bullet \boldsymbol{r}) + \vec{N}_2 \sin(\boldsymbol{Q} \bullet \boldsymbol{r})$$

$$\boldsymbol{Q} = \left(\frac{4\pi}{3}, \frac{4\pi}{\sqrt{3}} \right); \ \vec{N}_1^2 = \vec{N}_2^2 = 1 ; \ \vec{N}_1 \bullet \vec{N}_2 = 0$$

(α) Collinear spins, Berry phases, and bond-order

S=1/2 antiferromagnet on a bipartitie lattice

$$H = \sum_{i<j} J_{ij} \; \vec{S}_i \cdot \vec{S}_j$$

Include Berry phases after discretizing coherent state path integral on a cubic lattice in spacetime

$$Z = \prod_a \int d\boldsymbol{n}_a \delta\left(\boldsymbol{n}_a^2 - 1\right) \exp\left(\frac{1}{g} \sum_{a,\mu} \boldsymbol{n}_a \cdot \boldsymbol{n}_{a+\mu} - \frac{i}{2} \sum_a \eta_a A_{a\tau} \right)$$

$\eta_{\mathrm{a}} \to \pm 1$ on two sublattices ;

$\boldsymbol{n}_a \sim \eta_a \vec{S}_a \to$ Neel order parameter;

$A_{a\mu} \to$ oriented area of spherical triangle

formed by $\boldsymbol{n}_a$, $\boldsymbol{n}_{a+\mu}$, and an arbitrary reference point $\boldsymbol{n}_0$

Small g → Spin-wave theory about Neel state receives minor modifications from Berry phases.

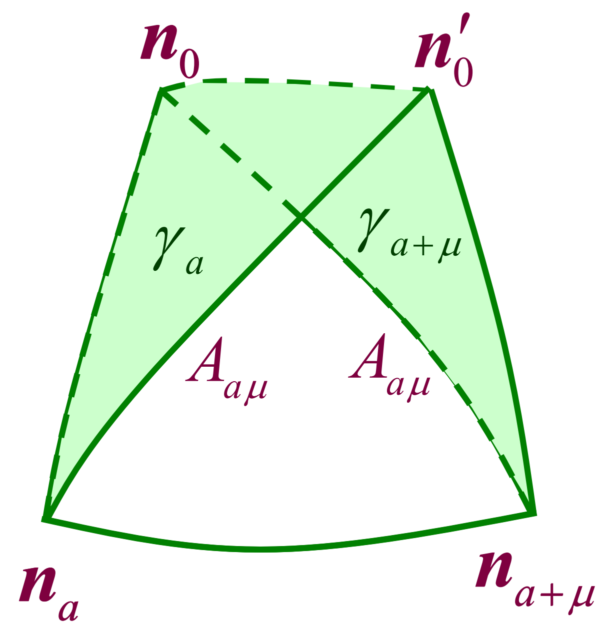
Large g → Berry phases are crucial in determining structure of "quantum-disordered" phase with $\langle \boldsymbol{n}_a \rangle = 0$

Integrate out $\boldsymbol{n}_a$ to obtain effective action for $A_{a\mu}$

Change in choice of $\boldsymbol{n}_0$ is like a "gauge transformation"

$$A_{a\mu} \to A_{a\mu} - \gamma_{a+\mu} + \gamma_a$$

(γ_a is the oriented area of the spherical triangle formed by $\boldsymbol{n}_a$ and the two choices for $\boldsymbol{n}_0$).



The area of the triangle is uncertain modulo 4π, and the action is invariant under

$$A_{a\mu} \to A_{a\mu} + 4\pi$$

These principles strongly constrain the effective action for $A_{a\mu}$

Simplest large g effective action for the $A_{a\mu}$

$$Z = \prod_{a,\mu} \int dA_{a\mu} \exp\left(-\frac{1}{2e^2} \sum_{\square} \cos\left(\frac{1}{2}\left(\Delta_\mu A_{a\nu} - \Delta_\nu A_{a\mu} \right) \right) - \frac{i}{2} \sum_a \eta_a A_{a\tau} \right)$$

with $e^2 \sim g^2$

This is compact QED in $d+1$ dimensions with Berry phases.

This theory can be reliably analyzed by a duality mapping.

(I) *d=2:*

The gauge theory is always in a confining phase. There is an energy gap and the ground state has bond order (induced by the Berry phases).

(II) *d=3:*

An additional "topologically ordered" Coulomb phase is also possible. There are deconfined spinons which are minimally coupled to a gapless U(1) photon.

