Strange metals
and
black holes

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Strange metals: no quasiparticles

Ordinary metals: quasiparticles

Black holes
Ordinary metals: quasiparticles

Strange metals: no quasiparticles

Black holes
Ordinary metals are shiny, and they conduct heat and electricity efficiently. Each atom donates electrons which are delocalized throughout the entire crystal.
Almost all many-electron systems are described by the quasiparticle concept: a quasiparticle is an “excited lump” in the many-electron state which responds just like an ordinary particle.
What are quasiparticles?

- Quasiparticles are additive excitations:
  The low-lying excitations of the many-body system can be identified as a set \( \{n_\alpha\} \) of quasiparticles with energy \( \varepsilon_\alpha \)

\[
E = \sum_\alpha n_\alpha \varepsilon_\alpha + \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \ldots
\]

In a lattice system of \( N \) sites, this parameterizes the energy of \( \sim e^{\alpha N} \) states in terms of poly(\( N \)) numbers.
**What are quasiparticles?**

- Quasiparticles eventually collide with each other. Such collisions eventually lead to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time diverges as

\[
\tau_{eq} \sim \frac{\hbar U^2 / E_F}{(k_B T)^2}, \quad \text{as } T \to 0,
\]

where \(U\) is the strength of interactions and \(E_F\) is the Fermi energy.
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- Similarly, a quasiparticle model implies a resistivity

\[ \rho = \frac{m^*}{ne^2} \frac{1}{\tau} \sim T^2 \quad \text{with} \quad \tau \sim \tau_{\text{eq}} \]
What are quasiparticles?

- These times are much longer than the ‘Planckian time’ $\hbar/(k_B T)$, which we will find in systems without quasiparticle excitations.

\[
\tau \sim \tau_{\text{eq}} \gg \frac{\hbar}{k_B T}, \quad \text{as } T \to 0.
\]
Ordinary metals: quasiparticles

Strange metals: no quasiparticles

Black holes
High temperature superconductors

$\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$
Remarkable recent observation of ‘Planckian’ strange metal transport in cuprates, pnictides, magic-angle graphene, and ultracold atoms: the resistivity is associated with a universal scattering time $\approx \hbar/(k_B T)$.

**Universal T-linear resistivity and Planckian dissipation in overdoped cuprates**

A. Legros$^{1,2}$, S. Benhabib$^3$, W. Tabis$^{3,4}$, F. Laliberté$^1$, M. Dion$^1$, M. Lizaire$^1$, B. Vignon$^3$, D. Vignonnes$^3$, H. Raffy$^5$, Z. Z. Li$^5$, P. Auban-Senzier$^5$, N. Doiron-Leyraud$^1$, P. Fournier$^{1,6}$, D. Colson$^2$, L. Taillefer$^1$$^{1,6}$* and C. Proust$^3$$^{3,6}$

**Planckian dissipation and scale invariance in a quantum-critical disordered pnictide**

Yasuyuki Nakajima$^{1,2}$, Tristin Metz$^2$, Christopher Eckberg$^2$, Kevin Kirshenbaum$^2$, Alex Hughes$^2$, Renxiong Wang$^2$, Limin Wang$^2$, Shanta R. Saha$^2$, I-Lin Liu$^{2,3,4}$, Nicholas P. Butch$^{2,4}$, Zhonghao Liu$^{5,6}$, Sergey V. Borisenko$^5$, Peter Y. Zavalij$^7$ and Johnpierre Paglione$^{2,8}$

**Strange metal in magic-angle graphene with near Planckian dissipation**

Yuan Cao$^{1,*}$, Debanjan Chowdhury$^{1,*}$, Daniel Rodan-Legrain$^1$, Oriol Rubies-Bigordà$^1$, Kenji Watanabe$^2$, Takashi Taniguchi$^2$, T. Senthil$^{1,†}$ and Pablo Jarillo-Herrero$^{1,†}$

