Damping of collective modes and quasiparticles in d-wave superconductors

•C. Buragohain
•Y. Zhang

Subir Sachdev

M. Vojta

Transparencies on-line at
http://pantheon.yale.edu/~subir

Review article: cond-mat/0005250
and references therein

Quantum Phase Transitions,
Cambridge University Press

Yale University

Elementary excitations of a d-wave superconductor

(A) *S=0* Cooper pairs, phase fluctuations

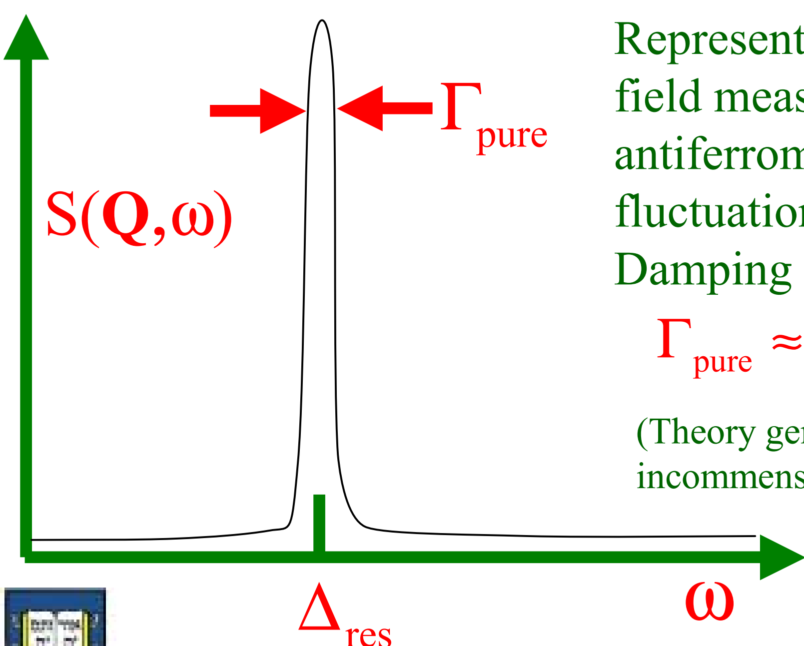
Negligible below T_c except near a T=0 superconductor-insulator transition.
Proliferate above T_c due to free vortex density.

(B) *S=1/2* Fermionic quasiparticles

Ψ_h : strongly paired fermions near $(\pi,0)$, $(0,\pi)$ have an energy gap ~ 30-40 meV

$\Psi_{1,2}$: gapless fermions near the nodes of the superconducting gap at $(\pm K, \pm K)$ with $K = 0.391\pi$

(C) *S=1* Bosonic, resonant collective mode

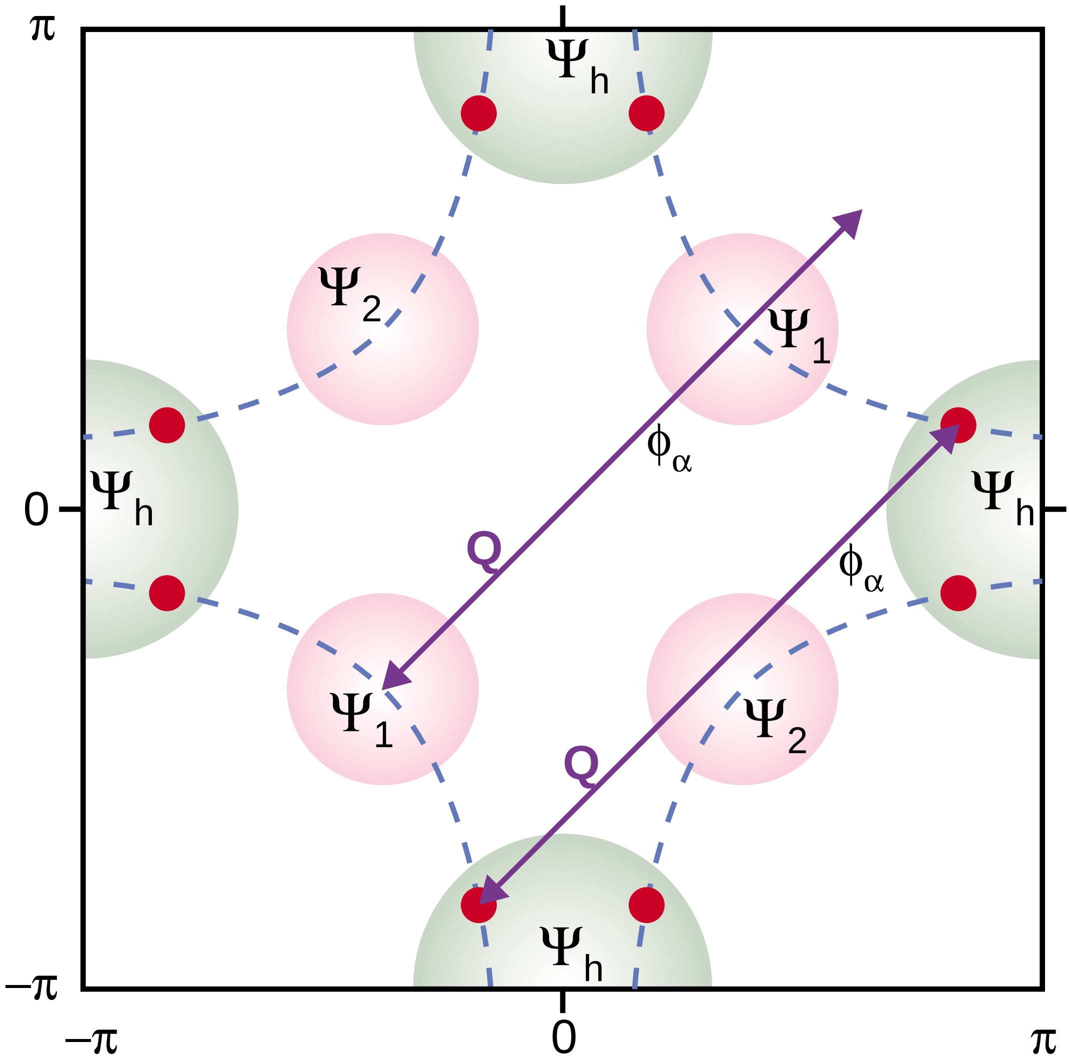


Represented by ϕ_α , a vector field measuring the strength of antiferromagnetic spin fluctuations near $\mathbf{Q} \approx (\pi,\pi)$
Damping is small at *T=0*

$$\Gamma_{pure} \approx 0 \text{ at } T = 0$$

(Theory generalizes to the cases with incommensurate **Q** and $\Gamma_{pure} \neq 0$)

Constraints from momentum conservation



Ψ_h : *strongly coupled* to ϕ_α and phase fluctuations

(leads to strong damping above T_c, and coherent pairing and gap formation below T_c)

$\Psi_{1,2}$: *decoupled* from ϕ_α and phase fluctuations

(absence of damping and pairing ?)

I. Zero temperature broadening of resonant collective mode ϕ_α by impurities: comparison with neutron scattering experiments of Fong *et al* Phys. Rev. Lett. **82**, 1939 (1999)

II. Intrinsic inelastic lifetime of nodal quasiparticles $\Psi_{1,2}$ (Valla *et al* Science **285**, 2110 (1999) and Corson *et al* cond-mat/0003243): critical survey of possible nearby quantum-critical points.

Independent low energy quantum field theories for the ϕ_α and the $\Psi_{1,2}$

I. Zero temperature broadening of resonant collective mode by impurities

Effect of *arbitrary localized deformations* ("impurities") of density n_{imp}

Each impurity is characterized by an integer/half-odd-integer S

As $\Delta_{\text{res}} \to 0$

$$\frac{\Gamma_{\text{imp}}}{\Delta_{\text{res}}} = n_{\text{imp}} \left(\frac{\hbar c}{\Delta_{\text{res}}} \right)^2 \left[C_S + O\left(\frac{\Delta_{\text{res}}}{J} \right) \right]$$

Correlation length ξ

$C_S \to$ Universal numbers dependent only on S

$$C_0 = 0 \; ; \; C_{1/2} \approx 1$$

Zn impurities in YBCO have S=1/2

"Swiss-cheese" model of quantum impurities (Uemura):

Inverse Q of resonance ~ fractional volume of holes in Swiss cheese.