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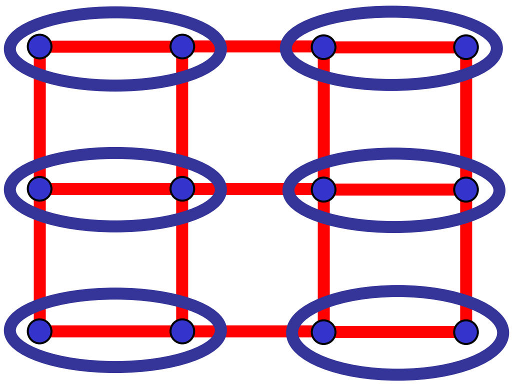
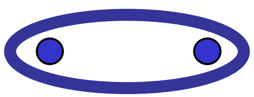
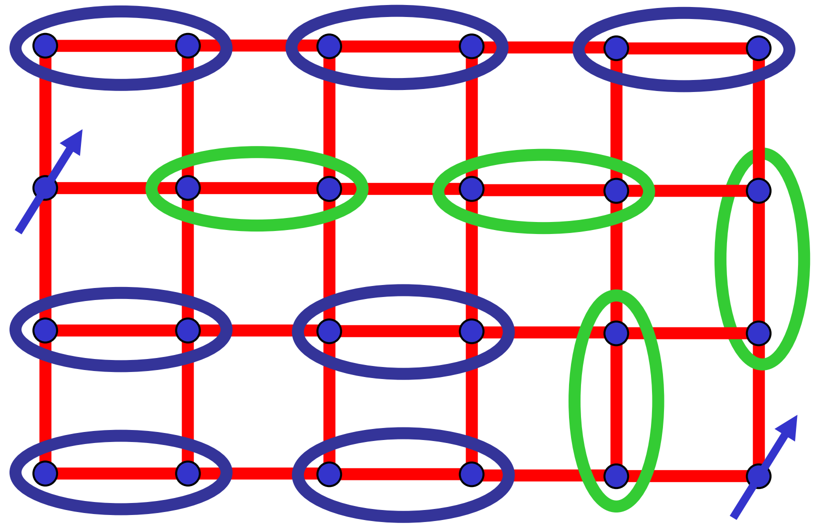
Paramagnetic states with $\left\langle \mathbf{S}_j \right\rangle = 0$

Bond order and confined spinons





$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle \right)$$

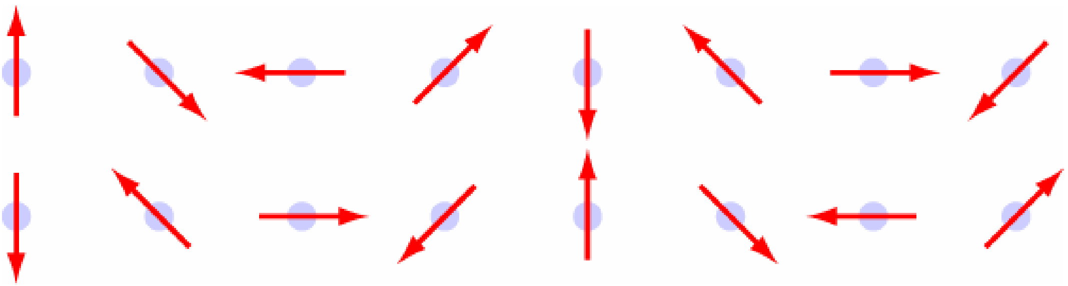


S=1/2 spinons are ***confined*** by a linear potential into a *S*=1 spin exciton

Confinement is required U(1) paramagnets in *d*=2

β. Noncollinear spins

Magnetic order $\left\langle \boldsymbol{S}_j \right\rangle = \boldsymbol{N}_1 \cos\left(\vec{K} \,.\, \vec{r}_j\right) + \boldsymbol{N}_2 \sin\left(\vec{K} \,.\, \vec{r}_j\right)$



$\vec{K} = (3\pi/4, \pi)$;

$\boldsymbol{N}_2^2 = \boldsymbol{N}_1^2 \,,\, \boldsymbol{N}_1 . \boldsymbol{N}_2 = 0$

Solve constraints by expressing $\boldsymbol{N}_{1,2}$ in terms of two complex numbers $z_\uparrow \,,\, z_\downarrow$

$$\boldsymbol{N}_1 + i\boldsymbol{N}_2 = \begin{pmatrix} z_\downarrow^2 - z_\uparrow^2 \\ i\left(z_\downarrow^2 + z_\uparrow^2\right) \\ 2z_\uparrow z_\downarrow \end{pmatrix}$$

Order in ground state specified by a spinor $\left(z_\uparrow, z_\downarrow\right)$ (modulo an overall sign).