**Bad metallic transport in a cold atom Fermi-Hubbard system**

*Peter T. Brown$^1$, Debayan Mitra$^1$, Elmer Guardado-Sanchez$^1$, Reza Nourafkan$^2$, Alexis Reymbaut$^2$, Charles-David Hébert$^2$, Simon Bergeron$^2$, A.-M. S. Tremblay$^{2,3}$, Jure Kokalj$^{4,5}$, David A. Huse$^6$, Peter Schauß$^7$, Waseem S. Bakr$^{1†}$
Remarkable recent observation of ‘Planckian’ strange metal transport in cuprates, pnictides, magic-angle graphene, and ultracold atoms: the resistivity, $\rho$, is

$$\rho = \frac{m^*}{n e^2} \frac{1}{\tau}$$

with a universal scattering rate

$$\frac{1}{\tau} \approx \frac{k_B T}{\hbar},$$

independent of the strength of interactions!
Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:

\[ \text{Hydrogen atom: } \quad | \uparrow \rangle \]

Hydrogen molecule:

\[ \text{Hydrogen molecule: } \quad | \uparrow \rangle - | \downarrow \rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]
Quantum Entanglement: quantum superposition with more than one particle
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Quantum Entanglement: quantum superposition with more than one particle
Einstein-Podolsky-Rosen “paradox” (1935): Measurement of one particle instantaneously determines the state of the other particle arbitrarily far away.
The Sachdev-Ye-Kitaev (SYK) model

Pick a set of random positions
Place electrons randomly on some sites
The SYK model

Place electrons randomly on some sites
The SYK model

Place electrons randomly on some sites
Entangle electrons pairwise randomly

The SYK model
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This describes both a strange metal and a black hole!
The SYK model

(See also: the “2-Body Random Ensemble” in nuclear physics; did not obtain the large $N$ limit; T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. 53, 385 (1981))

\[
H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;kl} c_i^\dagger c_j^\dagger c_k c_\ell + e \sum_i c_i^\dagger c_i
\]

\[
c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}
\]

\[
Q = \frac{1}{N} \sum_i c_i^\dagger c_i
\]

$U_{ij;kl}$ are independent random variables with $\overline{U_{ij;kl}} = 0$ and $|U_{ij;kl}|^2 = U^2$

$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL 70, 3339 (1993)
The SYK model

The large $N$ limit is given by the sum of “melon” Feynman graphs

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The large $N$ limit is given by the sum of “melon” Feynman graphs

For long times $\tau > 0$

$$\left\langle c_i(\tau)c_i^\dagger(0)\right\rangle = \frac{A}{\sqrt{\tau}}$$

$$\left\langle c_i^\dagger(\tau)c_i(0)\right\rangle = e^{-2\pi \mathcal{E}} \frac{A}{\sqrt{\tau}}$$

The parameter $\mathcal{E}$ determines the particle-hole asymmetry.

S. Sachdev and J. Ye, PRL 70, 3339 (1993)
The SYK model

There are $2^N$ many body levels with energy $E$. Shown are all values of $E$ for a single cluster of size $N = 12$. The $T \to 0$ state has an entropy $S_{GPS} = Ns_0$, where $s_0 < \ln 2$ is determined by integrating

$$\frac{ds_0}{dQ} = 2\pi\mathcal{E}.$$ 

At $Q = 1/2$,

$$s_0 = \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\ldots$$

where $G$ is Catalan’s constant.
The complex SYK model

Note: Peak width $\sim k_B T / \hbar$ and independent of $U$

$$-\text{Im} G^R(\omega)$$

$\epsilon = 0$
The complex SYK model

Note: Peak width \( \sim k_B T / \hbar \)
and independent of \( U \)

\[-\text{Im} G^R(\omega)\]

\[\mathcal{E} \propto \frac{e}{U}\]

\[\hbar \omega / (k_B T)\]
The complex SYK model

Note: Peak width $\sim \frac{k_B T}{\hbar}$
and independent of $U$

$$-\text{Im} G^R(\omega)$$

$$\mathcal{E} \propto \frac{e}{U}$$

$\mathcal{E} = 0$

$\mathcal{E} = 0.26$

$\mathcal{E} = -0.26$
The complex SYK model

Note: Peak width $\sim k_B T/\hbar$
and independent of $U$

$$E = 0.26$$

$$E = 0.1$$

$$E = -0.26$$

$$E = -0.1$$

$$E = 0$$
A generalized SYK model

\[ H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;kl} c_i^\dagger c_j^\dagger c_k c_\ell + e \sum_i c_i^\dagger c_i \]