As $\Delta_{res} \rightarrow 0$ there is a quantum phase transition to a magnetically ordered state

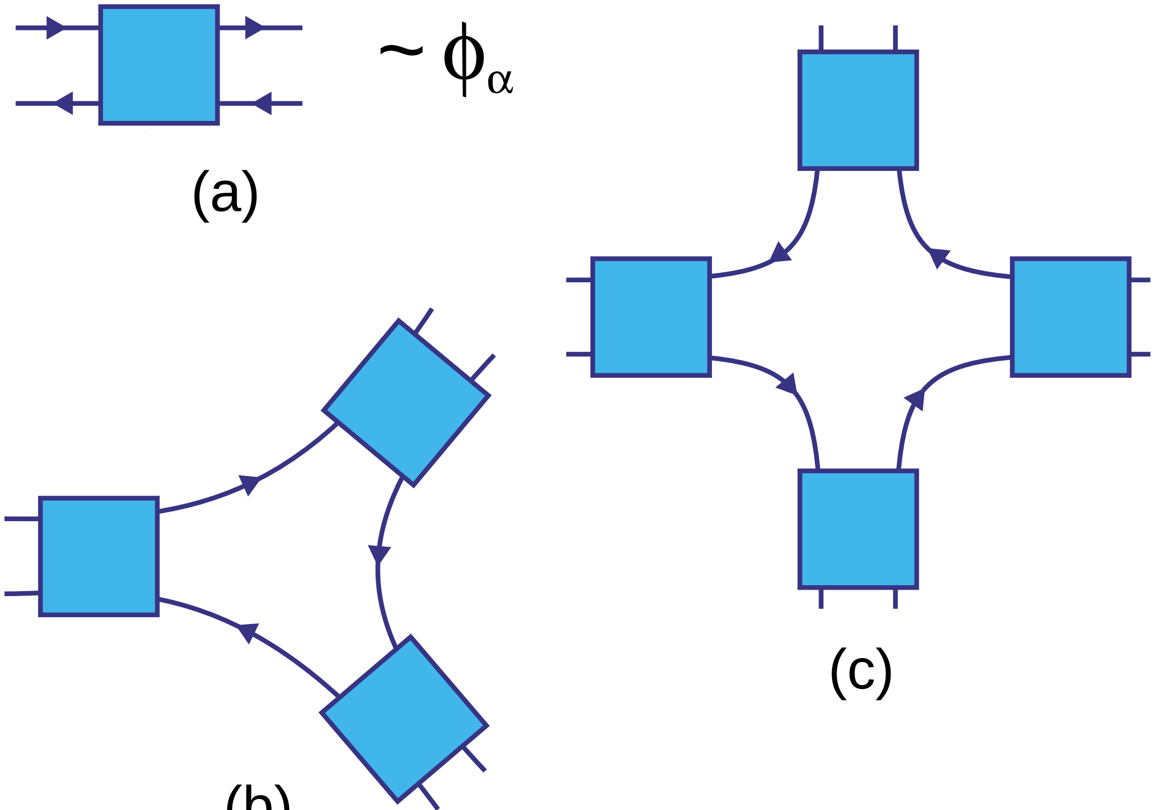
(A) Insulating Neel state (or collinear SDW at wavevector **Q**) $\Leftrightarrow$ insulating quantum paramagnet

(B) *d*-wave superconductor with collinear SDW at wavevector **Q** $\Leftrightarrow$ *d*-wave superconductor (paramagnet)

Transition (B) is in the same universality class as (A) provided Ψ_h fermions remain gapped at quantum-critical point.

Why appeal to proximity to a quantum phase transition ?

$\phi_\alpha \sim S=1$ bound state in particle-hole channel at the antiferromagnetic wavevector



Quantum field theory of critical point allows systematic treatment of the <u>*strongly relevant*</u> multi-point interactions in (b) and (c).

1. (A) Paramagnetic and Neel ground states in two dimensions --- coupled-ladder antiferromagnet.
   Field theory of quantum phase transition.

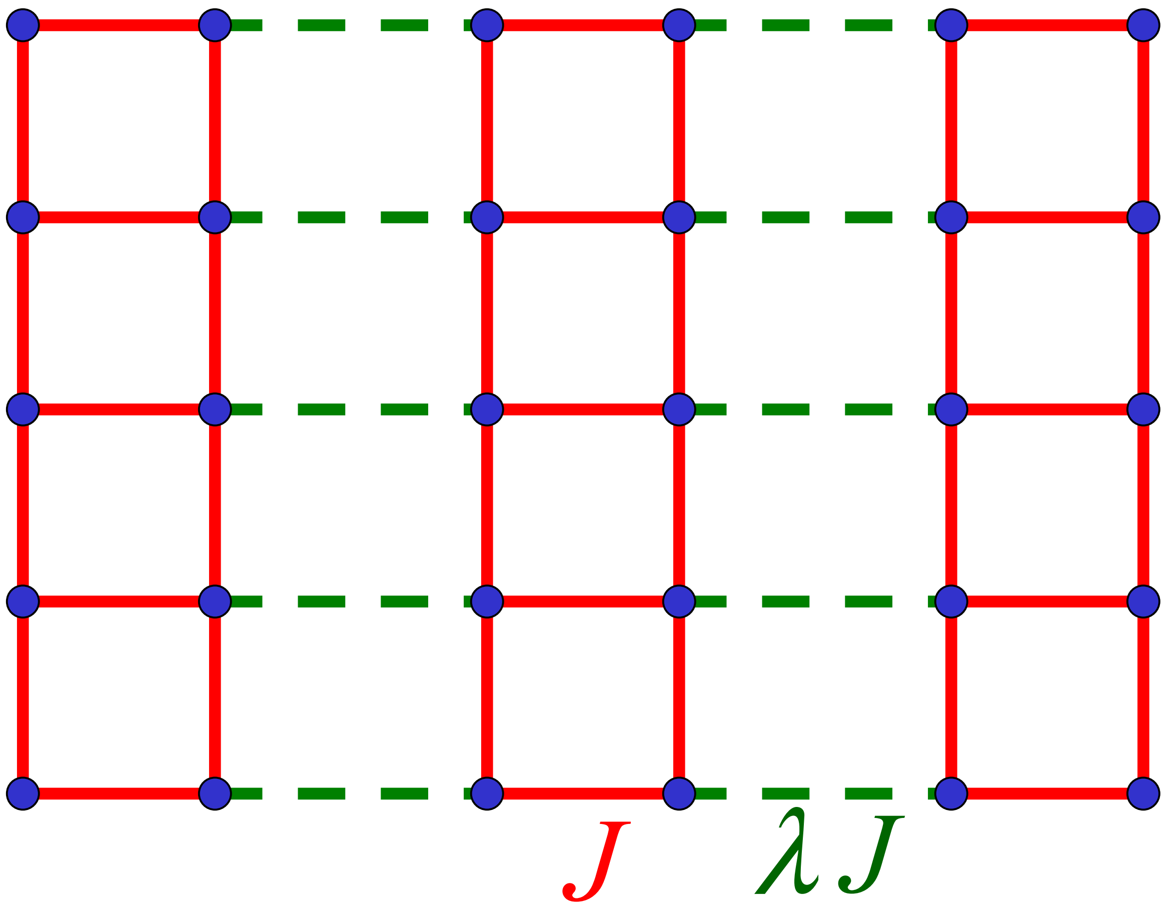
2. Non-magnetic impurities (Zn or Li) in two-dimensional paramagnets.

3. Application to (B) d-wave superconductors.
   Comparison with, and predictions for, expts

1. Paramagnetic and Neel states in insulators

(Katoh and Imada;
Tworzydlo, Osman, van Duin and Zaanen)

S=1/2 spins on coupled 2-leg ladders



$$H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

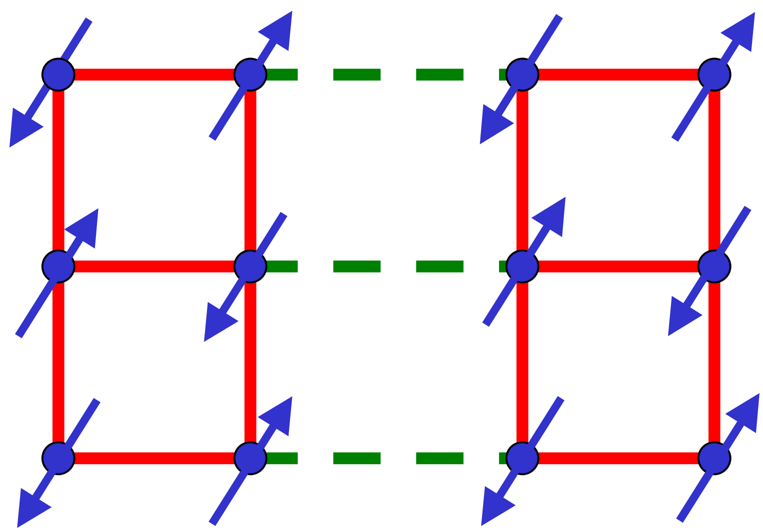
Follow ground state as a function of λ

$$0 \leq \lambda \leq 1$$

λ close to 1

Square lattice antiferromagnet

Experimental realization: La_2CuO_4



Ground state has long-range magnetic (Neel) order

$$\left\langle \vec{S}_i \right\rangle = \left(-1\right)^{i_x + i_y} N_0 \neq 0$$