This spinor could become a S=1/2 spinon in a quantum "disordered" state.

Order parameter space: S_3/Z_2

Physical observables are invariant under the Z_2 gauge transformation $z_a \to \pm z_a$

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III. Doping spin liquids

Reconsider Doniach phase diagram

It is more convenient to analyze the Kondo-Heiseberg model:

$$H = \sum_{i<j} t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + \sum_{i} \left(J_K c_{i\sigma}^{\dagger} \vec{\tau}_{\sigma\sigma'} c_{i\sigma} \cdot \vec{S}_{fi} \right) + \sum_{i<j} J_H (i,j) \vec{S}_{fi} \cdot \vec{S}_{fj}$$

Work in the regime $J_H > J_K$

Determine the ground state of the quantum antiferromagnet defined by J_H, and then couple to conduction electrons by J_K

Choose J_H so that ground state of antiferromagnet is a Z_2 or U(1) spin liquid

State of conduction electrons

At $J_K = 0$ the conduction electrons form a Fermi surface on their own with volume determined by n_c

Perturbation theory in J_K is regular, and topological order is robust, and so this state will be stable for finite J_K

So volume of Fermi surface is determined by **$(n_T - 1) = n_c (\text{mod } 2)$**, and Luttinger's theorem is violated.

The (U(1) or Z_2) FL* state

III. A new phase: FL*

This phase preserves spin rotation invariance, and has a Fermi surface of *sharp* electron-like quasiparticles.

The state has "*topological order*" and associated neutral excitations. The topological order can be easily detected by the violation of Luttinger's theorem. It can only appear in dimensions $d > 1$

$$2 \times \frac{v_0}{(2\pi)^d} \left(\text{Volume enclosed by Fermi surface} \right)$$