\( U_{ij;kl} \) are independent random variables with \( U_{ij;kl} = 0 \) and \( |U_{ij;kl}|^2 = U^2 \)
A generalized SYK model

\[ H = \frac{1}{(2N)^{3/2}} \sum_{\alpha, \beta, \gamma, \delta} \sum_{i, j, k, \ell=1}^{N} U_{i j \alpha \beta; k \ell \gamma \delta} c_{i \alpha}^{\dagger} c_{j \beta}^{\dagger} c_{k \gamma} c_{\ell \delta} \]

\[ + \sum_{i \alpha} e_{\alpha} c_{i \alpha}^{\dagger} c_{i \alpha} \]

\[ U_{i j \alpha \beta; k \ell \gamma \delta} \text{ is a random function of } i j k \ell \text{ (as before)} \]
\[ e_{\alpha} \text{ has a range of values of width } W. \]

\[ \hbar \omega/(k_B T) \text{ scaling behavior of SYK holds for } W^2/U \ll k_B T \ll U. \]
A generalized SYK model

\[ H = \frac{1}{(2N)^{3/2}} \sum_x \sum_{i,j,k,\ell=1}^N U_{ij; kl} c_{ix}^\dagger c_{jx}^\dagger c_{kx} c_{\ell x} - t \sum_{\langle xx' \rangle} \sum_i c_{ix}^\dagger c_{ix}' \]

Translationally-invariant model with \( \alpha \beta \gamma \delta \Rightarrow k \) and \( e_\alpha \Rightarrow e_k \), yields ‘incoherent metal’ for \( t^2 / U \ll k_B T \ll U \) with
\[ G(k, \omega) = G_{SYK}(\hbar \omega / (k_B T)) \]

independent of \( k \), and linear-in-\( T \) resistivity
\[ \rho \sim \frac{h}{e^2} \frac{k_B T}{t^2 / U} \]

Pengfei Zhang, PRB 96, 205138 (2017)
Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, PRX 8, 021049 (2018)
See also Antoine Georges and Olivier Parcollet PRB 59, 5341 (1999)
A generalized SYK model

\[ H = \frac{1}{(2N)^{3/2}} \sum_{\alpha, \beta, \gamma, \delta} \sum_{i,j,k,l=1}^{N} U_{ij\alpha\beta;kl\gamma\delta} c_{i\alpha}^\dagger c_{j\beta}^\dagger c_{k\gamma} c_{l\delta} \]

\[ + \sum_{i\alpha} e_\alpha c_{i\alpha}^\dagger c_{i\alpha} \]

\[ U_{ij\alpha\beta;kl\gamma\delta} \text{ is a random function of } ijk\ell \text{ (as before)} \]
\[ e_\alpha \text{ has a range of values of width } W. \]
\[ U_{ij\alpha\beta;kl\gamma\delta} \text{ is non-zero only for resonant interactions} \]
\[ \text{with } e_\alpha + e_\beta = e_\gamma + e_\delta. \]
A generalized SYK model

\[ H = \frac{1}{(2N)^{3/2}} \sum_{\alpha, \beta, \gamma, \delta} \sum_{i,j,k,l=1}^{N} U_{ij\alpha\beta;kl\gamma\delta} c_{i\alpha}^\dagger c_{j\beta}^\dagger c_{k\gamma} c_{l\delta} \]

\[ + \sum_{i\alpha} e_\alpha c_{i\alpha}^\dagger c_{i\alpha} \]

\( U_{ij\alpha\beta;kl\gamma\delta} \) is a random function of \( ijkl \) (as before)
\( e_\alpha \) has a range of values of width \( W \).
\( U_{ij\alpha\beta;kl\gamma\delta} \) is non-zero only for resonant interactions
with \( e_\alpha + e_\beta = e_\gamma + e_\delta \).