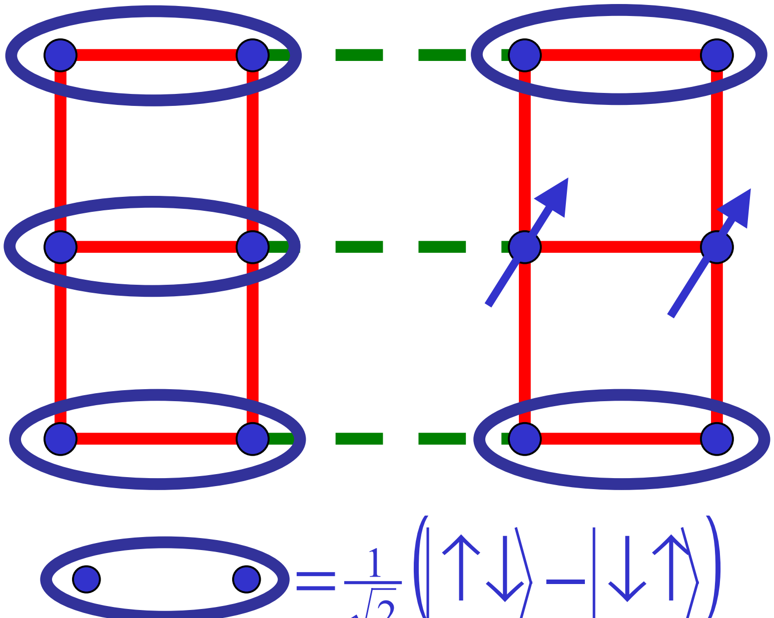
Excitations: 2 spin waves

Quasiclassical wave dynamics at low T

(Chakravarty et al, 1989;
Tyc et al, 1989)

λ close to 0

Weakly coupled ladders



$$=\frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle-\left|\downarrow\uparrow\right\rangle\right)$$

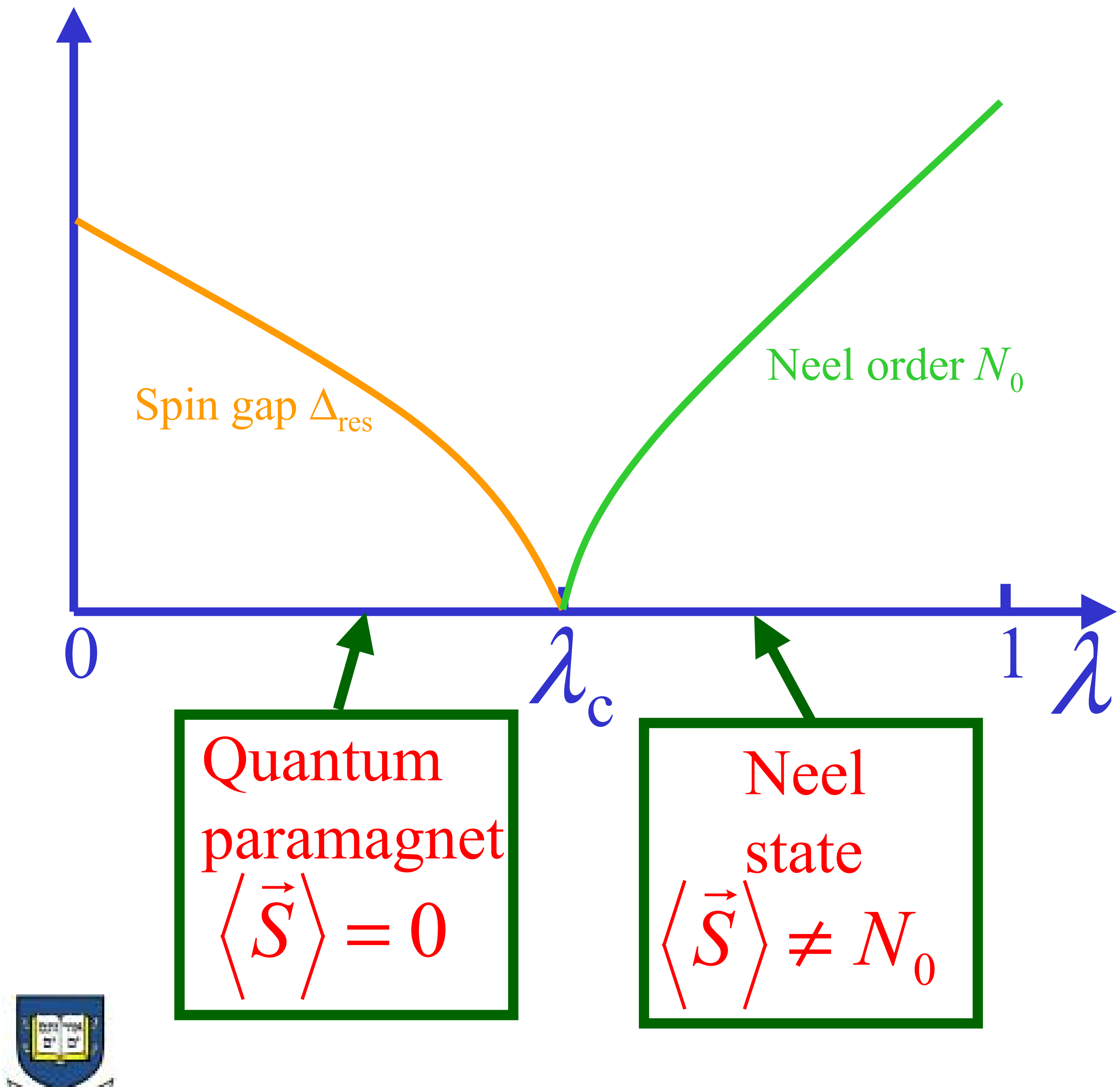
Paramagnetic ground state $\left\langle \vec{S}_i \right\rangle = 0$

Excitation: S=1, ϕ_α particle (collective mode)

Energy dispersion away from antiferromagnetic wavevector

$$\varepsilon = \Delta_{res} + \frac{c^2k^2}{2\Delta_{res}}$$

$\Delta_{res} \rightarrow$ Spin gap



Spin gap Δ_{res}
Neel order N_0
0
λ_c
1
λ
Quantum paramagnet
$\langle \vec{S} \rangle = 0$
Neel state
$\langle \vec{S} \rangle \neq N_0$

Nearly-critical paramagnets

λ is close to λ_c

Quantum field theory:

$$S_b = \int d^d x d\tau \left[\frac{1}{2} \left((\nabla_x \phi_\alpha)^2 + c^2 (\partial_\tau \phi_\alpha)^2 + r \phi_\alpha^2 \right) + \frac{g}{4!} (\phi_\alpha^2)^2 \right]$$

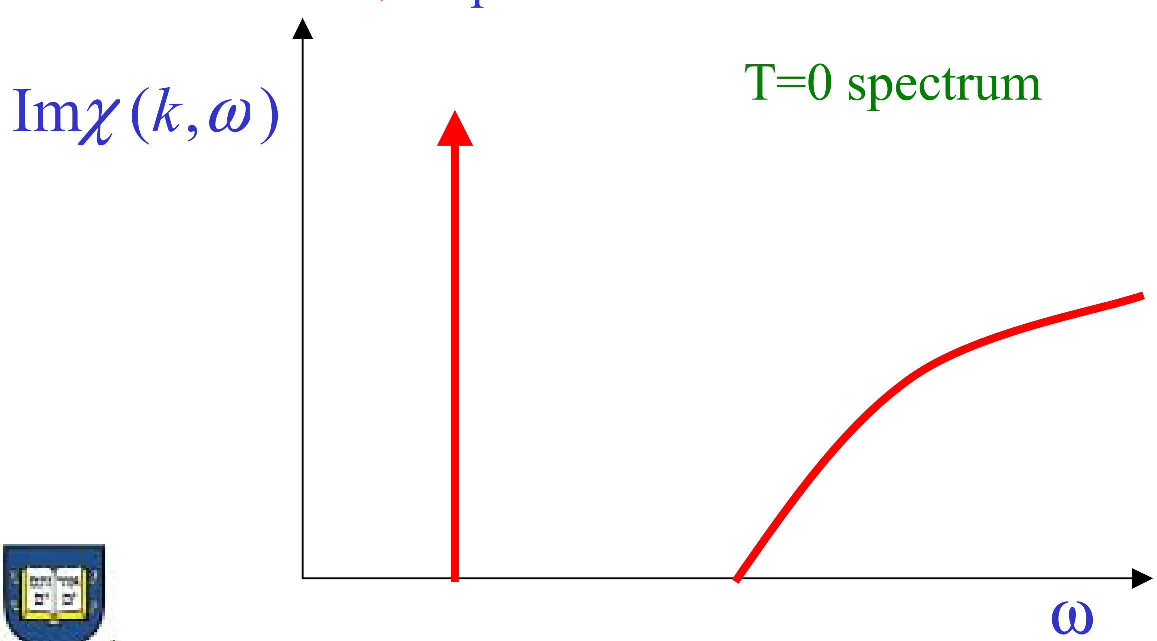
ϕ_α → 3-component antiferromagnetic order parameter