$$= (n_T - 1) (\bmod 2)$$

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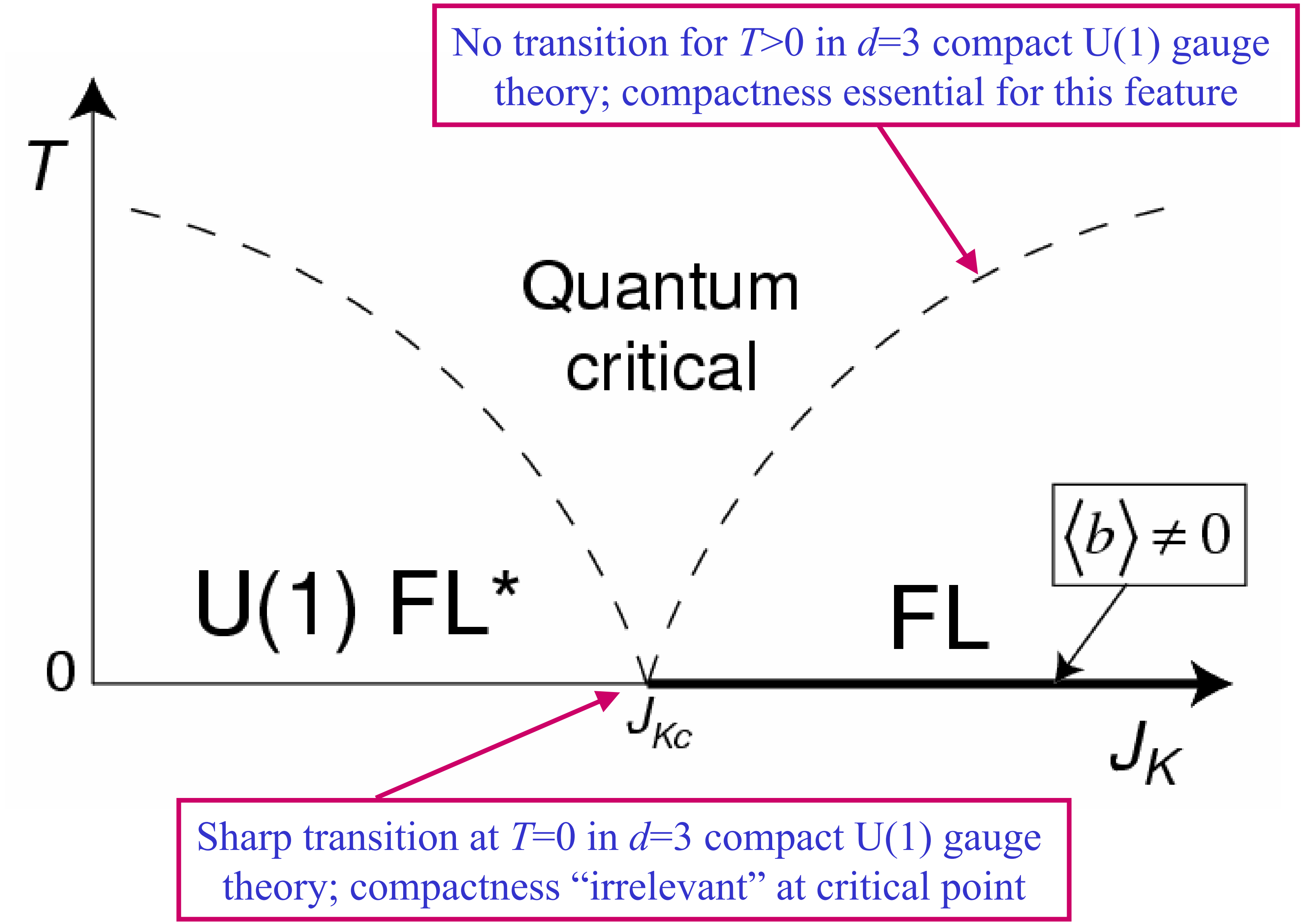
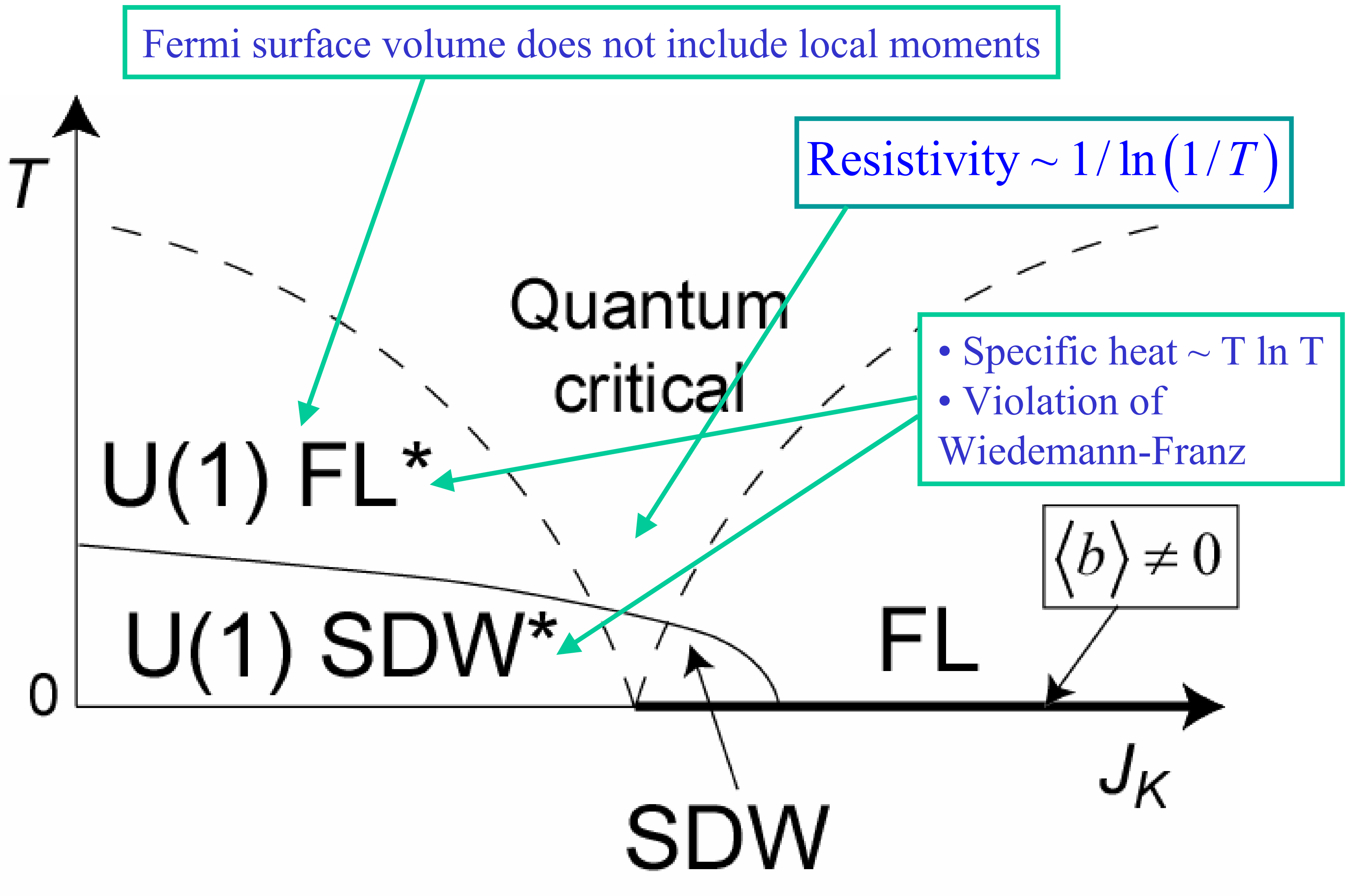
Phase diagram (U(1), d=3)