SYK behavior in a ‘Planckian metal’ as \( T \to 0 \),
with a remnant Fermi surface,
\( G(k, \omega) = G_{SYK}(\hbar \omega/(k_B T), \mathcal{E}_k) \), with \( \mathcal{E}_k \propto e_k/U \),
and resistivity \( \rho = \frac{m^*}{n e^2} \frac{1}{\tau} \),
with \( \frac{1}{\tau} \approx \frac{k_B T}{\hbar} \) insensitive to \( U \).
Ordinary metals: quasiparticles

Strange metals: no quasiparticles

Black holes
Black Holes

Objects so dense that light is gravitationally bound to them.

In Einstein’s theory, the region inside the black hole horizon is disconnected from the rest of the universe.
Quantum Entanglement across a black hole horizon
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Quantum Entanglement across a black hole horizon

There is quantum entanglement between the inside and outside of a black hole
Quantum Entanglement across a black hole horizon

Hawking used this to show that black hole horizons have an entropy and a temperature (because to an outside observer, the state of the electron inside the black hole is an unknown).
• Black holes have an entropy and a temperature, $T_H$

• The entropy is proportional to their surface area.

J. D. Bekenstein, PRD 7, 2333 (1973)
The ring-down is predicted by General Relativity to happen in a time \( \frac{8\pi GM}{c^3} \sim 8 \) milliseconds. Curiously this happens to equal \( k_B T_H \): so the ring down can also be viewed as the approach of a quantum system to thermal equilibrium at the fastest possible rate.


LIGO September 14, 2015
• The ring-down is predicted by General Relativity to happen in a time $8\pi GM/c^3 \sim 8$ milliseconds. Curiously this happens to equal $\hbar/k_B T_H$: so the ring down can also be viewed as the approach of a quantum system to thermal equilibrium at the fastest possible rate!
• Black holes have an entropy and a temperature, $T_H$

• The entropy is proportional to their surface area.

• They relax to thermal equilibrium in a Planckian time $\sim \hbar/(k_B T_H)$. 
Quantum Black holes

- Black holes have an entropy and a temperature, $T_H$
- The entropy is proportional to their surface area.
- They relax to thermal equilibrium in a Planckian time $\sim \hbar/(k_B T_H)$.

Holography:
Quantum black holes “look like” quantum many-particle systems without quasiparticle excitations, residing “on” the surface of the black hole.

Work with a theory of Maxwell’s electromagnetism and Einstein’s general relativity. Include a negative cosmological constant, and examine black hole solutions with a net charge
Work with a theory of Maxwell’s electromagnetism and Einstein’s general relativity. Include a negative cosmological constant, and examine black hole solutions with a net charge.

Zooming into the near-horizon region of a charged black hole at low temperature, yields a quantum theory in one space ($\vec{x}$) and one time dimension ($\zeta$).
The near-horizon region of a charged black hole has the geometry of (1+1)-dimensional anti-de Sitter spacetime. By holography, this should map to a zero-dimensional quantum system: this turns out to be the SYK model.
SYK model and charged black holes

\[ ds^2 = \left( d\zeta^2 - dt^2 \right) / \zeta^2 + d\bar{x}^2 \]

Gauge field: \( A = (E / \zeta) dt \)

\[ \zeta = \infty \quad \zeta \]

Bekenstein-Hawking entropy of AdS\(_2\) horizon at \( T = 0 \) \( \Leftrightarrow \) \( N s_0 \) entropy of SYK model.

\[ \frac{\partial s_0}{\partial Q} = 2\pi E \]
holds for both the black hole and the SYK model, where \( E \) determines identical fermion spectral functions.