$r > 0$ → $\lambda < \lambda_c$

$r < 0$ → $\lambda > \lambda_c$

Oscillations of ϕ_α about zero (for $r > 0$)

→ spin-1 collective mode



Coupling ***g*** approaches fixed-point value under renormalization group flow: beta function ($\varepsilon = 3\text{-}d$) :

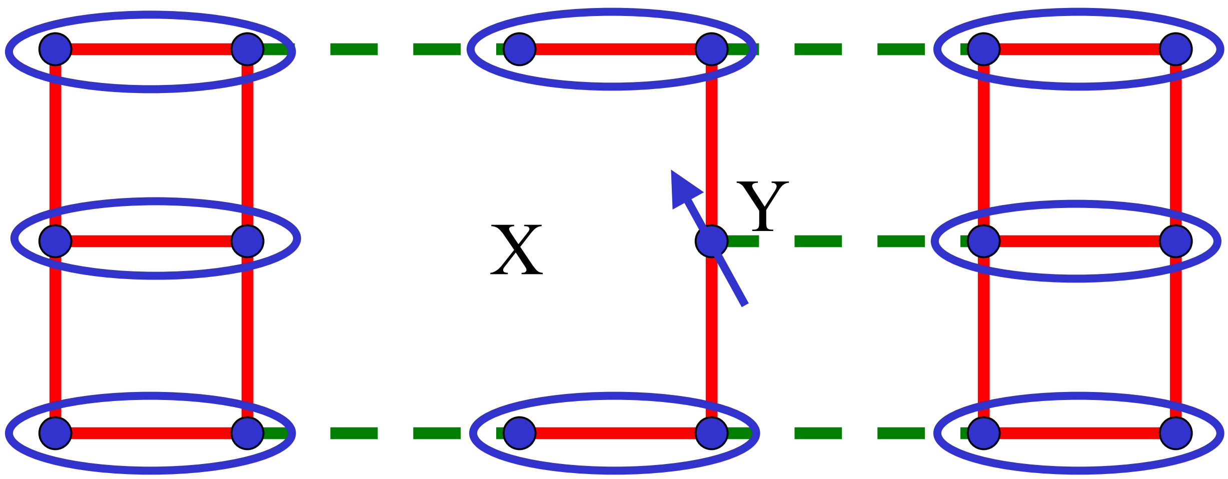
$$\beta(g) = -\epsilon g + \frac{11g^2}{6} - \frac{23g^3}{12} + \mathcal{O}(g^4)$$

Only relevant perturbation – ***r*** strength is measured by the spin gap Δ

Δ_{res} and c completely determine entire spectrum of quasi-particle peak and multiparticle continua, the S matrices for scattering between the excitations, and $T > 0$ modifications.

2. Quantum impurities in nearly-critical paramagnets

Make *any* localized deformation of antiferromagnet; e.g. remove a spin



Susceptibility $\chi = A\chi_b + \chi_{imp}$

(A = area of system)

In paramagnetic phase as $T \to 0$

$$\chi_b = \left(\frac{\Delta_{\mathrm{res}}}{\hbar^2 c^2 \pi}\right) e^{-\Delta_{\mathrm{res}}/k_B T} \quad ; \chi_{imp} = \frac{S(S+1)}{3k_B T}$$

For a general impurity χ_{imp} *defines* the value of S

Orientation of “impurity” spin -- $n_\alpha(\tau)$ (unit vector)

Action of “impurity” spin

$$S_{\text{imp}} = \int d\tau \left[iSA_\alpha(n)\frac{dn_\alpha}{d\tau} - \gamma S n_\alpha(\tau)\phi_\alpha(x=0,\tau) \right]$$

$A_\alpha(n)$ → Dirac monopole function

Boundary quantum field theory: $S_b + S_{\text{imp}}$

Recall -

$$S_b = \int d^d x d\tau \left[\frac{1}{2}\left((\nabla_x \phi_\alpha)^2 + c^2(\partial_\tau \phi_\alpha)^2 + r\phi_\alpha^2 \right) + \frac{g}{4!}(\phi_\alpha^2)^2 \right]$$

Coupling γ approaches *also* approaches a fixed-point value under the renormalization group flow

(Sengupta, 97
Sachdev+Ye, 93
Smith+Si 99)

Beta function:

$$\beta(\gamma) = -\frac{\epsilon\gamma}{2} + \gamma^3 - \gamma^5 + \frac{5g^2\gamma}{144} + \frac{\pi^2}{3}\left(S(S+1) - \frac{1}{3}\right) g\gamma^3 + \mathcal{O}\left((\gamma, \sqrt{g})^7\right)$$

No new relevant perturbations on the boundary;
All other boundary perturbations are irrelevant –

e.g. $\lambda \int d\tau \phi_\alpha^2(x=0,\tau)$

(This is the simplest allowed boundary perturbation for S=0 – its irrelevance implies $C_0 = 0$)

Δ_{res} and c completely determine spin dynamics near an impurity –
No new parameters are necessary !

Finite density of impurities n_{imp}

Relevant perturbation – strength determined by only energy scale that is linear in n_{imp} and contains only bulk parameters

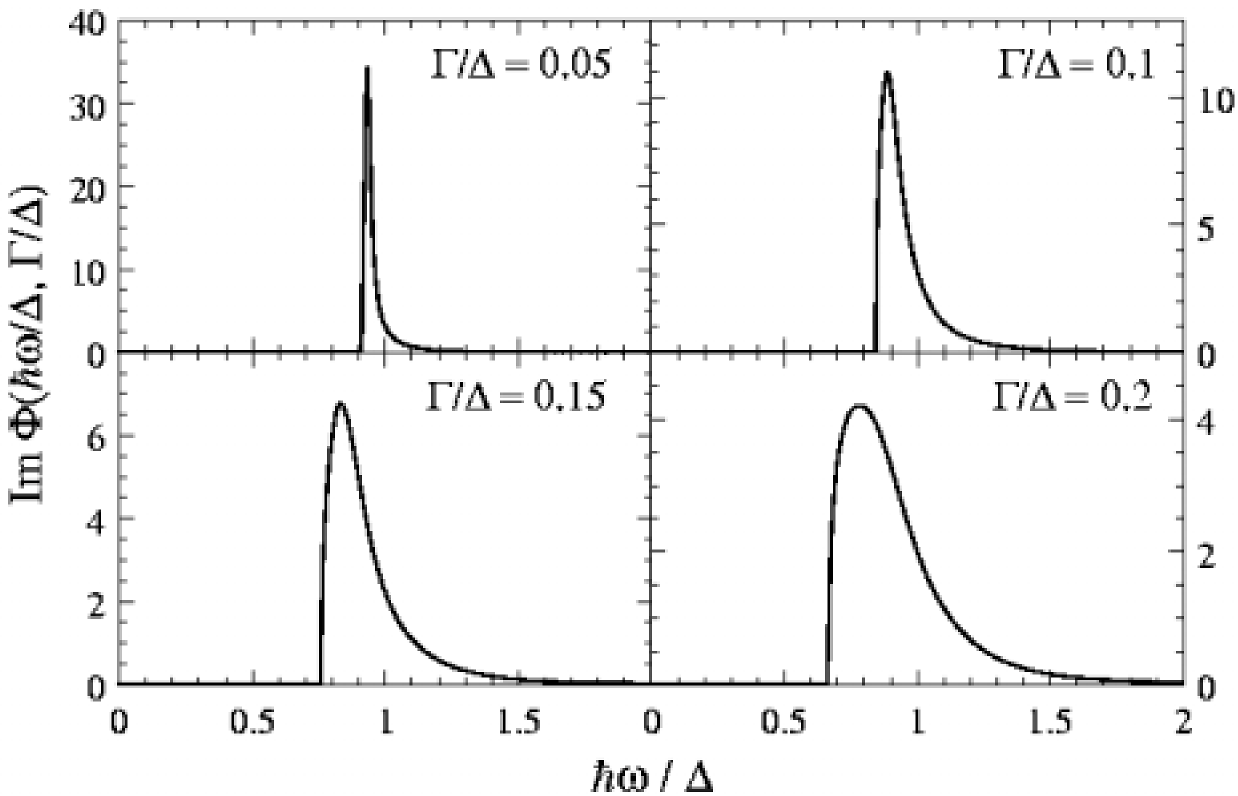
$$\Gamma \equiv \frac{n_{\text{imp}}(\hbar c)^2}{\Delta_{\text{res}}}$$