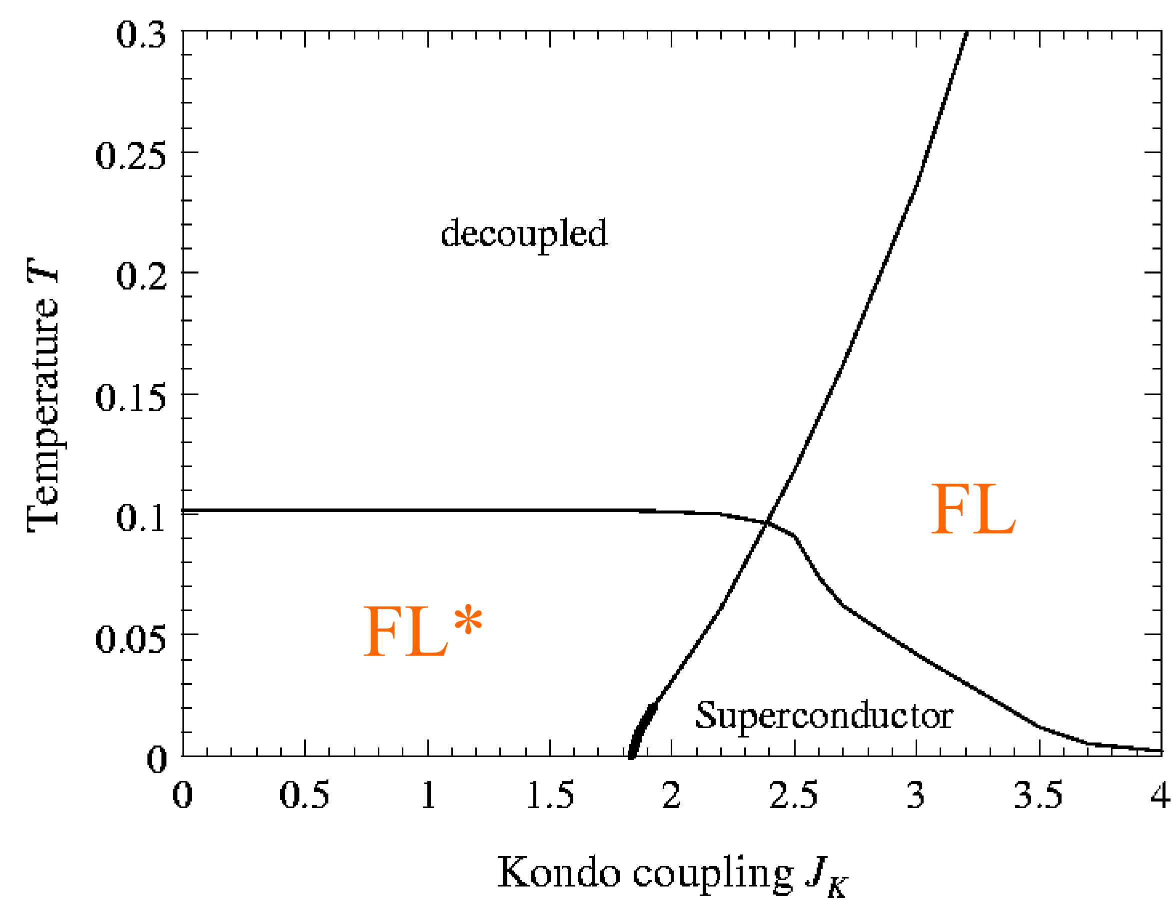


Phase diagram (U(1), d=3)



Z_2 fractionalization

Mean-field phase diagram



Pairing of spinons in small Fermi surface state induces superconductivity at the confinement transition

> Small Fermi surface state can also exhibit a second-order metamagnetic transition in an applied magnetic field, associated with vanishing of a spinon gap.

Conclusions

- New phase diagram as a paradigm for clean metals with local moments.
- Topologically ordered (*) phases lead to novel quantum criticality.
- New FL* allows easy detection of topological order by Fermi surface volume