Remarkably, the correspondence between charged black holes and the SYK model also holds for the leading fluctuations at higher temperatures: both are described by a ‘Schwarzian’ theory with emergent \( \text{SL}(2,\mathbb{R}) \) and \( \text{U}(1) \) gauge symmetries. For the black hole, the Schwarzian describes the fluctuations of the boundary between \( \text{AdS}_2 \) and \( \text{AdS}_4 \).
Main result

A. Kitaev (2015)


J. Maldacena, D. Stanford, and Zhenbin Yang, PTEP 12C104 (2016)

J. Engelsoy, T.G. Mertens, and H.Verlinde, JHEP 1607 (2016) 139


P. Chaturvedi, Yingfei Gu, Wei Song, Boyang Yu, arXiv:1808.08062
S. Sachdev, arXiv:1902.04078
Main result

SYK model of fermions with random interactions of mean-square-value $U$, with total fermion number $Q$, at temperatures $T \ll U$
**Main result**

SYK model of fermions with random interactions of mean-square-value $U$, with total fermion number $Q$, at temperatures $T \ll U$

and

Charged black holes in 3+1 dimensions of radius $R_h$, with total charge $Q$, at temperatures $T \ll 1/R_h$

are described by a common low energy quantum theory in $0+1$ dimensions.
Main result

The common low $T$ path integral is $Z = \int \mathcal{D}f \mathcal{D}\phi e^{-I}$. This can be exactly evaluated, and the action is

$$I = -s_0 + \int_0^{1/T} d\tau \left\{ \frac{K}{2} \left( \frac{\partial \phi}{\partial \tau} + i(2\pi \mathcal{E}T) \frac{\partial f}{\partial \tau} \right)^2 - \frac{\gamma}{4\pi^2} \text{Sch}\left[ \tan(\pi Tf(\tau)), \tau \right] \right\},$$

where $f(\tau)$ is a monotonic reparameterization of the temporal circle with

$$f(\tau + 1/T) = f(\tau) + 1/T,$$

$\phi$ is a phase conjugate to the charge density with

$$\phi(\tau + 1/T) = \phi(\tau) + 2\pi n, \ n \text{ integer},$$

$\text{Sch}[g[\tau], \tau]$ is the Schwarzian derivative of $g(\tau)$.

The couplings are related to the entropy $S(T, Q)$ and the chemical potential $\mu$ via

$$S(T \to 0, Q) = s_0 + \gamma T, \quad K = \left( \frac{dQ}{d\mu} \right)_{T \to 0}, \quad 2\pi \mathcal{E} = \frac{ds_0}{dQ}.$$
Main result

- Closely related to, but not the usual AdS/CFT correspondence, which involves only neutral black holes at $T > 0$.

- Unlike the AdS/CFT correspondence, both sides of the duality are fully solvable. This has enabled numerous recent studies of black holes quantum information.
Planckian dynamics (i.e. fastest possible local thermalization in a time $\hbar/(k_B T)$) is realized in the ‘solvable’ SYK models.

Momentum space SYK model yields Planckian transport with resistivity $\rho \sim \tau \sim \hbar/(m^* \nu^2)$.

Black holes thermalize in a Planckian time $\sim \tau \sim \hbar/(k_B T_H)$, where $T_H$ is the Hawking temperature.

A Schwarzian theory of a time reparameterization mode, with SL(2,R) symmetry, (along with a phase fluctuating mode) describes the quantum dynamics of

Quantum matter without quasiparticles
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- Planckian dynamics (i.e. fastest possible local thermalization in a time $\hbar/(k_B T)$) is realized in the ‘solvable’ SYK models.

- Momentum space SYK model yields Planckian transport with resistivity $\rho \approx (m^*/(ne^2))(k_B T/\hbar)$.
Quantum matter without quasiparticles

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- the SYK models
- black holes with near-extremal AdS$_2$ horizons