Fate of collective mode peak

Without impurities $\chi(G,\omega) = \dfrac{A}{\Delta_{\text{res}}^2 - \omega^2}$

With impurities $\chi(G,\omega) = \dfrac{A}{\Delta_{\text{res}}^2}\Phi\left(\dfrac{\hbar\omega}{\Delta_{\text{res}}}, \dfrac{\Gamma}{\Delta_{\text{res}}}\right)$

Φ → *Universal* scaling function. We computed it in a "self-consistent, non-crossing" approximation



Predictions: Half-width of line $\approx \Gamma$

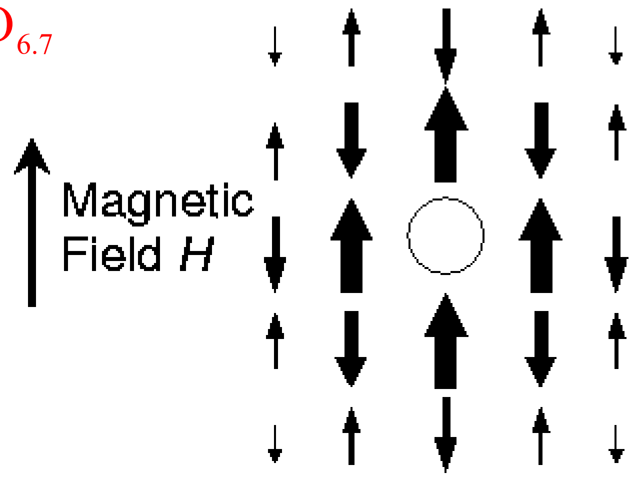
Universal asymmetric lineshape

3. Application to d-wave superconductors (YBCO)

Zn impurity in $YBa_2Cu_3O_{6.7}$

Moments measured by analysis of Knight shifts

M.-H. Julien, T. Feher, M. Horvatic, C. Berthier, O. N. Bakharev, P. Segransan, G. Collin, and J.-F. Marucco, Phys. Rev. Lett. **84**, 3422 (2000); also earlier work of the group of H. Alloul



Berry phases of precessing spins do not cancel between the sublattices in the vicinity of the impurity: net uncancelled phase of S=1/2

Pepin and Lee: Modeled Zn impurity as a potential scatterer in the unitarity limit, and obtained quasi-bound states at the Fermi level.
Our approach: Each bound state captures only one electron and this yields a Berry phase of S=1/2; residual potential scattering of quasiparticles is not in the unitarity limit.

Additional low-energy spin fluctuations in a *d*-wave superconductor

Nodal quasiparticles Ψ

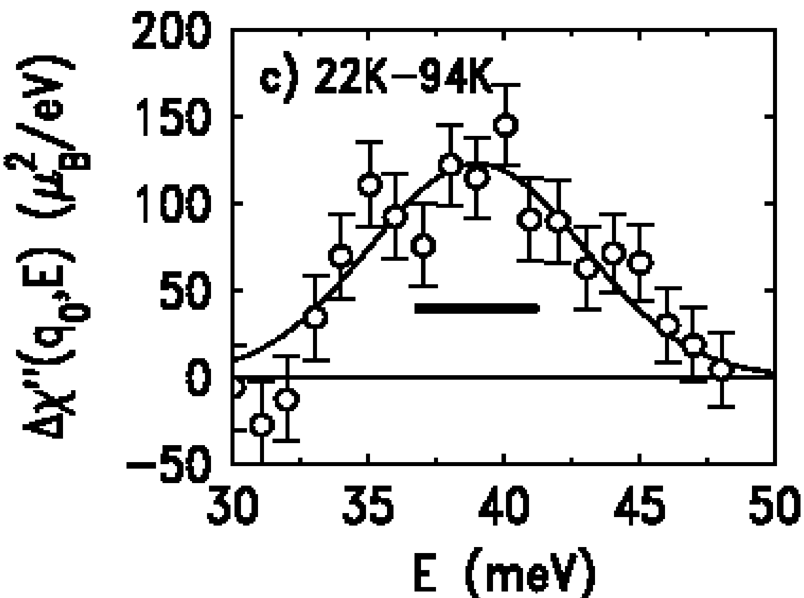
There is a Kondo coupling between moment around impurity and Ψ: $J_K S n_\alpha \Psi^* \sigma^\alpha \Psi$

However, because density of states vanishes linearly at the Fermi level, there is no Kondo screening for any finite J_K (below a finite J_K) with (without) particle-hole symmetry

(Withoff+Fradkin, Chen+Jayaprakash, Buxton+Ingersent)

$YBa_2Cu_3O_7 + 0.5\%$ Zn

H. F. Fong, P. Bourges, Y. Sidis, L. P. Regnault, J. Bossy, A. Ivanov, D.L. Milius, I. A. Aksay, and B. Keimer, Phys. Rev. Lett. **82**, 1939 (1999)



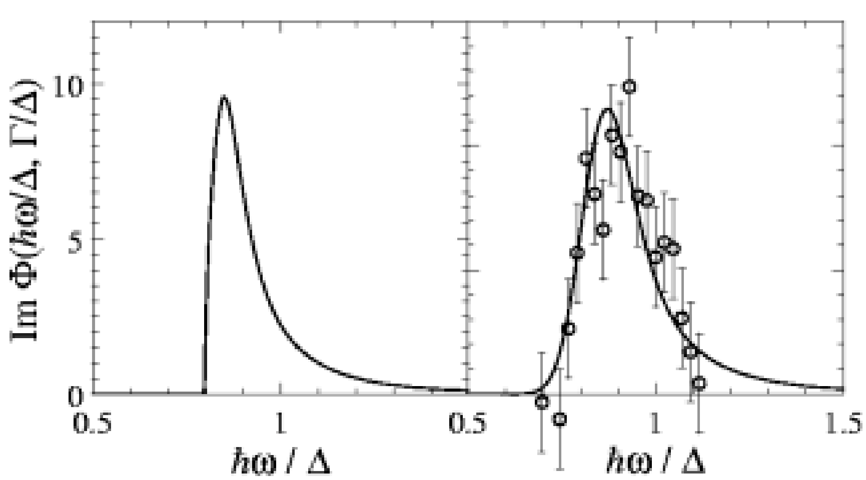
$$n_{\text{imp}} = 0.005$$

$$\Delta_{\text{res}} = 40\ \text{meV}$$

$$\hbar c = 0.2\ \text{eV}$$

$$\Rightarrow \quad \Gamma = 5\ \text{meV},\ \Gamma/\Delta_{\text{res}} = 0.125$$

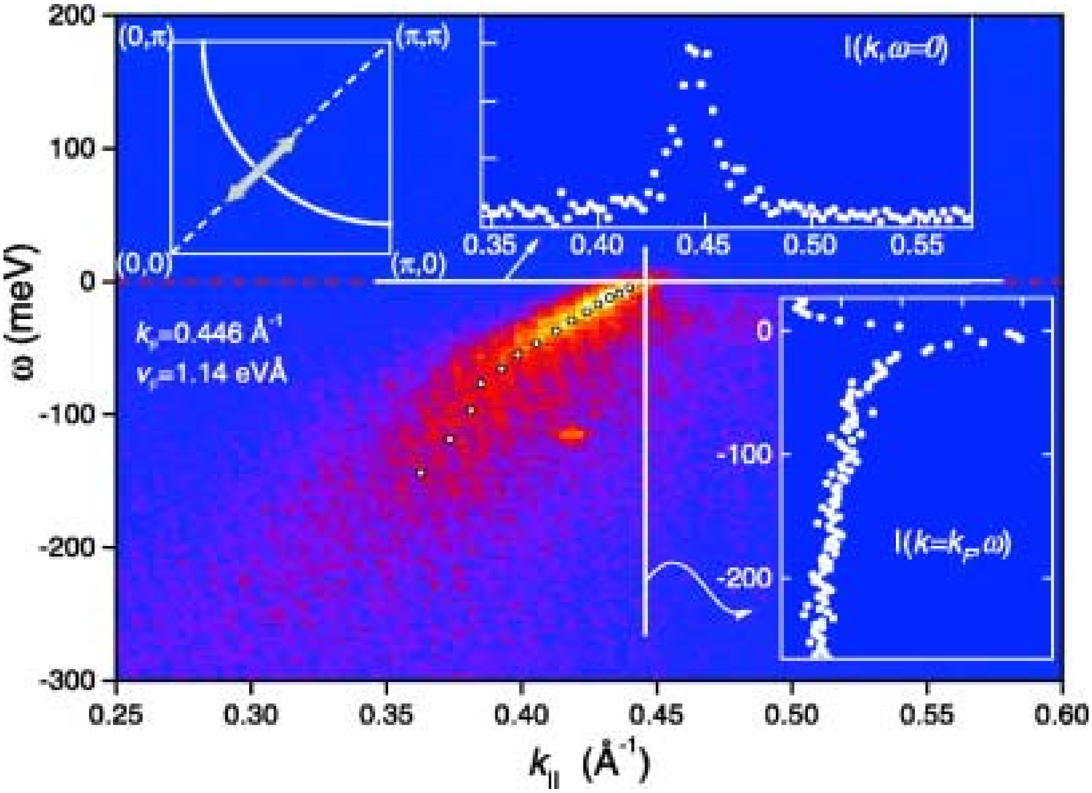
Quoted half-width = 4.25 meV



II. Intrinsic inelastic lifetime of nodal quasiparticles $\Psi_{1,2}$

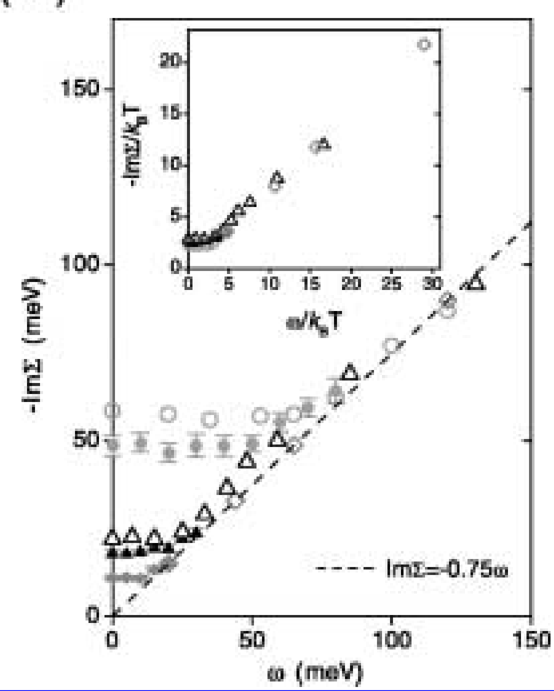
Photoemission on BSSCO

(Valla et al Science **285**, 2110 (1999))



Quantum-critical damping of quasi-particles along (1,1)

Quasi-particles sharp along (1,0)



$$\mathrm{Im}\Sigma \sim k_B T \text{ for } \hbar\omega < k_B T$$

$$\mathrm{Im}\Sigma \sim \hbar\omega \text{ for } \hbar\omega > k_B T$$

"Marginal Fermi liquid" (Varma *et al* 1989)
but only for nodal quasi-particles – strong ***k*** dependence at low temperatures

Origin of inelastic scattering ?

In a Fermi liquid

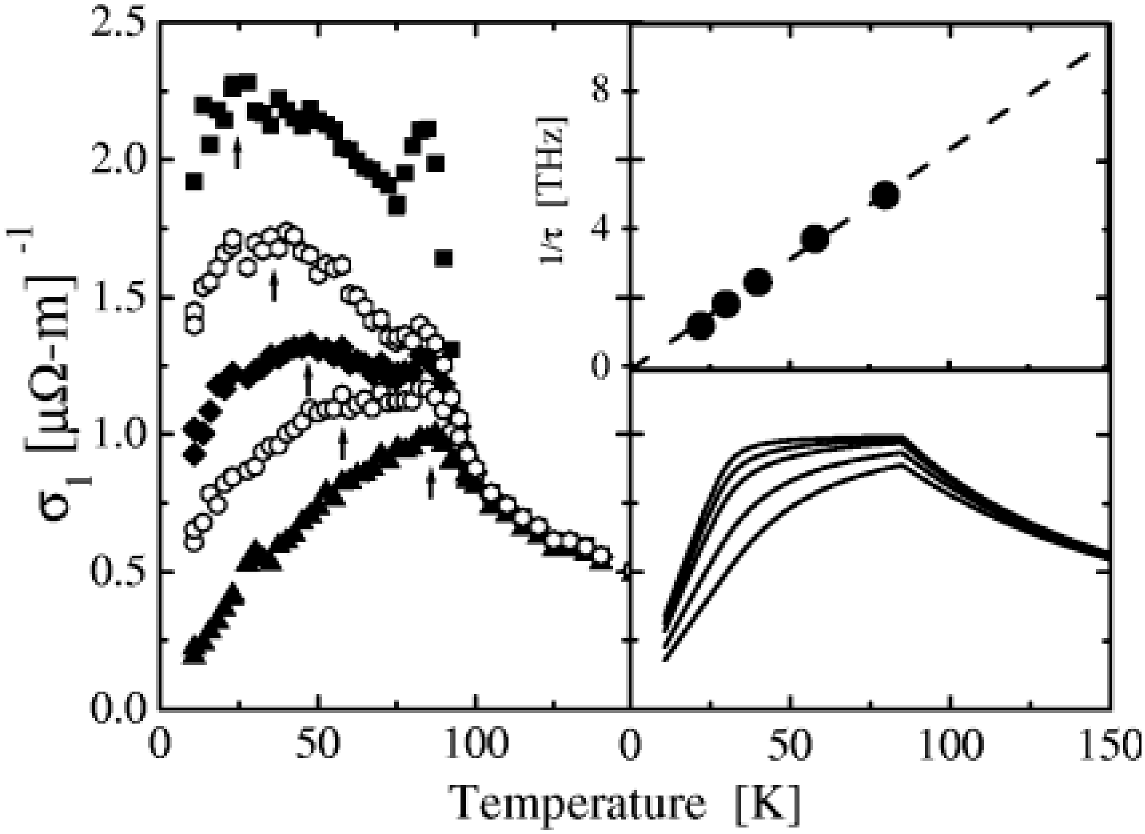
$$\mathrm{Im}\Sigma \sim T^2$$

In a BCS d-wave superconductor

$$\mathrm{Im}\Sigma \sim T^3$$

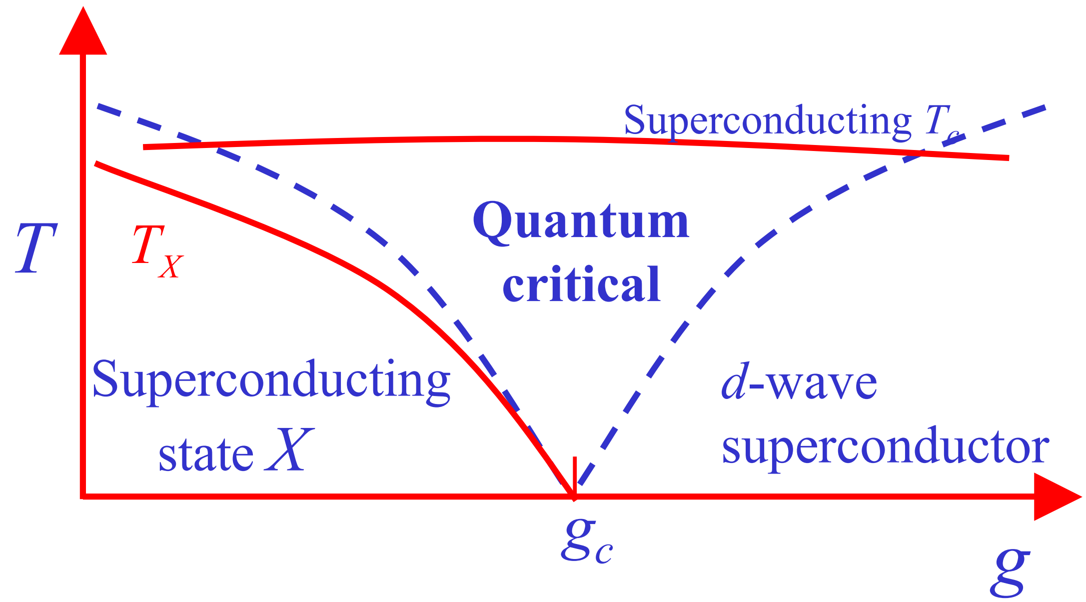
THz conductivity of BSCCO

(Corson et al cond-mat/0003243)



Quantum-critical damping of quasi-particles

Proximity to a quantum-critical point



(Crossovers analogous to those near quantum phase transitions in boson models
Weichmann *et al* 1986, Chakravarty *et al* 1989)

Relaxational dynamics in quantum critical region
(Sachdev+Ye, 1992)

$$G_F(k,\omega) = \frac{\Lambda^{-\eta_F}}{(k_B T)^{1-\eta_F}} \Phi\left(\frac{ck}{k_B T}, \frac{\hbar\omega}{k_B T}\right)$$

Nodal quasiparticle Green's function
k → wavevector separation from node

Necessary conditions

1. Quantum-critical point should be below its upper-critical dimension and obey hyperscaling.
2. Critical field theory should not be free – required to obtain damping in the scaling limit. Combined with (1) this implies that characteristic relaxation times ~ $\hbar / k_B T$
3. Nodal quasi-particles should be part of the critical-field theory.
4. Quasi-particles along (1,0), (0,1) should not couple to critical degrees of freedom.

1. *d*-wave superconductors

2. Candidates for *X:*

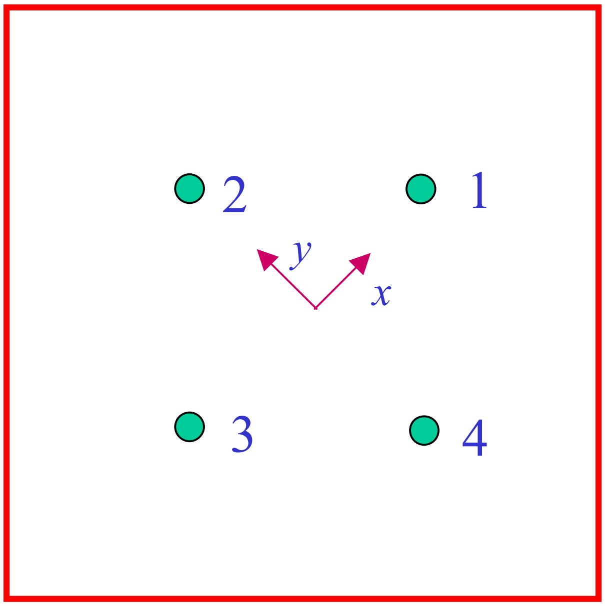
   a) Staggered-flux (or *orbital antiferromagnet*) order + *d*-wave superconductivity (breaks $\mathcal{T}$ – time-reversal symmetry).

   b) Superconductivity + charge density order (charge stripes)

   c) (*d*+*is*)-wave superconductivity (breaks $\mathcal{T}$)

   d) $d_{x^2-y^2} + id_{xy}$ wave superconductivity (breaks $\mathcal{T}$)

1. *d*-wave superconductors



Gapless Fermi Points in a *d*-wave superconductor at wavevectors $(\pm K, \pm K)$

$K=0.391\pi$

$$\Psi_1 = \begin{pmatrix} f_{1\uparrow} \\ f_{3\downarrow}^* \\ f_{1\downarrow} \\ -f_{3\uparrow}^* \end{pmatrix} \qquad \Psi_2 = \begin{pmatrix} f_{2\uparrow} \\ f_{4\downarrow}^* \\ f_{2\downarrow} \\ -f_{4\uparrow}^* \end{pmatrix}$$

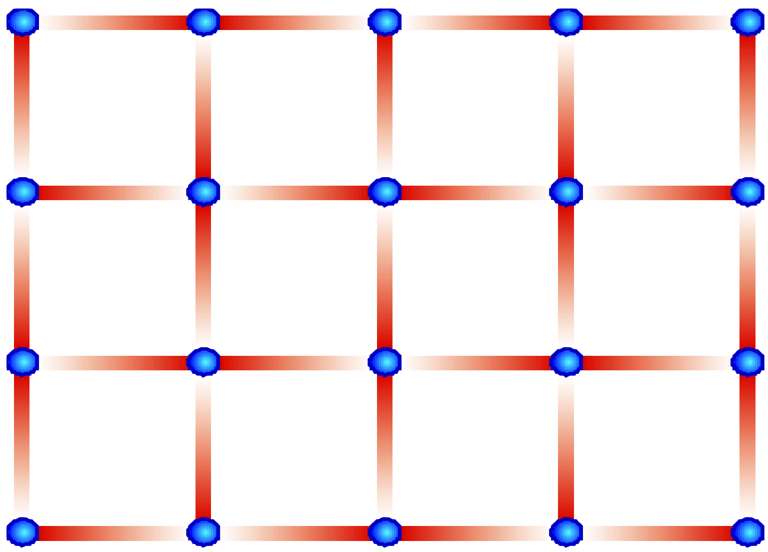
$$\begin{aligned} S_\Psi \;&=\; \int \frac{d^d k}{(2\pi)^d} T \sum_{\omega_n} \Psi_1^\dagger \left(-i\omega_n + v_F k_x \tau^z + v_\Delta k_y \tau^x\right) \Psi_1 \\ &+ \int \frac{d^d k}{(2\pi)^d} T \sum_{\omega_n} \Psi_2^\dagger \left(-i\omega_n + v_F k_y \tau^z + v_\Delta k_x \tau^x\right) \Psi_2 . \end{aligned}$$

τ^x, τ^z are Pauli matrices in Nambu space

2a. Orbital antiferromagnet

Checkerboard pattern of spontaneous currents:

(Affleck+Marston 1988, Schulz 1989,
Wang, Kotliar, Wang, 1990, Wen+Lee, 1996)



$\mathcal{T}$-breaking Ising order parameter ϕ

$$\langle c^{\dagger}_{k+G,a} c_{k,a} \rangle = i\phi(\cos k_x - \cos k_y) \quad ; \quad G = (\pi, \pi)$$

(Nayak, 2000)

$$S_\phi = \int d^d x d\tau \left[\frac{1}{2}(\partial_\tau \phi)^2 + \frac{c^2}{2}(\nabla \phi)^2 + \frac{s_0}{2}\phi^2 + \frac{u_0}{24}\phi^4 \right]$$

For K=π/2, only coupling to nodal quasiparticles is $\sim \zeta \int d^d x d\tau \phi^2 \Psi\Psi$; ζ is irrelevant and leads to

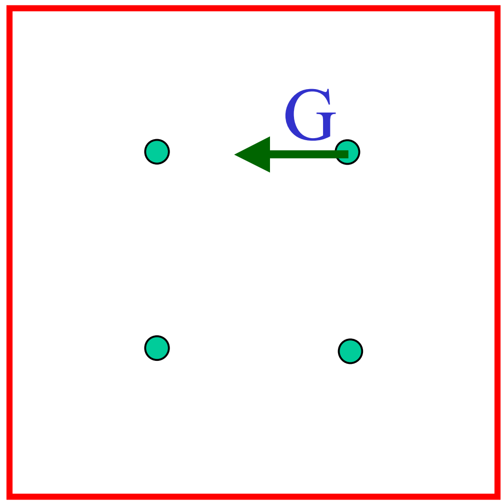
$$\mathrm{Im}\,\Sigma \sim T^{2d+1-2/\nu_{\mathrm{Ising}}} \sim T^{1.83}$$

2b. Charge stripe order

Charge density

$$\delta\rho \sim \mathrm{Re}\left[\Phi_x e^{iGx} + \Phi_y e^{iGy}\right]$$

If $G \neq 2K$ fermions do not couple efficiently to the order parameter and are not part of the critical theory



<u>Action for quantum fluctuations of order parameter</u>

$$\begin{aligned} S_\Phi &= \int d^d x d\tau \Big[|\partial_\tau \Phi_x|^2 + |\partial_\tau \Phi_y|^2 + |\nabla \Phi_x|^2 + |\nabla \Phi_y|^2 \\ &\quad + s_0 \left(|\Phi_x|^2 + |\Phi_y|^2 \right) + \frac{u_0}{2} \left(|\Phi_x|^4 + |\Phi_y|^4 \right) \\ &\quad + v_0 |\Phi_x|^2 |\Phi_y|^2 \Big] \end{aligned}$$

Coupling to fermions ~ $\lambda \int d^d x d\tau |\Phi_a|^2 \Psi\Psi$
and λ is irrelevant at the critical point

$$\mathrm{Im}\Sigma \sim T^{2d+1-2/\nu}$$

$$\sim T^{(\text{between 2 and 3})} \text{ for } 2/3 < \nu < 1$$

2c. (*d*+*is*)-wave superconductivity

(Kotliar, 1989)

$\mathcal{T}$-breaking Ising order parameter ϕ

$$\langle c_{k\uparrow} c_{-k\downarrow} \rangle = \Delta_0 (\cos k_x - \cos k_y) + i\phi(\cos k_x + \cos k_y).$$

Effective action:

$$S_\phi = \int d^d x d\tau \left[\frac{1}{2}(\partial_\tau \phi)^2 + \frac{c^2}{2}(\nabla \phi)^2 + \frac{s_0}{2}\phi^2 + \frac{u_0}{24}\phi^4 \right]$$

Efficient coupling to nodal quasi-particles (*generically*)

$$S_{\Psi\phi} = \int d^d x d\tau \left[\lambda_0 \phi \left(\Psi_1^\dagger \tau^y \Psi_1 + \Psi_2^\dagger \tau^y \Psi_2 \right) \right].$$

Coupling λ_0 takes a non-zero fixed-point value in the critical field theory

Strong inelastic scattering of nodal-quasiparticles in the scaling limit

Nodal quasiparticle lifetime ~ $\hbar / k_B T$

However: strong scattering of quasi-particles also along (1,0), (0,1) directions

2d. $d_{x^2-y^2} + id_{xy}$ -wave superconductivity

(Rokhsar 1993, Laughlin 1994)

$\mathcal{T}$-breaking Ising order parameter ϕ

$$\langle c_{k\uparrow} c_{-k\downarrow} \rangle = \Delta_0(\cos k_x - \cos k_y) + i\phi \sin k_x \sin k_y.$$

Effective action:

$$S_\phi = \int d^d x d\tau \left[\frac{1}{2}(\partial_\tau \phi)^2 + \frac{c^2}{2}(\nabla \phi)^2 + \frac{s_0}{2}\phi^2 + \frac{u_0}{24}\phi^4 \right]$$

Efficient coupling to nodal quasi-particles (*generically*)

$$S_{\Psi\phi} = \int d^d x d\tau \left[\lambda_0 \phi \left(\Psi_1^\dagger \tau^y \Psi_1 - \Psi_2^\dagger \tau^y \Psi_2 \right) \right].$$

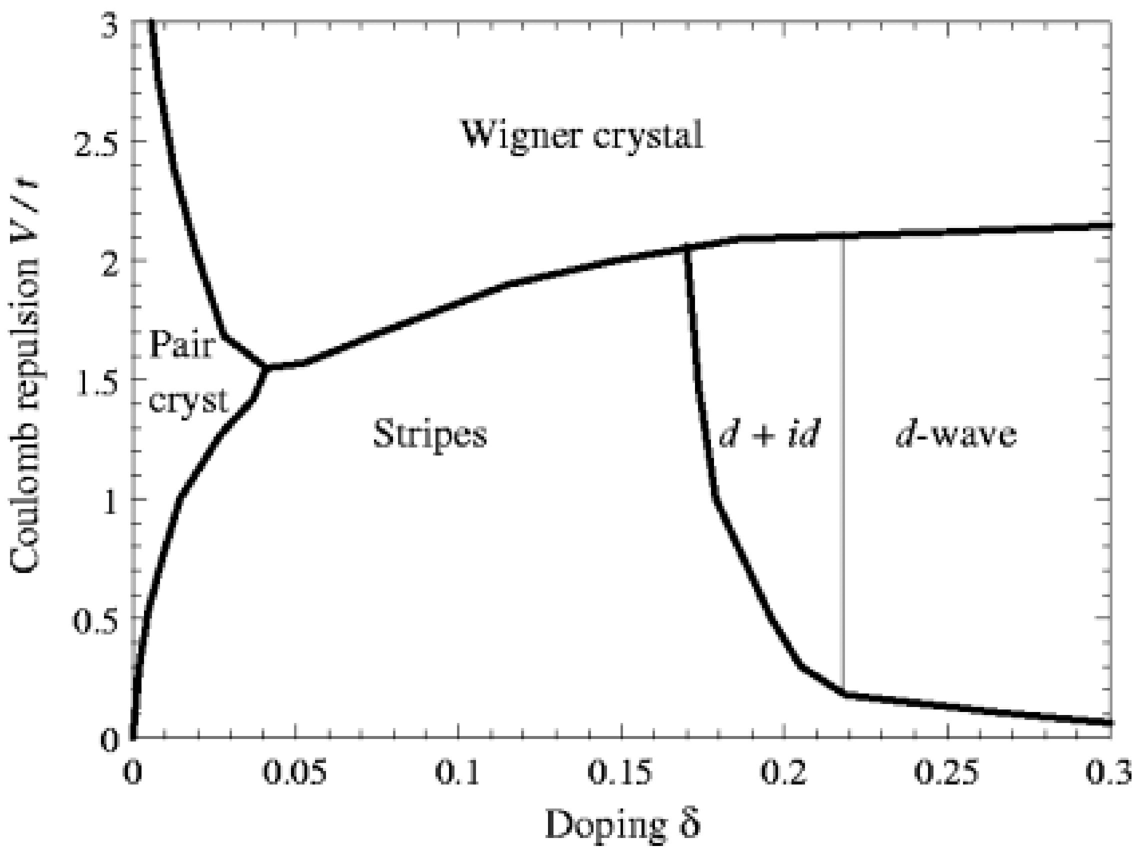
Coupling λ_0 takes a non-zero fixed-point value in the critical field theory

Strong inelastic scattering of nodal-quasiparticles in the scaling limit

Nodal quasiparticle lifetime ~ $\hbar / k_B T$

Moreover: no scattering of quasi-particles along (1,0), (0,1) directions !

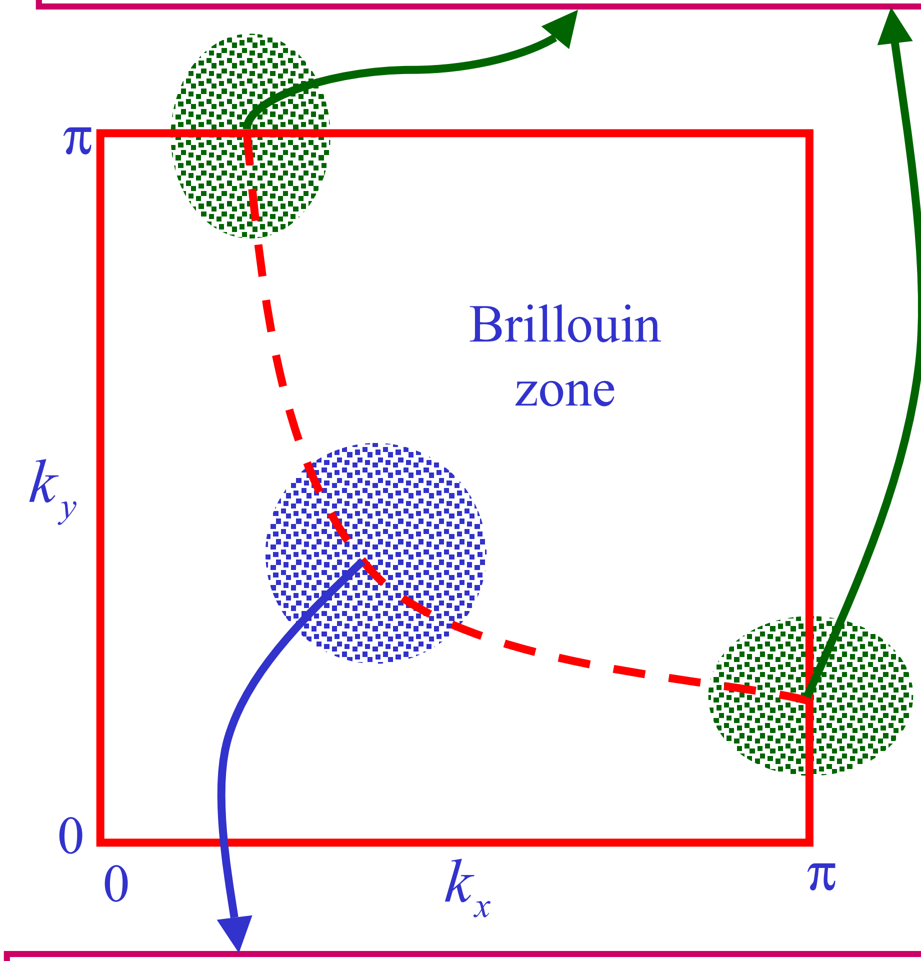
Large N (Sp(N)) phase diagram for time-reversal symmetry breaking and charge-ordering in a d-wave superconductor.



<u>Gapped quasiparticles</u>:

Below T_c : negligible damping

Above T_c: damping from strong coupling to superconducting phase and SDW fluctuations.



<u>Nodal quasiparticles</u>:

Below T_c : damping from fluctuations to $d_{x^2-y^2} + id_{xy}$ order

Above T_c: same mechanism applies as long as quantum-critical length < superconducting phase coherence length. Quasiparticles do not couple to phase or SDW fluctuations.

Conclusions: Part I

1. Universal *T=0* damping of S=1 collective mode by non-magnetic impurities.

   Linewidth: $$\Gamma \equiv \frac{n_{\text{imp}}(\hbar c)^2}{\Delta_{\text{res}}}$$

   independent of impurity parameters.

2. New interacting boundary conformal field theory in 2+1 dimensions

3. Universal irrational spin near the impurity at the critical point.

Conclusions: Part II

Classification of quantum-critical points leading to critical damping of quasiparticles in superconductor

Most attractive possibility: $\mathcal{T}$ breaking transition from a $d_{x^2-y^2}$ superconductor to a $d_{x^2-y^2} + id_{xy}$ superconductor

Leads to quantum-critical damping along (1,1), and no damping along (1,0), with no unnatural fine-tuning.

Note: stable ground state of cuprates can always be a $d_{x^2-y^2}$ superconductor; only need thermal/quantum fluctuations to $d_{x^2-y^2} + id_{xy}$ order in quantum-critical region.

Experimental update: Tafuri+Kirtley (cond-mat/0003106) claim signals of $\mathcal{T}$ breaking near non-magnetic impurities in YBCO films